Transition to perturbative QCD in two-photon collisions

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ABSTRACT

We propose that the different angular distributions in two-photon collisions observed at low and high center-of-mass energies $W_{\gamma\gamma}$ indicate the transition from nonperturbative to perturbative QCD. We calculate the differential cross sections of $\gamma\gamma \rightarrow \pi\pi, KK$ in the angle $\theta$ of one of the final-state mesons using QCD sum rules and the perturbative QCD approach based on $k_T$ factorization theorem. Our predictions from sum rules (perturbative QCD) decrease (increase) with $|\cos \theta|$, consistent with the Belle data of $\gamma\gamma \rightarrow K^+K^-$ for $W_{\gamma\gamma} \approx 1.5-1.7 \ (2.2-2.4)$ GeV.

It has remained as a controversy whether perturbative QCD (PQCD) is applicable to exclusive processes at moderate energies [1]. This issue has been widely discussed in the literature, but general agreement is still not available. The discussion usually focused on the simplest case, hadronic form factors, which have been computed in nonperturbative frameworks, such as QCD sum rules (QSR) [2], and in the PQCD formalism [3, 4, 5, 6]. Within theoretical uncertainty, the predictions from both approaches were claimed to be consistent with experimental data [7, 8]. The contradictory conclusions on the dominant dynamics, soft (Feynman mechanism [9]) or hard, in exclusive processes at moderate energies were drawn. We have explained this subtlety in [10], and argued that there is no contradiction, because the definitions of a soft contribution vary among different theoretical frameworks. With this subtlety, a hadronic form factor may not be the most appropriate quantity to discriminate the soft-dominance and hard-dominance pictures.

In this work we shall propose that the angular distribution in two-photon collisions is an appropriate quantity for the purpose. We take the processes $\gamma\gamma \rightarrow \pi\pi, KK$ as an example, and calculate the pion and kaon angular distributions using nonperturbative QSR, which are reliable at a low center-of-mass energy $W_{\gamma\gamma}$, and the PQCD approach based on $k_T$ factorization theorem [11, 12, 13, 14, 15], which is reliable at a high $W_{\gamma\gamma}$. It will be shown that QSR and PQCD give dramatically different predictions: the former decrease, while the latter increase with $|\cos \theta|$, where $\theta$ denotes the angle of one of the final-state mesons in the center-of-mass frame. In fact, this difference is a general feature of QSR and PQCD predictions for two-photon
collisions, regardless the final states being baryons or mesons. It is thus important to measure the dependence of the angular distribution on $W_{\gamma\gamma}$, and to see whether it evolves with $W_{\gamma\gamma}$ following the QSR and PQCD predictions. Such a transition has indeed been observed in the process $\gamma\gamma \to K^+K^-$ by Belle recently [16].

It will be shown that our predictions from QSR (PQCD) are consistent with the experimental data of the $\gamma\gamma \to K^+K^-$ differential cross section [16] for $W_{\gamma\gamma} \approx 1.5-1.7$ (2.2-2.4) GeV. Hence, we conclude that the transition from nonperturbative to perturbative QCD in $\gamma\gamma \to K^+K^-$ occurs at $W_{\gamma\gamma} \approx 2$ GeV, the same as that drawn from the analysis of the pion form factor [12]. The transition was not observed by ALEPH, since only the $\gamma\gamma \to \pi^+\pi^-$, $K^+K^-$ cross sections with $W_{\gamma\gamma} > 2$ GeV were measured [17]. The PQCD calculation of the processes $\gamma\gamma \to \pi\pi$, $KK$ has been performed in [18], but smaller cross sections were obtained. We shall point out that the difference between this work and [18] comes from the models of meson distribution amplitudes. We shall also compare the PQCD approach with another different type of factorization theorem formulated by means of two-meson distribution amplitudes [19], both of which give similar predictions. The same transition has been also observed in $\gamma\gamma \to p\bar{p}$ [20]: the proton angular distribution decreases with $|\cos \theta|$ for $W_{\gamma\gamma} < 2.5$ GeV and becomes increasing for $W_{\gamma\gamma} > 2.5$ GeV. The latter behavior is consistent with the predictions from the PQCD approach based on $k_T$ factorization theorem [21] and from the diquark model [22], in which a hard scattering takes place between the diquark and the third valence quark. Note that the transition scale was claimed to be about 3 GeV from the analysis of the proton form factor [23].

