Low-energy nuclear weak-interaction processes play important roles in many astrophysical contexts, and effective field theory is believed to be a highly useful framework for describing these processes in a model-independent manner. I present a brief account of the basic features of the nuclear effective theory approach, and some examples of actual calculations carried out in this method.
1 Introduction

Low-energy nuclear weak-interaction processes play important roles in many astrophysical phenomena and also in terrestrial experiments designed to detect the astrophysical neutrinos. Obviously, it is important to have reliable estimates of the cross sections for these processes. I wish to describe here some of the recent developments in our endeavor to obtain such estimates.\(^3\) My main emphasis will be placed on comparison between the traditional method, to be designated as the *standard nuclear physics approach* (SNPA), and the newly developed *nuclear effective field theory* (EFT) approach. I shall advocate the viewpoints that (i) nuclear EFT can indeed be a powerful framework for describing low-energy nuclear electroweak processes and (ii) that, in practical applications, EFT and SNPA can play complementary roles.

These points are nicely illustrated by the following three examples: (i) neutrino-deuteron reactions for solar neutrino energies; (ii) solar pp fusion; (iii) solar Hep fusion. Since the process (iii) and related topics will be discussed in detail by Dr. Tae-Sun Park at this Symposium, I shall concentrate on the first two reactions. Let me start with a brief explanation of why these processes are of particular current interest.

At SNO (Sudbury Neutrino Observatory), a one-kiloton heavy water Cerenkov counter is used to detect the solar neutrinos. SNO can monitor the neutrino-deuteron reactions:

\[
\nu_e + d \rightarrow e^- + p + p, \quad \nu_x + d \rightarrow \nu_x + p + n, \\
\bar{\nu}_e + d \rightarrow e^+ + n + n, \quad \bar{\nu}_x + d \rightarrow \bar{\nu}_x + p + n,
\]

as well as the pure leptonic reaction \(\nu_x + e^- \rightarrow \nu_x + e^-\). Here \(x\) stands for a neutrino of any flavor (\(e, \mu\) or \(\tau\)). The recent SNO experiments [2] have established that the total solar neutrino flux (counting all neutrino flavors) agrees with the prediction of the standard solar model [3], whereas the electron neutrino flux from the sun is significantly smaller than the total solar neutrino flux. The amount of deficit in the electron neutrino flux was found to be consistent with what had been known as the solar neutrino problem. These results of the SNO experiments have given clear evidence for the transmutation of solar electron neutrinos into neutrinos of other flavors. Obviously, a precise knowledge of the \(\nu\)-d reaction cross sections is important for the in-depth interpretation of the existing and future SNO data.

Meanwhile, the pp fusion reaction

\[
p + p \rightarrow d + e^+ + \nu_e
\]

is the primary solar thermomuclear reaction that essentially controls the luminosity of the sun, and therefore the exact value of its cross section is a crucial input for any elaborate solar models.

\(^3\)This talk has some overlap with the one I gave at NDM03 [1].
2 Calculational frameworks

2.1 Standard nuclear physics approach (SNPA)

The phenomenological potential picture has been highly successful in describing a vast variety of nuclear phenomena. In this picture an $A$-nucleon system is described by a non-relativistic Hamiltonian of the form

$$H = \sum_i t_i + \sum_{i<j} V_{ij} + \sum_{i<j<k} V_{ijk} + \cdots,$$  \hspace{1cm} (3)

where $t_i$ is the kinetic energy of the $i$-th nucleon, $V_{ij}$ is a phenomenological two-body potential between the $i$-th and $j$-th nucleons, $V_{ijk}$ is a phenomenological three-body potential, and so on. (Since potentials involving three or more nucleons play much less important roles than the two-body interactions, we shall be concerned here mainly with $V_{ij}$.) Once the model Hamiltonian $H$ is specified, the nuclear wave function $|\Psi>$ is obtained by solving the Shrödinger equation

$$H|\Psi> = E|\Psi>.$$ \hspace{1cm} (4)

It is fortunate that the progress of numerical techniques for solving eq.(4) has reached such a level [4] that the wave functions of low-lying levels for light nuclei can now be obtained with essentially no approximation (once the validity of the model Hamiltonian eq.(3) is accepted). This liberates us from the “familiar” nuclear physics complications that arise as a result of truncating nuclear Hilbert space down to certain model space (such as shell-model configurations within a limited number of major shells, cluster-model trial functions, etc.)

