Stabilization of Nanometre-Size Particle Beams in the Final Focus System of the Compact Linear Collider (CLIC)

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Abstract

The Compact Linear Collider (CLIC) study at the European Organization for Nuclear Research (CERN) is developing the design of a 3 TeV $e^+e^-$ linear collider. The discovery reach of this machine depends on obtaining a luminosity of $10^{35}$ cm$^{-2}$s$^{-1}$, which will be done by colliding beams with transverse spot sizes in the nanometre range ($\approx 60 \times 0.7$ nm$^2$). Tolerances on fast mechanical stability of the focusing quadrupoles reach the 0.2 nm level. The serious concern of magnet stabilization for future linear colliders has been addressed by building a CERN test stand on magnet stability, bringing together state-of-the-art stabilization technology, latest equipment for vibration measurements and realistic magnet prototypes. For the first time an accelerator magnet was successfully stabilized to the sub-nanometre level, reducing its vibrations level by one order of magnitude with respect to the supporting ground. The best measurements indicate transverse RMS vibration amplitudes (above 4 Hz) of $(0.79\pm0.08)$ nm horizontally and $(0.43\pm0.04)$ nm vertically, maintained to a maximum of less than $(1.47\pm0.15)$ nm and $(1.00\pm0.10)$ nm, respectively, over a period of several days. Detailed simulations of time-dependent luminosity, which use a model for magnet displacements based on measured vibration spectra, show that approximately 70% of the CLIC goal luminosity can be achieved with the demonstrated performance in the CERN test stand. This indicates the basic feasibility of colliding nanometre-size beams in CLIC.

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Abstract

The Compact Linear Collider (CLIC) study at the European Organization for Nuclear Research (CERN) is developing the design of a 3 TeV $e^+e^-$ linear collider. The discovery reach of this machine depends on obtaining a luminosity of $10^{35}$ cm$^{-2}$s$^{-1}$, which will be done by colliding beams with transverse spot sizes in the nanometre range ($\approx 60 \times 0.7$ nm$^2$). Transporting the CLIC beams over 15 km, focusing them to nanometre spot sizes and colliding the two separate beams head-on imposes extraordinary requirements on the stability of the magnetic guiding and focusing fields. Tolerances on fast mechanical stability reach the 0.2 nm level. These tolerances have been considered as one of the major obstacles for building high energy $e^+e^-$ linear colliders.

This serious concern has been addressed by building a CERN test stand on magnet stability, bringing together state-of-the-art stabilization technology, latest equipment for vibration measurements and realistic magnet prototypes. For the first time an accelerator magnet was successfully stabilized to the sub-nanometre level, reducing its vibrations level by one order of magnitude with respect to the supporting ground. Vibration spectra were recorded over four orders of magnitude. The best measurements indicate transverse RMS vibration amplitudes (above 4 Hz) of $(0.79\pm0.08)$ nm horizontally and $(0.43\pm0.04)$ nm vertically, maintained to a maximum of less than $(1.47\pm0.15)$ nm and $(1.00\pm0.10)$ nm, respectively, over a period of several days. The demanding tolerances on mechanical vibrations of accelerator magnets are basically achieved. Detailed simulations have been used to predict the achievable luminosity of the collider based on a realistic stability model. This set-up includes a 3D beam transport, a full dynamical 2D model of transverse magnet vibrations (based on experimental data), the measured vibration damping with stabilization technology, a beam-based position feedback and a quantum-electromagnetic model of the beam-beam interaction. The numerical simulation predicts that approximately 70% of the CLIC goal luminosity can be achieved with the demonstrated performance in the CERN test stand.

This indicates the basic feasibility of the nanometre-size colliding beams foreseen in the CLIC concept. Further improvements in technology and more advanced fast beam-based feedbacks will allow demonstrating the full design luminosity in the future.
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1 Introduction

The discovery reach of a future collider for particle physics depends on its energy and luminosity. Particle physics requirements beyond the Standard Model are demanding and impose many challenging research topics in the technology and design of future accelerators. It has been widely accepted by the international accelerator community (ECFA, ICFA, ACFA, HEPAP) that the next collider beyond the Large Hadron Collider (LHC) should be a high energy $e^+e^-$ linear collider. Among the various studies, the Compact Linear Collider (CLIC) is the most ambitious with a collision energy of 3 TeV and a target luminosity of $10^{35}\text{cm}^{-2}\text{s}^{-1}$, which is approximately 1000 times larger than the luminosity achieved by the Large Electron-Positron Collider (LEP). CLIC is a collaborative study centred at the European Organization for Particle Physics (CERN) with the aim to propose a possible future facility for basic research at the frontier of particle physics. The ambitious goals of the CLIC scheme can only be achieved with significant advances in accelerator technology. Various ongoing studies include the generation of high beam power, the high accelerating gradient in 30 GHz RF structures, the generation of low emittance beams and the stable collision of nanometre-size beams at the interaction point. The feasibility of the CLIC proposal depends on the successful completion of these studies.

The stable collision of nanometre-size beams has attracted particular interest over the last years. All linear colliders studies foresee vertical beam sizes at the interaction point from 0.7 nm (CLIC) to 5 nm, in order to achieve the required luminosity with an acceptable electrical power consumption. It must be noted that the beam size for CLIC is about the size of a water molecule. As the two independent particle beams must be transported over about 15 km and then steered into collision in a single point with sub-nanometre precision, serious feasibility concerns arise. In order to achieve stable collisions of the two independent beams, it must be shown in particular that the magnetic elements of the focusing and beam transport systems have mechanical stability in the 0.1 nm to 1 nm range, if vibration frequencies above about 1 Hz are considered.

Several experimental studies have been carried out in various accelerator laboratories and universities, in order to develop conceptional and technical solutions for this stability problem. A study at DESY in Germany demonstrated a quadrupole stability of about 10 nm in 1996. Studies in the US at the Stanford Linear Accelerator Center (SLAC) are ongoing with a present stability performance of approximately 10 nm. These studies are accompanied by an experimental program in Canada at the University of British Columbia (UBC). At CERN a “CLIC stability study” was started in 2001 with a test stand dedicated to magnet stability.

The work described in this PhD thesis was performed in the framework of this CLIC stability study. The experimental goals were to set up and equip a CERN test stand 1) with latest vibration measurement equipment, 2) prototype quadrupole magnets and 3) state-of-the-art stabilization devices. Similar equipment has been used in other scientific fields and in industry. For example, in transmission electron microscopy, a low energy electron beam (“nanobeam”) has been sent with sub-nanometre stability onto a target. The aim of this work is to show that the same technology could be used to demonstrate the feasibility of colliding “nanobeams” at high energy in CLIC. The experimental results obtained from this work can then be used to assess whether the proposed CLIC parameters are indeed realistic or not.

