The probabilistic model of parton distributions, previously developed by one of the authors, is generalized to include the transversity distribution. When interference effects are attributed to quark level only, the intrinsic quark motion produces the transversity, which is about twice as large as the usual polarized distribution. The applicability of such a picture is considered and possible corrections, accounting for interference effects at the parton-hadron transition stage are discussed.

I. INTRODUCTION

Nucleon spin functions represent a sensitive tool for understanding the nucleon internal structure in the language of QCD. Up to this day we have accumulated a very good knowledge of the nucleon spin functions $g_1$ and $g_2$, which were measured in deep inelastic scattering [1] - [8]. A further important and interesting quark spin distribution function is the transversity, the third non-diagonal element of the quark spin density matrix. Transversity is not accessible from the measuring of deep inelastic scattering, since it corresponds to the helicity flip amplitude. Its measuring is more complicated and that is the reason, why some more accurate and complete experimental data on the transversity are still missing. However, the recent and/or future data from the experiments HERMES (DESY-Hamburg), CLAS (JLab), COMPASS (CERN-Geneva) and RHIC (Brookhaven National Laboratory) could be interpreted also in terms of the transversity [9] - [11]. For the present status of research in both theory and experiment, see e.g. [12] and overview [13].

In Refs. [14], [15] the probabilistic, covariant quark-parton model (QPM), in which intrinsic quark motion with spheric symmetry is consistently taken into account, was developed by one of us (P.Z.). It was shown that such a model nicely reproduces some well-known sum rules and gives a very reasonable agreement with experimental data on the spin structure functions $g_1$ and $g_2$. Assuming $SU(6)$ symmetry, a calculation was done from the input on unpolarized valence quark distributions $q_V$. The aim of this paper is to extend this model also for description and calculation of the transversity distribution.

II. TRANSVERSITY

First, let us shortly summarize, how the spin structure functions $g_1, g_2$ were calculated in the paper [14]. The antisymmetric part of the tensor related to the photon absorption by a single quark reads:

$$t_{\alpha\beta} = m \varepsilon_{\alpha\beta\lambda\sigma} q^\lambda w^\sigma,$$

where $q, m, w$ are the photon momentum, quark mass and polarization vector; the corresponding handbag diagram is in Fig. 1a. Then it was shown that the corresponding tensor related to the target (proton) consisting of quasifree quarks can be written as

$$T_{\alpha\beta}^{(A)} = \varepsilon_{\alpha\beta\lambda\sigma} q^\lambda \frac{m}{2Pq} \int H \left( \frac{pP}{M} \right) \frac{w^\sigma}{Pq} \delta \left( \frac{pq}{Pq} - x \right) \frac{d^3p}{p_0}; \quad x = \frac{Q^2}{2Pq},$$

where $M$ is the proton mass, $p$ and $P$ are the quark and proton momenta. The distribution $H$ is the difference of the quark distributions with opposite spin projections. In the proton rest frame one can write

$$H(p_0) = G_+(p_0) - G_-(p_0).$$

For the time being, if not stated otherwise, we consider the quark charge equals unity. Further, we showed that the covariant form of the quark polarization vector reads
\[ w^\sigma = AP^\sigma + BS^\sigma + Cp^\sigma, \]  

where \( S \) is the proton polarization vector and

\[ A = -\frac{pS}{pP + mM}, \quad B = 1, \quad C = \frac{M}{m}A. \]  

Finally, in the last step the functions \( g_1, g_2 \) were extracted from the tensor (2). In the approximation

\[ Q^2 \gg 4M^2x^2 \]  

and identifying the beam direction with coordinate 1 in the proton rest frame, we obtain

\begin{align*}
  g_1(x) &= \frac{1}{2} \int H(p_0) \left( m + p_1 + \frac{p_1^2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \\
  g_2(x) &= -\frac{1}{2} \int H(p_0) \left( p_1 + \frac{p_1^2 - p_T^2/2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0},
\end{align*}

which implies

\[ g_T(x) = g_1(x) + g_2(x) = \frac{1}{2} \int H(p_0) \left( m + \frac{p_T^2/2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}. \]  