The differential cross section of pion Compton scattering $\pi\gamma \to \pi\gamma$ has been calculated in QSR and in PQCD [24, 25, 26, 27]. The $\gamma\gamma \to \pi\pi$ differential cross section can be easily derived by exchanging the Mandelstam invariants $s$ and $t$. Since the QSR formulas in [27] contain incomplete sub-leading terms in powers of $1/s$, $1/t$, the straightforward exchange of $s$ and $t$ does not respect the symmetry under the reflection between $\cos \theta$ and $-\cos \theta$. One must neglect the sub-leading terms first, and then exchange $s$ and $t$. The resultant differential cross section for two-photon collision $\gamma(q_1)\gamma(q_2) \to \pi(p_1)\pi(p_2)$ in QSR is written as,

$$\frac{d\sigma}{d\cos \theta} = \frac{|\mathcal{M}|^2}{32\pi s}, \quad |\mathcal{M}|^2 = \frac{|H_1|^2 + |H_2|^2}{2},$$

where the amplitudes $H_i$, $i = 1, 2$, come from the decomposition of the amplitude,

$$\mathcal{M}_{\mu\nu}(s, t, u) = H_1(s, t, u)e^{(1)}_\mu e^{(1)}_\nu + H_2(s, t, u)e^{(2)}_\mu e^{(2)}_\nu,$$

with the subscripts $\mu$ and $\nu$ corresponding to the two photon vertices. The helicity vectors $e^{(1)}$ and $e^{(2)}$ have been defined in [24, 26], which satisfy the orthogonality conditions $e^{(i)} \cdot e^{(j)} = -\delta_{ij}$.

The explicit expressions of $H_i$ are given by

$$f_\pi^2 H_i(s, t, u) \frac{tu}{s} = \left[ \int_0^{s_0} ds_1 \int_0^{s_0} ds_2 \rho_i^{\text{pert}}(s, t, u, s_1, s_2) 
+ \frac{\alpha_s}{\pi} (G^2) \int_0^{s_0} ds_1 \int_0^{s_0} ds_2 \rho_i^{\text{gluon}}(s, t, u, s_1, s_2) \right] \exp[-(s_1 + s_2)/M^2]$$

$$+ C_i^{\text{quark}}(s, t, u) \alpha_s \langle \bar{\psi} \psi \rangle,$$

$$f_\pi = 132 \text{ MeV}$$

being the pion decay constant. The Mandelstam invariants are given by

$$s = (q_1 + q_2)^2 = W_{\gamma\gamma}^2,$$
\[ t = (q_1 - p_1)^2 = -\frac{W_2^2}{2}(1 - \cos \theta), \]
\[ u = (q_1 - p_2)^2 = -\frac{W_2^2}{2}(1 + \cos \theta). \]
\[ (4) \]

For the process \( \gamma(q_1)\gamma(q_2) \to K(p_1)K(p_2) \), the differential cross section is obtained from Eq. (3) by applying the replacements,
\[ f_\pi \to f_K, \quad \exp[-(s_1 + s_2)/M^2] \to \exp[(2m_K^2 - s_1 - s_2)/M^2], \]
\[ (5) \]
with the kaon decay constant \( f_K = 160 \text{ MeV} \) and the kaon mass \( m_K = 0.49 \text{ GeV} \), which is not negligible compared to the value of \( s_0 \) determined below.