This report summarizes the work and results from 2001 to 2003. After a short introduction of the CLIC concept, the basic limitations to achieving high luminosity and possible restorative measures for enhanced luminosity performance in linear colliders are reviewed. After introducing the experimental set-up and techniques for magnet stabilization, the stability of prototype quadrupoles obtained under various conditions is presented. Finally, based on the experimental data, the achievable luminosity in CLIC is predicted and the feasibility of colliding CLIC “nanobeams” is assessed.
2 The CLIC design and its challenges

Linear colliders are one of the most promising options for near-future studies in high energy physics. After a general presentation of the main components of a linear collider, the design of the Compact Linear Collider (CLIC) is introduced. The beam delivery system is the section on which the interest of this thesis is focused and is presented in more detail.

2.1 Need for a linear collider

The Standard Model describes satisfactorily the experimental observation of all known particles. Nevertheless, it is generally expected that a more fundamental physics must lie beyond the Standard Model. Answers to the open questions on the origin of mass and whether super-symmetry exists can probably be found by looking at the production of new particles in the few TeV ($10^{15}$ electron-volt) energy range [1, 2]. Particle accelerators [3] are the only devices which offer physicists the opportunity of studying new particle production at the required energy with a high event rate. Collisions of accelerated particles produce new particles of masses up to the centre-of-mass collision energy, according to the known energy to mass relation $E = mc^2$. Over the last decades, the advance in accelerator technology has allowed achieving larger and larger collision energies [4].

The first important results towards a better understanding of the physics beyond the Standard Model will come within a few years from the Large Hadron Collider (LHC), presently under construction at CERN [5, 6], which will collide protons at a 14 TeV centre-of-mass energy. Lepton/anti-lepton colliders are then required to provide a complementary picture of the physics outcome of the LHC experiments. They provide a cleaner experimental environment and hence more precise measurements because they collide point-like elementary particles. They also offer the possibility of producing collisions between polarized beams and, in addition, $e^+\gamma$, $\gamma\gamma$ and $e^+e^-$ collisions can be obtained.

Circular $e^+e^-$ storage rings are limited in energy by the synchrotron radiation emission in bending sections. The emitted energy increases as the fourth power of the beam energy ($E_b$) as $E^4_b/\rho$ ($\rho$ is the bending radius). This limits the achievable beam energies to approximately 100 GeV, unless very large bending radii are used. For instance, in the CERN Large Electron Positron collider (LEP) [7] a few percent of the beam energy ($\approx 100$ GeV) was lost per turn due to synchrotron radiation.

On the other hand, if particles are accelerated along a linear path without bending, the radiation losses become important only for energy variations of the order of $mc^2$ over a distance $e^2/mc^2$ [8]. For electrons or positrons this corresponds to field gradient of $2 \times 10^{14}$ MV/m, orders of magnitude beyond accelerator gradients achievable to date. Therefore, radiation losses are not an issue for a linear collider. This makes linear colliders interesting for future high energy physics.

There is now a general consensus among the particle physics community that the next large-energy particle accelerator beyond the LHC should be an $e^+e^-$ linear collider [9]. Three design concepts are presently under investigation, with centre-of-mass energies between 1 TeV and 5 TeV [10, 11]:

1. The Compact Linear Collider (CLIC) is a CERN study for the post-LHC era, aiming at colliding positron and electron beams at a 3 TeV to 5 TeV centre-of-mass collision energy [12]. This study proposes a novel two-beam acceleration scheme that can produce large acceleration gradients (150 MV/m) with high frequency (30 GHz).

2. The Next Linear Collider (NLC) / Japan Linear Collider (JLC) study, carried out as a collaboration between SLAC and KEK, proposes a standard klystron-based beam acceleration up to

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This is true only as far as the particle acceleration is concerned. Synchrotron radiation effects have instead a relevant impact on the luminosity performance of linear colliders. See next Chapter.
collision centre-of-mass energy of 1.3 TeV [13].

(3) TESLA is a DESY study for a superconducting linear accelerator to provide $e^+e^-$ collisions up to 0.8 TeV centre-of-mass energy collisions [14].

Each design presents its peculiar technologies, features and challenges. It is worth mentioning that the sole linear collider built so far, is the Stanford Linear Collider (SLC) [15]. It was operating for more than 10 years until 1998 and delivered $e^+e^-$ beams at approximately 45.6 GeV with a maximum luminosity of $3 \times 10^{30}$ cm$^{-2}$s$^{-1}$.

After a general introduction of the main components of a linear collider, the attention is focused on specific features and challenges of CLIC.

2.2 The scheme of a linear collider

A linear collider consists of two linear accelerators that produce particle beam trains with the required energy and emittance. Once accelerated to the desired energy, the beams are steered into collision in the so-called interaction point (IP), where the particle detector is placed. Figure 1 shows the main components of a linear collider. The different designs presently under study foresee two opposing machines pointing at each other, with or without a crossing angle $^2$.

First come the particle injectors that provide electron and positron beams. Electrons are generated with a radio-frequency (RF) gun. Positrons are extracted from an electromagnetic shower by electrons impinging on a high-Z material target. The injector complex also includes pre-linacs that accelerate the beams up to the energy required for the next section, i.e. the damping rings. The large emittance of

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$^2$It is perhaps interesting to note that the Stanford Linear Collider (SLC), the sole linear collider built to date, did not actually consist of two opposing machines. Electron and positron beams were accelerated with a smart scheme in the same linac, sent in two separate arcs and then steered into collision. This scheme was possible for the SLC due to its small beam energy of approximately 30 GeV but is not feasible for beam energies of the order of hundreds of GeV.
the beam as they come out of the injectors must be reduced by several orders of magnitude to achieve a good luminosity performance. This is done by sending the beams into storage rings. The emittance reduction is obtained via the radiation damping of particles bent by dipole fields. The design of the damping ring lattice is one of the main challenges for future linear colliders because the aimed beam emittances are well below the values achieved to date and they must be produced within a few tens of milliseconds. Note that the damping rings also bunch the beam to the required RF frequency.

After nominal transverse emittances have been achieved, the longitudinal bunch length must also be reduced in order to limit the dilution of the vertical emittance in the linac due to transverse wakefields. This is provided by the bunch compressors. An energy spread correlated with longitudinal particle position is induced along the bunch. Dipole magnets are then used to compress longitudinally the bunch, making use of dispersion-induced path length changes to push the particles closer to the bunch centre. Length compression by a factor of 100 might be needed, which can possibly require a compression in more than one step (this is the case, for instance, for CLIC).