We note that there is large freedom in selecting possible forms of $V_{ij}$, apart from the well-established requirement that, for a large enough value of the inter-nucleon distance, $V_{ij}$ should agree with the one-pion exchange Yukawa potential. For the model-dependent short-range part of $V_{ij}$, the best we could do is to assume certain functional forms and fix the parameters contained in them by demanding that the solutions of eq.(4) for the $A=2$ case reproduce the nucleon-nucleon scattering data (typically up to the pion-production threshold energy) as well as the deuteron properties. There are by now several so-called modern high-precision phenomenological N-N potential that can reproduce all the existing two-nucleon data with normalized $\chi^2$ values close to 1. These potentials differ widely in the ways short-range physics is parametrized, and, as a consequence, they exhibit substantial difference in their off-shell behaviors. To what extent this arbitrariness may affect the observables of our concern is an important question, to which I will come back later.

In normal situations, nuclear responses to external electroweak probes are given, to good approximation, by one-body terms, which are also called the impulse approximation (IA) terms. To obtain higher accuracy, however, we must include exchange current (EXC) terms, which represent nuclear responses involving two or more nucleons. These exchange currents (usually taken to be two-body operators) are derived from one-boson exchange
diagrams, and the vertices featuring in the relevant diagrams are determined to satisfy
the low-energy theorems and current algebra [5]. We refer to a formalism based on this
picture as the standard nuclear physics approach (SNPA). (This is also called a potential
model in the literature.) Schematically, the nuclear matrix element in SNPA is given by

\[ M_{f_j}^{\text{SNPA}} = \langle \Psi_f^{\text{SNPA}} | \sum_{\ell} O_{\ell}^{\text{SNPA}} + \sum_{\ell<m} O_{\ell m}^{\text{SNPA}} | \Psi_i^{\text{SNPA}} \rangle, \]  

(5)

where the initial (final) nuclear wave function, \( \Psi_i^{\text{SNPA}} (\Psi_f^{\text{SNPA}}) \), is a solution of eq.(4); \( O_{\ell}^{\text{SNPA}} \) and \( O_{\ell m}^{\text{SNPA}} \) are, respectively, the one-body and two-body transition operators for a given
electroweak process.

SNPA has been used extensively to describe nuclear electroweak processes in light
nuclei, and generally good agreement between theory and experiment [4] gives a strong
indication that SNPA essentially captures much of the physics involved.

### 2.2 Effective field theory (EFT)

Even though SNPA has been extremely successful in correlating and explaining a wealth
of nuclear phenomenology, it is still important from a fundamental point of view to raise
the following issues. First, since the hadrons and hadronic systems (such as nuclei) are
governed by quantum chromodynamics (QCD), we should ultimately be able to relate
nuclear phenomena with QCD, but SNPA is reticent about this relation. In particular,
whereas chiral symmetry is known to be a fundamental symmetry of QCD, the SNPA is
largely disjoint from this symmetry. Second, even for describing low-energy phenomena,
SNPA starts with a “realistic” phenomenological potential which is tailored to encode
short-range (high-momentum) and long-range (low-momentum) physics simultaneously.
This mixing of the two different scales seems theoretically unsatisfactory. Third, as we
write down a phenomenological Lagrangian for describing the nuclear interaction and
nuclear responses to the electroweak currents, SNPA does not offer us a clear guiding
principle; it is not obvious whether SNPA is equipped with any identifiable expansion
parameter that helps us to control the possible forms of terms in the Lagrangian and
that provides a general measure of errors in our calculation. To address these and other
related issues, a new approach based on EFT was proposed [6] and it has been studied
with great intensity; for reviews, see Refs. [7]–[11].

The intuitive picture of EFT is quite simple. In describing phenomena characterized
by a typical energy-momentum scale \( Q \), we may expect that our Lagrangian need not
contain explicitly those degrees of freedom that belong to energy-momentum scales much
higher than \( Q \). This expectation motivates us to introduce a cut-off scale \( \Lambda \) that is
sufficiently large compared with \( Q \) and classify our fields (to be generically represented
by \( \phi \)) into two groups: high-frequency fields \( \phi_H \) whose frequencies are higher than \( \Lambda \),
and low-frequency fields \( \phi_L \) with frequencies lower than \( \Lambda \). By eliminating (or integrating out) \( \phi_H \), we arrive at an effective Lagrangian that only involves \( \phi_L \) as explicit dynamical
variables. In terms of path integrals, the effective Lagrangian \( \mathcal{L}_{\text{eff}} \) is related to the original
Lagrangian $\mathcal{L}$ as

$$\int [d\phi] \exp \{ i \int d^4x \mathcal{L}[\phi] \} = \int [d\phi_L] [d\phi_n] \exp \{ i \int d^4x \mathcal{L}[\phi_n, \phi_L] \}$$

(6)

$$\equiv \int [d\phi_L] \exp \{ i \int d^4x \mathcal{L}_{\text{eff}}[\phi_L] \} .$$

(7)

It can be shown that $\mathcal{L}_{\text{eff}}$ defined by eq.(7) inherits the symmetries (and the patterns of symmetry breaking, if there are any) of the underlying Lagrangian $\mathcal{L}$. It also follows that $\mathcal{L}_{\text{eff}}$ should be the sum of all possible monomials of $\phi_L$ and their derivatives that are consistent with the symmetry requirements of $\mathcal{L}$. Because a term involving $n$ derivatives scales like $(Q/\Lambda)^n$, we can organize terms in $\mathcal{L}_{\text{eff}}$ into a perturbative series in which $Q/\Lambda$ serves as an expansion parameter. The coefficients of terms in this expansion scheme are called the low-energy constants (LECs). Provided all the LEC’s up to a specified order $n$ can be fixed either from theory or from fitting to the experimental values of relevant observables, $\mathcal{L}_{\text{eff}}$ serves as a complete (and hence model-independent) Lagrangian to the given order of expansion.

Having sketched the basic idea of EFT, we now discuss the specific aspects of EFT as applied to nuclear physics. The underlying Lagrangian $\mathcal{L}$ in this case is the QCD Lagrangian $\mathcal{L}_{QCD}$, whereas, for a typical nuclear physics energy-momentum scale $Q \ll \Lambda_{\chi} \sim 1$ GeV, the effective degrees of freedom that feature in $\mathcal{L}_{\text{eff}}$ are the hadrons rather than the quarks and gluons. It is non-trivial to apply the formal definition in eq.(7) to derive $\mathcal{L}_{\text{eff}}$ written in terms of hadrons starting from $\mathcal{L}_{QCD}$, because the hadrons cannot be simply identified with the low-frequency field $\phi_L$ in $\mathcal{L}_{QCD}$. To proceed, we choose to be guided solely by symmetry considerations and the above-mentioned expansion scheme. Chiral symmetry plays an important role here. Chiral symmetry is known to be spontaneously broken, leading to the generation of the pions as Nambu-Goldstone bosons. We can incorporate this feature by assigning suitable chiral transformation properties to the Goldstone bosons and writing down all possible chiral-invariant terms up to a specified chiral order [12]. It is to be noted that the above consideration presupposes exact chiral symmetry in $\mathcal{L}_{QCD}$. In reality, $\mathcal{L}_{QCD}$ contains small but finite quark mass terms, which violate chiral symmetry explicitly and lead to a non-vanishing value of the pion mass $m_\pi$. Again, there is a well-defined framework to determine what terms are needed to represent the effect of explicit chiral symmetry breaking [12]. These considerations lead to an EFT called chiral perturbation theory (χPT) [13, 14]. The successes of χPT in the meson sector are well known; see, e.g., Ref. [7].

A difficulty we encounter in extending χPT to the nucleon sector is that, because the nucleon mass $m_N$ is comparable to the cut-off scale $\Lambda_{\chi}$, a simple application of expansion in $Q/\Lambda$ does not work. We can surmount this obstacle with the use of heavy-baryon chiral perturbation theory (HBχPT), which essentially consists in shifting the reference point of the nucleon energy from 0 to $m_N$ and integrating out the small component of the nucleon field as well as the anti-nucleon field. Thus an effective Lagrangian in HBχPT contains as explicit degrees of freedom the pions and the large components of the redefined nucleon field. HBχPT has as expansion parameters $Q/\Lambda_{\chi}, m_\pi/\Lambda_{\chi}$ and $Q/m_N$. Since $m_N \approx \Lambda_{\chi}$, it is convenient to combine chiral and heavy-baryon expansions and introduce the chiral index.
\( \bar{\nu} \) defined by \( \bar{\nu} = d + \frac{n}{2} - 2 \). Here \( n \) is the number of fermion lines participating in a given vertex, and \( d \) is the number of derivatives (with \( m_\pi \) counted as one derivative). A similar power counting scheme can be introduced for Feynman diagrams as well. According to Weinberg [6], a Feynman diagram that contains \( N_A \) nucleons, \( N_E \) external fields, \( L \) loops and \( N_C \) disjoint parts scales like \( \left( \frac{Q}{\Lambda} \right)^\nu \), where the chiral index \( \nu \) is defined as

\[
\nu = 2L + 2(N_C - 1) + 2 - (N_A + N_E) + \sum_i \bar{\nu}_i ,
\]

with the summation running over all the vertices.

Although HB\( \chi \)PT has been very successful in the one-nucleon sector [7], we cannot apply HB\( \chi \)PT in a straightforward manner to nuclei, which contain more than one nucleon. This is because nuclei allow very low-lying excited states, and the existence of this small energy scale invalidates chiral counting [6]. Weinberg avoided this difficulty by classifying Feynman diagrams into two groups, irreducible and reducible diagrams. Irreducible diagrams are those in which every intermediate state has at least one meson in flight; all others are categorized as reducible diagrams. The chiral counting rules should only be applied to irreducible diagrams. The contribution of all the two-body irreducible diagrams (up to a specified chiral order) is treated as an effective potential (to be denoted by \( V_{ij}^{EFT} \)) that acts on nuclear wave functions. Meanwhile, the contributions of reducible diagrams can be incorporated [6] by solving the Schrödinger equation

\[
H_{EFT}^{EFT} |\Psi_{EFT}^{EFT} > = E |\Psi_{EFT}^{EFT} > ,
\]

where

\[
H_{EFT}^{EFT} = \sum_i t_i + \sum_{i<j} V_{ij}^{EFT} ,
\]

We refer to this two-step procedure as nuclear \( \chi \)PT, or, to be more specific, nuclear \( \chi \)PT in the Weinberg scheme. (This is often called the \( \Lambda \)-counting scheme [9].)

To apply nuclear \( \chi \)PT to a process that involves (an) external current(s), we derive a nuclear transition operator \( T \) by calculating the contributions of all the irreducible diagrams (up to a given chiral order \( \nu \)) that involve the relevant external current(s). To maintain consistent chiral counting, the nuclear matrix element of \( T \) must be calculated with the use of nuclear wave functions which are governed by nuclear interactions that represent all the irreducible A-nucleon diagrams up to \( \nu \)-th order. Thus, a transition matrix in nuclear EFT is given by

\[
M_{f i}^{EFT} = < \Psi_f^{EFT} | \sum_{\ell} A_{\ell}^{EFT} + \sum_{\ell < m} A_{\ell m}^{EFT} | \Psi_i^{EFT} > ,
\]

where the superscript, “EFT”, implies that the relevant quantities are obtained according to EFT as described above. If this program is carried out exactly, it would constitute an \textit{ab initio} calculation. It is worth noting that EFT tells us exactly at what chiral order three-body operators start to contribute to \( T \), and that, to chiral orders of our present
concern, we do not need three-body operators. For this reason we have retained in eq.(11) only one- and two-body operators. This type of unambiguous classification of transition operators according to their chiral orders is a great advantage of EFT, which is missing in eq.(5).

I should mention that there exists an alternative form of nuclear EFT based on the power divergence subtraction (PDS) scheme. The PDS scheme proposed by Kaplan, Savage and Wise in their seminal papers [15] uses a counting scheme (often called Q-counting) that differs from the Weinberg scheme. An advantage of the PDS scheme is that it maintains formal chiral invariance, whereas the Weinberg scheme loses manifest chiral invariance. In many practical applications, however, this formal problem is not worrisome up to the chiral order under consideration, i.e., the chiral order up to which our irreducible diagrams are evaluated. Although the PDS scheme has produced many important results (for a review, see e.g. Ref. [10]), I concentrate here on the Weinberg scheme, because this is the framework in which our own work has been done.