The method to evaluate the perturbative spectral densities\( \rho_i^{\text{pert}} \) and the power corrections, \( \rho_i^{\text{gluon}} \) and \( C_i^{\text{quark}} \), has been described in [25]. The perturbative, gluonic and quark contributions to \( H_1 \) are written, respectively, as
\[ \rho_1^{\text{pert}} = \frac{640Q^{14}(Q^2 - t)(Q^2 - u)(s_1 + s_2)}{3\pi^2(4Q^4 - s_1s_2)^5}, \]
\[ \rho_1^{\text{gluon}} = \frac{-5120Q^{20}(4tu + 9Q^4)}{27(2Q^2 - s_1)^2(2Q^2 - s_2)^2(4Q^4 - s_1s_2)^5}, \]
\[ C_1^{\text{quark}} = -\frac{16(t - u)^2}{9M^4s}, \]
\[ (6) \]
for the variable,
\[ Q^2 = \frac{1}{4}(s_1 + s_2 - s + \sqrt{(s_1 + s_2 - s)^2 - 4s_1s_2}). \]
\[ (7) \]
The corresponding quantities associated with \( H_2 \) are given by
\[ \rho_2^{\text{pert}} = \frac{-640Q^{14}(Q^2 - t)(Q^2 - u)(s_1 + s_2)}{\pi^2(4Q^4 - s_1s_2)^5}, \]
\[ \rho_2^{\text{gluon}} = \frac{-5120Q^{20}(12tu + 7Q^4)}{27(2Q^2 - s_1)^2(2Q^2 - s_2)^2(4Q^4 - s_1s_2)^5}, \]
\[ C_2^{\text{quark}} = -\frac{128}{9}\frac{tu}{M^2s^2}. \]
\[ (8) \]
It is easy to find that Eqs. (6) and (8) respect the symmetry under the exchange of \( t \) and \( u \), i.e., of \( \cos \theta \) and \( -\cos \theta \). The gluon and quark condensates, \( \langle G^2 \rangle \) and \( \langle (\bar{\psi}\psi)^2 \rangle \), take the values
\[ \frac{\alpha_s}{\pi}\langle G^2 \rangle = 1.2 \times 10^{-2}\text{GeV}^4, \]
\[ \alpha_s\langle (\bar{\psi}\psi)^2 \rangle = 1.8 \times 10^{-4}\text{GeV}^6, \]
\[ (9) \]
respectively.

In the PQCD approach based on \( k_T \) factorization theorem, we have derived the factorization formulas for pion Compton scattering [27]. Compared to the derivation in collinear factorization theorem [28, 29, 30], parton transverse momenta have been retained. Note that there are minor mistakes in the factorization formulas for the pion Compton scattering presented in [27].
have taken this chance to correct them, and then applied the exchange of $s$ and $t$. The resultant expressions for $H_i$, $i = 1$ and 2, involved in the two-photon collision $\gamma\gamma \to \pi\pi$ are given by

$$H_i(s, t, u) = \int_0^1 dx_1 dx_2 \phi_\pi(x_1)\phi_\pi(x_2) \int_0^\infty db \left[ (e_u^2 + e_d^2)T_i(x_i, s, t, u, b) + 2e_ue_dT_i'(x_i, s, t, u, b) \right] \exp[-S(x_i, b, W_{\gamma\gamma}/\sqrt{2})] ,$$

where the variable $b$ denotes the transverse separation between the two valence quarks of the pion, and the charge factors are $e_u^2 + e_d^2 = 5/9$ and $e_u e_d = 2/9$. The corresponding hard kernels are written as

$$T_1 = -\frac{32\pi^2 C_F \alpha_s(w_1)}{1 - x_1}(1 - x_2) \left( \frac{u}{t} + \frac{t}{u} + 4 - 2x_1 - 2x_2 \right) \frac{i\pi}{2} H_0^{(1)}(\sqrt{r_1}b) ,$$

$$T_1' = 32\pi^2 C_F \alpha_s(w_2) \left[ \theta(-r_2)K_0(\sqrt{|r_2|b}) + \theta(r_2)\frac{i\pi}{2} H_0^{(1)}(\sqrt{|r_2|b}) \right]$$

