Laser frequency locking by direct measurement of detuning

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(Dated: June 5, 2005)

We present a new method of laser frequency locking in which the feedback signal is directly proportional to the detuning from an atomic transition, even at detunings many times the natural linewidth of the transition. Our method is a form of sub-Doppler polarization spectroscopy, based on measuring two Stokes parameters ($I_1$ and $I_2$) of light transmitted through a vapor cell. This extends the linear capture range of the lock loop by up to an order of magnitude and provides equivalent or improved frequency discrimination as other commonly used locking techniques.

PACS numbers:

Laser frequency locking is the process of controlling a laser’s frequency, relative to some physical reference, by means of feedback. Various physical references may be used, for example cavities or atomic transitions \cite{1}, and the methods described here apply equally well in both cases. In this work we focus on the spectroscopy of hyperfine transitions in Rb atomic vapor which is relevant to work on ultra-cold atoms.

Experiments in atomic physics often require lasers to be locked precisely to a known atomic transition. Saturated absorption, and polarization spectroscopy, are two well known techniques which allow this to be achieved \cite{2}. Both rely on counter propagating pump and probe beams to measure a single longitudinal velocity group of atoms within a Doppler broadened profile. In the case of saturation spectroscopy, the laser may be locked to the side of an absorption peak \cite{3} by direct comparison of a photodetector signal with a reference voltage, however it is generally desirable to lock to the top of a peak. This may be achieved by dithering the laser frequency around a peak and performing lock-in detection \cite{4}, which provides a derivative signal of the transmission. This has a zero crossing at top of the peak which is convenient for locking. Polarization spectroscopy is somewhat more attractive as it produces a narrow, sub-Doppler spectrum with a natural zero crossing at resonance \cite{5} without the need for dithering (which can broaden the laser linewidth) and lock-in detection \cite{6}.

Pearman et al. have studied a form of polarization spectroscopy locking in detail \cite{7}. Their method corresponds to a measure of the circular birefringence (differential refractive index for orthogonal polarizations) of an atomic sample which is induced by a circularly polarized pump beam. The locking signal is linear over a range approximately equal to the power broadened linewidth of the transition (typically 10-20 MHz in rubidium vapors).

The method we present incorporates this along with a measure of the circular dichroism (differential absorption for two orthogonal polarizations) which greatly extends the linear region of the lock signal. Our method stems from the proposal of Harvey and White \cite{8} which is best understood by analyzing the Stokes parameters of the transmitted probe light.

We wish to quantify the change in the polarization state of an input laser beam in terms of the anisotropic properties of the sample. As the anisotropy is defined in terms of circular polarizations, we choose to express the total electric field in the right-left (R-L) circular basis: $\mathbf{E} = [E_R \ E_L]$ where $E_R$ and $E_L$ are the right and left circular electric field components respectively. In this basis the (unnormalized) Stokes parameters are,

$$I_0 = |E_R|^2 + |E_L|^2,$$

$$I_1 = 2|E_R||E_L|\cos \phi,$$

$$I_2 = 2|E_R||E_L|\sin \phi,$$

$$I_3 = |E_R|^2 - |E_L|^2,$$

where $\phi = \phi_R - \phi_L$ is the phase difference between the right and left circular components. Equations 1-4 are equivalent to the familiar expressions in the horizontal-vertical ($H-V$) basis, and could also be written in the diagonal-antidiagonal ($D-A$) basis. The normalized Stokes parameters, $S_i$, are obtained from the ratio $I_i/I_0$. We recall the physical meaning of these, $I_0$ is the total irradiance, $I_1$, $I_2$ and $I_3$ are measures of the horizontal, diagonal and right circular polarizations, respectively \cite{9}.

With this in mind it is straightforward to design detectors for each of the Stokes parameters consisting of: for $I_1$ a polarizing beamsplitter cube and two photodetectors; $I_2$ a $\lambda/2$ plate and an $I_1$ detector; and $I_3$ a $\lambda/4$ plate and an $I_1$ detector. Figure \ref{fig:1} shows this schematically. $I_0$ is given by the sum of the two photodetector signals for any of the above $I_i$ measurements. In all cases the

![FIG. 1: Detector for measuring Stokes parameters $I_i$, $i=1,2,3$. PD = photodetector, PBS = polarizing beam splitter, $\lambda_i$-plate = 0, $\lambda/2$, $\lambda/4$ plate for $i=1,2,3$ respectively.](image)
waveplates convert \((D-A)\) to \((H-V)\) to measure \(I_2\) and \((R-L)\) to \((H-V)\) to measure \(I_3\) respectively.

We return now to the proposal of Harvey and White in which a measure of the phase difference between two orthogonal field components, provides a feedback signal which may be used to lock a laser \(\ddagger\). In the circular basis, the phase difference, \(\phi\), between the left and right handed components can be obtained from equations 2 and 3. Evaluating the arctangent of the ratio of the measured Stokes parameters \(S_2/S_1\) yields a direct measure of \(\phi\). In practice, calculating the arctangent in real time would require a digital signal processor. However, for small phase shifts, \(\tan \phi \approx \sin \phi \approx \phi \) and \(I_2\) provides a signal proportional to \(\phi\). In polarization spectroscopy \(\ddagger\), the small angle approximation is valid, and a signal directly proportional to \(\phi\) can be obtained simply by measuring the single Stokes component, \(I_2\). The shape of the expected \(I_2\) signal is given by \(\ddagger\),

\[
I_2(x) \propto \Delta n(x) = \frac{\Delta \alpha_0}{k} \frac{x}{1 + x^2},
\]

where \(x\) is the laser detuning from resonance in units of the power broadened transition linewidth, \(k\) is the (resonant) wavenumber of the laser, \(n(x)\) is the real part of the refractive index of the vapor and \(\Delta \alpha_0\) is the difference in absorption at line center of the two circular components of the probe. This signal is dispersion shaped and free of a Doppler broadened background (both left and right handed components experience equal Doppler broadened absorptions and this is removed in the subtraction).