Downstream of the bunch compressors, the heart of the machines comes: the linear accelerator (linac) section which provides the beam energy. This is the longest section of the machine. Structures of several kilometres are required to accelerate the beams up to the required energy. Unlike for circular accelerators, where the beams pass many times in the accelerating cavities, in a linear collider the full beam energy must be provided in a single pass. There are three different technologies presently under study to provide the required beam acceleration: (1) conventional, normal conducting RF cavities, proposed for the NLC/JLC design; (2) superconducting cavities, adopted by TESLA; (3) new two-beam acceleration scheme proposed for CLIC (see next section).

Once the beam reaches its nominal energy, it has to be focused to the small spot sizes required for obtaining the target luminosity. This is done in the so-called beam delivery system (BDS). This is the last section of the collider and has also the task of collimating the beam halos for cleaner collisions. The beam delivery system is described in more detail in Section 2.5. The problems related to the realization of the beam delivery systems are common to all the projects of future linear colliders. All designs aim at beam spot sizes at the interaction point in the nanometre range (from the 0.7 nm of CLIC to the 5 nm of TESLA) and hence similar challenges are imposed in all cases.

2.3 Definition of coordinate system

Before discussing specific features of the CLIC design, the coordinate system used throughout this thesis to describe the particle motion is introduced. A 3-dimensional coordinate system \( (s, x, y) \) is employed to describe the particle motion in the linear collider, as shown in Fig. 2. The longitudinal

![Figure 2: Schematic of the coordinate system used to measure the particle positions. The grey ellipse denotes a particle bunch.](image-url)
coordinate \( s \) gives the particle location along the lattice of the linear accelerator and the local tangent to \( s \) points in the direction of the beam line. The coordinates \( x \) and \( y \) measure the transverse horizontal and vertical particle deviation from the ideal particle trajectory, passing through the centre of perfectly aligned quadrupole magnets. Another longitudinal coordinate \( z = s - v_0 t \), where \( v_0 \) is the velocity of an ideal particle of nominal energy and \( t \) is the time, is conventionally introduced to measure deviations of the longitudinal particle position with respect to the ideal reference. The longitudinal reference is defined as the position of an ideal particle with nominal bunch energy, \( E_b \), located at the bunch centre. Deviations of particle energy \((E)\) with respect to \( E_b \) are measured with the coordinate \( \delta \equiv (E - E_b)/E_b \). Variations in slope of particle trajectories are denoted as \( x' \equiv dx/ds \) and \( y' \equiv dy/ds \). According to the majority of textbooks on accelerator physics theory, the metric MKS unit system is used to measure the particle coordinates.

The beam transport between two lattice locations is expressed in matrix notation as:

\[
\begin{pmatrix}
  x' \\
  x \\
  y' \\
  y \\
  z \\
  \delta
\end{pmatrix}_{\text{out}} =
\begin{pmatrix}
  R_{1,1} & R_{1,2} & R_{1,3} & R_{1,4} & R_{1,5} & R_{1,6} \\
  R_{2,1} & R_{2,2} & R_{2,3} & R_{2,4} & R_{2,5} & R_{2,6} \\
  R_{3,1} & R_{3,2} & R_{3,3} & R_{3,4} & R_{3,5} & R_{3,6} \\
  R_{4,1} & R_{4,2} & R_{4,3} & R_{4,4} & R_{4,5} & R_{4,6} \\
  R_{5,1} & R_{5,2} & R_{5,3} & R_{5,4} & R_{5,5} & R_{5,6} \\
  R_{6,1} & R_{6,2} & R_{6,3} & R_{6,4} & R_{6,5} & R_{6,6}
\end{pmatrix}
\begin{pmatrix}
  x \\
  x' \\
  y \\
  y' \\
  z \\
  \delta
\end{pmatrix}_{\text{in}},
\]

where the suffixes “in” and “out” refer to the incoming and outgoing beams for the lattice section of interest. The values of the \( R_{i,j} \) matrix elements can be expressed in terms of the lattice Twiss parameters, as in [16].

### 2.4 The design of the Compact Linear Collider

An overall view of the CLIC complex is shown in Fig. 3. The main parameters of CLIC are listed in Table 1. CLIC will accelerate trains of 154 bunches at a 100 Hz repetition frequency. The centre-of-mass collision energy considered for the CLIC baseline is 3 TeV and the goal luminosity \(10^{35} \text{ cm}^{-2} \text{s}^{-1}\). Energies up to 5 TeV can be achieved using the same RF technology, with longer acceleration sections. These larger beam energies are made possible by the novel two-beam acceleration scheme (see below). Increasing the collision energy demands higher luminosities in order to maintain a good event rate. Indeed the cross-section for interesting new physics processes decreases like the squared centre-of-mass collision energy \((E_{CM})^3\):

\[
\sigma_{\text{interesting}} \sim \frac{1}{E_{CM}^2}
\]

(2)

The CERN LEP collider, for instance, operated at a maximum luminosity of \(10^{32} \text{ cm}^{-2} \text{s}^{-1}\) at \( E_{CM} \approx 200 \text{ GeV} \) [7]. In order to deliver a comparable event rate, a luminosity of the order of \(10^{34}-10^{35} \text{ cm}^{-2} \text{s}^{-1}\) should be envisaged for an e\(^+\)e\(^-\) collider in the 3 TeV to 5 TeV centre-of-mass energy range.

A simplified estimate of luminosity in linear colliders, \( \mathcal{L} \), is given by

\[
\mathcal{L} = \frac{A}{\sigma_x^+\sigma_y^-},
\]

(3)

This cross-section reduction with increasing energy is induced by the fact that for point-like particles like leptons the Compton wavelength goes like \( \sim 1/E_{CM} \).
where $\sigma_x^*$ and $\sigma_y^*$ are the horizontal and vertical spot sizes at the interaction point. The constant $A$ depends on fixed machine parameters such as train repetition frequency, charge per bunch and number of bunches per train (see Section 3.1). For fixed beam power, to achieve high luminosity the spot sizes must be kept as small as possible [17]. For instance, the $3\,\text{TeV}$ CLIC base line aims at colliding beams with transverse spot sizes of about $0.7\,\text{nm}$ (vertical) times $60\,\text{nm}$ (horizontal). Corresponding to these small spot sizes, tight tolerances are imposed on the stability of colliding beam positions at the interaction point. In order to obtain a high luminosity, relative beam-beam offsets have to be kept within a fraction of the beam size in both directions. The stabilization of colliding beams is one of the major challenges for future linear colliders and in particular for CLIC, which aims at the smallest colliding beams. Stability issues for the final focus of CLIC are the main topic of this Thesis.

