Exclusive semileptonic $B_s$ decays to excited $D_s$ mesons: 
Search of $D_{sJ}(2317)$ and $D_{sJ}(2460)$

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We study the exclusive semileptonic decays $B_s \rightarrow D_s^* \ell \bar{\nu}$ and $B_s \rightarrow D_s^* \ell \bar{\nu}$, where $p$-wave excited $D_{s0}^*$ and $D_{s1}^*$ states are identified with the newly observed $D_{sJ}(2317)$ and $D_{sJ}(2460)$ states. Within the framework of HQET the Isgur-Wise functions up to the subleading order of the heavy quark expansion are calculated by QCD sum rules. The decay rates and branching ratios are computed with the inclusion of the order of $1/m_Q$ corrections. We point out that the investigation of the $B_s$ semileptonic decays to excited $D_s$ mesons may provide some information about the nature of the new $D_{sJ}^*$ mesons.

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I. INTRODUCTION

Recently BaBar collaboration reported a narrow state with $J^P = 0^+$ at a rather low mass $2317$ MeV decaying to $D_s \pi^0$ [1]. This observation was quickly confirmed by CLEO, and another narrow state with $J^P = 1^+$ at $2457$ MeV was found in the $D_s^* \pi^0$ channel [2]. Both states have been confirmed by Belle [3]. Since these states lie below $DK$ or $D^*K$ threshold, they decay instead via isospin-violating transitions.

These newly observed states have attracted much attention because their measured masses and widths do not match the predictions from potential-based quark models [4]. To resolve the discrepancy, many theoretical speculations have appeared in the literature. Bardeen et al. interpreted them as positive-parity $\bar{c}s$ $(0^+, 1^+)$ spin doublet of the $D_s$ and $D_{sJ}^*$ negative-parity $c\bar{s}$ $(0^-, 1^-)$ ground states in the framework of chiral symmetry [5]. Based on the quark-antiquark picture, various theoretical models are modified to accommodate the low masses and the narrow widths for the new states [6, 7, 8, 9, 10]. QCD sum rule analysis in [11] supports the quark-antiquark postulation, both new states, $D_{sJ}(2317)$ and $D_{sJ}(2460)$, are identified to be the $\bar{c}s$ excited $0^+$ and $1^+$ states in the $j_l = \frac{1}{2}^+$ doublet. Apart from the quark-antiquark interpretation, the $D_{sJ}^*(2317)$ meson has been interpreted as a $DK$ molecule [12], a $D_s \pi$ molecule [13], a four-quark state [14], and a mixing of the conventional state and the four-quark state [15].

Motivated by the interpretation that the two new states are the excited $p$-wave $D_s$ states belonging to the $(0^+, 1^+)$ doublet with $j_l = \frac{1}{2}^+$ in [11], it is worthwhile to investigate the $B_s$ exclusive semileptonic decays to this doublet, ie. $B_s \rightarrow D_{s0}^* \ell \bar{\nu}$ and $B_s \rightarrow D_{s1}^* \ell \bar{\nu}$.
The search of these decay modes may provide some information to understand the nature of these new states and to clarify the controversy. Although current $B$ factories do not produce $B_s$ meson, these decays can be studied at future hadron $B$ factories.

In this paper we shall use QCD sum rules [16, 17] in the framework of the heavy quark effective theory (HQET) [18, 19] to study exclusive semileptonic $B_s$ decays to excited spin symmetry doublet ($D_{s0}^*, D_{s1}^*$) mesons. HQET is a useful tool to describe the spectroscopy and weak decays of hadrons containing a single heavy quark, it provides a systematic method to compute the properties of heavy hadrons via the $1/m_Q$ expansion, where $m_Q$ is the heavy quark mass. The study for the non-strange $B$ semileptonic decays into charmed meson doublet ($0^+,1^+$) in HQET can be found in the literatures by using various approaches, including HQET-based considerations [20, 21], QCD sum rules [22, 23, 24] and various quark models [25, 26, 27, 28, 29, 30]. In this work, we shall calculate the weak decay elements and the heavy quark expansion, including HQET-based considerations [20, 21], QCD sum rules [22, 23, 24] and various quark models [25, 26, 27, 28, 29, 30]. In this work, we shall calculate the weak decay elements for $B_s \to D_{s0}^*, D_{s1}^*$ in HQET. Considering that most of the phase space for these decays is near zero recoil, we shall include the $\Lambda_{QCD}/m_Q$ corrections in the application.

The paper is organized as follows. In Sec. II we review the formulas for the matrix elements of the weak currents including the structure of the $\Lambda_{QCD}/m_Q$ corrections in HQET. The QCD sum rule analysis for the leading and subleading Isgur-Wise functions is presented in Sec. III. Section IV is devoted to numerical analysis and the applications to decay widths. This section also includes a brief summary.

II. DECAY MATRIX ELEMENTS AND THE HEAVY QUARK EXPANSION

The matrix elements of vector and axial vector currents ($V^\mu = \bar{c} \gamma^\mu b$ and $A^\mu = \bar{c} \gamma^\mu \gamma_5 b$) between $B_s$ meson and excited $D_{s0}^*$ or $D_{s1}^*$ mesons can be parameterized as

$$
\langle D_{s0}^*(v') | V^\mu | B_s(v) \rangle = 0,
\langle D_{s0}^*(v') | A^\mu | B_s(v) \rangle = g_+(v^\mu + v'^\mu) + g_-(v^\mu - v'^\mu),
\langle D_{s1}^*(v', \epsilon) | V^\mu | B_s(v) \rangle = g_V \epsilon^{\mu \nu} + (g_{V_2} v^\mu + g_{V_3} v'^\mu) \epsilon_\alpha^\nu \epsilon_\beta^\beta,
\langle D_{s1}^*(v', \epsilon) | A^\mu | B_s(v) \rangle = ig_A \epsilon^{\mu \nu} \epsilon_\alpha^\nu \epsilon_\beta^\beta v_\gamma.
$$

