The sum of (59) and (60) vanishes, as does the corresponding sum in the paper by Möller, only if \( \omega_1 < \epsilon \). In this case, \( \omega_+ \omega_1 - \omega_2 \) never vanishes in the physical interval \( (\mu, \infty) \) of the frequencies \( \omega_+ \omega_1 \) and the transition \( \nu + \epsilon = \nu + \epsilon' + \delta \) cannot occur on the energy shell. In the opposite case \( \omega > 2 \mu \), this transition makes a slight complication and we get

that the model suggested by T.D. Lee is in accordance with the physical probability concept only if a cut off is introduced and if the renormalized coupling constant is less than the critical value given.


It will be shown in appendix I that Eq. (36) has no non-real roots.
Appendix I.

In this appendix we will show by an explicit calculation how the indefinite metric is able to account for the negative sign on the right hand of the anticommutator

\[ \{ \psi^+_\nu(p), \psi_\nu(p') \} = \delta_{p,p'} \frac{1}{N^2} \]  

(A.1)

We compute the vacuum expectation value of this quantity for \( \nu > \nu_{\text{crit}} \) and \( p = p' \), and obtain

\[ \langle 0 | \{ \psi^+_\nu(p), \psi_\nu(p) \} | 0 \rangle = \sum_{\nu'} | \langle 0 | \psi_\nu(p) | \psi_{\nu'}(p) \rangle |^2 \langle \nu | \gamma_{\nu'} \rangle \]  

(A.2)

In (A.2) the summation is performed over any complete set of states. We can, e.g., sum over all physical states and get contributions from the physical \( \nu \)-particle state, the state \( | \nu_{\text{sc}} \rangle \) and the scattering states \( | \nu_{i,j} \rangle \). According to the result of paragraph II these contributions will be

\[ \langle 0 | \{ \psi^+_\nu(p), \psi_\nu(p) \} | 0 \rangle = \frac{1}{K} \sum_{\nu} | \beta(\nu) |^2 \frac{1}{|k_{\nu}(\nu) - k_{\nu}(\nu')|} = \frac{1}{K} \sum_{\nu} | \beta(\nu) |^2 \frac{1}{|k_{\nu}(\nu) - k_{\nu}(\nu')|} \]  

(A.3)

If there were no indefinite metric the right hand side would be positive and larger than one. This is also the usual proof\(^2\) that \( \nu^2 \) is a positive number less than one. In our case, the last term has a negative sign and there is no general principle according to which the right hand side of (A.3) has a definite sign. We will now show explicitly that this quantity has the correct value given by Eq. (35). The proof is essentially based on the fact that the function \( h(\alpha) \) defined by (36) and extended to the complex plane by

\[ h(\alpha) = \frac{1}{2} \left[ 1 + \frac{\alpha^2}{2V} \sum_{k} \frac{f^2(k) \alpha^2}{\omega^3(k - \omega)} \right] \]  

(A.4)

has zeros only on the real axis. Indeed, one has with \( z = x + iy \)

\[ J_m \frac{d}{dx} \frac{h(\alpha)}{x} = \frac{2}{2V} J_m \sum_{k} \frac{f^2(k) \omega^2}{\omega^3(k - \omega)} = \frac{1}{2V} \sum_{k} \frac{f^2(k) \omega^2}{\omega^3[(\omega - x)^2 + y^2]} \]  

(A.5)

which is always different from zero for \( y \neq 0 \).

Moreover, passing to the limit \( V \to \infty \) \( h(\alpha) \) transforms into an analytic function, given by

\[ h(\alpha) = \frac{1}{2} \left[ 1 + \gamma^2 \int f^2(\omega) \frac{\sqrt{\omega^2 - \alpha^2}}{\omega^3(\omega - \alpha)} d\omega \right] \]  

(A.5a)

(with the abbreviation \( \gamma = \frac{\alpha^2}{2V} \)) which is unique in the complex plane out along the real axis from \( \alpha \) to positive infinity. The imaginary part of \( h(\alpha) \) is discontinuous at this part of the real axis, having opposite signs in the upper and the lower half plane, whilst the real part is continuous. To this ambiguity of \( h(\alpha) \) corresponds the circumstance that \( \alpha = \alpha' \) is a branching point of the square root type of \( h(\alpha) \) (cf. the explicit form given in appendix II for the particular case \( f(\omega) = 1 \)).
These properties of $h(z)$ enable us to evaluate the integral
\[ \frac{1}{2\pi i} \oint_C \frac{dz}{h(z)} \]
along the path illustrated in Fig. 1 in two different ways. We first remark that
\[ \sum_K |\beta(K)|^2 = \gamma \int_0^\infty \omega \sqrt{\omega^2 + \lambda^2} \left[ h'(\omega) + \frac{\lambda^2}{\omega} h''(\omega) \right] \]
\[ = \frac{1}{\pi} \lim_{\varepsilon \to 0} \int_0^\infty \frac{d\omega}{h'(\omega + i\varepsilon)} = \frac{1}{\pi} \lim_{\varepsilon \to 0} \int_0^\infty \frac{d\omega}{h'(\omega - i\varepsilon)} \tag{A.6} \]