Among the challenges of the CLIC design, the proposed two-beam acceleration scheme should also be mentioned. In order to obtain multi-TeV energy collisions with a reasonable machine length, the CLIC design aims at a high-gradient ($150\,\text{MV/m}$), high-frequency ($30\,\text{GHz}$), high efficiency acceleration, which is not achievable with the klystron-based technology. The CLIC study proposes to extract the required radio-frequency power from a low-energy, high-intensity beam - the drive beam - which runs parallel to the main beam and is bunched to the desired $30\,\text{GHz}$ frequency (see Fig. 3 for some parameters of the drive beam). The main advantage of this acceleration scheme is that it allows achieving high RF frequencies, depending on the drive beam repetition frequency. It has been experimentally proven that the required repetition frequency can be achieved with an intra-bunching scheme, first proposed in [18] and then tested in the CLIC Test Facility 3 (CTF3).

The CLIC design depends on the feasibility of the two-beam acceleration scheme. The generation of the required drive beam, with the nominal beam current and bunch spacing, the generation of the $150\,\text{MV/m}$ accelerating gradient, are all topics of crucial importance for the CLIC design and still call for a satisfactory feasibility demonstration. This is not the topic of this Thesis. It is mentioned...
Table 1: The main parameters of CLIC.

<table>
<thead>
<tr>
<th>Main CLIC parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Centre of mass energy [TeV]</td>
<td>3</td>
</tr>
<tr>
<td>Design luminosity [10^{34} cm^{-2} s^{-1}]</td>
<td>8.0</td>
</tr>
<tr>
<td>RF frequency of main linac [GHz]</td>
<td>30</td>
</tr>
<tr>
<td>Bunch train repetition frequency [Hz]</td>
<td>100</td>
</tr>
<tr>
<td>Particles per bunch [10^{10}]</td>
<td>0.4</td>
</tr>
<tr>
<td>Number of bunches per train</td>
<td>154</td>
</tr>
<tr>
<td>Bunch separation [nsec]</td>
<td>0.67</td>
</tr>
<tr>
<td>Bunch train length [μsec]</td>
<td>0.102</td>
</tr>
<tr>
<td>Beam power per beam [MW]</td>
<td>14.8</td>
</tr>
<tr>
<td>Unloaded/loaded gradient [MV/m]</td>
<td>172/150</td>
</tr>
<tr>
<td>Total two-linac length [km]</td>
<td>28</td>
</tr>
<tr>
<td>Total beam delivery length [km]</td>
<td>5.2</td>
</tr>
</tbody>
</table>

that several important steps have recently been done towards the solution of some of the above issues. Very encouraging results have been obtained in the CLIC Test Facility 3 regarding the intra-bunching scheme for the generation of a drive beam with the correct repetition frequency [19, 20]. It has also been shown that the 150 MV/m accelerating gradient, which was long thought of as a major problem for the CLIC feasibility, can indeed be achieved without considerable damaging of the accelerating structures [21, 22].

2.5 Beam delivery systems of linear colliders

The term “Beam Delivery System” (BDS) refers to the optical transport line from the end of the linear accelerator to the interaction point (IP), which has the following two main functionalities [23, 24, 25]:

- collimation of the post-linac halo to reduce the detector background and to protect the machine;
- reduction of the beam transverse sizes at the interaction point down to the nanometre level.

In addition, the beam delivery system must also provide space for the beam phase-space diagnostic, like emittance measurements, and for the luminosity measurement and tuning, e.g. feedback correction of the beam-beam offsets at the interaction point and high order chromaticity corrections.

The collimation system is mainly designed to remove the halo particles which can increase the detector background due to the production of particles (electromagnetic showers, muons or neutrons) or due to the synchrotron radiation generated in the quadrupole field. Both the collimation of particles at large betatron amplitudes (betatron cleaning) and the collimation of off-energy particles (momentum cleaning) are required to obtain clean collisions. The collimation system should also protect the machine from mis-steered beams. The collimation system is not described in detail here. For a general overview, see for instance [24] and [25].

The optical transport system that provides the final beam focusing is referred to as the Final Focus System (FFS). The optics of this section is aimed at transporting to the interaction point a beam with the required β-functions, while correcting for chromatic aberrations. Vertical β-functions of tens of micrometres are needed to produce the required nanometre spot sizes at the interaction point. The strong focusing in both transverse directions is provided by two strong quadrupole magnets, located
a few meters upstream of the interaction point, which are referred to as Final Doublets (FD). In addition, the final focus system also contains sextupole magnets for the chromaticity correction and dipole magnets which generate dispersive regions (required for chromaticity correction and momentum collimation). Octupolar magnets have been proposed for the particle tail folding [26, 27, 28] as an option to reduce the detector background and hence are also envisaged in the final focus lattice.

In the free space downstream of the final focus quadrupole the vertical $\beta$-function is approximately given by:

$$\beta_y \approx \frac{f_y^2}{\beta_i^*},$$

(4)

where $\beta_y^*$ is the $\beta$-function at the interaction point and $f_y \equiv l^*$ is the quadrupole focal length (equal to the distance $l^*$ between the final doublet and the interaction point). For instance, in order to generate $\beta_y^* \lesssim 100 \mu$m, with $l^*$ typically a few meters long, the vertical $\beta$-function at the final quadrupole location must be of the order of 100 kilometres. Large $\beta$-functions are actually found all through the lattice of the final focus system and this results in large chromatic aberrations, which must be carefully compensated. Particles of different energies are focused differently because the kick from a magnetic field depends on the particle momentum. This is expressed by the so-called particle chromaticity, defined as $^4$

$$\xi \equiv \frac{1}{4\pi} \int \beta(s)K(s)ds,$$

(5)

where $K(s)$ is the normalized focusing strength

$$K = \frac{1}{B\rho} \frac{\partial B(y)}{\partial y},$$

(6)