Since

\[ m + p_1 + \frac{p_1^2}{p_0 + m} = m - p_0 + p_0 + p_1 + \frac{p_1^2}{p_0 + m} = p_0 + p_1 - \frac{(p_0 - m)(p_0 + m) - p_T^2}{p_0 + m}, \]

the \( \delta \)– function allows a useful alternative representation of \( g_1 \):

\[ g_1(x) = \frac{1}{2} \int H(p_0) \left( Mx - \frac{p_T^2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}. \]  

Let us also remark that in the paper [16] we showed that a similar approach for the corresponding symmetric part of the proton tensor gives the unpolarized structure function

\[ f_1(x) = \frac{F_2(x)}{x} = Mx \int (G_+(p_0) + G_-(p_0)) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}. \]  

Now, if one assumes the same spheric shape of the distributions \( G_\pm \) for both opposite polarizations, then the corresponding probabilities can be parameterized as

\[ G_+ = G(p_0) \cos^2(\eta/2), \quad G_- = G(p_0) \sin^2(\eta/2), \quad 0 \leq \eta \leq \pi, \]  

so for \( \eta = 0(\pi) \) we have a pure state with the polarization \(+(-)\). For example, in the case of \( SU(6) \) we have \( \cos \eta = 2/3(-1/3) \) for \( u(d) \) quarks. The last relations imply

\[ G_+(p_0) + G_-(p_0) = G(p_0), \quad G_+(p_0) - G_-(p_0) = G(p_0) \cos \eta, \]  

so the relations (8) - (11) can be rewritten as

\begin{align*}
  g_1(x) &= \frac{1}{2} \cos \eta \int G(p_0) \left( Mx - \frac{p_T^2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \\
  g_2(x) &= -\frac{1}{2} \cos \eta \int G(p_0) \left( p_1 + \frac{p_1^2 - p_T^2/2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \\
  g_T(x) &= \frac{1}{2} \cos \eta \int G(p_0) \left( m + \frac{p_T^2/2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \\
  f_1(x) &= Mx \int G(p_0) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}. \end{align*}
Here, in the same approach, we shall try to calculate the transversity. Generally, transversity may be related to the auxiliary polarized process described by the interference of vector and scalar currents \([17], [18]\), so that the respective quark tensor carries only one Lorentz index. The simplest handbag diagram in Fig. 1b corresponds to the expression

\[
T_\alpha = \varepsilon_{\alpha\beta\lambda\sigma} q^\lambda \eta^\beta \omega^\sigma,
\]

which will be used instead of the tensor (1). In the next step we integrate this vector equally as the tensor in (2). Here we assume for the time being, that due to rotational symmetry in the proton rest frame, the transversity distribution is generated by the same function \(H\) as that in the case of the longitudinal one:

\[
T_\alpha = \varepsilon_{\alpha\beta\lambda\sigma} q^\lambda \frac{1}{2pq} \int H \left(\frac{pP}{M}\right) p^\beta w^\sigma \delta \left(\frac{pq}{Fq} - x\right) \frac{d^3p}{p_0}.
\]

Obviously only the terms \(S^\sigma, P^\sigma\) from the vector (4) contribute here, i.e.,

\[
T_\alpha = \varepsilon_{\alpha\beta\lambda\sigma} q^\lambda \frac{1}{2pq} \int H \left(\frac{pP}{M}\right) \left(S^\sigma - \frac{pS}{pP + mM} P^\sigma\right) p^\beta \delta \left(\frac{pq}{Fq} - x\right) \frac{d^3p}{p_0}.
\]

Now we take the proton rest frame and assume

\[
S = (0, 0, 1, 0),
\]

\[
q = (v, |q|, 0, 0),
\]

\[
P = (M, 0, 0, 0),
\]

then one can check that only \(T_3 \neq 0\),

\[
T_3 = \frac{1}{2Mv} \int H(p_0) \left(\frac{p_0 - p_1}{M} - |q| \frac{p^2}{p_0 + m}\right) \delta \left(\frac{p_0 - p_1}{M} - \frac{|q|}{M} - x\right) \frac{d^3p}{p_0}.
\]