The path $C$ in Eq. (A.7).
We now divide the path $C$ into two parts. One of them $C_1$ starts from a point $z = Re + i\varepsilon$, with arbitrarily large $R$ and arbitrarily small, positive $\varepsilon$, goes below the real axis at a distance $\varepsilon$ from it, encircles the point $z = \mu$ in the negative direction, returns above the real axis at a distance $\varepsilon$ and ends at the point $z = Re + i\varepsilon$. The second part $C_2$ is a large circle with radius $R$ of which a small part near the positive real axis is omitted, with the contribution of the circular arc of $C_1$ going arbitrarily small, one first obtains
\[ \lim_{\varepsilon \to 0} \int_{C_1} \frac{dz}{h(z)} = -2\pi i \lim_{\varepsilon \to 0} \int_{C_2} \frac{dz}{h(z + i\varepsilon)} = -2\pi i \sum_K |\beta(K)|^2 \tag{A.7} \]
In this limit, the second part $C_2$ of $C$ goes over in the full circle $C$. The corresponding integral is easily evaluated with the aid of the asymptotic form of the function $h(z)$ (or, the remarks made before Eq. (43) ) and gives:
\[ \int_{C_2} \frac{dz}{h(z)} = 2\pi i \frac{1}{N^2} \tag{A.8} \]

Hence, we obtain in this way:
\[ \frac{1}{2\pi i} \oint_C \frac{dz}{h(z)} + \sum_K |\beta(K)|^2 = \frac{1}{N^2} \tag{A.9} \]
On the other hand, the absence of non real zeros of $h(z)$ and a knowledge of the residues of $h(z)^{-1}$ at the poles $z = 0$ and $z = -\lambda$ permits a direct evaluation of the integral
\[ \frac{1}{2\pi i} \oint_C \frac{dz}{h(z)} = 1 + \frac{1}{h'(-\lambda)} \tag{A.10} \]
Hence:
\[ 1 + \sum_K |\beta(K)|^2 + \frac{1}{h'(-\lambda)} = \frac{1}{N^2} \tag{A.11} \]
Eqn. (A.11) and (A.3) give together the expected result (A.1). If
the coupling constant is less than the critical value, the integrand
in (A.3) will have no pole at $\kappa = -\lambda$, and the last term in (A.10)
will be missing. Other matrix elements of the commutators and anti-
commutators can be treated in similar ways.

**Appendix II.**

In the particular case of no cut off $f(\omega) = 1$,
$1/\xi = 0$ the function $\beta(s)$ (cf. (A.6)) can be expressed in closed
form:

$$h(\omega\epsilon^i) = \omega + \gamma \left[ \omega + \frac{\epsilon_\omega^2}{2} - \sqrt{\omega^2 \mu^2 \left( \ln \frac{\omega + \sqrt{\omega^2 + \mu^2}}{\mu} + \pi i \right)} \right] \text{ if } \omega > \mu \text{ and } \epsilon > 0$$  \hspace{1cm} (A.12)

$$h(-\lambda) = -\lambda + \gamma \left[ -\lambda + \frac{\epsilon_\lambda^2}{2} \sqrt{\lambda^2 - \lambda^2 \mu^2} \text{ log } \frac{\lambda + \sqrt{\lambda^2 - \lambda^2 \mu^2}}{\mu} \right] \text{ if } \lambda > \mu$$  \hspace{1cm} (A.13)

Apart from the imaginary part in (A.12) these two cases can also be
represented by the same formula if an absolute value is taken for
the argument under the logarithm. For the third interval of the real
axis one has

$$h(\omega) = \omega + \gamma \left[ \omega - \sqrt{\omega^2 - \omega^2 \text{ arc sin } \frac{\omega}{\mu} + \frac{\omega^2}{2} \mu + \sqrt{\omega^2 - \omega^2 \text{ arc sin } \frac{\omega}{\mu}} \right] \text{ if } -\mu < \omega < \mu$$  \hspace{1cm} (A.14)

These expressions can be used to find the position of the most
$$h(-\lambda) = 0$$  \hspace{1cm} (A.15)

both in the weak and in the strong coupling limit. For weak coupling,
we find from (A.13)

$$\lambda \approx \frac{\mu}{2} \epsilon^2 \gamma \text{ if } \gamma \ll 1$$  \hspace{1cm} (A.16)

which excludes any kind of power series expansion. 9) In the strong
9) This is of some interest in connection with the failure to obtain
a power series with a finite radius of convergence by application
of perturbation methods to some examples of renormalizable field
3, 431 (1953).
coupling limit the application of (A.14) gives the following expression for the root

$$-\omega = \lambda \approx \frac{4}{\pi} \frac{f}{\delta} \quad \text{if } \gamma \gg 1$$

with a possibility of an expansion in powers of $\gamma^{-1}$. 

(A.17)