2.3 Hybrid EFT

In the above I emphasized the formal merits of nuclear EFT. In actual calculations, however, we face the following two problems. First, it is still a great challenge to generate, strictly within the EFT framework, nuclear wave functions whose accuracy is comparable to that of SNPA wave functions. Second, as mentioned earlier, the chiral Lagrangian, \( \mathcal{L}_{\text{eff}} \), is definite only when the values of all the relevant LECs are fixed, but there may be cases where this requirement cannot be readily met. A pragmatic solution to the first problem is to use in eq.(11) wave functions obtained in SNPA; we refer to this eclectic approach as hybrid EFT. A nuclear transition matrix element in hybrid EFT is given by

\[
\mathcal{M}_{fi}^{\text{hyb-EFT}} = \langle \Psi_{i}^{\text{SNPA}} | \sum_{\ell} \mathcal{O}_{\ell}^{\text{EFT}} + \sum_{\ell < m} \mathcal{O}_{\ell m}^{\text{EFT}} | \Psi_{f}^{\text{SNPA}} \rangle,
\]

(12)

Because, as mentioned, the NN interactions that generate SNPA wave functions reproduce accurately the entirety of the two-nucleon data, the adoption of eq.(12) is almost equivalent to using the empirical data themselves to control the initial and final nuclear wave functions. In the context of theoretically deriving the nuclear interactions based on EFT, hybrid EFT may look like “retrogression”. But, if our goal is to obtain a transition matrix element as accurately as possible with the maximum help of available empirical input, hybrid EFT is a justifiable approach insofar as the above-mentioned off-shell problem and the contributions of three-body (and higher-body) interactions are properly addressed. These points will be discussed later on.

The calculations reported in Refs. [19, 20] seem to support hybrid EFT. There, the nuclear matrix elements in the A=2 systems for one-body operators (or IA terms) calculated with the use of EFT-generated wave functions were found to be very close to those calculated with the SNPA wave functions. Thus EFT and hybrid EFT should give practically the same IA matrix elements. Meanwhile, we can generally expect that the ratio of the two-body EXC contributions to those of the IA operators should be much
less sensitive to the details of the nuclear wave functions than the absolute values are. It therefore seems reasonable to rely on $\chi$PT for deriving transition operators and evaluate their matrix elements using the realistic wave functions obtained in SNP A, and in this sense hybrid EFT is more than a mere expedient.

The issue of possible unknown LECs will be discussed in the next subsection.

2.4 MEEFT or EFT*

Hybrid EFT can be used for complex nuclei ($A = 3, 4, \ldots$) with essentially the same accuracy and ease as for the $A=2$ system. We should reemphasize in this connection that, in $A$-nucleon systems ($A \geq 3$), the contributions of transition operators involving three or more nucleons are intrinsically suppressed according to chiral counting, and hence, up to a certain chiral order, a transition operator in an $A$-nucleon system consists of the same EFT-based 1-body and 2-body terms as used for the two-nucleon system. Then, since SNP A provides high-quality wave functions for the $A$-nucleon system, one can calculate $M_{\text{hyb-EFT}}$ with precision comparable to that for the corresponding two-nucleon case.

Now, in most practical cases, the one-body operator, $O_{\ell}^{\text{EFT}}$, is free from unknown LECs. So let us concentrate on the two-body operator, $O_{\ell m}^{\text{EFT}}$, and suppose that $O_{\ell m}^{\text{EFT}}$ under consideration contains an LEC (call it $\kappa$) that cannot be determined with the use of $A=2$ data alone. It is possible that an observable (call it $\Omega$) in a $A$-body system ($A \geq 3$) is sensitive to $\kappa$ and that the experimental value of $\Omega$ is known with sufficient accuracy. Then we can determine $\kappa$ by calculating $M_{\text{hyb-EFT}}$ responsible for $\Omega$ and adjusting $\kappa$ to reproduce the empirical value of $\Omega$. Once $\kappa$ is fixed this way, we can make predictions for any other observables for any other nuclear systems that are controlled by the same transition operators. When hybrid EFT is used in this manner, we refer to it as MEEFT (more effective EFT) or EFT*.

MEEFT is the most efficient existing formalism for correlating various observables in different nuclei, using the transition operators controlled by EFT. A further notable advantage of MEEFT is that, since correlating the observables in neighboring nuclei is likely to serve as an additional renormalization, the possible effects of higher chiral order terms and/or off-shell ambiguities can be significantly suppressed by the use of MEEFT.\footnote{MEEFT should be distinguished from an earlier naive hybrid EFT model in which the short-range terms were dropped altogether using an intuitive argument based on short-range NN repulsion.}

I will come back to this point later, when I discuss concrete examples.

We need to recall here the important role of momentum cutoff in EFT. As emphasized before, the effective Lagrangian $\mathcal{L}_{\text{eff}}$ is, by construction, valid only below the specified cutoff scale $\Lambda$. This basic constraint must be respected in our nuclear EFT calculations; we must ensure that nuclear intermediate states involved in the computation of eq.(11) remain within this constrained regime. It is reasonable to implement this constraint by requiring that the two-nucleon relative momentum should be smaller than $\Lambda$. A possible choice of the cutoff function is the Gaussian form $\exp(-\vec{p}^2/\Lambda^2)$. (The detailed form of the cutoff function should not be very important.) As a reasonable range of the value of $\Lambda$ we may choose: $500$ MeV $\lesssim \Lambda \lesssim 800$ MeV, where the lower bound is dictated
by the requirement that $\Lambda$ should be sufficiently large compared with the pion mass (in order to accommodate pion physics), while the upper bound reflects the fact that our EFT is devoid of the $\rho$ meson.

3 Numerical results

We now discuss the applications of the above-described calculational methods to the two processes of our concern: $pp$ fusion and the $\nu\cdot d$ reaction. These reactions share the common feature that a precise knowledge of the Gamow-Teller (GT) transition matrix elements is crucial in estimating their cross sections. We therefore concentrate on the GT transitions. We will show here, following Refs. [23, 24], that MEEFT can be used very profitably for these reactions.

We can argue (see, e.g., Ref. [24]) that 1-body IA operators for the GT transition can be fixed unambiguously from the available 1-body data. As for the 2-body operators, to next-to-next-to-next-to-leading order ($N^3\text{LO}$) in chiral counting, there appears one unknown LEC that cannot be at present determined from data for the $A=2$ systems. This unknown LEC, denoted by $\hat{d}_R$ in Ref. [18], parametrizes the strength of contact-type four-nucleon coupling to the axial current. Park et al. [23, 24] noted that the same LEC, $\hat{d}_R$, also appears as a single unknown parameter in the calculation of the tritium $\beta$-decay rate $\Gamma^t_\beta$, and they used MEEFT to place a constraint on $\hat{d}_R$ from the experimental value of $\Gamma^t_\beta$. Since $\Gamma^t_\beta(\text{exp})$ is known with high precision, and since the accurate wave functions of $^3\text{H}$ and $^3\text{He}$ are available from a well-developed SNPA calculation [25], we can determine $\hat{d}_R$ with sufficient accuracy for our purposes. Once the value of $\hat{d}_R$ is determined this way, we can carry out parameter-free MEEFT calculations for $pp$-fusion [23, 24] and the $\nu\cdot d$ reactions [27]. I present here a brief summary of the results of these calculations.

For a given value of $\Lambda$ within the above-mentioned range ($500 \text{ MeV} \lesssim \Lambda \lesssim 800 \text{ MeV}$), $\hat{d}_R$ is adjusted to reproduce $\Gamma^t_\beta(\text{exp})$, and then the cross sections for $pp$-fusion and the $\nu\cdot d$ reactions are calculated. The results indicate that, although the best-fit value of $\hat{d}_R$ varies significantly as a function of $\Lambda$, the observables (in our case the above two reaction cross sections) exhibit a high degree of stability against the variation of $\Lambda$. This stability may be taken as an indication that the use of MEEFT for inter-correlating the observables in neighboring nuclei effectively renormalizes various effects, such as the contributions of higher-chiral order terms, mismatch between the SNPA and EFT wave functions, etc. This stability is essential in order for MEEFT to maintain its predictive power.