Here form factors $g_i$ are functions of the dot-product, $y = v \cdot v'$, of the initial and final meson four-velocities. The differential decay rates expressed in terms of the form factors are given by

$$
\frac{d\Gamma_{D_{s0}^*}}{dy} = \frac{G_F^2 |V_{cb}|^2 m_B^5}{4\pi^3} r_0^3 (y^2 - 1)^{3/2} \left[ (1 + r_0) g_+ - (1 - r_0) g_- \right]^2,
$$

$$
\frac{d\Gamma_{D_{s1}^*}}{dy} = \frac{G_F^2 |V_{cb}|^2 m_B^5}{4\pi^3} r_1^3 \sqrt{y^2 - 1} \left\{ \left[ (y - r_1) g_V + (y^2 - 1)(g_{V_2} + r_1 g_{V_3}) \right]^2 + 2(1 - 2r_1 y + r_1^2) g_A^2 \right\}.
$$

where $r_0 = m_{D_{s0}^*}/m_{B_s}$ and $r_1 = m_{D_{s1}^*}/m_{B_s}$. 
The form factors $g_i$ can be expressed by a set of Isgur-Wise functions at each order in $\Lambda_{\text{QCD}}/m_{c,b}$. This is achieved by evaluating the matrix elements of the effective current operators arising from the HQET expansion of the weak currents. This problem has been discussed previously in Ref. [21], we outline here the analysis for the sake of completeness.

One introduces the matrix representations

$$H_v = \frac{1 + \gamma_5}{2} \left[ P_v^\mu \gamma_\mu - P_v \Gamma_5 \right],$$
$$K_v = \frac{1 + \gamma_5}{2} \left[ P'_v^\mu \gamma_5 \gamma_\mu + P'_v \Gamma_5 \right],$$

composed from the fields $P_v, P'^\mu_v$ and $P'_v, P'^\mu_v$ that destroy mesons in the doublets $j^P_i = \frac{1}{2}^-$ and $\frac{3}{2}^+$ with four-velocity $v$ in HQET, respectively. At leading order of the heavy quark expansion the hadronic matrix elements of weak current between the states annihilated by the fields in $H_v$ and $K_{\nu'}$ are written as

$$\bar{h}^{(c)}_{\nu'} \Gamma h^{(b)}_v = \zeta(w) \text{Tr} \left\{ \bar{K}_{\nu'} \Gamma H_v \right\}.$$  \hspace{1cm} (4)

where $h^{(Q)}_v$ is the heavy quark field in the effective theory and $\zeta$ is a universal Isgur-Wise function of $y$.

At order $\Lambda_{\text{QCD}}/m_Q$ there are contributions to the decay matrix elements originating from corrections to the HQET Lagrangian

$$\delta L = \frac{1}{2m_Q} \left[ O^{(Q)}_{\text{kin},v} + O^{(Q)}_{\text{mag},v} \right],$$

$O^{(Q)}_{\text{kin},v} = \bar{h}^{(Q)}_v (iD)^2 h^{(Q)}_v$, $O^{(Q)}_{\text{mag},v} = \bar{h}^{(Q)}_v \frac{g_s}{2} \sigma^{\alpha\beta} G_{\alpha\beta} h^{(Q)}_v$ \hspace{1cm} (5)

and originating from the matching of the $b \rightarrow c$ flavor changing current onto those in the effective theory

$$\bar{c} \Gamma b = \bar{h}^{(c)}_{\nu'} \left( \Gamma - \frac{i}{2m_c} \overleftrightarrow{D} \Gamma + \frac{i}{2m_b} \overleftrightarrow{D} \Gamma \right) h^{(b)}_v,$$ \hspace{1cm} (6)

where $D$ is the covariant derivative. The matrix elements of the later operators can be parameterized as

$$\bar{h}^{(c)}_{\nu'} i\overleftrightarrow{D}_\lambda \Gamma h^{(b)}_v = \text{Tr} \left\{ S^{(c)}_\lambda \bar{K}_{\nu'} \Gamma H_v \right\},$$
$$\bar{h}^{(c)}_{\nu'} \Gamma i\overleftrightarrow{D}_\lambda h^{(b)}_v = \text{Tr} \left\{ S^{(b)}_\lambda \bar{K}_{\nu'} \Gamma H_v \right\}.$$ \hspace{1cm} (7)

The most general decomposition for $S^{(Q)}_\lambda$ is

$$S^{(Q)}_\lambda = \zeta_1^{(Q)} v_\lambda + \zeta_2^{(Q)} v'_\lambda + \zeta_3^{(Q)} \gamma_\lambda.$$  \hspace{1cm} (8)

The functions $\zeta_i$ depend on $y$ and have mass dimension one. The translation invariance, $i\partial_\nu (\bar{h}^{(c)}_{\nu'} \Gamma h^{(b)}_v) = (\bar{A} v_\nu - \bar{A}' v'_\nu) \bar{h}^{(c)}_{\nu'} \Gamma h^{(b)}_v$, and the motion equation for the heavy quark,
The functions $\chi_{ke}^{c,b}(\xi)$ have mass dimension and effectively correct the leading order Isgur-Wise function $\zeta(\xi)$ since the kinetic energy operator does not violate heavy quark spin symmetry.

There are $\Lambda_{QCD}/m_Q$ corrections associated with the insertion of chromomagnetic operator $O_{\text{kin}}$. The QCD sum rule approach for the semileptonic $B$ decays to ground state and excited $D$ mesons shows that the functions parameterizing the time-ordered products of the chromomagnetic term in the HQET Lagrangian with the leading order currents are negligibly small \cite{24, 31}. This is in agreement with the results obtained from the HQET-motivated considerations \cite{21} and relativistic quark model \cite{30}. Therefore, we shall neglect the chromomagnetic correction hereafter.