The phase shift of equation \(\ddagger\) depends only on \(\Delta \alpha_0\), the differential absorption at line center. This difference originates from an anisotropy induced in atoms which have interacted with the circularly polarized pump laser. Atoms are optically pumped into one of the extreme magnetic states, say \(m_F = +F\). The differential absorption, \(\Delta \alpha\) of the two circular components of the probe field is maximized in this case. In the \(5S_{1/2} F = 2, m_F = 2\) ground state of \(^{87}\text{Rb}\) the difference in oscillator strengths for \(\sigma^+\) and \(\sigma^-\) transitions is \(15:1\). The \(\sigma^+\) component experiences enhanced absorption and the \(\sigma^-\) decreased absorption with respect to the Doppler background. As the Doppler broadened absorption is the same for both components the differential absorption profile is given by the Lorentzian,

\[
I_3(x) \propto \Delta \alpha(x) = \frac{\Delta \alpha_0}{1 + x^2},
\]

which is related to equation \(\ddagger\) by the Kramers-Kronig dispersion relation \(\ddagger\). Equation \(\ddagger\) is simply the difference in absorption of left and right handed circular component (circular dichroism) which is equivalent to \(I_3\) of equation 4. Inspection of equations \(\ddagger\) and \(\ddagger\) reveals that their quotient, \(Q(x)\)

\[
Q(x) = \frac{I_2(x)}{I_3(x)} = \frac{x}{k},
\]

provides a signal directly proportional to \(x\) which does not decay as \(1/(1 + x^2)\) at large detunings. In the polarization locking of \(\ddagger\), \(I_2\) is used as the error signal and a controller provides feedback signal to the laser which drives this to zero. \(I_2(x)\) is linear for small \(x\) but not at large detunings where it approaches zero. Our locking method compensates for this by dividing \(\ddagger\) by \(\ddagger\) which greatly extends the linear region of the locking signal.

We have implemented this method spectroscopically and constructed a simple control circuit for locking. Figure \(\ddagger\) is a schematic of our apparatus. We choose for our experiments the \(5S_{1/2} F = 2 \rightarrow 5P_{3/2} F' = 1, 2, 3\) transition at 780.1nm in \(^{87}\text{Rb}\). Light is provided by a 90mW Rohm RLD78PZW1 diode laser in an external cavity configuration similar to that described in \(\ddagger\). Four photodetectors at the output of the optical apparatus are connected to an analogue circuit which performs the necessary subtractions and division in real time. The output of this device, either the polarization spectroscopy signal \(I_2\), or the quotient \(I_2/I_3\), is fed into the laser lock circuit which provides feedback to the laser frequency via the injection current and piezoelectric transducer (PZT). Additionally, the lock circuit has a differential input to which we can apply an offset voltage which shifts the lock point away from zero volts. Spectra were obtained as the laser frequency was scanned across the transition by applying a triangular voltage ramp to the PZT. Figure \(\ddagger\) shows the results of such a scan. The upper trace retains this value at large detunings.

The normalized gradients of the quotient \(I_2/I_3 = S_2/S_3\) and \(S_2 = I_2/I_0\) spectrum are the same at \(x = 0\) but \(Q(x)\) retains this value at large detunings.

As \(I_3\) approaches zero \(Q(x)\) becomes sensitive to electronic noise on the photodetector signals. We overcome this by low-pass filtering the output of the analogue di-

![FIG. 2: Experimental setup for polarization quotient locking. A small fraction of light is split off the laser output and sent into a polarization spectrometer. The \(I_2\) and \(I_3\) Stokes parameters of the probe light are measured and used to provide feedback to lock the laser. BS = beam splitter, PBS = polarizing BS, NPBS = 50/50 non-polarizing BS.](image-url)
FIG. 3: Spectra for the $^{87}$Rb $5S_{1/2} F=2 \rightarrow 5P_{3/2} F'=1,2,3$ transition. Top trace is a regular saturated absorption spectrum (SA), the second is the $I_2$ spectrum and third from the top is $I_3$. The trace below shows the quotient $Q(x)$. The shaded regions indicate the monotonic capture range of $I_2$ (upper) and $Q(x)$ locking (lower).

FIG. 4: Step responses of a laser locked using $I_2$ and $Q(x)$ locking after switching the DC lock point voltage at $t=0$. Solid lines show the measured error signal after switching and the dashed lines show the ideal response. $Q(x)$ locking allows much larger jumps to be achieved without unlocking the laser.

In summary, we have seen that for a Lorentzian transition, the ratio of the dispersion and absorption spectra yields a measurement of detuning. Polarization spectroscopy obtains a Doppler-free dispersion spectrum by measuring the unnormalized Stokes parameter $I_2$. A similar measurement can yield the parameter $I_3$ which provides a Doppler-free measurement of the differential absorption. Combining these produces a spectroscopic measurement proportional to detuning ideal for use in laser locking which extends the capture region of the lock by up to a factor of ten. This results in a very robust lock and allows us to make rapid and precise jumps of the laser frequency of up to 70 MHz either side of resonance.

This work is supported by the Australian Research Council.

References:

