Splashing and evaporation of nucleons from excited nuclei

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Abstract

The energy spectrum and the emission rate of particles emitted from excited nucleus due to both the evaporation and the splashing (emission from a cold vibrating nucleus) are calculated. We show that the collective motion of the nuclear Fermi liquid is accompanied by direct non-statistical emission of nucleons via the dynamical distortion of the Fermi surface.

Keywords: Nuclear Fermi liquid, particle emission, monopole giant resonance

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I. INTRODUCTION

The emission of particles from a collectively excited state of the nucleus as a Fermi liquid drop can occur in two ways. First, due to the relaxation processes the collective energy is transferred to the intrinsic degrees of freedom with subsequent evaporation of particles. On the other hand, a direct non-statistical emission (splashing) of nucleons is also possible via the dynamical distortion of the Fermi surface accompanying the collective motion. In general the relative contributions of these mechanisms depend upon the magnitude of the nuclear friction coefficient. In this work, the limiting cases of the direct (non-statistical) particle emission from the non-damped giant multipole resonance (GMR) and the particle evaporation from the heated nucleus are compared. The direct particle emission from the GMR has been extensively studied within pure quantum mechanical approaches (see, e.g. [1, 2, 3, 4, 5, 6]). We exploit here a semiclassical phase-space theory to study the energy spectra and the escape width for nucleons directly emitted from a collective state (splash out effect). Within this approach, the particle emission is describing in a more qualitative way than in microscopic quantum mechanical models. However an advantage of this approach is the possibility to take into account the statistical, direct and cascade emissions of the particles from the heated nucleus which is simultaneously involved in the collective motion. In particular, this approach can be used to describe both the evaporation of neutrons and the direct (non-statistical) neutron emission at the nuclear descent from the fission barrier to the scission point. Below, we will restrict ourselves to the case of particle emission from the giant monopole resonance. A generalization of our approach to the case of an arbitrary multipole giant resonances can be done in a straightforward way.

II. EMISSION OF PARTICLES CAUSED BY THE FERMI-SURFACE DISTORTION

For an excited nucleus, the particle emission rate can be calculated from the assumption that the nucleons localized in a single-particle mean field of depth $V_0$ and described by the phase-space distribution function $f(\vec{r}, \vec{p}, t)$ escape out the nuclear surface if the energy of their radial motion exceeds $V_0$. The flux density of neutrons emitted from the nuclear surface
is given by, see also \cite{7},

\[ J_{\text{emis}}(t) = \Theta(E) \frac{g_s d\vec{q}}{(2\pi\hbar)^3} (\vec{n} \cdot \vec{v}) \Theta(\vec{n} \cdot \vec{n}_q) f(\vec{r}, \vec{p}, t) \bigg|_{\text{surf}} \]  

(1)

where \( \Theta(x) \) is the function which equals unity for \( x > 0 \) and is zero otherwise, \( g_s = 2 \) is the spin degeneracy factor of the emitted neutron, \( \vec{n} = \vec{r}/r \) is the normal to the surface, \( \vec{n}_q = \vec{q}/q = \vec{p}/p = \vec{n}_p \), \( \vec{v} = \vec{q}/m \), \( m \) is the particle mass, and \( E = q^2/2m = p^2/2m - V_0 \) is the kinetic energy of the emitted neutrons. We assume that the nucleons are confined in a spherical well potential with a depth \( V_0 \) and time-dependent radius \( R(t) \). The integrand in Eq. (1) is taken at \( r = R(t) \). In the phase-space approach, the distribution function \( f(\vec{r}, \vec{p}, t) \) in Eq. (1) differs from the equilibrium one due to the dynamical distortion of the Fermi surface. We use the local-equilibrium approximation and assume the following form for the distribution function in nuclear interior \cite{8}

\[ f(\vec{r}, \vec{p}, t) = \frac{1}{\exp \{ [ (\vec{p} - m \vec{u}(\vec{r}, t))^2 / 2m - \mu(t) ] / T \} + 1} \]  