$B\rho \equiv p/e$ is the particle rigidity ($e = 1.6022 \times 10^{-19} \text{ C}$ is the electron charge). Note that $K(s)$ depends on the particle energy, and this is where the chromatic aberrations come from. The chromaticity induces an increase of the beam size at the interaction point ($\sigma_y^*$) because off-energy particles have a different quadrupole focal point. If $\sigma_{y,0}^* = \sqrt{\beta_y^* \epsilon_y}$ is the theoretical beam size from the optical functions ($\epsilon_y$ is the vertical emittance), the additional increase of vertical spot size due to chromaticity reads

$$\sigma_y^* \approx \sigma_{y,0}^* \sqrt{1 + \xi^2 \delta_{\text{RMS}}^2},$$

(7)

where $\delta_{\text{RMS}}$ is the RMS momentum spread of the beam. A similar expression holds for the horizontal beam size. In thin lens approximation, $\xi_y \approx \beta_y / f_y$. Therefore, the RMS variation of the beam size from chromaticity, $\Delta y_{\text{RMS}}$ reads:

$$\frac{\Delta y_{\text{RMS}}}{\sigma_{y,0}^*} \approx \left( \frac{\beta_y}{f_y} \right) \delta_{\text{RMS}}$$

(8)

For the CLIC typical parameters of $\beta_y \approx 7.2 \times 10^5 \text{ m}$, $l^* = 4.3 \text{ m}$ and $\delta_{\text{RMS}} = 0.28\%$, an increase in the vertical beam size of about 50 times would be expected if the chromatic aberration were not be corrected.

$^4$In the right-hand-side of Eq. (5) only chromatic aberrations from the quadrupoles are taken into account (natural chromaticity).
Two different conceptual designs for the final focus system have been proposed, which differ in the scheme used for the chromatic correction: (1) a non-local correction scheme, experimentally verified for the SLC [29] and for the Final Focus Test Beam (FFTb) [30] at SLAC, and (2) a compact local correction scheme recently proposed [32] for the next generation of linear colliders. The two concepts are qualitatively shown in Fig. 4.

The design (1) (top of Fig. 4) foresees a dedicated chromatic correction section upstream of the final telescope. Sextupoles combined with dipoles (to generate horizontal dispersion) are used to compensate the chromaticity of the lattice. On the other hand, the beam focusing is carried out in a downstream, non-dispersive section. This scheme with separate sections is meant to avoid generating unwanted high order chromatic terms. Pairs of sextupoles separated by minus unit optical transformations \((-I)\) are used to correct the chromaticity without introducing second-order geometric aberrations [24].

This system is conceptually simple and has a solid experimental validation. However, it has the main disadvantage that it is quite long. The chromatic correction section requires large \(\beta\)-functions and dispersion, which moreover imposes tight tolerances on the magnet alignment. In addition, the chromatic kick is not local but it has to be transported to the interaction point through many lattice elements. This induces higher order aberration terms.

In order to counteract the drawbacks of the long system, another design for the final focus system has been developed [32, 33]. This novel scheme, often referred to as compact design because of its much shorter length, is given in Fig. 4 bottom part. It relies on a local correction of the chromaticity, to be applied in the vicinities of the final doublet where a sextupole is to be placed. In order to correct the pure geometrical chromaticity, another sextupole must be placed upstream, at a \(-I\) optical transformation. The optics must be adjusted so as to have a zero dispersion at the interaction point. This type of local correction reduces dramatically the total length of the system. In the case of CLIC, the compact scheme reduces the length of the final focus section by a factor 6 [34]. More details on the CLIC design are given in the next section.
Another advantage of the compact system is that the local correction allows compensating larger chromaticities, so that \( l^* \) can be increased (as shown in Eq. (8), the aberration-driven beam size is inversely proportional to \( l^* \equiv f_y \)). In addition, since in the local system the chromatic correction is carried out close to the interaction point, no transport of the sextupolar kick is required and therefore high order aberrations are limited. This results in a larger bandwidth of the system. On the other hand, one of the main disadvantages of the compact system is that it does not show modular optics of non local system. Therefore, it is more difficult to find a small set of parameters for the machine tuning. Unlike the other design, it has not yet had any experimental verification. Nevertheless, all the projects of future linear colliders presently are considering the compact design as the baseline for the final focus system [32, 35].

2.6 Baseline design of the CLIC beam delivery system

The two final focus design concepts described in Section 2.5 have been considered for the CLIC beam delivery system. The non-local correction scheme was first proposed for the 3 TeV CLIC design in [36]. Later the new local correction scheme was also considered [37]. The comparison between the two systems [34] shows that in spite of similar performances in terms of tunability, tolerances and luminosity, the compact design has a larger momentum bandwidth and is about 6 times shorter (550 m instead of 3300 m), which has an important impact on the cost of the overall machine. Thus the compact scheme appears to be a preferable choice. Further developments of the beam delivery system optics were therefore focused on the compact design. The present baseline, first presented in [38], is described in detail in [39]. A reduction of the collimation system led to a further shortening of the total system to about 2.6 km.

The main design parameters of the CLIC beam and the optics functions at the entrance and at the end of the beam delivery system are listed in Table 2. The horizontal and vertical \( \beta \)-functions and the horizontal dispersion of the whole beam delivery section are given in Fig. 5, as calculated with MAD8 [40]. The overall length of the system is 2557 m. The first 1392 meters are dedicated to the energy collimation. Here, a non-zero dispersion is required for collimating the off-energy particles. After a short, about 40 m long matching section, the 500 meter-long betatron collimation is installed. The total length of the collimation section, including additional beam transport and matching sections, is approximately 2000 m. The last 548.4 meters of the beam delivery system are dedicated to the final focus system. The horizontal and vertical \( \beta \)-functions and phase advances of this section are shown in Figures 6 and 7, respectively.
Table 2: Parameters of the CLIC beam and main optical functions at the entrance of the beam delivery system (BDS) and at the interaction point. The energy spread is a square distribution with a 1% full width. The theoretical beam sizes are calculated as $\sqrt{\epsilon_i \beta_i}$.

<table>
<thead>
<tr>
<th>Entrance of the beam delivery system</th>
<th>Normal. emittances $\epsilon_x$</th>
<th>680 nm</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\epsilon_y$</td>
<td>10 nm</td>
</tr>
<tr>
<td>Horizontal beta functions</td>
<td>$\beta_x$</td>
<td>64.171 m</td>
</tr>
<tr>
<td></td>
<td>$\alpha_x$</td>
<td>-1.95 m</td>
</tr>
<tr>
<td>Vertical beta functions</td>
<td>$\beta_y$</td>
<td>18.0 m</td>
</tr>
<tr>
<td></td>
<td>$\alpha_y$</td>
<td>0.61 m</td>
</tr>
<tr>
<td>Bunch length</td>
<td>$\sigma_z$</td>
<td>35 $\mu$m</td>
</tr>
</tbody>
</table>

| Interaction point                   | Beta functions $\beta_x^*$       | 6 mm   |
|                                     | $\beta_y^*$                      | 0.07 mm |
| Theoretical beam sizes, $\sqrt{\beta_i \epsilon_i}$ | $\sigma_x$ | 37.28 nm |
|                                     | $\sigma_y$                       | 0.49 nm |

Figure 5: Horizontal and vertical $\beta$-functions ($\beta_{x,y}$) and horizontal dispersion ($D_x$) versus longitudinal coordinate $s$ for the CLIC beam delivery system.
Figure 6: Horizontal beta function ($\beta_x$, solid line, left axis) and betatron phase ($\phi_x$, dotted line, right axis) versus longitudinal coordinate $s$, as calculated with MAD [40] for the CLIC final focus system. The dashed lines show the points at $\pi$ and $2\pi$ betatron phases from the interaction point (IP).