Summing up all the contributions the resulting structure of the decay form factors is

$$
\begin{align*}
g_+ &= \varepsilon_c \left[ 2(y-1)\zeta_1 - 3\zeta \frac{y\Lambda' - \Lambda}{y + 1} \right] - \varepsilon_b \left[ \frac{\Lambda'(2y + 1) - \Lambda(y + 2)}{y + 1} \zeta - 2(y - 1)\zeta_1 \right], \\
g_- &= \zeta + \varepsilon_c \chi_{ke}^{c} + \varepsilon_b \chi_{ke}^{b}, \\
g_A &= \zeta + \varepsilon_c \left[ \frac{y\Lambda' - \Lambda}{y + 1} \zeta + \chi_{ke}^{c} \right] - \varepsilon_b \left[ \frac{\Lambda'(2y + 1) - \Lambda(y + 2)}{y + 1} \zeta - 2(y - 1)\zeta_1 - \chi_{ke}^{b} \right], \\
g_{V_1} &= (y - 1)\zeta + \varepsilon_c \left[ (y\Lambda' - \Lambda)\zeta + (y - 1)\chi_{ke}^{c} \right] - \varepsilon_b \left\{ \left[ \frac{\Lambda'(2y + 1) - \Lambda(y + 2)}{y + 1} \zeta - 2(y^2 - 1)\zeta_1 - (y - 1)\chi_{ke}^{b} \right] \right\}, \\
g_{V_2} &= 2\varepsilon_c \zeta_1, \\
g_{V_3} &= -\zeta - \varepsilon_c \left[ \frac{y\Lambda' - \Lambda}{y + 1} \zeta + 2\zeta_1 + \chi_{ke}^{c} \right] + \\
&\quad \varepsilon_b \left[ \frac{\Lambda'(2y + 1) - \Lambda(y + 2)}{y + 1} \zeta - 2(y - 1)\zeta_1 - \chi_{ke}^{b} \right].
\end{align*}
$$

where $\bar{\Lambda}(\Lambda') = m_M(m_{M'}) - m_Q$ is the difference between heavy ground state $j_f^P = \frac{1}{2}^-$ (excited $j_f^P = \frac{1}{2}^+$) meson and heavy quark masses in the $m_Q \to \infty$ limit. These relations show that all corrections to the form factors coming from the matching of the weak currents in QCD onto those in the effective theory are expressible in terms of $\zeta$ and $\zeta^{(c)}$. 

The matrix elements of $\Lambda_{QCD}/m_Q$ corrections from the insertions of the kinetic energy operator $O_{\text{kin}}$ have the structure

$$
\begin{align*}
i \int d^4x T \left\{ O^c_{\text{kin},v}(x) \left[ \bar{H}^{(c)}_v H^{(b)}_v \right] (0) \right\} &= \chi_{ke}^{c} \text{Tr} \left\{ \bar{K}_v \Gamma H_v \right\}, \\
i \int d^4x T \left\{ O^b_{\text{kin},v}(x) \left[ \bar{H}^{(c)}_v H^{(b)}_v \right] (0) \right\} &= \chi_{ke}^{b} \text{Tr} \left\{ \bar{K}_v \Gamma H_v \right\}.
\end{align*}
$$

where $m_M = m_{M'} - m_Q$ is the difference between heavy ground state $j_f^P = \frac{1}{2}^-$ (excited $j_f^P = \frac{1}{2}^+$) meson and heavy quark masses in the $m_Q \to \infty$ limit.
where $\varepsilon_Q = 1/(2m_Q)$ and the superscript on $\zeta^{(c)}$ is dropped. In the following sections we shall employ the QCD sum rule approach to calculate the leading and subleading Isgur-Wise functions.

### III. FORM FACTORS FROM QCD SUM RULES

#### A. Leading Isgur-Wise function $\zeta$

The proper interpolating current $J_{\alpha_1 \ldots \alpha_j}$ for a heavy mesonic state with the quantum number $j, P, j_\ell$ in HQET was given in \[32\]. These currents were proved to satisfy the following conditions

$$
\langle 0 | J_{\alpha_1 \ldots \alpha_j}^{(a)} (0) | j', P', j'_\ell \rangle = f_{Pj} \delta_{j'j} \delta_{PP'} \delta_{j'_\ell j_\ell} \eta_{\alpha_1 \ldots \alpha_j},
$$

$$
i \langle 0 | \bar{J}_{\alpha_1 \ldots \alpha_j}^{(a)} (x) J_{\alpha_1 \ldots \alpha_j}^{(a)} (0) | 0 \rangle = \delta_{j'j} \delta_{PP'} \delta_{j'_\ell j_\ell} (-1)^j S g_{\alpha_1 \beta_1} \cdots g_{\alpha_j \beta_j}
$$

$$
\times \int dt \delta (x - vt) \Pi_{P,j_\ell} (x)
$$

in the $m_Q \to \infty$ limit. Where $\eta_{\alpha_1 \ldots \alpha_j}$ is the polarization tensor for the spin $j$ state, $g_{\alpha \beta} = g^{\alpha \beta} - v^\alpha v^\beta$ is the transverse metric tensor, $S$ denotes symmetrizing the indices and subtracting the trace terms separately in the sets $(\alpha_1 \cdots \alpha_j)$ and $(\beta_1 \cdots \beta_j)$, $f_{Pj}$ and $\Pi_{P,j_\ell}$ are a constant and a function of $x$ respectively which depend only on $P$ and $j_\ell$.

The local interpolating current for creating $0^-$ pseudoscalar $B_s$ meson is taken as

$$
J_{0,-1/2}^\dagger = \sqrt{\frac{1}{2}} \bar{h}_v \gamma_5 s,
$$

and the local interpolating currents for creating $0^+$ and $1^+ \ (D_{s0}^*, D_{s1}^*)$ mesons are taken as

$$
J_{0,+1/2}^\dagger = \frac{1}{\sqrt{2}} \bar{h}_v (-i) D_t s,
$$

$$
J_{1,+1/2}^\dagger = \frac{1}{\sqrt{2}} \bar{h}_v \gamma_5 \gamma_t^\alpha (-i) D_t s,
$$

In order to calculate this form factor using QCD sum rules, we follow the same procedure as \[33\] to study the analytic properties of the three-point correlators

$$
i^2 \int d^4 x d^4 z e^{i(k' - k - z)} \langle 0 | T \left( J_{0,+1/2}^\dagger (x) J_A^\mu (0) J_{0,-1/2}^\dagger (z) \right) | 0 \rangle = \Xi (\omega, \omega', y) \mathcal{L}_A^\mu,
$$

$$
i^2 \int d^4 x d^4 z e^{i(k' - k - z)} \langle 0 | T \left( J_{1,+1/2}^\dagger (x) J_{V,A}^\mu (0) J_{0,-1/2}^\dagger (z) \right) | 0 \rangle = \Xi (\omega, \omega', y) \mathcal{L}_{V,A}^\mu,
$$

where $J_A^\mu = \bar{h}(v') \gamma_5 h(v)$ and $J_{V,A}^\mu = \bar{h}(v') \gamma_\mu \gamma_5 h(v)$ are leading order vector and axial vector currents, respectively. The variables $k, k'$ denote residual “off-shell” momenta which are
related to the momenta \( p \) of the heavy quark in the initial state and \( p' \) in the final state by \( k = p - m_Q v, \ k' = p' - m_Q v' \), respectively. The Lorentz structures, \( \mathcal{L}_{V,A} \), have the forms

\[
\mathcal{L}_A^\mu = v_\mu - v'_\mu, \quad \mathcal{L}_V^{\mu\nu} = -i\epsilon^{\mu\nu\alpha\beta} v_\alpha v'_\beta, \quad \mathcal{L}_V^{\mu\nu} = (y - 1) q_\mu^\nu - v_\mu v'_\nu,
\]

where \( v_\alpha = g_\alpha^\beta v_\beta = v_\alpha - yv'_\alpha \).