Further, if we again assume approximation (6), then

\[
\frac{|q|}{\nu} = \sqrt{1 + 4M^2x^2/Q^2} \to 1
\]

and

\[
T_3 = \frac{1}{2} \int H(p_0) \left(\frac{p_0 - p_1}{M} - \frac{p^2}{p_0 + m}\right) \delta \left(\frac{p_0 - p_1}{M} - x\right) \frac{d^3p}{p_0}
\]

\[
= \frac{1}{2} \int H(p_0) \left(x - \frac{p^2/2}{M(p_0 + m)}\right) \delta \left(\frac{p_0 - p_1}{M} - x\right) \frac{d^3p}{p_0}
\]

\[
= \frac{1}{2} \cos \eta \int G(p_0) \left(x - \frac{p^2/2}{M(p_0 + m)}\right) \delta \left(\frac{p_0 + p_1}{M} - x\right) \frac{d^3p}{p_0}.
\]

So now we shall try to identify the transversity with the dimensionless function

\[
\delta q(x) = \cos \eta \int G(p_0) \left(Mx - \frac{p^2/2}{p_0 + m}\right) \delta \left(\frac{p_0 + p_1}{M} - x\right) \frac{d^3p}{p_0}.
\]

If we use the alternative notation

\[
q(x) = f_1(x), \quad \Delta q(x) = 2g_1(x), \quad \Delta q_T(x) = 2g_T(x),
\]

then combination of the relations (14) and (16) assuming \(m \to 0\) gives

\[
\delta q(x) = \Delta q(x) + \Delta q_T(x).
\]

Note that for the first moments, assuming the validity of the Burkhardt-Cottingham sum rule, this implies

\[
\int_0^1 \delta q(x) dx = 2 \int_0^1 \Delta q(x) dx.
\]
Further, using the Wanzura-Wilczek [19] relation, which was proved for our \( g_1, g_2 \) in [15],

\[
g_1(x) + g_2(x) = \int_x^1 \frac{g_1(y)}{y} \, dy, \tag{31}
\]

the rule (29) can be represented as

\[
\delta q(x) = \Delta q(x) + \int_x^1 \frac{\Delta q(y)}{y} \, dy.
\tag{32}
\]

Moreover, in the same paper we suggested the relations between the spin functions and valence quark distributions:

\[
g_1^q(x) = \frac{\cos \eta_q}{2} \left[ q_V(x) - 2x^2 \int_x^1 \frac{q_V(y)}{y^3} \, dy \right], \quad g_2^q(x) = \frac{\cos \eta_q}{2} \left[ -q_V(x) + 3x^2 \int_x^1 \frac{q_V(y)}{y^3} \, dy \right]; \quad q = u, d, \tag{33}
\]

which imply

\[
\delta q(x) = \cos \eta_q \left( q_V(x) - x^2 \int_x^1 \frac{q_V(y)}{y^3} \, dy \right). \tag{34}
\]

So now one can calculate \( \delta q \) either using experimental input on \( \Delta q \) or from some fit on the valence distribution \( q_V \). In Fig. 2 we show transversities calculated with the use of formulae (32) and (34). The spin functions \( \Delta u \) and \( \Delta d \) are here extracted from the parameterization of the world data on the proton spin function \( g_1 \) [4] assuming the \( SU(6) \) approach,

\[
\Delta u(x) : \Delta d(x) = \frac{4}{3} : (-\frac{1}{3}); \quad g_1(x) = \frac{1}{2} \left( \frac{4}{3} \Delta u(x) + \frac{1}{3} \Delta d(x) \right). \tag{35}
\]

For the valence functions \( xu_V(x) \) and \( xd_V(x) \) we use the parametrization obtained by the standard global analysis in [20]. All the parameterizations are taken for \( Q^2 = 4 \text{GeV}^2/c^2 \). The reason why in this figure the (dashed) curve based on the experimental input on \( g_1 \) is above the (solid) curve calculated from fitted \( q_V \), can be the same, as it was discussed in [15] directly for the \( g_1 \).