Figure 7: Vertical beta function ($\beta_y$, solid line, left axis) and betatron phase ($\phi_y$, dotted line, right axis) versus longitudinal coordinate $s$, as calculated with MAD [40] for the CLIC final focus system. The dashed lines show the points at $\pi$ and $2\pi$ betatron phases from the interaction point (IP).
3 Theory and simulations of CLIC luminosity performance

Together with the energy, the luminosity is one of the two primary parameters for a collider. In this section the basic theory and simulations of the CLIC luminosity performance are presented. The parameters limiting the nominal luminosity performance and the luminosity reductions due to mis-steered beam are discussed. Possible cures to counteract degradations of the design performance, such as beam-based feedback correction and mechanical stabilization of magnets, are considered.

3.1 Design luminosity of CLIC

The luminosity, $L$, is the ratio between the event rate and the cross-section of an event of interest. For a test particle crossing head-on a bunch of charge $N_b$, the luminosity is proportional to the bunch charge and inversely proportional to the effective transverse beam area. For two trains with $n_b$ bunches each, colliding at a repetition frequency $f_{rep}$, the luminosity is given by:

$$L = \frac{n_b f_{rep} n_b^2 (x,y) \psi_+(x,y) \psi_-(x,y)}{\mathcal{H}_D},$$  \hspace{1cm} (9)

where $\psi(x,y)$ is the normalized particle distribution in the transverse plane. The indexes + and − refer to positron and electron beams, respectively. It is assumed here that the two colliding bunches have the same transverse spot sizes, charge and a zero longitudinal length. The expression between the curly brackets is the purely geometric contribution to the luminosity given by the particle distribution overlapping. The so-called luminosity enhancement factor, $\mathcal{H}_D$, has also been included in Eq. (9). It accounts for the luminosity enhancement due to the mutual beam-beam interaction during the collision.

For Gaussian beams the transverse particle distributions read:

$$\psi(x,y; \sigma_x, \sigma_y) = \frac{1}{2\pi \sigma_x \sigma_y} \exp \left[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right],$$  \hspace{1cm} (10)

where $\sigma_x$ and $\sigma_y$ are the horizontal and vertical RMS beam sizes. If there is no mutual transverse tilt between the two distributions, the integral of Eq. (9) gives the well-known expression for the luminosity of linear colliders:

$$L = \frac{f_{rep} n_b N_b^2}{4\pi \sigma_x^* \sigma_y^*} \mathcal{H}_D,$$  \hspace{1cm} (11)

where $\sigma_x^*$ and $\sigma_y^*$ are the transverse RMS spot sizes at the interaction point.

In the more general case of colliding bunches with different transverse dimensions and charge the luminosity reads:

$$L = \frac{f_{rep} n_b N_b^+ N_b^-}{2\pi \Sigma_x \Sigma_y} \mathcal{H}_D,$$  \hspace{1cm} (12)

where $\Sigma_i = \sigma_{i,+}^2 + \sigma_{i,-}^2$ ($i = x, y$) are the effective beam sizes at the interaction point. The indexes + and − refer to positron and electron beams, respectively.

The simple estimate of design luminosity is expanded to include several advanced effects.
3.1.1 **Hourglass Effect**

The short-bunch expression of Eq. (11) overestimates the actual achievable luminosity because it does not take into account effects related to the finite bunch length $\sigma_z$. In particular, for bunch lengths comparable to the $\beta$-function values at the interaction point ($\beta^*_x, \beta^*_y$), the beam size variation along the opposing bunch reduces the luminosity. This effect is referred to as the *hourglass effect* because the longitudinal beam profile around the focusing point reminds the shape of a 90 degree tilted hourglass (see Fig. 8). For colliding bunches with the same beam parameters ($\sigma^*_t, \beta^*_t$) the transverse RMS beam sizes around the interaction point depend on $s$ as

$$\frac{\sigma^2_{x,y}}{\sigma^2_{x,y}} = 1 + \frac{s^2}{\beta^2_{x,y}}$$

(13)

See [41] for a more general case. As an example, Fig. 8 shows the vertical variations of a CLIC bunch when the beam-beam effects are neglected.

The luminosity reduction from hourglass effect must be calculated taking into account the longitudinal motion of the colliding beams. Equation (9) must be generalized to the 4-dimensional case [41]. Disregarding the disruption effect $\mathcal{H}_D$, the corresponding luminosity reduction can be expressed by the factor

$$R_{\text{hg}} \equiv \frac{\mathcal{L}}{\mathcal{L}_0} = \int_{-\infty}^{\infty} \frac{du}{\sqrt{\pi}} \frac{e^{-u^2}}{\sqrt{1 + \frac{\sigma^2 u^2}{\beta_x^2}} \sqrt{1 + \frac{\sigma^2 u^2}{\beta_y^2}}}$$

(14)

where $\mathcal{L}_0$ is the luminosity for infinitely short bunches. For the CLIC case the integral of Eq. (14) simplifies because $\sigma^2_x/\beta_x^2 = (0.035/6)^2 \ll 1$. Thus the first factor in the integral denominator can be neglected. A numerical calculation gives $R_{\text{hg}} = 0.95$, i.e. the CLIC design luminosity is reduced by 5% due to the hourglass effect.

Figure 8: Relative vertical beam size around the interaction point (located at the $s = 0$ position) calculated for design CLIC parameters and neglecting the beam-beam interaction. The dotted lines show the beam envelope at one sigma as calculated with a Gaussian fit to the vertical particle distribution. The arrow indicates the beam direction. The bunch length is $35 \, \mu m$. 