$$\times \left[ \frac{1}{x_1(1 - x_1)} + \frac{1}{x_2(1 - x_2)} + \frac{s}{x_2(1 - x_1)t} + \frac{s}{x_1(1 - x_2)u} \right] ,$$

$$T_2 = T - T_1, \quad T_2' = T' - T_1' ,$$

$$T = \frac{64\pi^2 C_F \alpha_s(w_1)}{1 - x_1}(1 - x_2) 2H_0^{(1)}(\sqrt{r_1}b) \left\{ \left[ (1 - x_1)s + u \right] \left[ (1 - x_2)s + u \right] \right\}$$

$$\times \frac{1}{t^2} - \frac{2(1 - x_1) - 2(1 - x_2)}{u^2} \right\} ,$$

$$T' = 64\pi^2 C_F \alpha_s(w_2) \left[ \theta(-r_2)K_0(\sqrt{|r_2|b}) + \theta(r_2)\frac{i\pi}{2} H_0^{(1)}(\sqrt{|r_2|b}) \right]$$

$$\times \left[ \frac{1}{x_1(1 - x_1)} - \frac{(1 + x_2 - x_1 x_2)s^2 + (1 + x_2 - x_1)su}{x_2(1 - x_1)t^2} \right]$$

$$+ \frac{1}{x_2(1 - x_2)} - \frac{(1 + x_1 - x_1 x_2)s^2 + (1 + x_2 - x_1)st}{x_1(1 - x_2)u^2} \right\} ,$$

with the constant $\alpha = 1/137$, the color factor $C_F = 4/3$, the Bessel functions $K_0$ and $H_0^{(1)}$, and the invariants,

$$r_1 = x_1 x_2 s, \quad r_2 = x_1 x_2 s + x_1 u + x_2 t .$$

The arguments $w_i$ of $\alpha_s$ are chosen as the largest mass scales in the hard scattering,

$$w_1 = \max \left( \sqrt{r_1}, \frac{1}{b} \right), \quad w_2 = \max \left( \sqrt{r_2}, \frac{1}{b} \right) .$$

The extra Sudakov factor $\exp(-S)$ compared to the standard collinear factorization formula [3] arises from the all-order summation of the large logarithms in $k_T$ factorization theorem [31]. Simply speaking, it describes the extrinsic $b$ dependence of a parton in the pion, and decreases quickly in the large-$b$ region. This is how the Sudakov suppression improves the perturbative expansion. There also exists an intrinsic $b$ dependence [32], which is less essential compared to the Sudakov effect in the processes involving only light mesons. For the explicit expression of $\exp(-S)$ and the values of the QCD scale $\Lambda_{\text{QCD}} (= 0.25 \text{ GeV})$ and of the quark flavor number $n_f (= 3)$, refer to [33].
The PQCD factorization formulas for $\gamma\gamma \rightarrow K^+K^−$ are slightly different. Since the kaon distribution amplitude is not symmetric under the exchange of $x$ and $1−x$, the contributions from the diagrams with the photons attaching the $u$ quark and the $s$ quark can not be combined. The amplitudes are expressed as

$$H_i(s, t, u) = \int_0^1 dx_1dx_2 \int_0^\infty db \{[e_u^2T_i(x_i, s, t, u, b) + e_u e_s T'_i(x_i, s, t, u, b)]\phi_K(x_1)\phi_K(x_2) + [e_u^2T_i(x_i, s, t, u, b) + e_u e_s T'_i(x_i, s, t, u, b)]\phi_K(1−x_1)\phi_K(1−x_2)\} \times \exp[-S(x_i, b, W_{\gamma\gamma}/\sqrt{2})] \, ,$$

with $\phi_K$ being the kaon distribution amplitude. The pion and kaon distribution amplitudes are adopted as [34, 35],