The coefficient \( \Xi(\omega, \omega', y) \) in (16) is an analytic scalar function in the “off-shell energies” \( \omega = 2v \cdot k \) and \( \omega' = 2v' \cdot k' \) with discontinuities for positive values of these variables. By saturating the double dispersion integrals for the correlators in (16) with physical intermediate states in HQET, one finds the hadronic representation of the correlator as following

\[
\Xi_{\text{hadro}}(\omega, \omega', y) = \frac{f_{-\frac{1}{2}} f_{+\frac{1}{2}} \xi(y)}{(2\Lambda - \omega - i\epsilon)(2\Lambda' - \omega' - i\epsilon)} + \text{higher resonances}.
\]

As the result of equation (12), only one state with \( j^P = 1^+ \) contributes to (17), the other resonance with the same quantum number \( j^P \) and different \( j^\ell \) does not contribute.

On the other hand, the correlator can be calculated in QCD in the Euclidean region, i.e., for large negative values of \( \omega \) and \( \omega' \), in terms of perturbative and nonperturbative contributions. Furthermore, the nonperturbative effects are able to be encoded in vacuum expectation values of local operators, the condensates. Hence one has

\[
\Pi(\omega, \omega', y) = \int d\nu d\nu' \frac{\rho_{\text{pert}}(\nu, \nu', y)}{(\nu - \omega - i\epsilon)(\nu' - \omega' - i\epsilon)} + \Pi_{\text{cond}} + \text{subtractions}.
\]

The QCD sum rule is obtained by equating the phenomenological and theoretical expressions for \( \Xi \). In doing this the quark-hadron duality needs to be assumed to model the contributions of higher resonance part of Eq. (17). Generally speaking, the duality is to simulate the resonance contribution by the perturbative part above some threshold energies. In the QCD sum rule analysis for \( B \) semileptonic decays into ground state \( D \) mesons, it is argued by Blok and Shifman in [34] that the perturbative and the hadronic spectral densities can not be locally dual to each other, the necessary way to restore duality is to integrate the spectral densities over the “off-diagonal” variable \( \omega_\perp = (\nu - \nu')/2 \), keeping the “diagonal” variable \( \omega_\parallel = (\nu + \nu')/2 \) fixed. It is in \( \omega_\parallel \) that the quark-hadron duality is assumed for the integrated spectral densities. We shall use the same prescription in our application.

In order to suppress the contributions of higher resonance states a double Borel transformation in \( \omega \) and \( \omega' \) is performed to both sides of the sum rule, which introduces two Borel parameters \( T_1 \) and \( T_2 \). For simplicity we shall take the two Borel parameters equal: \( T_1 = T_2 = 2T \).

The calculations of \( \rho_{\text{pert}} \) and \( \Pi_{\text{cond}} \) in HQET are straightforward. In doing this, for simplicity, the residual momentum \( k \) is chosen to be parallel to \( v \) such that \( k_\mu = (k \cdot v) v_\mu \) (and similar for \( k' \)) in the theoretical calculation. Since we deal with \( \bar{Q}s \) states, the light
quark mass shall be included in calculating the spectral function and condensates. For the condensates we confine us to the operators with dimension $D \leq 5$ in OPE. By performing the Taylor expansion around $x_\mu = 0$, we obtain the expansion for the quark-condensate

\[ \langle \bar{\psi}(x) D_\alpha \psi(y) \rangle = \frac{\langle \bar{q}q \rangle}{16} [im_\mu (\gamma_\mu)_\rho\sigma - \delta_{\rho\sigma} m_\mu^2 (x_\alpha - y_\alpha)] + \frac{\langle \bar{g}g \sigma \cdot Gq \rangle}{32} \left\{ (g_{\alpha\mu} - \frac{i}{3} \sigma_{\alpha\mu}) \rho \sigma \times (x^\mu - y^\mu) + \frac{i}{12} m_\mu [ (\gamma_\alpha)_\rho\sigma (x - y)^2 + 2 (\not{x} - \not{y})_{\rho\sigma} (x_\alpha - y_\alpha)] - m_\mu^2 \left[ \frac{\delta_{\rho\sigma}}{6} (x - y)^2 (x_\alpha - y_\alpha) - \frac{i}{36} \sigma_{\rho\sigma}^{\mu\nu} ((x - y)^2 y_\mu g_{\nu\alpha}}
+ 2(x_\alpha - y_\alpha) y_\mu x_\nu \right] \right\}. \] (19)

After making double Borel transformations in the variables $\omega$ and $\omega'$ and changing the integral variables $\omega_+ = (\nu + \nu')/2$, $\omega_- = (y + 1)/(y - 1)^{1/2}(\nu - \nu')/2$, one obtains the sum rule for $\zeta$ as follows

\[ \zeta(y) f_{-\frac{1}{2}} f_{+\frac{1}{2}} \sum_{n=0}^{\infty} \frac{\langle \bar{s}s \rangle}{8 T^2 \pi^2} (y + 1)^2 \int_0^{\omega_+} d\omega e^{-\omega/T} \left[ \omega_+^3 + 3(1 + y)(m_s \omega_+^2 + m_s^2 \omega_+) \right] \]
\[ \quad - \langle \bar{s}s \rangle \left[ \frac{3 m_s - m_s^2 y + 1}{T} \right] - \frac{1}{12} m_0^2 \langle \bar{s}s \rangle \frac{1 + y}{T} \left( 1 - \frac{5 m_s}{8 T} + \frac{m_s^2}{48 T^2} \right) \]
\[ \quad - \frac{1}{192} \frac{\alpha_s}{\pi} G G \frac{y - 1}{y + 1} + \frac{m_s}{32 T} \frac{\alpha_s}{\pi} G G \left( 2 \gamma_E - \ln \frac{T^2}{\mu^2} + \ln \frac{T^2}{\mu^2} \right), \] (20)

where $m_0^2 \langle \bar{s}s \rangle = \langle \bar{s}g \sigma_{\mu\nu} G_{\mu\nu} s \rangle$ with $m_0^2 = 0.8$ GeV$^2$.