(2)

where \( \mu(t) \) is the chemical potential, \( T \) is the temperature and \( \vec{u}(\vec{r}, t) \) is the velocity field associated with the collective excitation of the nucleus. At small temperature \( T \) such excitation corresponds to the zero sound regime in a Fermi liquid \cite{9, 10} within the scaling model \cite{11}. The distribution function of Eq. (2) implies the distortion of the Fermi surface in momentum space of multipolarities \( l = 0 \) and \( l = 1 \). This is consistent with the scaling model, where the distortions of the Fermi surface of multipolarities \( l \geq 2 \) are absent due to the particular choice of the displacement field \( \vec{\chi}(\vec{r}, t) \) in the following form

\[ \vec{\chi}(\vec{r}, t) = \alpha(t) \vec{\nabla} \phi(r), \quad \vec{u}(\vec{r}, t) = \frac{\partial}{\partial t} \vec{\chi}(\vec{r}, t) \]  

(3)

with

\[ \alpha(t) = \alpha_0 \cos(\omega t) \]  

(4)

and \( \alpha_0 \) is the (small) amplitude of oscillations. Considering only monopole density vibrations, we put \( \phi(r) = r^2 \). The displacement field of Eq. (3) provides the disappearance of the highest multipolarities of the Fermi surface distortion with \( l \geq 2 \). Thus, the Fermi surface preserves the spherical symmetry under the assumption that the displacement field is given by Eq. (3).

Let us relate the time dependence of the chemical potential \( \mu(t) \) and the radius \( R(t) \) to the amplitude \( \alpha(t) \). Neglecting the change in the total number of particles \( A \) during the emission process \( (A \gg 1) \) we have
\[ A = \int \frac{g \, d\vec{r} \, d\vec{p}}{(2\pi\hbar)^3} f(\vec{r}, \vec{p}, t) = \text{const}, \]  
(5)

where \( g = 4 \) is the spin-isospin degeneracy factor. Substituting the distribution function \( f(\vec{r}, \vec{p}, t) \) from Eq. (2) into Eq. (5) and using the boundary condition on the velocity radial component
\[ u_r |_{r=R(t)} = \frac{\partial R(t)}{\partial t}, \]  
(6)

we obtain the approximate relations
\[ R(t) \approx R \left[ 1 + 2 \alpha(t) + 2 \alpha^2(t) \right], \quad \mu(t) \approx \mu \left[ 1 - 4 \alpha(t) + 8 \alpha^2(t) \right]. \]  
(7)

Here, both the equilibrium values \( R \) and \( \mu \) are temperature dependent
\[ R = R_0 \left[ 1 + \frac{\pi^2}{24} \left( \frac{T}{\epsilon_F} \right)^2 \right], \quad \mu = \epsilon_F \left[ 1 - \frac{\pi^2}{12} \left( \frac{T}{\epsilon_F} \right)^2 \right], \]  
(8)

where \( R_0 \) and \( \epsilon_F \) are the corresponding ground state values of the nuclear radius and the Fermi energy, respectively.

Taking into account that the inequality \( |m \, \vec{u}(\vec{r}, t)| \ll p_F \) is satisfied (see below Fig. 2), using Eqs. (2) and (3) and keeping the lowest non-zero order of the amplitude \( \alpha_0 \), the basic Eq. (1) can be reduced to the following form
\[ J_{\text{emis}}(t) = \frac{m}{\pi^2 \hbar^3} \int_0^1 dx \int_0^\infty dE \, E \, \Theta(E) \int_0^1 dx \exp \left\{ \left[ E + \eta(t) - q(E) s_F(t) x \right]/T \right\} + 1. \]  
(9)

Here, the following notations are used
\[ s_F(t) = 2 \dot{\alpha}(t) R_0/v_F, \quad \eta(t) = V_0 - \mu(t) \approx \lambda + 4 \epsilon_F \alpha(t), \quad q(E) = 2 \sqrt{\epsilon_F (\epsilon_F + \lambda + E)}, \]  
(10)

\( \lambda = V_0 - \epsilon_F \) is the neutron separation energy, \( p_F \) is the Fermi momentum and \( v_F = p_F/m \). The energy distribution of the density current of the emitted particles is given from Eq. (9) by the following expression
\[ \frac{dJ_{\text{emis}}(E, t)}{dE} = \frac{m}{\pi^2 \hbar^3} E \Theta(E) \int_0^1 dx \frac{x}{\exp \left\{ \left[ E + \eta(t) - q(E) s_F(t) x \right]/T \right\} + 1}. \]  
(11)

We will also consider the spectral density of emission rate \( d^2 N/dE \, dt \) derived as
\[ \frac{d^2 N}{dE \, dt} = 4 \pi R^2 \frac{dJ_{\text{emis}}(E, t)}{dE}. \]  
(12)
The derived quantity (12) should be averaged over time to smear out rapid oscillations, so the observable spectrum has the form
\[
\bar{d^2 N \over dE dt} = {1 \over (2\pi/\omega)} \int_0^{2\pi/\omega} dt \; d^2 N \over dE dt.
\] (13)

Here and below the bar means time-averaging.