![Figure 8: Relative vertical beam size around the interaction point](image)
3.1.2 BEAM-BEAM EFFECTS

 Beam-beam forces

During collision the particles of one beam interact with the electric and magnetic fields generated by the opposing beam and this has a relevant impact on the particle dynamics. A test particle travelling through a flat, Gaussian beam with \( N_b \) particles and with transverse RMS sizes \( \sigma_x \) and \( \sigma_y \) (\( \sigma_x \ll \sigma_y \)) is considered, see Eq. (10). The electric field in the bunch frame system, \( E' = E_y' + iE_x' \), is given by [42, 43]:

\[
E' = E_y' + iE_x' = \frac{eN_b\lambda(z)}{4\pi\epsilon_0} \sqrt{\frac{2\pi}{\sigma_x^2 - \sigma_y^2}} \times \left[ w \left( \frac{x + iy}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right. \\
\left. - \exp \left( - \frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right) w \left( \frac{x\sigma_y/\sigma_x + iy\sigma_x/\sigma_y}{\sqrt{2(\sigma_x^2 - \sigma_y^2)}} \right) \right],
\]

where \( \epsilon_0 = 8.8542 \times 10^{-12} \text{ F/m} \) (farad per meter) is the electric permittivity of vacuum. \( w(z) \) is the complex error function defined for \( z = x + iy \) as \( w(z) = \exp(-z^2)(1 - \text{erf}(-iz)) \), with \( \text{erf}(\eta) = 2/\sqrt{\pi} \int_0^\eta \exp(-t^2)dt \). \( N_b\lambda(z) \) is the charge line density. Assuming for the bunch a Gaussian distribution also in the longitudinal direction, \( N_b\lambda(z) \) is given by:

\[
\lambda(z) = \frac{1}{\sqrt{2\pi\sigma_z}} \exp \left( - \frac{z^2}{2\sigma_z^2} \right),
\]

where \( z = s \pm ct \) is the longitudinal coordinate and \( \sigma_z \) is the bunch length. It is assumed here that \( \sigma_z \) is much larger than the transverse beam sizes and that the beam angles are small (\( x'\sigma_z \ll \sigma_x \), \( y'\sigma_z \ll \sigma_y \)) [42]. As an example, the field generated by a CLIC bunch along the horizontal and vertical axis is shown in Fig. 9 left and right parts, respectively. The field is calculated in the bunch centre, i.e. at the longitudinal position \( z = 0 \). Electric fields as large as 10^{12} \text{ V/m} are generated, which correspond to magnetic fields larger than 3000 Tesla.

Close to the beam centre, where the electromagnetic field increases approximately linearly, the mutual beam-beam focusing is equivalent to a quadrupole-like force that focuses the beam and does not kick it transversally, i.e. without changing its trajectory. The corresponding focal length reads:

\[
\frac{1}{f_{x,y}} = \frac{2N_br_e}{\gamma} \frac{1}{\sigma_{x,y}(\sigma_x + \sigma_y)},
\]

where \( r_e = e^2/(4\pi\epsilon_0 m_e c^2) = 2.81794 \times 10^{-15} \text{ m} \) is the classical electron radius. For the typical CLIC parameters (effective beam sizes at the interaction point of \( \sigma_x \approx 60 \text{ nm} \) and \( \sigma_y \approx 0.8 \text{ nm} \)) the equivalent vertical focal length is of the order of 6 \( \mu \text{m} \), i.e. shorter than the longitudinal bunch length (35 \( \mu \text{m} \)). The particles of one bunch are not only focused but can also undergo oscillations inside the field of the opposite bunch. This particle dynamics has a relevant impact on the luminosity.

 Disruption

The beam-beam force is usually parametrized with the disruption, \( D_{x,y} \), defined for the horizontal and vertical directions as the ratio of the focal length of Eq. (17) to the bunch length:

\[
D_{x,y} = \frac{2N_br_e}{\gamma} \frac{\sigma_z}{\sigma_{x,y}(\sigma_x + \sigma_y)}.
\]
For the CLIC design, \( D_x \approx 0.08 \) and \( D_y \approx 6 \). For \( D_y \gg 1 \) the beam particles can undergo some oscillations while crossing the opposing bunch and the oscillation number is proportional to \( \sqrt{D_i} \) [44]. Disruption effects were observed in the Stanford Linear Collider (SLC) [45].

**Pinch effect**

The primary effect of the disruption is a luminosity enhancement. The particles of the colliding beams are mutually focused towards the beam centre and this results in a smaller effective beam size. This enhancement is commonly referred to as the *pinch effect* and it is expressed by the variable \( H_D \) in Eq. (11). There are no precise analytical calculations of \( H_D \) and its estimates rely on the results of dedicated simulation codes for the beam-beam interaction [46, 47, 48, 49]. The values of \( H_D \) are typically between 1 and 3. Note that the luminosity enhancement is mainly driven by the pinch in the vertical direction for flat beams with \( \sigma_x \gg \sigma_y \). Figure 10 shows the variation of the vertical beam size around the CLIC interaction point with disruption.

In principle, the luminosity could be increased by changing carefully the disruption parameter. However, in practice \( D_i \) cannot be larger than about 20 because otherwise a two-stream instability arises. Initial small beam-beam offsets can grow exponentially due to the oscillation of the particles of one bunch inside the field of the opposing bunch. This effect is referred to as the *single-bunch kink instability* [50]. The beam oscillations can, in addition, couple with density modulations of the bunches, resulting in a further luminosity reduction [51, 44].

**Beamstrahlung**

The disruption does not only induce a beneficial enhancement of the luminosity. The beam particles are bent by the field of the opposing beam charge and emit *synchrotron radiation* (see [8, 52] for a general overview or [53] for Accelerator Physics related issues). The radiation of the beam particles during the collision is referred to as *beamstrahlung* [54, 55]. For a linear collider, beamstrahlung was first observed at the SLC [56]. Due to the high fields experienced by the particles, the emitted energy can be a significant fraction of the total beam energy. The consequences of such an energy loss are:

1. Unwanted collisions at lower energy. Thus, the luminosity is not peaked at the mean nominal
Figure 10: Relative vertical beam size around the interaction point \((s = 0)\) position for a nominal CLIC bunch when the beam-beam interaction is taken into account. The dotted lines show the beam envelope at one sigma as calculated with a Gaussian fit to the particle distribution. The beam moves towards positive \(s\), as indicated by the arrow.

Energy \(E_b\) but a luminosity spectrum is induced [57]. An example of a luminosity spectrum simulated for CLIC is shown in Fig. 11. (2) The emitted photons increase the background in the detector. (3) The beam after collision, the so-called spent beam has a large, virtually 100\%, energy spread. High energy photons can produce \(e^+e^-\) pairs, further increasing the detector background. In the case of CLIC, for instance, the average particle energy after collision is reduced from 1500 GeV to 1130 GeV and the energy spread is increased from a few percent to about 40\%.