$$\phi_\pi(x) = \frac{3f_\pi}{\sqrt{2N_c}} x(1−x) \left[1 + 0.44C_{2}^{3/2}(1−2x) + 0.25C_{4}^{3/2}(1−2x)\right] \, ,$$

$$\phi_K(x) = \frac{3f_K}{\sqrt{2N_c}} x(1−x)[1−0.54(1−2x) + 0.16C_{2}^{3/2}(1−2x)] \, ,$$

with $N_c = 3$ being the number of colors, and the Gegenbauer polynomials,

$$C_{2}^{3/2}(t) = \frac{3}{2}(5t^2 − 1) \, , \quad C_{4}^{3/2}(t) = \frac{15}{8}(21t^4 − 14t^2 + 1) \, .$$

We then perform the numerical analysis of the pion and kaon angular distributions for various center-of-mass energy $W_{\gamma\gamma}$ in two-photon collisions. For QSR, the duality interval $s_0$ is determined in the way that the amplitudes $H$ are most stable with respect to the variation of the Borel mass $M^2$. The best choice of $s_0$ are found to be $s_0 = 0.54, 0.57, 0.59, 0.615$, and $0.675$ GeV$^2$ at $W_{\gamma\gamma} = 1.50, 1.54, 1.58, 1.62$, and $1.70$ GeV, respectively, for which the QSR results become approximately constant as $M^2 > 2$ GeV$^2$. Hence, we have chosen $M^2 = 4$ GeV$^2$ in Eq. (3). Since $s_0$ increases with $W_{\gamma\gamma}$, higher excitations beyond the pion pole will contribute to the left-hand sides of Eq. (3) at some large value of $W_{\gamma\gamma}$, such that the sum rules fail. Therefore, we regard the QSR results for $W_{\gamma\gamma} < 1.7$ GeV as being reliable. Note that the best choice of $s_0$ slightly depends on $|\cos \theta|$ for a fixed $W_{\gamma\gamma}$, varying in the range of $±0.005$ GeV$^2$.

The QSR curves for the $\gamma\gamma \rightarrow \pi\pi$, $KK$ differential cross section $d\sigma/d|\cos \theta|$ are shown in Fig. 1, which exhibit a decrease with $|\cos \theta|$, consistent with the tendency of the Belle data [16]. Because the pion decay constant is smaller than the kaon one, the $\gamma\gamma \rightarrow \pi\pi$ cross sections are larger according to Eq. (3). We have noticed that the QSR results are very sensitive to the variation of $s_0$, since, as indicated in Eqs. (6) and (8), the perturbative spectral densities behave like $s_0^2$ for $Q^2 \sim s_1 \sim s_2 \sim s_0$. $Q^2$ is small in two-photon collisions, but of order $t$ in Compton scattering, which can be realized from Eq. (7) by substituting the negative $t$ for $s$. This is the reason the QSR results for the latter are insensitive to $s_0$ [27]. To demonstrate the sensitivity, we present three curves in Fig. 1, corresponding to the best $s_0$ plus 0.005 GeV$^2$, to the best $s_0$, and to the best $s_0$ minus 0.005 GeV$^2$ from top to bottom, respectively, which survive the stability analysis for different $\theta$ as mentioned above. The range enclosed by these three curves represents the theoretical uncertainty, within which the QSR predictions are in agreement with the $\gamma\gamma \rightarrow K^+K^−$ data [16]. We stress that as long as a stability window exists, QSR results should be regarded as being reliable. Therefore, the results presented here are solid, though they exhibit larger theoretical uncertainty. It is found that the deviation from the data becomes, as expected, more obvious as $W_{\gamma\gamma} > 1.7$ GeV.
Figure 1: $d\sigma/d\cos\theta$ derived from QCD sum rules for $W_{\gamma\gamma} = 1.50, 1.54, 1.58, 1.62, \text{ and } 1.70$ GeV with the dashed (solid) lines corresponding to $\gamma\gamma \rightarrow \pi^+\pi^- \ (\gamma\gamma \rightarrow K^+K^-)$. The three curves in each plot correspond to the best $s_0$ plus 0.005 GeV$^2$, to the best $s_0$, and to the best $s_0$ minus 0.005 GeV$^2$ from top to bottom, respectively. The area enclosed by the three curves for $\gamma\gamma \rightarrow K^+K^-$ has been shaded. The data points arise from the range of $W_{\gamma\gamma} \pm 0.02$ GeV.
The $\gamma\gamma \to \pi\pi$, $KK$ differential cross section derived from PQCD for $W_{\gamma\gamma} = 2.22, 2.26, 2.30, 2.34,$ and $2.38$ GeV are shown in Fig. 2, which exhibit an increase with $|\cos \theta|$, also consistent with the tendency of the Belle data [16]. The ascending of the PQCD results is understood through the hard kernels, which are proportional to $1/(tu) \propto 1/(1 - \cos^2 \theta)$. The $\gamma\gamma \to KK$ cross sections are smaller despite of the decay constants $f_K > f_\pi$, since the amplitudes with the photons attaching the $s$ quark are suppressed by the shape of the kaon distribution amplitude. It is found that the PQCD predictions are in good agreement with the $\gamma\gamma \to K^+K^-$ data [16], and that the deviation from the data becomes more obvious at lower $W_{\gamma\gamma}$: Figure 2 shows that the data descend a bit first before ascending with $|\cos \theta|$ for $W_{\gamma\gamma} = 2.22$ GeV, a behavior whose explanation requires a theoretical framework more sophisticated than QSR and PQCD. We conclude that the high-energy behavior of two-photon collisions can be explained perfectly by PQCD.

