Quantum Gravity Phenomenology and Lorentz Violation

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In most fields of physics it goes without saying that observation and prediction play a central role, but unfortunately quantum gravity (QG) has so far not fit that mold. Many intriguing and ingenious ideas have been explored, but it seems safe to say that without both observing phenomena that depend on QG, and extracting reliable predictions from candidate theories that can be compared with observations, the goal of a theory capable of incorporating quantum mechanics and general relativity will remain unattainable.

Besides the classical limit, there is one observed phenomenon for which quantum gravity makes a prediction that has received encouraging support: the scale invariant spectrum of primordial cosmological perturbations. The quantized longitudinal linearized gravitational mode, albeit slave to the inflaton and not a dynamically independent degree of freedom, plays an essential role in this story [1].

What other types of phenomena might be characteristic of a quantum gravity theory? Motivated by tentative theories, partial calculations, intimations of symmetry violation, hunches, philosophy, etc, some of the proposed ideas are: loss of quantum coherence or state collapse, QG imprint on initial cosmological perturbations, scalar moduli or other new fields, extra dimensions and low-scale QG, deviations from Newton’s law, black holes produced in colliders, violation of global internal symmetries, and violation of spacetime symmetries. It is this last item, more specifically the possibility of Lorentz violation (LV), that is the focus of this article.

From the observational point of view, technological developments are encouraging a new look at the possibility of LV. Increased detector size, space-borne instruments, technological improvement, and technique refinement are permitting observations to probe higher energies, weaker interactions, lower fluxes, lower temperatures, shorter time resolution, and longer distances. It comes as a welcome surprise that the day of true quantum gravity observations may not be so far off [2]

1 Lorentz violation?

Lorentz symmetry is linked to a scale-free nature of spacetime: unbounded boosts expose ultra-short distances, and yet nothing changes. However, suggestions for Lorentz violation have come from: the need to cut off UV divergences of quantum field theory and of black hole entropy, tentative calculations in various QG scenarios (e.g. semiclassical spin-network calculations in Loop QG, string theory tensor VEVs, non-commutative geometry, some brane-world backgrounds), and the possibly missing GZK cutoff on ultra high energy (UHE) cosmic rays.

The GZK question has generated a lot of interest, and is to date the only phenomenon that might point to a breakdown of standard physics in quantum gravity, hence we take a moment to discuss it. The idea [3] is that in collisions of ultra high energy protons with cosmic microwave background photons there can be sufficient energy in the center of mass frame to create a pion, leading to the reaction

\[ p + \gamma_{\text{CMB}} \rightarrow p + \pi. \]

In this way, the initial proton energy is degraded with an attenuation length of about 50 Mpc. Since plausible astrophysical sources for UHE particles are located at distances larger than 50 Mpc, one expects a cutoff in the cosmic ray proton energy spectrum at around \( 5 \times 10^{19} \) eV for protons coming from beyond a few megaparsecs. If Lorentz symmetry is violated, then the energy threshold for this reaction could be lowered, raised, or removed entirely, or an upper threshold where the reaction cuts off could even be introduced (see e.g. [4] and references therein).

One of the experiments measuring the UHE cosmic ray spectrum, the AGASA experiment, has not seen the cutoff. An analysis [5] from January 2003 concluded that the cutoff was absent at the 2.5 sigma level, while another experiment, Hi-Res, is consistent with the cutoff but at a lower confidence level. The question should be answered in the near future by the AUGER observatory, a combined array of 1600 water Čerenkov detectors and 24 telescopic air fluorescence detectors under construction on the Argentine pampas [6]. The new observatory will see an event rate one hundred times higher, with better systematics.