In the case of particle emission from the vibrating cold nucleus, the expression (11) reads
\[
d_{J_{\text{emis}}} (E, t) \over dE = m \pi \hbar^3 \Theta(E) \int_0^1 dx \; x \; \Theta \left[ q(E) s_F(t) \; x - \eta(t) - E \right].
\] (14)

We will consider below the particle emission from the giant monopole resonance. In this case, the amplitude \(\alpha_0\) in Eq. (4) is small enough to satisfy the condition \(\eta(t) > 0\) (see next section). That means that the increase of the Fermi-sphere radius due to the oscillations of the nuclear surface is not sufficient to cause the emission of particles, i.e., the particle emission appears only because of the dynamic Fermi-surface shift. In this case, Eq. (11) takes the following explicit form
\[
d_{J_{\text{emis}}} (E, t) \over dE = m \pi \hbar^3 \Theta(E) \Theta[s_F(t)] \Theta[1 - \xi(t, E)] \left[ 1 - \xi^2(t, E) \right],
\] (15)

where
\[
\xi(t, E) = \eta(t) \over s_F(t) q(E).
\] (16)

Eq. (15) allows us to evaluate the maximal value of the kinetic energy, \(E_{\text{max}}\), of the emitted particles. The result reads
\[
E_{\text{max}} \approx -\lambda + {10 \over 3} \; E^* \left[ 1 + \sqrt{1 + {3 \over 5} A \; {\varepsilon_F \over E^*} + {12 \over 5} \left( {\varepsilon_F \over E^*} \right)^3} \right],
\] (17)

where \(k_F = p_F / \hbar\) and \(E^*\) is the energy of the collective excitation.

III. NUMERICAL CALCULATIONS AND DISCUSSION

We will apply Eqs. (11) and (15) to the particle emission from the isoscalar giant monopole resonance (ISGMR). The eigenfrequency \(\omega\) of the ISGMR in Eq. (4) can be found from the classical derivation \(\omega = \sqrt{C/B}\), where \(C\) and \(B\) are the stiffness and the
mass coefficient, respectively. Both transport coefficients $C$ and $B$, can be obtained evaluating the collective potential, $E_{pot}$, and kinetic, $E_{kin}$, energy of the nucleus within the scaling approximation \(3\). Using Eq. \(3\), we obtain

\[
E_{kin} = \frac{1}{2} m \int d^3 r \rho_{eq} u^2 \approx \frac{6}{5} A m R_0^2 \dot{\alpha}^2(t) = \frac{1}{2} B \dot{\alpha}^2(t) \tag{18}
\]

and

\[
B = (12/5) A m R_0^2. \tag{19}
\]

Here, $\rho_{eq}$ is the equilibrium particle density.

The collective potential energy $E_{pot}$ is derived as

\[
E_{pot} = \frac{1}{2} \int d^3 r \left( \frac{\delta^2 \mathcal{E}}{\delta \rho^2} \right)_{eq} (\delta \rho)^2, \tag{20}
\]

where $\epsilon$ is the particle energy density which is related to the static incompressibility $K$ by

\[
K = 9 \left( \frac{\delta^2 \mathcal{E}}{\delta \rho^2} \right)_{eq}. \tag{21}
\]

Using the continuity equation, $\delta \rho = -\nabla \cdot \rho_{eq} \chi$, we obtain from Eqs. \(3\) and \(20\)

\[
E_{pot} \approx 2 K A \alpha^2(t) = \frac{1}{2} C \alpha^2(t), \quad C = 4 K A \tag{22}
\]

and

\[
\omega = \sqrt{\frac{K}{m \langle r^2 \rangle}}, \tag{23}
\]

where $\langle r^2 \rangle = (3/5) R_0^2$ is the mean square radius of the nucleus. We point out that in the case of finite nuclei the incompressibility $K$ is $A$-dependent.

Using Eqs. \(11\) and \(18\), one can evaluate the amplitude $\alpha_0$ of the Fermi-surface oscillations. It can be done using the expression for the collective kinetic energy $E_{kin}$ of the monopole vibrations. Averaging Eq. \(18\) over time within the period of oscillations $2\pi/\omega$ and using the virial theorem we find

\[
\alpha_0^2 = \frac{5}{3} \frac{1}{A} \frac{1}{(k_F R_0)^2} \frac{\epsilon_F E^*}{(\hbar \omega)^2}. \tag{24}
\]

In the case of particle emission from the GMR, we put $E^* = \hbar \omega = E_{GMR}$, where $E_{GMR}$ is the eigenenergy of the ISGMR. In this work we adopt the values of $R_0 = 1.12 \cdot A^{1/3}$ fm, $\epsilon_F = 37$ MeV, $k_F = 1.36$ fm$^{-1}$, $E_{GMR} = 82 \cdot A^{-1/3}$ MeV and $\lambda = 7$ MeV.
FIG. 1: The spectral density of emission rate $\hbar d^2N/dE dt$ for splashing of neutrons from the giant monopole resonance (GMR) in cold nucleus (solid curve) and for particle evaporation from thermal equilibrated nucleus (dashed curve) versus particle kinetic energy $E$ of emitted neutrons. We have used the values of GMR energy $E_{GMR} = 82 \cdot A^{-1/3}$ MeV, thermal excitation energy $E_T = E_{GMR}$ and mass number $A = 90$.

In Fig. 1 the dependence of $d^2N/dE dt$, obtained from Eqs. (14), (12) and (13), on the kinetic energy of the emitted particle $E$ is shown (solid line) for the nucleus with $A = 90$ and $T = 0$. We will compare this result of particle emission from the ISGMR in the cold nucleus with the one for the evaporation of neutrons from the thermal equilibrated nucleus with the thermal excitation energy $E_T = E_{GMR}$. For the case of thermal evaporation of particles, we take $\mu(t) = \lambda$ and $\vec{u}(\vec{r},t) = 0$ in the distribution function $f(\vec{r}, \vec{p}, t)$ of Eq. (2). Using Eqs. (11) and (12), we obtain, see also Ref. [1],

$$\left. \frac{d^2N}{dE dt} \right|_{\text{evap}} = \frac{(k_F R_0)^2}{\pi \hbar \epsilon_F} E \Theta(E) \frac{1}{1 + \exp[(E + V_0 - \lambda)/T]}, \quad (25)$$

The dashed curve in Fig. 1 represents the spectral density of evaporation rate (25) from a heated nucleus with the temperature $T = \sqrt{8 E_T/A}$. As seen from Fig. 1, the thermal
The emission rate $dN/dt$ of neutrons from the giant monopole resonance in cold nucleus with $A = 90$ (solid line). The dashed line shows the time dependence of the dimensionless parameter, $s_F(t)$, of the Fermi surface distortion, see Eq. (10).

spectrum (dashed line) is much broader than the cold emission spectrum (solid line) and has its maximum at a larger energy.

Fig. 1 shows that the kinetic energy of the emitted particle from a cold nucleus is much smaller than the Fermi energy $\epsilon_F$, so in (16) we can neglect the dependence of $q(E)$ upon $E$. Integrating (15) with $q(E) = q_0 = 2\sqrt{\epsilon_F (\epsilon_F + \lambda)}$ over energy $E$, we obtain an analytical expression for the cold particle emission rate $dN/dt$ as

$$\frac{dN}{dt} = 4\pi R_0^2 \ J_{\text{emis}}(t)$$

$$= \frac{(k_F R_0)^2}{4\pi \hbar \epsilon_F} \ [s_F(t) \ q_0]^2 \ \Theta [s_F(t)] \ \Theta [1 - \beta(t)] \ \left[1 - \frac{8}{3} \beta(t) + 2 \beta^2(t) - \frac{1}{3} \beta^4(t) \right],$$

(26)

where $\beta(t) = \eta(t)/q_0 s_F(t)$. Fig. 2 shows the emission rate $dN/dt$ of Eq. (26) for the interval of time $\Delta t \leq 2\pi/\omega$. As seen from Fig. 2 the particle emission from ISGMR occurs as a short time splashing.