Park et al. [23, 24] used MEEFT to calculate the rate of $pp$ fusion, $pp \rightarrow e^+\nu_e\cdot d$. The result expressed in terms of the threshold $S$-factor is

$$S_{pp}(0) = 3.94 \times (1 \pm 0.005) \times 10^{-25} \text{ MeV b}.$$  \hspace{1cm} (13)

It has been found that $S_{pp}(0)$ changes only by $\sim 0.1\%$ against changes in $\Lambda$, assuring thereby the robustness of the MEEFT prediction. The MEEFT result, eq.(13), is consistent with that obtained in SNPA by Schiavilla et al. [26]. Meanwhile, the fact that MEEFT allows us to make an error estimate [as given in eq.(13)] is a notable advantage over SNPA.
The details on how we arrive at this error estimate can be found in Refs. [23, 24]. Here I just remark that the error indicated in eq.(13) represents an improvement by a factor of $\sim 10$ over the previous results based on a naive hybrid EFT [18].

We now move to the $\nu$-$d$ reactions, eq.(1), and give a brief survey of all the recent results obtained in SNP A, EFT and MEEFT. Within SNP A a detailed calculation of the $\nu$-$d$ cross sections, $\sigma(\nu d)$, was carried out by Nakamura, Sato, Gudkov and myself [28], and this calculation has recently been updated by Nakamura et al. (NETAL) [30]. As demonstrated in Ref.[31], the SNP A exchange currents for the GT transition are dominated by the $\Delta$-particle excitation diagram, and the reliability of estimation of this diagram depends on the precision with which the coupling constant $g_{\pi N\Delta}$ is known. NETAL fixed $g_{\pi N\Delta}$ by fitting $\Gamma_5^\text{\theta}(\text{exp})$, and proceeded to calculate $\sigma(\nu d)$. Meanwhile, Butler, Chen and Kong (BCK) [32] carried out an EFT calculation of the $\nu$-$d$ cross sections, using the PDS scheme [15]. The results of BCK agree with those of NETAL in the following sense. BCK’s calculation involves one unknown LEC (denoted by $L_{1A}$), which like $d_R$ in Ref.[24], represents the strength of a four-nucleon axial-current coupling term. BCK determined $L_{1A}$ by requiring that the $\nu d$ cross sections of NETAL be reproduced by their EFT calculation. With the value of $L_{1A}$ adjusted this way, $\sigma(\nu d)$’s obtained by BCK show a perfect agreement with those of NETAL for all the four reactions in eq.(1) and for the entire solar neutrino energy range, $E_\nu \lesssim 20$ MeV. Moreover, the best-fit value, $L_{1A} = 5.6\, \text{fm}^3$, found by BCK [32] is consistent with its magnitude expected from the naturalness argument (based on a dimensional analysis), $|L_{1A}| \leq 6\, \text{fm}^3$. The fact that an EFT calculation (with one parameter fine-tuned) reproduces the results of SNP A very well strongly suggests the robustness of the SNP A calculation of $\sigma(\nu d)$.

Even though it is reassuring that the $\nu$-$d$ cross sections calculated in SNP A and EFT agree with each other (in the above-explained sense), it is desirable to carry out an EFT calculation that is free from any adjustable LEC. Fortunately, MEEFT allows us to carry out an EFT-controlled parameter-free calculation of the $\nu$-$d$ cross sections, and such a calculation was carried out by Ando et al. [27]. The $\sigma(\nu d)$’s obtained in Ref. [27] are found to agree within 1% with $\sigma(\nu d)$’s obtained by NETAL using SNP A [30]. These results show that the $\nu$-$d$ cross sections used in interpreting the SNO experiments [2] are reliable at the 1% precision level.

We remark that, as PDS [15] is built on an expansion scheme for transition amplitudes themselves, it does not employ the concept of wave functions. This feature is an advantage in some contexts, but its disadvantage in the present context is that we cannot readily relate the transition matrix elements for an A-nucleon system with those for the neighboring nuclei; in PDS, each nuclear system requires a separate parametrization. This feature underlies the fact that, in the work of BCK [32], $L_{1A}$ remained undetermined, because no experimental data is available to fix $L_{1A}$ within the two-nucleon systems.