Note that our results are larger than those obtained from a similar PQCD analysis based on $k_T$ factorization theorem in [18], due to the different models of meson distribution amplitudes. If employing the asymptotic model of the pion distribution amplitude favored by [18, 36],

$$\phi^{\text{AS}}_\pi(x) = \frac{3f_\pi}{\sqrt{2N_c}}x(1-x), \quad (19)$$

our numerical results shown in Fig. 2 will be close to those in [18]. We would like to mention that the above asymptotic model has been excluded by [37]. The completely contradictory conclusions are attributed to the different treatments of the subleading contributions, when the pion distribution amplitude was extracted from the data of the pion transition form factor. The authors in [36] considered only the leading-order hard kernel and the leading-twist (twist-2) pion distribution amplitude, and included the Sudakov factor. However, those in [37] further considered the next-to-leading-order hard kernel and the twist-4 pion distribution amplitude, but did not take into account the Sudakov factor (due to the fast decreasing behavior of their distribution amplitudes at the end points of the momentum fraction). For other postulations for the models of the pion distribution amplitude, refer to [38].

The Gegenbauer coefficients of the kaon distribution amplitude in Eq. (17) also exhibit uncertainty from the sum-rule analysis [35]. We have examined that our predictions are insensitive (within 10%) to the variation of the first coefficient between 0.54 ± 0.27, but are to the variation of the second coefficient between 0.16 ± 0.10. Therefore, we display three solid curves for the $\gamma\gamma \to KK$ differential cross section corresponding to the second coefficient 0.26, 0.16 and 0.06 from top to bottom, respectively. The area enclosed by these three curves can be regarded as part of the theoretical uncertainty in the PQCD calculation. Strictly speaking, a Gegenbauer coefficient evolves with the scale $1/b$ in $k_T$ factorization theorem governed by $[\alpha_s(1/b)/\alpha_s(\mu_0)]^\gamma$, where $\mu_0 = 1$ GeV represents the initial scale the evolution starts with, and $\gamma$ is an anomalous dimension [35]. We have investigated this evolution effect, and found that it is not essential and can be covered by the theoretical uncertainty from the variation of the Gegenbauer coefficient.

Below we compare the PQCD approach with another different type of factorization theorem formulated by means of two-meson distribution amplitudes [19]. In the latter formalism the hard kernel is represented by the $\gamma\gamma \to qq$ scattering, and the nonperturbative input is a two-meson distribution amplitude, which collects soft gluon exchanges between the energetic $qq$ pair. Since this factorization theorem involves the hard scattering represented by quark diagrams, its prediction for the angular distributions should also show the characteristic of the spin-half particle production, i.e., increase with $|\cos \theta|$ as in PQCD. It has been claimed
Figure 2: $d\sigma/d\cos\theta$ derived from PQCD based on $k_T$ factorization theorem for $W_{\gamma\gamma} = 2.22, 2.26, 2.30, 2.34, \text{ and } 2.38$ GeV. The dashed (dotted) lines for $\gamma\gamma \rightarrow \pi^+\pi^-$ correspond to the pion distribution amplitude in Eq. (16) [Eq. (19)]. The three solid curves for $\gamma\gamma \rightarrow K^+K^-$ correspond to the second Gegenbauer coefficients 0.26, 0.16 and 0.06 of the kaon distribution amplitude from top to bottom, respectively.
that the resultant bag-diagram contribution dominates over that from PQCD. First, we point out that the bag-diagram contribution is sufficient to account for the \( \gamma \gamma \rightarrow \pi \pi, KK \) cross sections, because the two-meson distribution amplitudes have been tuned to fit the data. The PQCD contribution is small, because the asymptotic model of the meson distribution amplitudes was adopted in [18]. Therefore, the dominance of the bag-diagram contribution requires an independent check, that is, an independent study of the behavior of the two-meson distribution amplitudes using nonperturbative methods. Second, there is an overlap between the bag-diagram and PQCD predictions, and a comparison between them should be made carefully. The quark diagrams in PQCD, in which the two photons attach the same quark lines and the exchanged gluon attaches the two valence quarks of one of the mesons, can be factorized following [19], when the spectator quarks carry small momentum fractions. It is not necessary to specify a small momentum fraction, at which the bag-diagram factorization holds, since it is related to a factorization scheme, and quite arbitrary. We have investigated the contribution from these diagrams and found that it exceeds half of the differential cross section only in the large \(| \cos \theta | > 0.5 \) region. Anyway, an experimental discrimination of the bag-diagram and PQCD approaches has been proposed in [19]: the equality of the \( \gamma \gamma \rightarrow \pi^+ \pi^- \) and \( \gamma \gamma \rightarrow \pi^0 \pi^0 \) cross sections would favor the former.

In this paper we have calculated the pion and kaon angular distributions \( d\sigma/d|\cos \theta| \) in the two-photon collisions \( \gamma \gamma \rightarrow \pi \pi, KK \) using both the nonperturbative QSR and the PQCD approach based on \( k_T \) factorization theorem. It has been shown that the predicted angular distributions in the two approaches differ dramatically: the former decrease, while the latter increase with \( | \cos \theta | \). Therefore, the change from the descending behavior at a low center-of-mass energy \( W_{\gamma \gamma} \) to the ascending behavior at a high \( W_{\gamma \gamma} \) indicates the transition from nonperturbative dynamics to perturbative dynamics. An angular distribution in two-photon collisions thus serves the purpose for discriminating the soft-dominance and hard-dominance pictures, which is more unambiguous than a hadronic form factor usually considered in the literature. Such a transition has indeed been observed by Belle at \( W_{\gamma \gamma} \approx 2 \text{ GeV} \) in \( \gamma \gamma \rightarrow K^+ K^- \) [16], and at \( W_{\gamma \gamma} \approx 2.5 \text{ GeV} \) in \( \gamma \gamma \rightarrow p \bar{p} \) [20]. Our results from QSR (PQCD) are consistent with the \( \gamma \gamma \rightarrow K^+ K^- \) data for \( W_{\gamma \gamma} \approx 1.5-1.7 \) (2.2-2.4) GeV.

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