### B. Subleading Isgur-Wise function $\zeta_1$

In order to derive the QCD sum rule for the subleading form factor $\zeta_1(y)$ defined in [8], we consider the following three-point correlation functions

\[ \Xi_{0A}^{\mu}(\omega, \omega', y) = i^2 \int d^4 x d^4 z \epsilon^{i(k' - k - z)} \langle 0 | T \left( J_{0,+,1/2}(x) \bar{h}_\nu^{(c)} i \bar{D} \gamma^\mu \gamma_5 h_\nu^{(b)}(0) \right) J_{0,+,1/2}(z) \rangle | 0 \rangle \]
\[ = \Xi_0(\omega, \omega', y) \mathcal{L}_{0A}^{\mu}, \] (21a)

\[ \Xi_{1A}^{\mu}(\omega, \omega', y) = i^2 \int d^4 x d^4 z \epsilon^{i(k' - k - z)} \langle 0 | T \left( J_{1,+,1/2}(x) \bar{h}_\nu^{(c)} i \bar{D} \gamma^\mu \gamma_5 h_\nu^{(b)}(0) \right) J_{0,+,1/2}(z) \rangle | 0 \rangle \]
\[ = \Xi_1(\omega, \omega', y) \mathcal{L}_{1A}^{\mu}, \] (21b)

\[ \Xi_{1V}^{\mu}(\omega, \omega', y) = i^2 \int d^4 x d^4 z \epsilon^{i(k' - k - z)} \langle 0 | T \left( J_{1,+,1/2}(x) \bar{h}_\nu^{(c)} i \bar{D} \gamma^\mu h_\nu^{(b)}(0) \right) J_{0,+,1/2}(z) \rangle | 0 \rangle \]
\[ = \Xi_2(\omega, \omega', y) \mathcal{L}_{1V}^{\mu} \Xi_3(\omega, \omega', y) \mathcal{L}_{1V}^{\mu}, \] (21c)

where the Lorentz structures have the forms

\[ \mathcal{L}_{0A}^{\mu} = \bar{v}_\mu + v_\mu \]
\[ \mathcal{L}_{1V}^{\mu} = g_1^{\mu\nu} (y + 1) - v_\mu v_\nu, \quad \mathcal{L}_{1V}^{\mu} = \bar{v}_\mu v_\nu - v_\mu v_\nu. \]
The coefficient functions \( \Xi_i(\omega, \omega', y) \) can be expressed in terms of perturbative and non-perturbative contributions in QCD theoretical calculation. These functions are used to construct the sum rules needed.

By saturating the double dispersion integral for the three-point functions in (21) with hadron states and using Eqs. (17)–(22), one can isolate the contributions from the double pole at \( \omega = 2\bar{\Lambda}, \omega' = 2\bar{\Lambda}' \):

\[
\Xi_{0A}^\mu(\omega, \omega', y) = \frac{f_{-\frac{3}{2}} f_{+1/2}}{(2\bar{\Lambda} - \omega - ie)(2\bar{\Lambda}' - \omega' - ie)} \left[ 2(y - 1)\zeta_1(y) - 3\frac{y\bar{\Lambda}' - \bar{\Lambda}}{y + 1}\zeta(y) \right] L_{0A}^\mu + \cdots ,
\]

\[
\Xi_{1A}^\mu(\omega, \omega', y) = \frac{f_{-\frac{1}{2}} f_{+1/2}}{(2\bar{\Lambda} - \omega - ie)(2\bar{\Lambda}' - \omega' - ie)} \frac{y\bar{\Lambda}' - \bar{\Lambda}}{y + 1}\zeta(y) L_{1A}^\mu + \cdots ,
\]

\[
\Xi_{1V}^\mu(\omega, \omega', y) = \frac{f_{-\frac{1}{2}} f_{+1/2}}{(2\bar{\Lambda} - \omega - ie)(2\bar{\Lambda}' - \omega' - ie)} \left[ -\frac{y\bar{\Lambda}' - \bar{\Lambda}}{y + 1}\zeta(y) L_{1V}^\mu 
- 2\zeta_1(y) L_{1V}^\mu \right] + \cdots .
\]

From (21) and (22) one can see that in the case of \( \Xi_3 \), the residue of the pole is proportional to the universal function \( \zeta_1(y) \), the pole contribution to \( \Xi_1 \) and \( \Xi_2 \) is related to \( \zeta(y) \). While for \( \Xi_0 \) the residue of the pole is proportional to both \( \zeta(y) \) and \( \zeta_1(y) \). QCD sum rule is obtained by equating the phenomenological and theoretical expressions for \( \Xi \). Therefore, the sum rule for the subleading form factor \( \zeta_1(y) \) can be constructed either from \( \Xi_3 \) or from \( \Xi_0 \). One can also yield the sum rule for the leading form factor in the form \((y\bar{\Lambda}' - \bar{\Lambda})\zeta(y)\) from \(\Xi_0, \Xi_1 \) and \(\Xi_2\), respectively.

We shall focus on the coefficient function \( \Xi_3(\omega, \omega', y) \) to construct the sum rule for the subleading form factors \( \zeta_1(y) \). One obtains the sum rule for \( \zeta_1(y) \) as follows

\[
\zeta_1(y) f_{-\frac{3}{2}} f_{+1/2} e^{-(\bar{\Lambda}+\bar{\Lambda}')/T} = \frac{1}{16\pi^2} \frac{1}{(y + 1)^3} \int_{2m_s}^{\infty} d\omega_+ e^{-\omega_+/T} \left[ \omega_+^4 + 4m_s(y + 1)(2y + 1)^2\omega_+^3 \right]
+ \frac{5}{96} m_s m_0^2 \frac{\langle ss \rangle}{T} - \frac{1}{96} \frac{\alpha_s}{\pi} GG \frac{y}{(y + 1)^2} T .
\]

Moreover, the sum rule for the combination \((y\bar{\Lambda}' - \bar{\Lambda})\zeta(y)\) can be obtained independently from the coefficient function \( \Xi_1 \) and \( \Xi_2 \) in (21) together with (22), respectively. We have double checked that the resulted sum rule for \( \zeta(y) \) has the same form as (20). The above consistency checks confirm that our method is consistent with the general analysis of Ref. [21] described in Sec. [II].