The particle emission rate $dN/dt$ can be used to derive the life time, $\tau_{\text{cold}}$, with respect
to the splashing of neutron from the ISGMR. The quantity (26) must be averaged over time as in (13), resulting in

$$\tau_{\text{splash}} = \frac{dN}{dt} = \frac{1}{(2\pi/\omega)} \int_0^{2\pi/\omega} dt \frac{dN}{dt}.$$  (27)

For the nucleus with $A = 90$, one obtains from Eqs. (26) and (27) that $\tau_{\text{splash}} = 6.0 \cdot 10^{-20}$ s. The analogous quantity for particle evaporation, calculated by means of (25), is $\tau_{\text{evap}} = 5.3 \cdot 10^{-19}$ s. The calculations of $\tau_{\text{splash}}$ can be improved if the higher multipolarities of the Fermi surface distortion, at least the quadrupole, are taken into account, see Appendix.

IV. CONCLUSIONS

By use of phase space approach to the particle emission from the nucleus, we have connected the particle emission rate to the dynamical distortion of the Fermi surface of the cold vibrating nucleus. The particle emission occurs here as a classical splashing effect from the vibrating liquid drop. We have shown that cold particle emission (splashing) may result in the deviation of the observed spectra from the usual statistical ones in the small energy region, see Fig. 1. In particular, for the cold particle emission, the emission rate, $(dN/dt)_{\text{splash}}$, (averaged over time) is significantly large then the corresponding value, $(dN/dt)_{\text{evap}}$, for the particle evaporation (for the ISGMR $(dN/dt)_{\text{splash}}$ exceeds $(dN/dt)_{\text{evap}}$ by factor about 10).

The general conclusion of this paper is that the collective motion in a finite nuclear Fermi liquid is accompanied by direct (non-statistical) emission of nucleons via the dynamical distortion of the Fermi surface. This mechanism of cold particle emission can be also applied to the large amplitude motion like a descent of the nucleus from the fission barrier to the scission point or to the first stage of the heavy ion collision.

V. ACKNOWLEDGMENTS

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APPENDIX A

We will generalize the local-equilibrium approximation of Eq. (2) and assume the following non-equilibrium form for the distribution function in nuclear interior at $T = 0$, see Ref. [8],

$$ f(\vec{r}, \vec{p}, t) = \Theta \left[ \frac{1}{2m} p_F^2(\vec{r}, t) - \frac{1}{2m} [\vec{p} - m \vec{u}(\vec{r}, t)]^2 - \nu(\vec{r}, \vec{p}, t) \right], \quad (A1) $$

where $p_F(\vec{r}, t)$ is the Fermi momentum and $\nu(\vec{r}, \vec{p}, t)$ is associated with the distortion of the Fermi-surface in the momentum space.

To describe the collective excitations with the multipolarity $L$ in the spherical well potential we will consider $\nu(\vec{r}, \vec{p}, t)$ as a superposition of plane wave to create a state with a good angular moment $L$, see also [12, 13]. Namely,

$$ \nu(\vec{r}, \vec{p}, t) = \int d\Omega_k Y_{L0}(\vec{n}_k) F(\vec{n}_p \cdot \vec{n}_k) \exp[i (\vec{k} \cdot \vec{r} - \omega t)] + c.c., \quad (A2) $$

where

$$ F(\vec{n}_p \cdot \vec{n}_k) = \sum_{l \neq 1} \nu_l^{(0)} Y_{l0}(\vec{n}_p \cdot \vec{n}_k) \quad (A3) $$

and $\nu_l(t) = \nu_l^{(0)} (e^{-i\omega t} + c.c.)$ is the amplitude of the Fermi-surface distortion of multipolarity $l$ in the momentum space. We point out that the condition $l \neq 1$ in Eq. (A3) is because the shift of the Fermi surface was already extracted in the distribution function $f(\vec{r}, \vec{p}, t)$, taken in the form of Eq. (A1). We will follow the nuclear fluid dynamic approach, and take into account the dynamic Fermi-surface distortion up to multipolarity $l = 2$ in the expansion of Eq. (A3). In this case, the amplitudes $\nu_0$ and $\nu_2$ are related to each other as $\nu_2 = \sqrt{4/5} \nu_0$, see Ref. [14]. Finally, putting $Y_{L=0,0} = 1/\sqrt{4\pi}$ in Eq. (A2), we obtain the distortion amplitude $\nu(\vec{r}, \vec{p}, t)$ for monopole mode in the following form

$$ \nu(\vec{r}, \vec{p}, t) = \nu_0 \left[ j_0(kr) - 2 j_2(kr) P_2(x) \right] e^{-i\omega t} + c.c., \quad \text{for } L = 0, \quad (A4) $$

where $j_l(kr)$ is the spherical Bessel function, $P_l(x)$ is the Legendre polynomial and $x = \vec{n} \cdot \vec{n}_p$.

The amplitude $\nu_0$ is related to the bulk density variation $\delta \rho(\vec{r}, t)$. Namely, using Eqs. (A1) and (A4), one obtains the following expression in the nuclear interior

$$ \delta \rho(\vec{r}, t) = \int \frac{d\vec{p}}{(2\pi \hbar)^3} f(\vec{r}, \vec{p}, t) - \rho_{eq}(\vec{r}) = \nu_0^{(0)} N_0 j_0(kr) e^{-i\omega t} + c.c., \quad (r < R, L = 0), \quad (A5) $$
where $N_0 = 2 m p_F/\pi^2 h^3$ is the density of single particle states on the Fermi-surface. The velocity field $\vec{u}(\vec{r}, t)$ can be evaluated from the continuity equation

$$\frac{\partial}{\partial t} \delta \rho(\vec{r}, t) = -\vec{\nabla} \cdot \rho_{eq} \vec{u}(\vec{r}, t). \quad (A6)$$

Using Eqs. (A5) and (A6), one obtains

$$\vec{u}(\vec{r}, t) = 2 \nu_0 N_0 \omega \frac{j_1(kr)}{\rho_0 kr} \vec{r} \sin(\omega t), \quad (L = 0). \quad (A7)$$

The wave number $k$ in Eqs. (A4), (A5) and (A7) is derived from the boundary condition. The boundary condition can be taken as a condition for the balance, at the nuclear surface, between the compressional pressure and the surface tension pressure, see \[15, 17\]. Taking into account the consistent change of the compressional pressure due to the Fermi-surface distortion, the boundary condition reads \[17\]

$$z_n j_0(z_n) - (f_\sigma + f_\mu) j_1(z_n) = 0. \quad (A8)$$

Here, $z = kR$,

$$f_\sigma = \frac{18 \sigma}{\rho_0 R_0 K'}, \quad f_\mu = \frac{36 \mu}{K'}, \quad (A9)$$

where $\sigma$ is the surface tension coefficient, $K'$ is the dynamic incompressibility given by

$$K' = K + K_\mu \quad (A10)$$

and $K$ is the commonly used static incompressibility

$$K = R^2 \frac{\delta^2 E/A}{\delta R^2} \bigg|_{R=R_0} \quad (A11)$$

The additional contribution, $K_\mu$, to the incompressibility $K'$ in Eq. (A10) is due to the dynamic Fermi-surface distortion effect. The quantity $K_\mu$ can be evaluated in a general case of arbitrary multipolarity of the Fermi-surface distortion and is given by \[8, 16, 17\]

$$K_\mu = 12 \epsilon_F \Omega_{20}(s) \Omega_{00}(s) \quad \text{with} \quad \Omega_{00}(s) = \frac{1}{2} \int_{-1}^{1} dx \frac{x P_l(x) P_0(x)}{x-s}, \quad (A12)$$

where the dimensionless zero-sound velocity $s = h\omega/p_F k$ is found from the Landau’s dispersion relation \[10, 18\]. In the case of an isotropic Landau’s interaction amplitude, i.e., $F_0 \neq 0, \quad F_{l\neq0} = 0$, the dispersion relation reads

$$\Omega_{00}(s) = -\frac{1}{F_0}. \quad (A13)$$
For realistic nuclear forces, \( F_0 \sim 0 \), Eqs. (A12) and (A13) lead to large renormalization of the incompressibility with \( K_\mu \approx 2K \). The giant monopole resonance (GMR) corresponds to the lowest solution, \( z_0 > 1 \), to the secular equation (A8).

Using Eqs. (A1), (A4) and (A7), the flux density \( J_{\text{emis}}(t) \) of Eq. (1) can be evaluated beyond the scaling approximation used in Sect. II.