\footnote{For a review of the earlier SNP A calculations, see Ref. [29].}
4 Discussion

In introducing hybrid EFT, we replace $|\Psi^{EFT}\rangle$ for the initial and final nuclear states in eq.(11) with the corresponding $|\Psi^{SNPA}\rangle$’s; see eq.(12). This replacement may bring in a certain degree of model dependence, called the off-shell effect, because the phenomenological NN interactions are constrained only by the on-shell two-nucleon observables.\(^6\) This off-shell effect, however, is expected to be small for the reactions under consideration, since they involve low momentum transfers and hence are not extremely sensitive to the short-range behavior of the nuclear wave functions. One way to quantify this expectation is to compare a two-nucleon relative wave function generated by the phenomenological potential with that generated by an EFT-motivated potential. Phillips and Cohen [20] made such a comparison in their analysis of the 1-body operators responsible for electron-deuteron Compton scattering, and showed that a hybrid EFT works well up to momentum transfer 700 MeV. A similar conclusion is expected to hold for a two-body operator, so long as its radial dependence has a “smeared-out” structure reflecting the finite momentum cutoff. We can therefore expect that hybrid EFT as applied to low energy should be practically free from the off-shell ambiguities. The off-shell effect should be even less significant in MEEFT, wherein an additional “effective” renormalization is likely to be at work (see subsection 2.4).

Another indication of the stability of the MEEFT results comes from a recently proposed idea of the low-momentum nuclear potential [33]. As mentioned, a “realistic phenomenological” nuclear interaction, $V_{ij}$ in eq.(3), is determined by fitting to the full set of two-nucleon data up to the pion production threshold energy. So, physically, $V_{ij}$ should reside in a momentum regime below a certain cutoff, $\Lambda_c$. In the conventional treatment, however, the existence of this cutoff scale is ignored, and eq.(4) is solved in such a manner that the entire momentum range is allowed to participate. Bogner et al. proposed to construct an effective low-momentum potential $V_{low-k}$ by eliminating (or integrating out) from $V_{ij}$ the momentum components higher than $\Lambda_c$, and calculated $V_{low-k}$’s corresponding to a number of well-established $V_{ij}$’s. It was found that all these $V_{low-k}$’s lead to identical half-off-shell T-matrices, even though the ways short-range physics is encoded in them are highly diverse. This implies that the $V_{low-k}$’s are free from the off-shell ambiguities, and therefore the use of $V_{low-k}$’s is essentially equivalent to employing $V_{ij}^{EFT}$ (appearing in eq.(10)), which by construction should be model-independent. Now, as mentioned, our MEEFT calculation has a momentum-cutoff regulator built in, and this essentially ensures that the matrix element, $\mathcal{M}_{fj}^{hyb-EFT}$, in eq.(12) is only sensitive to the half-off-shell T-matrices that are controlled by $V_{low-k}$ instead of $V_{ij}$. Therefore, we can expect that the MEEFT results reported here are essentially free from the off-shell ambiguities.

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\(^6\)In a fully consistent theory, physical observables are independent of field transformations that lead to different off-shell behaviors, and therefore the so-called off-shell effect is not really a physical effect. In an approximate theory, observables may exhibit superficial dependence on the off-shell behavior, and it is customary to refer to this dependence as an off-shell “effect”.

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5 Summary

After giving a very limited survey of nuclear $\chi$PT, I must repeat my disclaimer that I have left out many important topics belonging to nuclear $\chi$PT. Among others, I did not discuss very important studies by Epelbaum, Glöckle and Meißner [34] to construct a formally consistent framework for applying $\chi$PT to complex nuclei. It should be highly informative to apply this type of formalism to electroweak processes and compare the results with those of MEEFT. In this connection I find it noteworthy that the range of the cutoff parameter favored in Ref. [34] is consistent with the range used by Park et al. [23, 24]

Despite the highly limited scope of topics covered, I hope I have succeeded in demonstrating that MEEFT is a powerful framework for computing the transition amplitudes of low-energy electroweak processes in light nuclei. I also wish to emphasize that, in each of the cases for which both SNPA and MEEFT calculations have been performed, it has been found that the result of MEEFT supports and improves the SNPA result.

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References