Trans-GZK cosmic rays are not the only window of opportunity we have to detect or constrain Lorentz violation induced by QG effects. In fact, many
phenomena accessible to current observations/experiments are sensitive to possible violations of Lorentz invariance. A partial list is

- sidereal variation of LV couplings as the lab moves with respect to a preferred frame or directions
- long baseline dispersion and vacuum birefringence (e.g. of signals from gamma ray bursts, active galactic nuclei, pulsars, galaxies)
- new reaction thresholds (e.g. photon decay, vacuum Čerenkov effect)
- shifted thresholds (e.g. photon annihilation from blazars, GZK reaction)
- maximum velocity (e.g. synchrotron peak from supernova remnants)
- dynamical effects of LV background fields (e.g. gravitational coupling and additional wave modes)

We conclude this section with a brief historical overview including some of the more influential papers but by no means complete. Suggestions of possible LV in particle physics go back at least to the 1960’s, when a number of authors wrote on that idea [7]. The possibility of LV in a metric theory of gravity was explored beginning at least as early as the 1970’s [9]. Such theoretical ideas were pursued in the ’70’s and ’80’s notably by Nielsen and several other authors on the particle theory side [10], and by Gasperini [11] on the gravity side. A number of observational limits were obtained during this period [12].

Towards the end of the 80’s Kostelecky and Samuel [13] presented evidence for possible spontaneous LV in string theory, and motivated by this explored LV effects in gravitation. The role of Lorentz invariance in the “trans-Planckian puzzle” of black hole redshifts and the Hawking effect was emphasized in the early 90’s [14]. This led to study of the Hawking effect for quantum fields with LV dispersion relations commenced by Unruh [15] and followed up by others. Early in the third millenium this line of research led to work on the related question of the possible imprint of trans-Planckian frequencies on the primordial fluctuation spectrum [16].

Meanwhile the consequences of LV for particle physics were being explored using LV dispersion relations e.g. by Gonzalez-Mestres [17], and a systematic extension of the standard model of particle physics incorporating all possible LV in the renormalizable sector was developed by Colladay and Kostelecký [18]. This latter work provided a framework for computing the observable consequences for any experiment and led to much experimental work setting limits on the LV parameters in the lagrangian [19]. Around the same time Coleman and Glashow suggested the possibility that LV was the culprit in the possibly missing GZK cutoff [20], and explored many other consequences of renormalizable, isotropic LV leading to different limiting speeds for different particles [21].

It is amusing to note that Kirzhnits and Chechin in [7] explore the possibility that an apparent missing cutoff in the UHE cosmic ray spectrum could be explained by something that looks very similar to the recently proposed “doubly special relativity” [8].
Also at that time it was pointed out by Amelino-Camelia et al \[22\] that the sharp high energy signals of gamma ray bursts could reveal LV photon dispersion suppressed by one power of energy over the mass $M \sim 10^{-3}M_{\text{P}}$, tantalizingly close to the Planck mass. Shortly afterwards Gambini and Pullin \[23\] argued that semiclassical loop quantum gravity suggests just such LV. (Some later work supported this notion, but a recent paper by Kozameh and Parisi \[24\] argues the other way.) In any case the theory is not under enough control at this time to make any definite statements.

A very strong constraint on photon birefringence was obtained by Gleiser and Kozameh \[25\] using UV light from distant galaxies, and if the recent measurement of polarized gamma rays from a GRB hold up to further scrutiny this constraint will be further strengthened dramatically \[26, 27\]. Further stimulus came from the suggestion \[28\] that an LV threshold shift might explain the apparent under-absorption on the cosmic IR background of TeV gamma rays from the blazar Mkn501, however it is now believed by many that this anomaly goes away when a corrected IR background is used \[29\].

The extension of the effective field theory framework to include LV dimension 5 operators was introduced by Myers and Pospelov \[30\], and used to strengthen prior constraints. Also this framework was used to deduce a very strong constraint \[31\] on the possibility of a maximum electron speed less than the speed of light from observations of synchrotron radiation from the Crab Nebula.

**2 Theoretical framework for LV**

Various different theoretical approaches to LV have been taken to further pursue the ideas summarized above. Some researchers restrict attention to LV described in the framework of effective field theory (EFT), while others allow for effects not describable in this way, such as those that might be due to stochastic fluctuations of a “space-time foam”. Some restrict to rotationally invariant LV, while others consider also rotational symmetry breaking. Both true LV as well as “deformed” Lorentz symmetry (in the context of so-called “doubly special relativity” \[8\]) have been pursued. Another difference in approaches is whether one allows for distinct LV parameters for different particle types, or proposes a more universal form of LV.