C. QCD sum rules for \( \eta_{ke}^{c,b} \)

For the determination of the form factor \( \eta_{ke}^{c,b} \), which relates to the insertion of \( \Lambda_{QCD}/m_c \) kinetic operator of the HQET Lagrangian, one studies the analytic properties of the three-
point correlators
\[ i^2 \int d^4x \, d^4x' \, d^4z \, e^{i(k', x' - k \cdot x)} \langle 0 | T \left( J_{0, +1/2}(x') \, O_{\text{kin}, \nu}^{(c)}(z) \, J_{0, -1/2}^\mu(0) \right) | 0 \rangle = \Xi(\omega, \omega', y) \, \mathcal{L}_0^\mu, \] (24a)

\[ i^2 \int d^4x \, d^4x' \, d^4z \, e^{i(k', x' - k \cdot x)} \langle 0 | T \left( J_{0, +1/2}(x') \, O_{\text{kin}, \nu}^{(c)}(z) \, J_{0, -1/2}^\mu(0) \right) | 0 \rangle = \Xi(\omega, \omega', y) \, \mathcal{L}_0^\mu. \] (24b)

By saturating (24) with physical intermediate states in HQET, one can isolate the contribution of interest as the one having poles at \( \omega = 2\bar{\Lambda}, \omega' = 2\bar{\Lambda}'. \) Notice that the insertions of the kinetic operator not only renormalize the leading Isgur-Wise function, but also the meson coupling constants and the physical masses of the heavy mesons which define the position of the poles. The correct hadronic representation of the correlator is

\[ \Xi_{\text{hadro}}(\omega, \omega', y) = \frac{f_{-1/2} f_{+1/2}}{(2\Lambda - \omega - i\epsilon)(2\Lambda' - \omega' - i\epsilon)} \left( \eta_{ke}^c(y) + (G_{+1/2}^K + \frac{K_{+1/2}}{2\Lambda - \omega' - i\epsilon}) \zeta(y) \right) + \text{higher resonance}, \] (25)

where \( K_{P,j} \) and \( G_{P,j}^K \) are defined by [32, 39] \[ \langle j, P, j| O_{\text{kin}, \nu}^{(Q)} | j, P, j \rangle = K_{P,j}, \]

\[ (0) i \int d^4x \, O_{\text{kin}, \nu}^{(Q)}(x) \, J_{j, P,j}^{c_\alpha_1 \cdots \alpha_j}(0) | j, P, j \rangle = f_{P,j} \, G_{P,j}^K \, \eta_0^{\alpha_1 \cdots \alpha_j}. \] (26)

Within the same procedure one finds the sum rule for \( \eta_{ke}^b \) as

\[ \left[ \eta_{ke}^c(y) + (G_{+1/2}^K + \frac{K_{+1/2}}{2T}) \zeta(y) \right] f_{-1/2} f_{+1/2} e^{-(\bar{\Lambda} + \bar{\Lambda}')/T} = \]

\[ -\frac{3}{16\pi^2} \frac{3y + 2}{(y + 1)^3} \int_{2m_s}^{\infty} d\omega_+ e^{-\omega_+/T}[\omega_+^3 + \frac{4}{3}m_s(1 + y)\omega_+] \]

\[ + \frac{5}{32} \frac{m_s m_s^2}{T} + \frac{1}{96} \left( \frac{\alpha_s}{\pi} \right) GG \frac{3y + 1}{(y + 1)^2} T, \] (27)

From the consideration of symmetry, the sum rule for \( \eta_{ke}^b \) that originates from the insertion of \( \Lambda_{\text{QCD}}/m_b \) kinetic operator of the HQET Lagrangian is of the same form as in (27), but with the HQET parameters \( G_{+1/2}^K \) and \( K_{+1/2} \) replaced by \( G_{-1/2}^K \) and \( K_{-1/2} \), respectively. The definitions of \( G_{-1/2}^K \) and \( K_{-1/2} \) can be found in Eq. (26).

We end this subsection by noting that the QCD \( O(\alpha_s) \) corrections have not been included in the sum rule calculations. However, the Isgur-Wise functions obtained from the QCD sum rule actually are a ratio of the three-point correlator to the two-point correlator results. While both of these correlators subject to large perturbative QCD corrections, the remaining corrections to the form factors themselves may in fact be small because of cancellation. This is what happens in the case of analysis for \( B \) semileptonic decay to ground state and excited charmed mesons [36, 37].
IV. NUMERICAL RESULTS AND PREDICTIONS FOR SEMILEPTONIC DECAY WIDTHS

We now turn to the numerical evaluation of these sum rules and the phenomenological implications. In order to obtain information for $\zeta(y)$, $\zeta_1(y)$ and $\eta_{c,b}(y)$ from the sum rules, we need the related decay constants, $f_{-1/2}$ and $f_{+1/2}$, defined in [12], as input. The QCD sum rule calculations for the correlators of two heavy-light currents give [11, 38]:

$$f_{-1/2}' e^{-2\bar{\Lambda}'/T} = \frac{3}{16\pi^2} \int_{2m_s}^{\omega_0} d\omega (\omega^2 + 2m_s\omega - 2m_s^2)e^{-\omega/T} - \frac{1}{2} \langle \bar{s}s \rangle (1 - \frac{m_s}{2T} + \frac{m_s^2}{2T^2})$$

$$+ \frac{m_0^2}{8T^2} \langle \bar{s}s \rangle \left(1 - \frac{m_s}{3T} + \frac{m_s^2}{3T^2}\right) - \frac{m_s}{16T^2} \langle \alpha_s \pi GG \rangle \left(2\gamma_E - 1 - \ln \frac{T^2}{\mu^2}\right).$$

These two-point sum rules can be used to eliminate the explicit dependence of three-point sum rules on $f_{-1/2}$ and $f_{+1/2}$, as well as on $\bar{\Lambda}$ and $\bar{\Lambda}'$. This procedure may help to reduce the uncertainties in the calculation.