Now we check if the obtained transversities satisfy the Soffer inequality [22]:

\[
|\delta q(x)| \leq \frac{1}{2} (q(x) + \Delta q(x)) = q^+(x). \tag{36}
\]

After inserting \( \delta q, f_1, g_1 \) from the relations (27),(17),(14) we get

\[
\left| \cos \eta_q \int G(p_0) \left( Mx - \frac{p_T^2/2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0} \right| \leq \frac{1}{2} \int \left( MxG(p_0) + \cos \eta_q MxG(p_0) \right) \cos \eta_q \frac{p_T^2}{p_0 + m} \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \tag{37}
\]

and after rearranging the r.h.s. one obtains

\[
\left| \cos \eta_q \int G(p_0) \left( Mx - \frac{p_T^2/2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0} \right| \leq \frac{1}{2} Mx (1 - \cos \eta_q) \int G(p_0) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0} + \cos \eta_q \int G(p_0) \left( Mx - \frac{p_T^2/2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \tag{38}
\]

which means that

\[
|\delta q(x)| - \delta q(x) \leq Mx \sin^2 \left( \eta_q/2 \right) \int G(p_0) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \tag{39}
\]

which is apparently correct for \( \delta q(x) \geq 0 \). Further, since
definite relation to the functions \( G \) constructed in Eqs. (14)−(17), where the combinations of amplitudes shall modify the transversity definition as follows. The structure functions are proportional (see e.g. [17]) to the polarization takes place for \( \cos \eta \rightarrow -1 \) and \( \sin^2 (\eta/2) = 2/3 \). Then the combination of Eq. (27) and relation (39) gives

\[
\int G(p_0) \left( \frac{Mx - p_T^2/2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0} \geq 0, \tag{40}
\]

which means that the transversity sign is controlled only by the sign of \( \cos \eta \), which is determined by the sign of \( G_+(p_0) - G_-(p_0) \). So in our \( SU(6) \) approach the inequality (39) is safely satisfied for \( u \)−quarks. Now let us consider negative \( \delta q \), for \( d \)−quarks in the \( SU(6) \) approach, when \( \cos \eta = -1/3 \) and \( \sin^2 (\eta/2) = 2/3 \). Then the combination of Eq. (27) and relation (39) gives

\[
\int G(p_0) \left( \frac{Mx - p_T^2/2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0} \leq 0, \tag{41}
\]

which contradicts the inequality (40), so in this limit also the transversity (27) contradicts the Soffer inequality. Why?

The reason is in the assumption that transversity is generated by the same function \( H = G \cos \eta \) as the spin functions \( g_1 \) and \( g_2 \). The resulting transversity contradicts the Soffer inequality in the case of large negative quark polarization. Indeed, inequality (36) means that \( |\delta q(x)| \) cannot exceed \( q^+(x) \). At the same time, large negative polarization takes place for \( \cos \eta \rightarrow -1 \); then \( q^+(x) \) becomes small, while \( \delta q(x) \) is large (and negative). Below we shall modify the transversity definition as follows. The structure functions are proportional (see e.g. [17]) to the combinations of amplitudes:

\[
f_1 \propto \sum_X \left( a_{++}^*(X)a_{++}(X) + a_{+-}^*(X)a_{+-}(X) \right) \tag{42}
\]

\[
g_1 \propto \sum_X \left( a_{++}^*(X)a_{++}(X) - a_{+-}^*(X)a_{+-}(X) \right) \tag{43}
\]

\[
\delta q \propto \sum_X \left( a_{++}^*(X)a_{--}(X) + a_{--}^*(X)a_{++}(X) \right). \tag{44}
\]