The rest of this article will focus on just one of these approaches, namely LV describable by standard EFT, assuming rotational invariance, and allowing distinct LV parameters for different particles. In exploring the possible phenomenology of new physics, it seems useful to retain enough standard physics so that a) clear predictions can be made, and b) the possibilities are narrow enough to be meaningfully constrained.

This approach is not universally favored. For example a sharp critique appears in \[32\]. Therefore we think it is important to spell out the motivation for the choices we have made. First, while of course it may be that EFT is not
adequate for describing the leading quantum gravity phenomenology effects, it has proven itself very effective and flexible in the past. It produces local energy and momentum conservation laws, and seems to require for its validity just locality and local spacetime translation invariance above some length scale. It describes the standard model and general relativity (which are presumably not fundamental theories), a myriad of condensed matter systems at appropriate length and energy scales, and even string theory (as perhaps most impressively verified in the calculations of black hole entropy and Hawking radiation rates). It is true that, e.g., non-commutative geometry (NCG) seems to lead to EFT with problematic IR/UV mixing, however this more likely indicates a physically unacceptable feature of such NCG rather than a physical limitation of EFT.

The assumption of rotational invariance is motivated by the idea that LV may arise in QG from the presence of a short distance cutoff. This suggests a breaking of boost invariance, with a preferred rest frame, but not necessarily rotational invariance. Since a constraint on pure boost violation is, barring a conspiracy, also a constraint on boost plus rotation violation, it is sensible to simplify with the assumption of rotation invariance at this stage.

Finally why do we choose to complicate matters by allowing for different LV parameters for different particles? First, EFT for first order Planck suppressed LV (see section 2.1) requires this for different polarizations or spin states, so it is unavoidable in that sense. Second, we see no reason a priori to expect these parameters to coincide. The term “equivalence principle” has been used to motivate the equality of the parameters. However, in the presence of LV dispersion relations, particles with different masses travel on different trajectories even if they have the same LV parameters [33, 4]. Moreover, different particles would presumably interact differently with the spacetime microstructure since they interact differently with themselves and with each other. An example of this occurs in the braneworld model discussed in Ref. [34], and an extreme version occurs in the proposal of Ref. [35] in which only certain particles feel the spacetime foam effects. (Note however that in this proposal the LV parameters fluctuate even for a given kind of particle, so EFT would not be a valid description.)

2.1 Deformed dispersion relations

A simple approach to a phenomenological description of LV is via deformed dispersion relations. If rotation invariance and integer powers of momentum are assumed in the expansion of $E^2(p)$, the dispersion relation for a given particle type can be written as

$$E^2 = p^2 + m^2 + \Delta(p),$$

(2)

where $p$ is the magnitude of the three-momentum, and

$$\Delta(p) = \tilde{\eta}_1 p^1 + \tilde{\eta}_2 p^2 + \tilde{\eta}_3 p^3 + \tilde{\eta}_4 p^4 + \cdots$$

(3)
Let us introduce two mass scales, \( M = 10^{19} \text{GeV} \approx M_{\text{Planck}} \), the putative scale of quantum gravity, and \( \mu \), a particle physics mass scale. To keep mass dimensions explicit we factor out possibly appropriate powers of these scales, defining the dimensionful \( \eta \)'s in terms of corresponding dimensionless parameters. It might seem natural that the \( p^n \) term with \( n \geq 3 \) be suppressed by \( 1/M^{n-2} \), and indeed this has been assumed in most work. But following this pattern one would expect the \( n = 2 \) term to be unsuppressed and the \( n = 1 \) term to be even more important. Since any LV at low energies must be small, such a pattern is untenable. Thus either there is a symmetry or some other mechanism protecting the lower dimension operators from large LV, or the suppression of the higher dimension operators is greater than \( 1/M^{n-2} \). This is an important issue to which we return later in this article.