For the QCD parameters entering the theoretical expressions, we take the standard values

$$\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3 \text{GeV}^3,$$
$$\langle \frac{\alpha_s}{\pi} GG \rangle = (0.012 \pm 0.004) \text{GeV}^4. \quad (30)$$

The strange/nonstrange condensate ratio is adopted as $\langle \bar{s}s \rangle = (0.8 \pm 0.1)\langle \bar{q}q \rangle$. The strange quark mass is taken as $m_s(1 \text{GeV}) = 0.15 \text{ GeV}$, and the cut-off parameter is chosen as $\mu = 1 \text{ GeV}$.

Let us evaluate numerically the sum rules for $\zeta(y)$ and $\zeta_1(y)$. The continuum thresholds $\omega_0$ and $\omega_1$ in [28] and [29] are determined by requiring stability of these sum rules. One finds that $1.7 \text{ GeV} < \omega_0 < 2.2 \text{ GeV}$ and $2.6 \text{ GeV} < \omega_1 < 3.1 \text{ GeV}$ [11, 33, 38]. Imposing usual criterion on the ratio of contribution of the higher-order power corrections and that of the continuum, we find that for the central values of the condensates given in [30], if the threshold parameter $\omega_c$ lies in the range $2.3 < \omega_c < 2.7 \text{ GeV}$, there is an acceptable "stability window" $T = 0.8 - 1.2 \text{ GeV}$ in which the calculation results do not change appreciably.

The values of the form factors $\zeta(y)$ and $\zeta_1(y)$ at zero recoil as functions of the Borel parameter are shown in Fig. 1(a) and 2(a), for three different values of the continuum threshold $\omega_c$. One can see that the variation is quit moderate in the range $0.8 < T < 1.2 \text{ GeV}$. The numerical results for $\zeta(y)$ and $\zeta_1(y)$ are shown in Fig. 1(b) and 2(b), where the curves refer to three different values of $\omega_c$ and $T$ is fixed at $T = 1.0 \text{ GeV}$. 

$$f_{+1/2}^2 e^{-2\bar{\Lambda}/T} = \frac{3}{64\pi^2} \int_{2m_s}^{\omega_1} d\omega (\omega^4 + 2m_s\omega^3 - 6m_s^2\omega^2 - 12m_s^3\omega)e^{-\omega/T} d\omega$$

$$- \frac{1}{16} m_0^2 \langle \bar{s}s \rangle (1 - \frac{m_s}{T} + \frac{4m_s^2}{3T^2}) + \frac{3}{8} m_s^2 \langle \bar{s}s \rangle - \frac{m_s}{16\pi} \langle \alpha_s G^2 \rangle. \quad (29)$$
FIG. 1: Fig. 1(a) shows the dependence of $\zeta(1)$ on the Borel parameter $T$ for the continuum threshold in the range $2.3 < \omega_c < 2.7$ GeV; Fig. 1(b) shows Isgur-Wise function $\zeta(y)$ with $T = 1.0$ GeV.

In order to evaluate numerically the sum rules for $\eta_{ke}^c(y)$ and $\eta_{ke}^b(y)$, we need to specify the following HQET parameters as input, which are obtained by QCD sum rules [11, 38, 40]:

$$K_{+3/2} = -(1.6 \pm 0.30) \text{ GeV}^2, \quad G_{+3/2}^K = -(1.0 \pm 0.45) \text{ GeV}$$
$$K_{-1/2} = -(1.2 \pm 0.20) \text{ GeV}^2, \quad G_{-1/2}^K = -(1.6 \pm 0.6) \text{ GeV}.$$

In Fig. 3(a), the sum rule for $\eta_{ke}^c(y)$ is plotted at zero recoil as a function of Borel parameter for various choices of the continuum thresholds in the range $2.5 < \omega_c < 2.9$. Fig. 3(b) shows the $y$ dependence of the form factor $\eta_{ke}^c(y)$ for the central value of HQET parameters and $T = 1.2$ GeV. It should be noted that apart from the uncertainty from the sum rule working window, there is uncertainty to a large extent due to the variation of $K$’s and $G$’s in the numerical analysis. The numerical evaluation for the sum rule of $\eta_{ke}^b(y)$ can follow the same procedure.

The numerical analysis shows that all Isgur-Wise functions $\zeta(y)$, $\zeta_1(y)$ and $\eta_{ke}^{cb}(y)$ are

FIG. 2: Numerical result for the sum rule [25]: (a) dependence of $\zeta_1(1)$ on the Borel parameter $T$ for the continuum threshold in the range $2.3 < \omega_c < 2.7$ GeV; (b) The form factor $\zeta_1(y)$ with $T = 1.0$ GeV.
slowly varying functions in the allowed kinematic range for $B_s \to D^*_0 \ell \bar{\nu}$ and $B_s \to D^*_{s1} \ell \bar{\nu}$ decays. They can be well fitted by the linear approximation

$$
\zeta(y) = \zeta(1) (1 - 0.5(y - 1)) , \quad \zeta(1) = 0.45 \pm 0.05 \text{ GeV}
$$
$$
\zeta_1(y) = -\zeta_1(1) (1 + 0.4(y - 1)) , \quad \zeta_1(1) = 0.65 \pm 0.06 \text{ GeV}
$$
$$
\eta^{c}_{ke}(y) = \eta^{c}_{ke}(1) (1 - 0.9(y - 1)) , \quad \eta^{c}(1) = 1.7 \pm 0.2 \text{ GeV}
$$
$$
\eta^{b}_{ke}(y) = \eta^{b}_{ke}(1) (1 - 0.9(y - 1)) , \quad \eta^{b}(1) = 1.6 \pm 0.2 \text{ GeV} .
$$

The errors reflect the uncertainty due to $\omega$’s and $T$. The uncertainty due to the variation of the QCD and HQET parameters is not included, which may reach 5% or more. The systematic error resulted from the use of quark-hadron duality above $\omega_c$ is difficult to estimate. Conservatively speaking, there is a 10% systematic error.

Above parameterizations of the Isgur-Wise functions can be used to calculate the total semileptonic rates and decay branching ratios by integrating Eqs. (2a) and (2b). The values of $\bar{\Lambda}$ and $\bar{\Lambda}'$ can be obtained from two-point sum rules (28) and (29), respectively. They are: $\bar{\Lambda} = 0.62 \text{ GeV}$ [38] and $\bar{\Lambda}' = 0.86 \text{ GeV}$ [11]. The quark masses are taken to be $m_b = 4.7 \text{ GeV}$, $m_c = 1.4 \text{ GeV}$. We use the physical masses, $m_{B_s} = 5.369$ [41], $m_{D^*_{s0}} = 2.317$ and $m_{D^*_{s1}} = 2.457$ [3], for $B_s$, $D^*_{s0}$ and $D^*_{s1}$ mesons. The maximal values of $y$ in the present case are $y_{0\text{max}} = (1 + r^2_0)/2r_0 = 1.374$ and $y_{1\text{max}} = (1 + r^2_1)/2r_1 = 1.321$.

In Table I we present our results for decay rates and branching ratios, as well as those in the infinitely heavy quark limit. We have taken $\tau_{B_s} = 1.46 \text{ ps}$ [41]. In the calculation, the central values for the Isgur-Wise functions in (32) are taken, and the theoretical uncertainties are not included. The ratios of the two semileptonic rates for $B_s$ decays into $D^*_{s0}$ and $D^*_{s1}$ mesons both in taking account of the $1/m_Q$ corrections and in the infinitely heavy quark mass limit are

$$
R_{Br} \equiv \frac{B(B_s \to D^*_{s0} \ell \bar{\nu})}{B(B_s \to D^*_{s1} \ell \bar{\nu})} = \begin{cases} 
2.05 & \text{with } 1/m_Q , \\
1.16 & m_Q \to \infty .
\end{cases}
$$
The branching ratio for $B_s \to D_{s0}^* \ell \bar{\nu}$ decay exceeds the one for $B_s \to D_{s1}^* \ell \bar{\nu}$ in both cases.

<table>
<thead>
<tr>
<th>Decay</th>
<th>With $1/m_Q$</th>
<th>$m_Q \to \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B \to D_{s1}^* \ell \bar{\nu}$</td>
<td>0.38</td>
<td>0.31</td>
</tr>
<tr>
<td>$B \to D_{s0}^* \ell \bar{\nu}$</td>
<td>0.78</td>
<td>0.36</td>
</tr>
</tbody>
</table>

TABLE I: Decay rates $\Gamma$ (in units of $|V_{cb}|/0.04^2 \times 10^{-15}$ GeV) and branching ratios $\text{BR}$ (in %) for $B_s \to D_{s}^{(*\ast)} \ell \bar{\nu}$ decays in taking account of the $1/m_Q$ corrections and in the infinitely heavy quark mass limit. $R$ is a ratio of branching ratios including $O(1/m_Q)$ corrections to branching ratios in the infinitely heavy quark mass limit.

The numerical predictions in Table I indicate that a substantial part of the inclusive semileptonic $B_s$ decays should go to excited $D_s$ meson states. In future hadron $B$ factories the $D_s$ resonant states can be produced directly in a considerable amount of branching ratio from the weak decay of the $B_s$ meson. The study of the semileptonic $B_s$ decays to excited $D_s$ states can provide some information about the structure and the properties for the newly observed $D_{sJ}^*$ states.

From Table I, we see that the $B \to D_{s0}^* \ell \bar{\nu}$ decay rate receives large $1/m_Q$ contributions and gets a sharp increase, while the $B \to D_{s1}^* \ell \bar{\nu}$ decay rate is only moderately increased by subleading $1/m_Q$ corrections. The reason for this is as following. From Eqs. (11) and (111) we see that the decay matrix elements at zero recoil are determined by form factors $g_+ (1)$ and $g_{V1} (1)$, which receive non-vanishing contributions from first order heavy quark mass corrections. Explicitly,

\[ g_+ (1) = -\frac{3}{2} (\varepsilon_c + \varepsilon_b) (\Lambda' - \bar{\Lambda}) \zeta (1), \]
\[ g_{V1} (1) = (\varepsilon_c - 3 \varepsilon_b) (\Lambda' - \bar{\Lambda}) \zeta (1). \]  

(34)

At zero recoil the form factor $g_{V1}$ is suppressed by a very small factor $\varepsilon_c - 3 \varepsilon_b \approx 0.04 \text{GeV}^{-1}$. As a result the $B_s \to D_{s1}^* e \nu$ decay rate is only slightly increased by subleading $1/m_Q$ corrections. On the other hand, $B_s \to D_{s0}^* e \nu$ decay rate receives a large enhancement from $1/m_Q$ corrections. Note that the sharp increase of $B_s \to D_{s1}^* e \nu$ decay rate by $1/m_Q$ corrections does not imply the breakdown of the heavy quark expansion. This is because the allowed kinematic ranges for $B \to D_{s0} \ell \bar{\nu}$ is fairly small, the contribution to the decay rate of the rather small $1/m_Q$ corrections is substantially increased. Hence it is rather a result of kinematical and dynamical effects.

In summary, we have presented the investigation for semileptonic $B_s$ decays into excited $D_s$ mesons. Within the framework of HQET we have applied the QCD sum rules to calculate the universal Isgur-Wise functions up to the subleading order of the heavy quark expansion. The differential decay widths and the branching ratios for the decays $B_s \to$
$D_{s0}^*\ell\bar{\nu}$ and $B_s \rightarrow D_{s1}^*\ell\bar{\nu}$ are computed up to the order of $1/m_Q$ corrections. The decay rates are substantially influenced by the inclusion of the first order $1/m_Q$ corrections.

With the assumption that the newly discovered states $D_{sJ}^*(2317)$ and $D_{sJ}^*(2460)$ can be identified with spin symmetry doublet ($D_{s0}^*, D_{s1}^*$), it is worthwhile to study the semileptonic $B_s$ decays to these newly observed states. In hadron $B$ factories these $p$-wave excited $D_s$ states can be produced directly from the semileptonic decay of the $B_s$ meson with a considerable amount of branching ratio. A measurement of the $B_s \rightarrow D_{sJ}^*$ can provide some information on the nature of the new $D_{sJ}^*$ mesons.

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