Top quark spin polarization in ep collision

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We discuss the degree of spin polarization of single top quarks produced via $Wg$ fusion process in $ep$ collision at TESLA+HERAp and CLIC+LHC energies $\sqrt{s} = 1.6$ and 5.3 TeV. For $c\bar{b} \rightarrow t\bar{\nu}$ subprocess we show that the top quark spin is completely polarized when the spin basis is chosen in the direction of the incoming positron beam in the rest frame of top quark. A description on how to combine the cross sections of $e^+b \rightarrow t\bar{\nu}$ and $e^+g \rightarrow t\bar{\nu}$ processes is given. $e^+$-beam direction is taken to be the favorite top quark spin decomposition axis in its rest frame and it is found to be comparable with the ones in $pp$ collision. It is argued that theoretical simplicity and experimental clearness are the advantage of $ep$ collision.


PACS numbers: 14.65.Ha, 13.88.+e

I. INTRODUCTION

The large mass of the top quark implies that its weak decay time is much shorter than the typical time for the strong interactions to affect its spin [1]. Within the standard model, the dominant decay chain of the top quark is $t \rightarrow W^+b$ ($W \rightarrow \ell^+\nu$, $d\bar{u}$). The angular distribution of the top decay given below is a simple expression to clarify the correlations between the top quark decay products and the top spin:

$$\frac{1}{\Gamma_T} \frac{d\Gamma}{d\cos\theta} = \frac{1}{2} (1 + \alpha \cos\theta)$$

Here $\theta$ is defined as the angle between top quark decay products and top quark spin quantization axis in the rest frame of the top quark. For $\ell$ or $d$ the correlation coefficient $\alpha = 1$ which leads to the strongest correlation. Therefore, it is expected that the study of the single production of top quark will give a useful tool to understand the standard model mechanism for electroweak symmetry and coupling of the top quark to other particles. There are many detailed discussions in the literature for the single top production and spin correlations in $pp$ and $p\bar{p}$ collisions [2,3].

The purpose of this work is to describe polarization of the top quarks along the direction of various spin bases for the single production in $ep$ collision via $e^+b \rightarrow t\bar{\nu}$ and $e^+g \rightarrow t\bar{\nu}$ processes.

It is hoped that the linear $e^+e^-$ collider can be converted into an $ep$ collider as an additional option when linear collider is constructed on the same base as the proton ring. Linear collider design TESLA at DESY is the one which has this option called TESLA+HERAp $ep$ collider [4]. Similar option would be considered for CLIC+LHC at CERN. We will take into account the estimations about the energy and luminosity of these colliders as $\sqrt{s_{ep}} = 1.6$ TeV ($E_p = 800, E_e = 800$ GeV), $L_{ep} = 10^{31}$ cm$^{-2}$s$^{-1}$ for TESLA+HERAp and $\sqrt{s_{ep}} = 5.3$ TeV ($E_p = 7000, E_e = 1000$ GeV), $L_{ep} = 10^{32}$ cm$^{-2}$s$^{-1}$ for CLIC+LHC.

At $pp(\bar{p})$ colliders top quark decay products occur in both the $t\bar{t}$ and the single $t$ production modes. In the case of pair production, there is potentially observable angular correlations among the decay products arising from the fact that spin up top quark is more likely to be produced along with a spin down top antiquark. The size of these 2-particle correlations tends to be smaller in $gg$ collisions than in $q\bar{q}$ collisions. Because the dominant contribution to top quark pair production is due to the $gg$ initial state, the predicted 2-particle correlations are rather small [5]. The single top quark production has the electroweak process with produced top quark couples to a $W$ boson. That is why we expect strong spin polarization of top quarks. But single top decay products in the final states give smaller statistics when compared to pair production. Therefore, a sophisticated signal analysis needs to be performed at $pp(\bar{p})$ colliders.

In $ep$ collision top quark decay products in the final states are dominated by the single $t$ production due to absence of the $t\bar{t}$ production. This is a great advantage that the single top production is a result of primarily electroweak process. Therefore, it is anticipated that the $ep$ colliders with linear electron (or positron) beams are appropriate choice for investigating top quark spin polarization.