For the moment we simply follow the observational lead and insert at least one inverse power of \( M \) in each term, viz.

\[
\begin{align*}
\tilde{\eta}_1 &= \eta_1 \frac{\mu^2}{M}, \\
\tilde{\eta}_2 &= \eta_2 \frac{\mu}{M}, \\
\tilde{\eta}_3 &= \eta_3 \frac{1}{M}, \\
\tilde{\eta}_4 &= \eta_4 \frac{1}{M^2},
\end{align*}
\]

(4)

In characterizing the strength of a constraint we refer to the \( \eta_\tilde{n} \) without the tilde, so we are comparing to what might be expected from Planck-suppressed LV. We allow the LV parameters \( \eta_i \) to depend on the particle type, and indeed it turns out that they \textit{must} sometimes be different but related in certain ways for photon polarization states, and for particle and antiparticle states, if the framework of effective field theory is adopted. In an even more general setting, Lehnert [36] studied theoretical constraints on this type of LV and deduced the necessity of some of these parameter relations.

This general framework allows for superluminal propagation, and spacelike 4-momentum relative to a fixed background metric. It has been argued [37] that this may lead to problems with causality and stability, but we do not share this opinion. In the context of a LV theory, there can be a preferred reference frame. As long as the physics is guaranteed to be causal and the states all have positive energy in the preferred frame, we cannot see any room for such problems to arise.

### 2.2 Effective field theory and LV

The standard model extension (SME) of Colladay and Kostelecký [18] consists of the standard model of particle physics plus all Lorentz violating renormalizable operators (i.e. of mass dimension \( \leq 4 \)) that can be written without changing the field content or violating the gauge symmetry. For illustration, the leading order terms in the QED sector are the dimension three terms

\[
- b_a \bar{\psi} \gamma_5 \gamma^a \psi - \frac{1}{2} H_{ab} \bar{\psi} \sigma^{ab} \psi
\]

(5)

and the dimension four terms
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\[-\frac{1}{4} k_{abcd} F_{ab} F_{cd} + \frac{i}{2} \bar{\psi} (c_{ab} + d_{ab} \gamma_5) \gamma^a D^b \psi, \tag{6}\]

where the dimension one coefficients $b_a$, $H_{ab}$ and dimensionless $k_{abcd}$, $c_{ab}$, and $d_{ab}$ are constant tensors characterizing the LV. If we assume rotational invariance then these must all be constructed from a given unit timelike vector $u^a$ and the Minkowski metric $\eta_{ab}$, hence $b_a \propto u_a$, $H_{ab} = 0$, $k_{abcd} \propto u^{[a} \eta^{b]c} u^d$, $c_{ab} \propto u_a u_b$. Such LV is thus characterized by just four numbers.

The study of Lorentz violating EFT in the higher mass dimension sector was initiated by Myers and Pospelov [30]. They classified all LV dimension five operators that can be added to the QED Lagrangian and are quadratic in the fields, rotation invariant, gauge invariant, not reducible to lower and/or higher dimension operators using the field equations, and contribute $p^4$ terms to the dispersion relation. Again, just three parameters arise:

\[\frac{\xi}{M} u^m F_{mn} (u \cdot \partial) (u_n \tilde{F}^{na}) + \frac{1}{M} u^m \bar{\psi} \gamma_m (\zeta_1 + \zeta_2 \gamma_5) (u \cdot \partial)^2 \psi \tag{7}\]

where $\tilde{F}$ denotes the dual of $F$. All of these terms violate CPT symmetry as well as Lorentz invariance. Thus if one knew CPT were preserved, these LV operators would be forbidden.

In the limit of high energy $E \gg m$, the photon and electron dispersion relations following from QED with the above terms are [30, 26]

\[\omega^2_{R,L} = k^2 \pm \frac{2\xi}{M} k^3 \tag{8}\]
\[E^2_{\pm} = p^2 + m^2 + \frac{2(\zeta_1 \pm \zeta_2)}{M} p^3. \tag{9}\]

The photon subscripts $R$ and $L$ refer to right and left circular polarization, hence these necessarily have opposite LV parameters. The electron subscripts $\pm$ refer to the helicity, which can be shown to be a good quantum number in the presence of these LV terms [26]. Moreover, if we write $\eta_{\pm} = 2(\zeta_1 \pm \zeta_2)$ for the LV parameters of the two electron helicities, those for positrons are given by [26]

\[\eta_{\pm}^{\text{positron}} = -\eta_{\mp}^{\text{electron}}. \tag{10}\]

2.3 Un-naturalness of small LV at low energy

As discussed above in subsection 2.1, if LV operators of dimension $n > 4$ are suppressed, as we have imagined, by $1/M^{n-2}$, LV would feed down to the lower dimension operators and be strong at low energies [21, 30, 38, 39], unless there is a symmetry or some other mechanism that protects operators of dimension four and less from strong LV. What symmetry (other than Lorentz invariance, of course) could that possibly be?