Now, in our approach we identify

\[
\sum_X a_{++}^*(X)a_{++}(X) = G_+(p_0), \quad \sum_X a_{--}^*(X)a_{--}(X) = G_-(p_0), \quad \sum_X a_{+-}^*(X)a_{+-}(X) = G_{\mp}(p_0), \tag{45}
\]

where \( G_+ \pm G_- \) are the distributions in relations (13), from which the structure functions \( g_1, g_2, g_T \) and \( f_1 \) are constructed in Eqs. (14)−(17). But what about the remaining interference function

\[
G_T(p_0) \equiv \sum_X \left( a_{++}^*(X)a_{--}(X) + a_{--}^*(X)a_{++}(X) \right), \tag{46}
\]

which we are going to insert into Eq. (27) instead of \( G_+ - G_- = G \cos \eta \)? The \( G_T \) is a new function, which has no definite relation to the functions \( G_{\mp} \). However, as a consequence of

\[
\int G(p_0) \left( \frac{Mx - p_T^2/2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0} \leq 0, \tag{41}
\]
\[ \sum_X |a_{++}(X) \pm a_{--}(X)|^2 \geq 0 \]  

one gets

\[ |G_T(p_0)| \leq G_+(p_0) = G(p_0) \cos^2 (\eta/2). \]  

So in the first step we check the Soffer inequality for both corresponding extremes \( \pm \delta q_{\text{max}}(x) \):

\[ \delta q_{\text{max}}(x) = \cos^2 (\eta_q/2) \int G(p_0) \left( M x - \frac{p_T^2/2}{p_0 + m} \right) \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}. \]  

After inserting into the Soffer inequality\(^1\) one gets for both extremes

\[ 0 \leq \sin^2 (\eta_q/2) \int G(p_0) \frac{p_T^2/2}{p_0 + m} \delta \left( \frac{p_0 + p_1}{M} - x \right) \frac{d^3p}{p_0}, \]  

so the inequality is satisfied for any transversity \( \delta q(x) \) in the band \( \pm \delta q_{\text{max}}(x) \) given by Eq. (49) with any \( \eta_q \). In fact, two inequalities are now satisfied:

\[ |\delta q(x)| \leq \delta q_{\text{max}}(x) \leq \frac{1}{2} (q(x) + \Delta q(x)) \]  

and in this way we have shown that taking into account the interference nature of the transversity at the stage of parton-hadron transition is quite substantial for general compliance with the Soffer inequality.

Obviously, nothing can be said about saturation of the inequality

\[ |\delta q(x)| \leq \delta q_{\text{max}}(x) \]  

within this simple approach. Concerning the sign of the transversity \( \delta q \), let us note, that now there is no simple correspondence with the sign of \( \Delta q \).

Since the relations (27) and (49) differ only in the \( \eta \)--dependent factor ahead of the integral, the relations (29),(32) and (34) imply for the second approach

\[ \delta q_{\text{max}}(x) = \kappa \cdot (\Delta q(x) + \Delta q_T(x)) = \kappa \cdot \left( \Delta q(x) + \int_x^1 \frac{\Delta q(y)}{y} dy \right); \quad \kappa = \frac{\cos^2 (\eta_q/2)}{\cos \eta_q}, \]  

\[ \delta q_{\text{max}}(x) = \cos^2 (\eta_q/2) \left( q_v(x) - x^2 \int_x^1 \frac{q_v(y)}{y^3} dy \right). \]  

This approach for the transversity is compared with the previous one in Fig. 3, again assuming \( SU(6) \) approximation for contributions from \( u \) and \( d \) valence quarks. However, one should point out that curves corresponding to the second approach represent only upper limits \( \delta q_{\text{max}} \) for transversities, in the sense of the relation (51). The left part of the figure shows results for \( d \)-quarks. The relations (53) - dashed and (54) - solid curves are compared with those in Fig. 2, calculated from Eqs. (32),(34). It follows, that curves of the second approach are enhanced by the factor \( \cos^2 (\eta_u/2) / \cos \eta_u = 5/4 \) with respect to the first one. The right part of the figure demonstrates similar curves for \( u \)-quarks, but since \( \cos^2 (\eta_d/2) = - \cos \eta_d = 1/3 \), both the corresponding pairs of curves are equal up to sign. So only the second pair is displayed. The dotted curves in both parts represent transversities calculated in Ref. [21] in the LO and evolved from the initial scale \( 0.6 GeV^2/c^2 \) to \( 4 GeV^2/c^2 \). Obviously, our results are well compatible with them.