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II. CROSS SECTIONS FOR SINGLE PRODUCTION OF POLARIZED TOP QUARK

The calculation of single top quark production cross section is based on the $Wg$ fusion process with $e^+ g \rightarrow t\bar{b}\nu$ diagram. These diagrams are dominated by the configuration where the final state $b$ quark is nearly collinear with the incoming gluon. If $b$ quark mass is neglected the cross section becomes singular. Furthermore, at each order in the strong coupling, there are logarithmically enhanced contributions, converting the perturbation expansion caused by the large logarithms can be absorbed into the $b$ quark distribution function. These diagrams are dominated by the configuration where the final state $\bar{b}\nu\nu$ is first defined in CM system where the cross section is calculated, one should apply Lorentz boost to the rest frame of the top quark with the expression below

\[ [\sigma(eb \rightarrow t\bar{b}\nu) + \sigma(eg \rightarrow t\bar{b}\nu) - \sigma(g \rightarrow bb* eb \rightarrow t\bar{b}\nu)] \]

where the subtracted term is the gluon splitting piece of the cross section for correction to the $2 \rightarrow 2$ process, 

For the main process $e^+ b \rightarrow t\bar{b}\nu$ spin dependent squared amplitude is given by

\[ |M|^2 = \frac{2g^4_w}{(q^2 - M^2_{Wb})^2} (p_b \cdot p_e)[p_c \cdot p_t - m_t p_c \cdot s_t] \]

with R.S. stands for rest system of the top quark.

In the case of $2 \rightarrow 2$ process we will show that, the top quark spin is completely polarized along the positron beam direction (spin up) in top rest frame. The definition of the spin axis in the rest frame of the top quark does not depend on the coordinate system where the cross section is performed. So it is more convenient to calculate cross section in the center of mass system of incoming positron and $b$ quark. The momentum definitions of incoming and outgoing fermions are

\[ p_b = \frac{\sqrt{s}}{2}(1, 0, 0, 1) \quad p_c = \frac{\sqrt{s}}{2}(1, 0, 0, -1) \]

\[ p_t = (E_t, |\vec{p}_t| \sin \theta, 0, |\vec{p}_t| \cos \theta) \]

\[ p_\nu = (|\vec{p}_\nu|, -|\vec{p}_\nu| \sin \theta, 0, -|\vec{p}_\nu| \cos \theta) \]

Here Mandelstam variable $s$ belongs to subprocess of $eb$ scattering. Top quark energy $E_t$ and momentum $|\vec{p}_t|$ can be written in terms of $s$ and top mass $m_t$

\[ |\vec{p}_t| = \frac{s - m_t^2}{2\sqrt{s}}, \quad E_t = \frac{s + m_t^2}{2\sqrt{s}} \]

Let us define top spin direction along the positron beam as follows

\[ s^2 = \lambda \frac{p^2_\nu}{|p^2_\nu|}, \quad \lambda = \pm 1. \]

where $p^2_\nu$ is the positron momentum observed in the rest frame of the top quark. Since positron momentum $p_\nu$ is first defined in CM system where the cross section is calculated, one should apply Lorentz boost to the rest frame of the top quark with the expression below
\[ \vec{p}_e' = \vec{p}_e + \frac{\gamma - 1}{\beta^2}(\vec{\beta} \cdot \vec{p}_e)\vec{\beta} - E_e \gamma \vec{\beta} \quad (10) \]

where \(\vec{\beta}\) is the velocity of the top quark in the CM system and \(\beta^2\) can be written in terms of \(s\) and \(m_t\)

\[ \beta = \frac{\vec{p}_t}{E_t}, \quad \beta^2 = \frac{(s - m_t^2)^2}{(s + m_t^2)^2}, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad (11) \]

Using the above expressions we obtain scalar products of four vectors \(p_c \cdot p_t\) and \(p_c \cdot s_t\) inside the square bracket of the squared amplitude

\[ p_c \cdot p_t = \frac{1}{4}[(s - m_t^2) \cos \theta + s + m_t^2] \quad (12) \]
\[ p_c \cdot s_t = -\frac{\lambda}{4m_t}[(s - m_t^2) \cos \theta + s + m_t^2] \quad (13) \]

When these results are put into the squared amplitude of process \(e^+ b \rightarrow t\bar{\nu}\) it is clear that the amplitude gives zero for \(\lambda = -1\) (spin down). Therefore, top spin is 100% polarized when the spin direction is chosen along the incoming e-beam (spin up) in the top quark rest frame.

For the correction to \(2 \rightarrow 2\) process \(W g\) fusion process \(e^+ g \rightarrow t\bar{\nu}\) we write the amplitude as a sum of the contribution from each diagram:

\[ |M|^2 = |M_1|^2 + |M_2|^2 + |M_{int}|^2 \quad (14) \]
\[ |M_1|^2 = \frac{g_W^4 g_s^2}{(q_1^2 - M_W^2)^2(q_t^2 - m_b^2)^2} A_1 \quad (15) \]
\[ |M_2|^2 = \frac{g_W^4 g_s^2}{(q_1^2 - M_W^2)^2(q_2^2 - m_t^2)^2} A_2 \quad (16) \]
\[ |M_{int}|^2 = \frac{g_W^4 g_s^2}{(q_1^2 - M_W^2)^2(q_2^2 - m_t^2)(q_3^2 - m_t^2)} A_{int} \quad (17) \]

where \(M_1, M_2\) and \(M_{int}\) correspond diagrams with \(b\) quark exchange, \(\bar{t}\) quark exchange and their interference, respectively. Reduced amplitudes \(A_1, A_2\) and \(A_{int}\) are

\[ A_1 = (p_{\nu} \cdot p_b)(-p_b \cdot p_g p_t \cdot p_e + m_t p_b \cdot p_g p_e \cdot s_t + m_b^2 p_t \cdot p_e + m_4 m_b^2 p_e \cdot s_t) \]
\[ + (p_{\nu} \cdot p_g)(p_b \cdot p_g p_t \cdot p_e - m_t p_b \cdot p_g p_e \cdot s_t + m_b^2 p_t \cdot p_e - m_4 m_b^2 p_e \cdot s_t) \quad (19) \]

\[ A_2 = (p_{\nu} \cdot p_b)(-p_t \cdot p_e p_t \cdot p_g + m_t p_t \cdot p_g p_e \cdot s_t - m_b^2 p_t \cdot p_e + p_t \cdot p_g p_e \cdot p_g \]
\[ - m_4 p_e \cdot p_g p_e \cdot s_t + m_b^2 p_e \cdot p_g + m_4^2 p_e \cdot s_t) \quad (20) \]

\[ A_{int} = (p_c \cdot p_b)(2 p_b \cdot p_t \cdot p_e - p_b \cdot p_t p_e \cdot p_g - 2 m_t p_b \cdot p_t p_e \cdot s_t \]
\[ + p_b \cdot p_c p_t \cdot p_e - m_t p_b \cdot p_g p_e \cdot s_t + m_4 p_b \cdot p_c p_e \cdot p_g) \]
\[ + (p_{\nu} \cdot p_t)(p_b \cdot p_g p_t \cdot p_e - m_t p_b \cdot p_g p_e \cdot s_t) \]
\[ + (p_{\nu} \cdot p_g)(-p_b \cdot p_t p_e \cdot p_c + m_t p_b \cdot p_t p_e \cdot s_t) \quad (21) \]

\[ q_1 = p_e - p_{\nu}, \quad q_2 = p_g - p_b, \quad q_3 = p_t - p_g \quad (22) \]
Gluon splitting part of the cross section that we should subtract can be written as follows

\[ \sigma(g \rightarrow b\bar{b} \ast e b \rightarrow t\bar{\nu}) = \int_{m_t^2/s}^{1} \tilde{\sigma}(e b \rightarrow t\bar{\nu}) f^{LO}_{b/p}(x, \mu^2) dx \]

where \( f^{LO}_{b/p} \) is the probability for a gluon to split into \( b\bar{b} \) pair at leading order

\[ f^{LO}_{b/p}(x, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \ln(\mu^2/m_b^2) \int_{x}^{1} \frac{dz}{z} P_{b/g}(z) f_{g/p}(x/z, \mu^2) \]

with splitting function

\[ P_{b/g}(z) = \frac{1}{2} [z^2 + (1 - z)^2]. \]

In splitting procedure the same QCD factorization scale should be considered as the one used in the parton distribution functions. All of the cross sections presented in this paper have been computed using MRST2002 parton distribution functions \([8]\) and running \( \alpha_s \). Considering combination, we obtain unpolarized total cross section of single top quark production 3.2 pb at TESLA+HERAp and 32.5 pb at CLIC+LHC.

III. RESULTS AND DISCUSSION

Let us now discuss the spin of the top quarks produced with \( e^+b \rightarrow t\bar{\nu} \) process (2 → 2 contribution) at TESLA+HERAp and CLIC+LHC energies \( \sqrt{s} = 1.6 \) and 5.3 TeV. In the zero momentum frame (ZMF) of the initial state partons t quark and antineutrino go out back to back. Since they couple to a W boson, in the initial state b quark has left handed chirality and positron right handed chirality. Their chiralities imply left and right handed helicities due to their high speed. Because of the angular momentum conservation outgoing t quark has both left and right handed helicity component. In this frame, it is shown from Table II and Table III that t quark is left handed 95% and 97% of the time for TESLA+HERAp and CLIC+LHC, respectively. Helicity of the particle with large mass is highly frame dependent because the speed of t quark is not ultrarelativistic at the energy region of ep colliders. So, in the laboratory frame of TESLA+HERAp and CLIC+LHC it is right handed 80% and 62% of the time.

In the case of \( e^+g \rightarrow t\bar{\nu} \) process (2 → 3) additional third particle in the final state shares the momentum, then the fraction of left handed helicity component decreases in the ZMF which is left handed only 76%(77%) of the time at TESLA+HERAp (CLIC+LHC) energies. It is right handed 77% and 62% of the time in the laboratory (LAB) frame of TESLA+HERAp and CLIC+LHC.

It should be pointed out that the overlap region needs an extra discussion. There are two possibilities to define ZMF which are in terms of either \( eg \) or \( eb \) initial states. As explained above helicity of the top quark is not invariant under longitudinal boosts connecting these two frames. Therefore, it is not possible to define ZMF unqiuely. A further discussion concerning its experimental side can be found in Ref. [2] for pp collision.

For a spin basis whose definition does not depend on the existence of a ZMF, we would think of antineutrino direction as the decomposition axis of the top quark spin in theoretical point of view. Since the spin direction is defined in the rest frame of the top quark, choosing antineutrino axis is almost equivalent to observing the helicity of the t quark in the frame where antineutrino and t quark are back to back for 2 → 2 process. In this case we obtain spin up contribution with 95% of the time which has the same degree of polarization as the ZMF helicity basis for TESLA+HERAp. At CLIC+LHC energy, fractions with left handed helicity in ZMF and with spin up in antineutrino direction are both 97% . In the 2 → 3 case 90% of top quarks are produced with spin up in this basis for TESLA+HERAp and 89% for CLIC+LHC. Combination of 2 → 2 and 2 → 3 processes with subtraction gives 92%(91%) fraction of spin up quarks at TESLA+HERAp (CLIC+LHC) energy. In experimental point of view, the antineutrino is invisible and we need somthing visible to make the basis definition. Fortunately, such an alternate definition is possible. The outgoing antineutrino is only slightly deflected from the incoming positron direction in 2 → 2 or 2 → 3 process, and so it is interesting to consider \( e^+ - beam \) line to define the spin basis. Then we find overall fraction of the spin up top quarks as 96% which is close to the one in the antineutrino axis at TESLA+HERAp collider. This becomes 93% at CLIC+LHC collider. In tables, we have kept results from the antineutrino direction for theoretical motivation. From the rest frame of the top quark e-beam and p-beam are not back to back. Then, it is also interesting to discuss the proton-beam (p-beam) as a spin decomposition axis for comparison. The last lines in the tables represent the results of p-beam axis which show the spin down most of the time as expected.
TABLE I: Dominant spin fractions and asymmetries for the various top quark spin bases in the production of single top process with $Wg$ fusion channel at $\sqrt{s} = 1.6$ TeV TESLA+HERAp energy. Contributions from each $2 \rightarrow 2$ and $2 \rightarrow 3$ processes and combination of them are listed.

<table>
<thead>
<tr>
<th>basis</th>
<th>$2 \rightarrow 2$</th>
<th>$2 \rightarrow 3$</th>
<th>overlap</th>
<th>total</th>
<th>$\frac{N_1 - N_\uparrow}{N_1 + N_\uparrow}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LAB helicity</td>
<td>80%(R)</td>
<td>77%(R)</td>
<td>79%(R)</td>
<td>77%(R)</td>
<td>0.54</td>
</tr>
<tr>
<td>ZMF helicity</td>
<td>95%(L)</td>
<td>76%(L)</td>
<td>undefined</td>
<td>undefined</td>
<td>undefined</td>
</tr>
<tr>
<td>e-beam</td>
<td>100%↑</td>
<td>93%↑</td>
<td>98%↑</td>
<td>96%↑</td>
<td>0.92</td>
</tr>
<tr>
<td>Antineutrino</td>
<td>95%↑</td>
<td>90%↑</td>
<td>93%↑</td>
<td>92%↑</td>
<td>0.84</td>
</tr>
<tr>
<td>p-beam</td>
<td>90%↓</td>
<td>86%↓</td>
<td>88%↓</td>
<td>87%↓</td>
<td>-0.75</td>
</tr>
</tbody>
</table>

Another quantity concerning the spin-induced angular correlations is the spin asymmetry

$$A_{\uparrow\downarrow} = \frac{N_1 - N_\downarrow}{N_1 + N_\downarrow}$$ (26)

In the case where spin-up and spin-down top quarks are both present spin asymmetry appears in the differential distribution of the decay angle presented in the first section

$$\frac{1}{\Gamma_T} \frac{d\Gamma}{d \cos \theta} = \frac{1}{2} (1 + A_{\uparrow\downarrow} \alpha \cos \theta)$$ (27)

The last column of the Table I shows that antineutrino or $e^+\text{-}beam$ basis improves the asymmetry a factor of 1.7 when compared to LAB helicity system. From Table I we see that this factor takes the value of 4. Clearly, it is easier to observe angular correlation as the asymmetry increases. Comparison of Table I and Table II provides that the fraction of top quarks produced with right handed helicity in LAB helicity frame gets smaller values as the center of mass energy of $ep$ system gets larger.

It is useful to plot the top quark $P_T$ distributions to compare different spin bases. Contributions from dominant spin components in LAB helicity right, e-beam up, p-beam down states and unpolarized case(Total) are shown in Fig. 1 for TESLA+HERAp energy and Fig. 2 for CLIC+LHC energy, using the combination of $2 \rightarrow 2$ and $2 \rightarrow 3$ processes. Similar features in tables are reflected in figures too.

In conclusion, we have shown that the single top quarks produced in the $Wg$ fusion channel in $ep$ collision at energies $\sqrt{s} = 1.6$ and 5.3 TeV are used to observe their high degree spin polarization along the direction of positron beam in the top quark rest frame. The TESLA+HERAp gives better results leading higher spin polarization than CLIC+LHC. Although $ep$ and $pp$ colliders are comparable in terms of high spin polarization, the $Wg$ fusion channel in $ep$ collision is expected to be much less complicated theoretically and experimentally than the case in $pp$ collision.

It is anticipated that next-to-leading order corrections to $Wg$ fusion will improve the results given in Table I and Table II.

The experimental conditions to observe the spin induced angular correlations with cuts and detector environments still need to be discussed before a firm decision.

FIG. 1: Transverse momentum $P_T$ distributions of the singly produced top quarks via $Wg$ fusion at TESLA+HERA energy $\sqrt{s} = 1.6$ TeV. Dominant spin bases LAB helicity right, e-beam up, antineutrino up and unpolarized(Total) case are drawn. Combination of $2 \rightarrow 2$ and $2 \rightarrow 3$ processes are considered.

FIG. 2: Transverse momentum $P_T$ distributions of the singly produced top quarks via $Wg$ fusion at CLIC+LHC energy $\sqrt{s} = 5.3$ TeV. Dominant spin bases LAB helicity right, e-beam up, antineutrino up and unpolarized (Total) case are drawn. Combination of $2 \rightarrow 2$ and $2 \rightarrow 3$ processes are considered.