In the Euclidean context, a discrete subgroup of the Euclidean rotation group suffices to protect the operators of dimension four and less from violation of rotation symmetry. For example [40], consider the “kinetic” term in
the EFT for a scalar field with hypercubic symmetry, $M^{\mu\nu}\partial_\mu \phi \partial_\nu \phi$. The only tensor $M^{\mu\nu}$ with hypercubic symmetry is proportional to the Kronecker delta $\delta^{\mu\nu}$, so full rotational invariance is an “accidental” symmetry of the kinetic operator.

If one tries to mimic this construction on a Minkowski lattice admitting a discrete subgroup of the Lorentz group, one faces the problem that each point has an infinite number of neighbors related by the Lorentz boosts. For the action to share the discrete symmetry each point would have to appear in infinitely many terms of the discrete action, presumably rendering the equations of motion meaningless.

Another symmetry that could do the trick is three dimensional rotational symmetry together with a symmetry between different particle types. For example, rotational symmetry would imply that the kinetic term for a scalar field takes the form $(\partial_t \phi)^2 - c^2 (\nabla \phi)^2$, for some constant $c$. Then for multiple scalar fields, a symmetry relating the fields would imply that the constant $c$ is the same for all, hence the kinetic term would be Lorentz invariant with $c$ playing the role of the speed of light. Unfortunately this mechanism does not work in nature, since there is no symmetry relating all the physical fields. Perhaps under some conditions a partial symmetry could be adequate, e.g. grand unified gauge and/or super symmetry.

We are thus in the uncomfortable position of lacking any theoretical realization of the Lorentz symmetry breaking scheme upon which constraints are being imposed. This does not mean that no realization exists, but it is worrisome. If none exists, then our parametrization (4) is misleading, since there should be more powers of $1/M$ suppressing the higher dimension terms, likely rendering any constraints on those terms uninteresting.

### 3 Constraints

Observable effects of LV arise, among other things, from 1) sidereal variation of LV couplings due to motion of the laboratory relative to the preferred frame, 2) dispersion and birefringence of signals over long travel times, 3) anomalous reaction thresholds. We will often express the constraints in terms of the dimensionless parameters $\eta_n$ introduced in (4). An order unity value might be considered to be expected in Planck suppressed LV.

The possibility of interesting constraints in spite of Planck suppression arises in different ways for the different types of observations. In the laboratory experiments looking for sidereal variations, the enormous number of atoms allow a resonance frequency to be measured extremely accurately. In the case of dispersion or birefringence, the enormous propagation distances would allow a tiny effect to accumulate. In the anomalous threshold case, the creation of a particle with mass $m$ would be strongly affected by a LV term when the momentum becomes large enough for this term to be comparable to the mass term in the dispersion relation.
Consider first the case \( n = 2 \). For the \( n = 2 \) term in (3,4), the absence of a strong threshold effect yields a constraint \( \eta_2 \lesssim (m/p)^2 (M/\mu) \). If we consider protons and put \( \mu = m = m_p \sim 1 \) GeV, this gives an order unity constraint when \( p \sim \sqrt{mM} \sim 10^{19} \) eV. Thus the GZK threshold, if confirmed, can give an order unity constraint, but multi-TeV astrophysics yields much weaker constraints. The strongest laboratory constraints on dimension three and four operators come from clock comparison experiments using noble gas masers [41]. The constraints limit a combination of the coefficients for dimension three and four operators for the neutron to be below \( 10^{-31} \) GeV (the dimension four coefficients are weighted by the neutron mass, yielding a constraint in units of energy). Astrophysical limits on photon vacuum birefringence give a bound on the coefficients of dimension four operators of \( 10^{-32} \) [46].

For \( n = 3 \) the constraint from the absence of a strong effect on energy thresholds involving only electrons and photons is of order

\[
\eta_3 \lesssim (10 \text{ TeV}/p)^3. \tag{11}
\]

Thus we can obtain order unity and even much stronger constraints from high energy astrophysics, as discussed shortly.

### 3.1 Summary of constraints on LV in QED at \( O(E/M) \)

Since we do not assume universal LV coefficients, different constraints cannot be combined unless they involve just the same particle types. To achieve the strongest combined constraints it is thus preferable to focus on processes involving a small number of particle types. It also helps if the particles are very common and easy to observe. This selects electron-photon physics, i.e. QED, as a useful arena.

The current constraints on the three LV parameters at order \( E/M \)—one in the photon dispersion relation and two in the electron dispersion relation—will now be summarized. These are equivalent to the parameters in the dimension five operators (7) written down by Myers and Pospelov.

First, the constraint \( |\eta_+ - \eta_-| < 4 \) on the difference between the positive and negative electron helicity parameters was deduced by Myers and Pospelov [30] using a previous spin-polarized torsion pendulum experiment [44] that looked for diurnal changes in resonance frequency. (They also determined a numerically stronger constraint using nuclear spins, however this involves four different LV parameters, one for the photon, one for the up-down quark doublet, and one each for the right handed up and down quark singlets. It also requires a model of nuclear structure.)

In Fig. 1 (from Ref. [26]) constraints on the photon (\( \xi \)) and electron (\( \eta \)) LV parameters are plotted on a logarithmic scale to allow the vastly differing strengths to be simultaneously displayed. For negative parameters minus the logarithm of the absolute value is plotted, and a region of width \( 10^{-18} \) is excised around each axis. The synchrotron and Čerenkov constraints are known
to apply only for at least one $\eta_\pm$. The IC and synchrotron Čerenkov lines are truncated where they cross. Prior photon decay and absorption constraints are shown in dashed lines since they do not account for the EFT relations between the LV parameters.

**Fig. 1.** Constraints on LV in QED at $O(E/M)$ (figure from Ref. [26]).

**Vacuum birefringence**

The birefringence constraint arises from the fact that the LV parameters for left and right circular polarized photons are opposite (9). The phase velocity thus depends on both the wavevector and the helicity. Linear polarization is therefore rotated through an energy dependent angle as a signal propagates, which depolarizes any initially linearly polarized signal. Hence the observation of linearly polarized radiation coming from far away can constrain the magnitude of the LV parameter. This effect has been used to constrain LV in the dimension three (Chern-Simons) [45], four [46] and five [25, 26, 27] terms. The constraint shown in the figure derives from the recent report [47] of a high degree of polarization of MeV photons from GRB021206. The data analysis has been questioned [48] and defended [49], so we shall have to wait and see if it is confirmed. The next best constraint on the dimension five term is $|\xi| \lesssim 2 \times 10^{-4}$, and was deduced by Gleiser and Kozameh [25] using UV light from distant galaxies.

**Photon time of flight**

The $\gamma$ time of flight constraint arises from an energy dependent dispersion in the arrival time at Earth for photons originating in a distant event [50, 22], which was previously exploited for constraints [51, 52, 53]. The dispersion of
the two polarizations is larger since the difference in group velocity is then $2|\xi|p/M$ rather than $\xi(p_2 - p_1)/M$, but the time of flight constraint remains many orders of magnitude weaker than the birefringence one from polarization rotation. In Fig. 1 we use the EFT improvement of the constraint of [52] which yields $|\xi| < 63$.

Vacuum Čerenkov effect, inverse Compton electrons

In the presence of LV the process of vacuum Čerenkov radiation $e \rightarrow e\gamma$ can occur. The inverse Compton (IC) Čerenkov constraint uses the electrons of energy up to 50 TeV inferred via the observation of 50 TeV gamma rays from the Crab nebula which are explained by IC scattering. Since the vacuum Čerenkov rate is orders of magnitude higher than the IC scattering rate, that process must not occur for these electrons [21, 4]. The threshold for vacuum Čerenkov radiation depends in general on both $\xi$ and $\eta$, however in part of the parameter plane the threshold occurs with emission of a soft photon, so $\xi$ is irrelevant. This produces the vertical IC Čerenkov line in Fig. 1. One can see from (11) that this yields a constraint on $\eta$ of order $(10\ \text{TeV}/50\ \text{TeV})^3 \sim 10^{-2}$. It could be that only one electron helicity produces the IC photons and the other loses energy by vacuum Čerenkov radiation. Hence we can infer only that at least one of $\eta_+$ and $\eta_-$ satisfies the bound.

Crab synchrotron emission

A complementary constraint was derived in [31] by making use of the very high energy electrons that produce the highest frequency synchrotron radiation in the Crab nebula. For negative values of $\eta$ the electron has a maximal group velocity less than the speed of light, hence there is a maximal synchrotron frequency that can be produced regardless of the electron energy [31]. Observations of the Crab nebula reveal synchrotron radiation at least out to 100 MeV (requiring electrons of energy 1500 TeV in the Lorentz invariant case), which implies that at least one of the two parameters $\eta_\pm$ must be greater than $-7 \times 10^{-8}$ (this constraint is independent of the value of $\xi$). We cannot constrain both $\eta$ parameters in this way since it could be that all the蟹 synchrotron radiation is produced by electrons of one helicity. Hence for the rest of this discussion let $\eta$ stand for whichever of the two $\eta$'s satisfies the synchrotron constraint.

This must be the same $\eta$ as satisfies the IC Čerenkov constraint discussed above, since otherwise the energy of these synchrotron electrons would be below 50 TeV rather than the Lorentz invariant value of 1500 TeV. The Crab spectrum is well accounted for with a single population of electrons responsible for both the synchrotron radiation and the IC $\gamma$-rays. If there were enough extra electrons to produce the observed synchrotron flux with thirty times less energy per electron, then the electrons of the other helicity which would be producing the IC $\gamma$-rays would be too numerous [26]. It is important that the same $\eta$, i.e. either $\eta_+$ or $\eta_-$, satisfies both the synchrotron and the IC
Čerenkov constraints. Otherwise, both constraints could have been satisfied by having one of these two parameters arbitrarily large and negative, and the other arbitrarily large and positive.

**Vacuum Čerenkov effect, synchrotron electrons**

The existence of these synchrotron producing electrons can be exploited to improve on the vacuum Čerenkov constraint. For a given \( \eta \) satisfying the synchrotron bound, some definite electron energy \( E_{\text{synch}}(\eta) \) must be present to produce the observed synchrotron radiation. (This is higher for negative \( \eta \) and lower for positive \( \eta \) than the Lorentz invariant value [31].) Values of \(|\xi|\) for which the vacuum Čerenkov threshold is lower than \( E_{\text{synch}}(\eta) \) for either photon helicity can therefore be excluded [26]. (This is always a hard photon threshold, since the soft photon threshold occurs when the electron group velocity reaches the low energy speed of light, whereas the velocity required to produce any finite synchrotron frequency is smaller than this.) For negative \( \eta \), the Čerenkov process occurs only when \( \xi < \eta \) [4, 54], so the excluded parameters lie in the region \(|\xi| > -\eta\).

**Photon decay and photon absorption**

Previously obtained constraints from photon decay \( \gamma \to e^+e^- \) and absorption \( \gamma\gamma \to e^+e^- \) must be re-analyzed to take into account the different dispersion for the two photon helicities, and the different parameters for the two electron helicities, but there is a further complication: both these processes involve positrons in addition to electrons. Previous constraint derivations have assumed that these have the same dispersion, but that need not be the case [36]. As discussed above, for the \( O(E/M) \) corrections this is indeed not so [26]. Taking into account the above factors could not significantly improve the strength of the constraints (which is mainly determined by the energy of the photons). We indicate here only what the helicity dependence of the photon dispersion implies, thus neglecting the important role of differing parameters for electrons, positrons and their helicity states.

The strongest limit on photon decay came from the highest energy photons known to propagate, which at the moment are the 50 TeV photons observed from the Crab nebula [4, 54]. Since their helicity is not measured, only those values of \(|\xi|\) for which both helicities decay could be ruled out. The photon absorption constraint came from the fact that LV can shift the standard QED threshold for annihilation of multi-TeV \( \gamma \)-rays from nearby blazars such as Mkn 501 with the ambient infrared extragalactic photons [55, 56, 57, 4, 54, 58, 59]. LV depresses the rate of absorption of one photon helicity and increases it for the other. Although the polarization of the \( \gamma \)-rays is not measured, the possibility that one of the polarizations is essentially unabsorbed appears to be ruled out by the observations which show the predicted attenuation [59].
3.2 Constraints at $O(E^2/M^2)$?

As previously mentioned, CPT symmetry alone could exclude the dimension five LV operators that give $O(E/M)$ modifications to particle dispersion relation, and in any case the constraints on those have become nearly definitive. Hence it is of interest to ask about the dimension five and six operators that give $O(E^2/M^2)$ corrections. We close with a brief discussion of the constraints that might be possible on those, i.e. constraints at $O(E^2/M^2)$.

As discussed above, the strength of constraints can be estimated by the requirement $\eta_4 p^4/M^2 \lesssim m^2$, which yields

$$\eta_4 \lesssim \left( \frac{m}{1 \text{ eV}} \frac{100 \text{ TeV}}{p} \right)^4. \quad (12)$$

Thus for electrons, an energy around $10^{17}$ eV is needed and we are probably not going to see any effects directly from such electrons. For protons an energy $\sim 10^{18}$ eV is needed. This is well below the UHE cosmic ray energy cutoff, hence if and when Auger [6] confirms the identity of UHE cosmic rays as protons at the GZK cutoff, we will obtain a constraint of order $\eta_4 \lesssim 10^{-5}$ from the absence of vacuum Čerenkov radiation for $10^{20}$ eV protons! Also, from the fact that the GZK threshold is not shifted, we will obtain a constraint of order $\eta_4 \gtrsim -10^{-2}$, assuming equal $\eta_4$ values for proton and pion.

Impressive constraints might also be obtained from the absence of neutrino vacuum Čerenkov radiation: putting in 1 eV for the mass in (12) yields an order unity constraint from 100 TeV neutrinos, but only if the Čerenkov rate is high enough. The rate will be low, since it proceeds only via the non-local charge structure of the neutrino. Recent calculations [60] have shown that the rate is not high enough at that energy. However, for $10^{20}$ eV UHE neutrinos, which may be observed by the EUSO and OWL planned satellite observatories, the emission rate will be high enough to derive a strong constraint. The exact value depends on the emission rate, which has not yet been computed. For a gravitational Čerenkov reaction, the rate (which is lower but easier to compute than the electromagnetic rate) would be high enough for a neutrino from a distant source provided $\eta_4 \gtrsim 10^{-2}$. Hence in this case one might obtain a constraint of order $\eta_4 \lesssim 10^{-2}$, or stronger in the electromagnetic case.

A time of flight constraint at order $(E/M)^2$ might be possible [61] if gamma ray bursts produce UHE ($\sim 10^{19}$ eV) neutrinos, as some models predict, via limits on time of arrival differences of such UHE neutrinos vs. soft photons (or gravitational) waves. Another possibility is to obtain a vacuum birefringence constraint with higher energy photons [27] (although such a constraint would be less powerful since the parameters for opposite polarizations need not be opposite at order $(E/M)^2$). If future GRB’s are found to be polarized at $\sim 100$ MeV, that could provide a birefringence constraint $|\xi_{4+} - \xi_{4-}| \lesssim 1$. 
Conclusion

At present there are only hints, but no compelling evidence for Lorentz violation from quantum gravity. Moreover, even if LV is present, the use of EFT for its low energy parametrization is not necessarily valid. Nevertheless, we believe that the constraints derived from the simple ideas discussed here are still important. They allow tremendous advances in observational reach to be applied in a straightforward manner to limit reasonable possibilities that might arise from fundamental Planck scale physics. Such guidance is especially welcome for the field of quantum gravity, which until the past few years has had little connection with observed phenomena.

References

11. See, for example, M. Gasperini, Class. Quantum Grav. 4, 485 (1987); Gen. Rel. Grav. 30, 1703 (1998); and references therein.
50. See Pavlopoulos in [7].
60. D. Mattingly and B. McElrath, To be published.