Further, let us analyze the relation

\[ \delta q_{\text{max}}(x) \leq \frac{1}{2} (q(x) + \Delta q(x)) = q^+(x). \]  

\(^1\)One can start from relation (38), where on the l.h.s. \( \cos \eta_q \) is substituted by \( \cos^2 (\eta_q/2) \).
Obviously, its saturation is equivalent to the equality in relation (50), which takes place either for \( \eta = 0 \) (pure state of the quark with polarization +) or for static quarks \( (p_T^2 = 0) \). On the other hand, in the limit \( m \to 0 \) and with the use of Eq. (33) one can write

\[
q^+(x) = \frac{1}{2} \left[ q_V(x) + \cos \eta_q \left( q_V(x) - 2x^2 \int_x^1 \frac{q_V(y)}{y^3} dy \right) \right] = \cos^2 \left( \frac{\eta_q}{2} \right) \cdot q_V(x) - x^2 \cos \eta_q \int_x^1 \frac{q_V(y)}{y^3} dy. \quad (56)
\]

In this way, both sides of relation (55) are displayed in Fig. 4. It is seen that particularly the transversity of d-quarks in this approach is constrained considerably more than only by the Soffer bound \( q^+ \). The relationship between both bounds depends on \( \cos \eta_q \). One can check that

\[
\frac{\int_0^1 \delta \delta_{\text{max}}(x) dx}{\int_0^1 q^+(x) dx} = \frac{2 + 2 \cos \eta_q}{3 + \cos \eta_q}, \quad (57)
\]

which in the \( SU(6) \) approach gives fractions 10/11 and 1/2 for \( u \) and \( d \) quarks.

**III. SUMMARY**

The covariant QPM, which takes into account intrinsic quark motion, was generalized to involve the transversity distribution. Two ways of generalization were considered:

1) The interference effects were assumed on a quark level only (interference of vector and scalar currents produce the quark trace with one Lorentz index) and the generic quark polarized distribution \( H = G_+ - G_- \) was assumed the same as that for the structure functions \( q_1 \) and \( q_2 \). We derived the relation between the transversity \( \delta q \) and the usual polarized distribution \( \Delta q \), which implies that the resulting transversity is roughly twice as large as the usual distribution function. We discussed the compatibility of the obtained transversity with the Soffer inequality and we have found that the inequality is violated in the case of large negative quark polarization.

2) In the second approach we accounted also for interference effects at the quark-hadron transition stage, which is formally represented by the interference sum (46). In this approach, assuming the validity of Soffer inequality at the stage of parton-hadron transition (48), we obtained a new bound on the transversity, rather than the transversity in itself. This bound is more strict, than the Soffer one. Roughly speaking, the new bound is more restrictive for quarks with a greater proportion of intrinsic motion and/or smaller (or negative) portion in the resulting polarization.

For both approaches, with the use of the obtained sum rules, we have done the numeric calculations, in which either experimentally measured spin function \( g_1(x) \) or the valence quark distributions \( q_V(x) \) were used as an input. To conclude, in general our calculations suggest that the quark intrinsic motion plays an important role for the spin functions - including the transversity.

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P. Schweitzer, private communication.
FIG. 1. Diagram related to deep-inelastic scattering (a) and the transversity (b), see text.

FIG. 2. Transversities of the $u$ and $d$ valence quarks calculated from the valence distributions (solid lines) and extracted from the experimental data on proton spin function $g_1$ (dashed lines) – the first approach, see text.

FIG. 3. Transversities of the $u$ valence quarks (left) calculated from the valence distributions (solid lines) and extracted from the experimental data on the proton spin function $g_1$ (dashed lines). Lower curves correspond to the first approach from Fig. 2, upper curves represent the second approach given by $\delta q_{\text{max}}$ calculated from Eqs. (53), (54). The corresponding transversities $\delta q_{\text{max}}$ of the second approach for the $d$ valence quarks (right) coincide, up to sign, with the first approach from Fig. 2. The dotted lines represent the calculation [21], with opposite sign for $d$ quarks.
FIG. 4. Bounds on the transversities of the $u$ and $d$ valence quarks: $\delta q_{\text{max}}$ (solid lines) and $q^+$ (dashed lines).