LOCALIZED STATES ("qubits")
ENTANGLEMENT AND DECOHERENCE
FROM WIGNER ZOO

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Abstract

We present the application of the variational-wavelet approach to the construction and analysis of solutions of Wigner/von Neumann/Moyal and related equations corresponding to the nonlinear (polynomial) dynamical problems. (Naive) deformation quantization, the multiresolution representation (exact multiscale decomposition) and the variational approach are the key points. We construct the solutions via the high-localized nonlinear eigenmodes in the base of the compactly supported wavelets and the wavelet packets. We demonstrate the appearance of (stable) localized patterns (waveletons) and consider entanglement and decoherence as possible applications.

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Localized states ("qubits"), entanglement and decoherence from Wigner zoo

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We present the application of the variational-wavelet analysis to the calculations and analysis of the solutions of Wigner/von Neumann/Moyal and related equations corresponding to the nonlinear (polynomial) dynamical problems [1], [2]. (Naive) deformation quantization, the multiresolution representations and the variational approach are the key points. We construct the solutions via the multiscale expansions in the high-localized nonlinear eigenmodes in the base of the compactly supported wavelets and the wavelet packets. We demonstrate the appearance of (stable) localized patterns (waveletons) and consider entanglement and decoherence as possible applications. Our goals are some attempt of classification and the explicit numerical-analytical constructions of the existing zoo of possible/realizable quantum states. There is a hope on the understanding of relation between the structure of initial Hamiltonians and the possible types of quantum states and the qualitative type of their behaviour. Inside the full spectrum there are at least four possibilities which are the most important from our point of view for possible realization of quantum-like computations: localized states, chaotic-like or/and entangled patterns, localized (stable) patterns (waveletons). We consider the calculations of the Wigner functions $W(p,q,t)$ (WF) corresponding to the classical polynomial Hamiltonians $H(p,q,t)$ as the solution of the Wigner-von Neumann equation:

$$i\hbar \frac{\partial}{\partial t} W = H \ast W - W \ast H$$

and related Wigner-like equations, e.g. Wigner transform of master equation (Lindblad-like) describing the decoherence

$$W = \{H, W\} + \sum_{n \geq 1} \frac{\hbar^{2n}(-1)^n}{2^{2n}(2n+1)!} \partial_x^{2n+1} U(x) \partial_p^{2n+1} W + 2\gamma \partial_p \partial_p W + D \partial_p^2 W$$

We have constructed the following quantum states: localized (as a model for qubit), entangled patterns, localized (stable) patterns (e.g., modeling the decoherence as a result of interaction with environment). The obtained solutions of these equations have the following multiscale or multiresolution decomposition.
via nonlinear high-localized eigenmodes

\[
W(t, x_1, x_2, \ldots) = \sum_{(i,j) \in \mathbb{Z}^2} a_{ij} U^i \otimes V^j(t, x_1, x_2, \ldots),
\]

\[
V^j(t) = V_{N}^{j, \text{slow}}(t) + \sum_{l \geq N} V_{l}^{j}(\omega_l t), \quad \omega_l \sim 2^l, \quad (3)
\]

\[
U^i(x_s) = U_{M}^{i, \text{slow}}(x_s) + \sum_{m \geq M} U_{m}^{i}(k_{m}^{s} x_s), \quad k_{m}^{s} \sim 2^m,
\]

which correspond to the full multiresolution expansion in all underlying phase space or space-time scales. The formulae (3) give the expansion into a slow part and fast oscillating parts for arbitrary \(N, M\). So, we may move from the coarse scales of resolution to the finest ones for obtaining more detailed information about the dynamical process. In this way one obtains contributions to the full solution from each scale of resolution or each time/space scale or from each nonlinear eigenmode. It should be noted that such representations give the best possible localization properties in the corresponding (phase)space/time coordinates. Formulae (3) do not use perturbation techniques or linearization procedures. The modeling demonstrates the appearance of different (stable) pattern formation from high-localized coherent structures or chaotic/entangled behaviour. Our (nonlinear) eigenmodes (Fig. 1) are more realistic for the modeling of nonlinear classical/quantum dynamical process than the corresponding linear gaussian-like coherent states. Here we mention only the best convergence properties of the expansions based on wavelet packets, which realize the minimal Shannon entropy property and the exponential control of convergence of expansions like (3). As an example, waveleton (Fig. 3) corresponds to result of superselection (einselection) after decoherence process started from entanglement state demonstrated by Fig. 2 and constructed from localized “qubit-like” states (Fig. 1) [2]. It should be noted that we can control the type of behaviour on the level of the reduced algebraic system [2].

References


Figure 1: Localized modes.

Figure 2: Entangled-like Wigner function.

Figure 3: Localized pattern: waveleton Wigner function.