Combination of CDF and DØ Results on the Top-Quark Mass

The CDF Collaboration, the DØ Collaboration, and the Tevatron Electroweak Working Group

Abstract

The results on the measurements of the top-quark mass, based on the data collected by the Tevatron experiments CDF and DØ at Fermilab during Run I from 1992 to 1996, are summarized. The combination of the published results, taking correlated uncertainties properly into account, is presented. The resulting world average for the mass of the top quark is: \( M_{\text{top}} = 178.0 \pm 4.3 \text{ GeV}/c^2 \), where the total error of consists of a statistical part of 2.7 GeV/c\(^2\) and a systematic part of 3.3 GeV/c\(^2\).
1 Introduction

The experiments CDF and DØ, taking data at the Tevatron proton-antiproton collider located at the Fermi National Accelerator Laboratory, have published several direct experimental measurements of the pole mass, $M_{\text{top}}$, of the top quark $t$ [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]. The measurements are based on Run I data (1992-1996) and utilize all decay topologies arising in $t\bar{t}$ production given by the leptonic or hadronic decay of the W boson occurring in top-quark decay: the di-lepton channel (di-l) [1, 2, 3, 4, 5], the lepton+jets channel (l+j) [6, 7, 8, 9, 10], and the all-jets channel (all-j) analysed by CDF only [11]. The lepton+jets channel yields the most precise determination of $M_{\text{top}}$. The recently published new measurement in this channel by the DØ collaboration [10] is based on a powerful analysis technique yielding a much smaller measurement uncertainty.

This note reports on the combination of the most recent final and comprehensive mass measurements in each channel by CDF and DØ [2, 3, 5, 7, 10, 11]. The combination takes into account the statistical and systematic uncertainties as well as the correlations between systematic uncertainties, and replaces the previous combination [12]. The new DØ measurement [10] is the single most precise top-quark mass measurement and has the largest weight in this new combination.

2 Measurements

The five measurements of $M_{\text{top}}$ to be combined are listed in Table 1. Besides central values and statistical uncertainties, the systematic errors arising from various sources are reported. In order of decreasing importance, the systematic error sources are:

- Jet energy scale (JES): The systematics for jet energy scale include the uncertainties on the absolute jet energy corrections, calorimeter stability, underlying event and relative jet energy corrections.

- Model for signal (signal): The systematics for the signal model include initial and final state radiation effects, b-tagging bias, dependence upon parton distribution functions as well as variations in $\Lambda_{\text{QCD}}$.

- Model for background (BG): The background model includes estimates of the effect of setting $Q^2 = \langle p_t \rangle^2$ instead of $Q^2 = M_W^2$ in VECBOS [13] simulations of W+jets production, the use of ISAJET [14] fragmentation instead of HERWIG [15] fragmentation as well as the effect of varying the background fraction attributed to QCD.

- Uranium noise and multiple interactions (UN/MI): This uncertainty includes uncertainties arising from uranium noise in the DØ calorimeter and from multiple interactions...
overlapping signal events. CDF includes the systematic uncertainty due to multiple interactions in the JES contribution.

- **Method for mass fitting (fit):** This systematic uncertainty takes into account the finite sizes of Monte Carlo samples used for fitting, impact of jet permutations, and other fitting biases. In the CDF lepton+jets analysis, the systematic uncertainty due to finite Monte Carlo statistics is included in the statistical uncertainty.

- **Monte Carlo generator (MC):** The systematic uncertainty on the Monte Carlo generator provides an estimate of the sensitivity to the simulated physics model by comparing HERWIG to PYTHIA [16] or to ISAJET. In the DØ analyses, the systematic uncertainty associated with the comparison of HERWIG to ISAJET is included in the signal model uncertainty.

For each measurement, the individual error contributions are combined in quadrature.

<table>
<thead>
<tr>
<th>Run I</th>
<th>CDF l+j</th>
<th>CDF di-l</th>
<th>CDF all-j</th>
<th>DØ l+j</th>
<th>DØ di-l</th>
</tr>
</thead>
<tbody>
<tr>
<td>Result</td>
<td>176.1</td>
<td>167.4</td>
<td>186.0</td>
<td>180.1</td>
<td>168.4</td>
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<td>Stat.</td>
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<td>10.3</td>
<td>10.0</td>
<td>3.6</td>
<td>12.3</td>
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<td>JES</td>
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<td>3.8</td>
<td>5.0</td>
<td>3.3</td>
<td>2.4</td>
</tr>
<tr>
<td>Signal</td>
<td>2.6</td>
<td>2.8</td>
<td>1.8</td>
<td>1.1</td>
<td>1.8</td>
</tr>
<tr>
<td>BG</td>
<td>1.3</td>
<td>0.3</td>
<td>1.7</td>
<td>1.0</td>
<td>1.1</td>
</tr>
<tr>
<td>UN/MI</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td>Fit</td>
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<td>0.7</td>
<td>0.6</td>
<td>0.6</td>
<td>1.1</td>
</tr>
<tr>
<td>MC</td>
<td>0.1</td>
<td>0.6</td>
<td>0.8</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Syst.</td>
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<td>4.8</td>
<td>5.7</td>
<td>3.9</td>
<td>3.6</td>
</tr>
<tr>
<td>Total</td>
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<td>11.4</td>
<td>11.5</td>
<td>5.3</td>
<td>12.8</td>
</tr>
</tbody>
</table>

Table 1: Summary of the five measurements of \( M_{\text{top}} \) performed by CDF and DØ. All numbers are in GeV/c\(^2\). For each measurement, the corresponding column lists experiment and channel, central value and contributions to the total error, namely statistical error and systematic errors arising from various sources defined in the text. Overall systematic errors and total errors are obtained by combining individual errors in quadrature.

### 3 Combination

In the combination, the error contributions arising from different sources are uncorrelated between measurements. The correlations of error contributions arising from the same source
are as follows:

- uncorrelated: statistical error, fit error;
- 100% correlated within each experiment: JES error, UN/MI error;
- 100% correlated within each channel: BG error;
- 100% correlated between all measurements: signal error, MC error.

The resulting matrix of global correlation coefficients is listed in Table 2.

<table>
<thead>
<tr>
<th>Run I</th>
<th>CDF l+j</th>
<th>CDF di-l</th>
<th>CDF all-j</th>
<th>DØ l+j</th>
<th>DØ di-l</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF l+j</td>
<td>1.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CDF di-l</td>
<td>0.29</td>
<td>1.00</td>
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</tr>
<tr>
<td>CDF all-j</td>
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<td>0.19</td>
<td>1.00</td>
<td></td>
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</tr>
<tr>
<td>DØ l+j</td>
<td>0.11</td>
<td>0.05</td>
<td>0.03</td>
<td>1.00</td>
<td></td>
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<tr>
<td>DØ di-l</td>
<td>0.05</td>
<td>0.04</td>
<td>0.02</td>
<td>0.17</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Table 2: Matrix of global correlation coefficients between the five measurements of $M_{\text{top}}$.

The five measurements are combined using two independent programs: one which has already been used for the previous combination [12], and one implementing a numerical $\chi^2$ minimisation as well as the analytic BLUE method [18, 19]. The three methods used are mathematically equivalent, and give identical results for the combination. In addition, the BLUE method yields the decomposition of the error on the average in terms of the error categories specified for the input measurements [19].

4 Results

The combined value for the top-quark mass is:

$$M_{\text{top}} = 178.0 \pm 4.3 \text{ GeV}/c^2,$$

(1)

where the total error of 4.3 GeV/c$^2$ contains the following components: a statistical error of 2.7 GeV/c$^2$; and systematic error contributions of: JES 2.6 GeV/c$^2$, signal 1.6 GeV/c$^2$, background 0.88 GeV/c$^2$, UN/MI 0.83 GeV/c$^2$, fit 0.35 GeV/c$^2$, and MC 0.12 GeV/c$^2$, for a total systematic error of 3.3 GeV/c$^2$. 

4
The mass of the top quark is measured by various experiments, and the results are summarized below:

- CDF di-l: $167.4 \pm 11.4$ GeV/c$^2$
- DØ di-l: $168.4 \pm 12.8$ GeV/c$^2$
- CDF l+j: $176.1 \pm 7.3$ GeV/c$^2$
- DØ l+j: $180.1 \pm 5.3$ GeV/c$^2$
- CDF all-j: $186.0 \pm 11.5$ GeV/c$^2$
- TEVATRON Run-I: $178.0 \pm 4.3$ GeV/c$^2$

The chi-square of this average is $2.6$ for 4 degrees of freedom, corresponding to a probability of 63%, showing that all measurements are in good agreement with each other which can also be seen in Figure 1. The pull of each measurement with respect to the average and the weight of each measurement in the average are reported in Table 3.

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5 Summary

An updated combination of the five published measurements of $M_{\text{top}}$ from CDF and DØ based on Run I data is presented. Taking into account statistical and systematic errors including their correlations, the Tevatron and thus the world-average result is: $M_{\text{top}} = 178.0 \pm 4.3 \text{ GeV}/c^2$. The mass of the top quark is now known with an accuracy of 2.4%.

References