The rate of energy loss by radiation (emitted power), \(P_\gamma\), is given by:

\[
P_\gamma = 2 \frac{e^2 c}{3} \frac{\beta^4 \gamma^4}{4 \pi \epsilon_0} \frac{1}{\rho^2},
\]

where \(\epsilon_0\) is the electric permittivity of vacuum [8, 53]. Half of the power is emitted below the so-called critical photon energy,

\[
\epsilon_c = \hbar \omega_c = \frac{3}{2} \frac{c}{\rho} \gamma^3,
\]

where \(\omega_c\) is the so-called critical photon frequency.

The classical results discussed above apply if the critical photon energy is small compared to the particle energy. This is typically the case for the synchrotron radiation emitted in the lattice magnets of a linear collider, where the bending fields are moderately small. The corresponding critical energies for CLIC are of the order of a few MeV, i.e. a million times smaller than the beam energy. This is not the case at the interaction point, where the particles experience a field up to the order of \(10^3\) Tesla. The classical results are no longer correct because they predict energies of the emitted photons larger than the particle energy. A proper quantum mechanical calculations must be used [58, 59, 60].

The beamstrahlung is characterized by the parameter \(\Upsilon\):

\[
\Upsilon = \frac{2 \hbar \omega_c}{3 E_b} \approx \frac{5}{6} \frac{\gamma^2 \gamma^2 N_b}{\alpha \sigma_z (\sigma_x + \sigma_y)},
\]
Figure 11: Luminosity spectrum for a typical CLIC collision. $s$ is the particle energy and $E_{cm}$ is the centre-of-mass collision energy. Courtesy of Daniel Schulte [57].

The expression on the right-hand-side of Eq. (21) is the mean value of $\Upsilon$ calculated for an average bunch [46]. Critical photon energies much larger than the beam energy correspond to $\Upsilon \gg 1$ and require a quantum treatment of the radiation emission. In the case of CLIC, $\Upsilon$ equals around 5 units. In this regime, the number of beamstrahlung photons emitted per electron, $N_\gamma$, is approximately given by [24]

$$N_\gamma \approx \frac{5 \alpha^2 \sigma_x}{2 r e \gamma} \frac{\Upsilon}{\sqrt{1 + \Upsilon^{2/3}}} \approx 2 \frac{\alpha r e N_b}{(\sigma_x + \sigma_y)}.$$  \hspace{1cm} (22)

The resulting fractional luminosity reduction depends on $N_\gamma$ as

$$\frac{\Delta L}{L} \approx \frac{(1 - e^{-N_\gamma})^2}{N_\gamma^2}.$$  \hspace{1cm} (23)

The reduction of design luminosity scales as $L \propto N_b N_\gamma (1 + r)/r$, with $r = \sigma_y/\sigma_x$. The luminosity profits of large horizontal to vertical beam size ratios. All projects of future linear colliders foresee flat beams, with $\sigma_y/\sigma_x \lesssim 0.01$.

The beamstrahlung radiation can produce $e^+/e^-$ pairs since the emitted photons have energies larger than twice the electron mass (0.51 MeV). The primary sources for pair production are [61] (1) the incoherent pairs due to the scattering between real or virtual photons with the particles. This effect dominates for $\Upsilon \lesssim 0.6$. (2) The direct interaction between the photons and the collective field of the opposing bunch ($0.6 \lesssim \Upsilon \lesssim 100$). (3) The coherent direct production $e^\pm \rightarrow e^\pm e^+e^-$. In all cases, the additional positrons and electrons can enhance the background in the detector.

3.1.3 Emittance increase from synchrotron radiation

Synchrotron radiation from dipole fields

Particles bent by a dipole field radiate photons and lose correspondingly energy, as discussed in the previous section. This results in an additional longitudinal emittance (beam energy spread) which
is not chromatically corrected by the lattice sextupoles and can enlarge both horizontal and vertical beam sizes. The luminosity is then reduced.

The beam delivery system of a linear collider requires horizontal bending dipoles to generate dispersive sections for the momentum collimation and for the chromatic correction. Dipole magnets are the main source of synchrotron radiation emission. However, the beam particles that have an offset with respect to the centre of quadrupole and sextupole magnets experience a dipole bending field. Thus, quadrupoles and sextupoles also contribute to the photon emission.

The beam particles that enter a quadrupole with a transverse offset are subjected to a bending field that is proportional to their distance from the magnetic centre. For high beam energies and strong quadrupole gradients the synchrotron radiation emission in quadrupoles can become significant. This phenomenon, first studied by Oide [62], is referred to as the Oide effect.

The Oide effect introduces an energy spread and an increase of the beam spot size. This effect arises in all quadrupoles of the beam line but the largest contribution is expected in the final doublets. The final focus quadrupoles have the largest field gradient and at their locations the beam particles also have the largest amplitudes, because large $\beta$-functions are required for the final beam demagnification.

The Oide effect limits the achievable beam size at the interaction point. For decreasing $\beta^*$, a value below which a further reduction of $\beta^*$ does no longer result in a spot size reduction. The beam size is dominated by the Oide effect and tends to increase for decreasing $\beta^*$. The reduction of $\beta^*$ is achieved by increasing the $\beta$-function at the final focus quadrupole and this increases the Oide emission. The minimum Oide beam size is [46]

$$\sigma \approx 1.83(r_c \lambda_e F)^{1/7} e_n^{5/7}$$

and is obtained for

$$\beta^* \approx 2.39(r_c \lambda_e F)^{2/7} e_n^{3/7}.$$  \hspace{1cm} (25)

$\lambda_e = 3.862 \times 10^{-13}$ m is the electron Compton length and $F$ is a constant that depends on the details of the final focus design [24].

3.1.4 CROSSING ANGLE

The baseline design of CLIC foresees a horizontal crossing angle scheme. With a crossing angle, bunches from the two opposing beams collide only at the interaction point, as illustrated in Fig. 12. A so-called multi-bunch kink instability can be induced due to parasitic encounters between spent
bunches of one train and the bunches of the opposing train which have not yet reached the collision point [50]. The multi-bunch kink instability induces instabilities in the particle trajectories and reduces the luminosity performance if the opposing bunches are not properly separated. In addition, the crossing angle scheme also helps in avoiding the outgoing bunches after collision to hit the final doublet of the opposing machine.

A crossing angle induced a reduction in luminosity with respect to the head-on collision, because the overlapping between the colliding bunches is reduced [63]. For small crossing angles \( \theta_c \) the relative reduction with respect to the head-on luminosity, \( \mathcal{L}_0 \), is given by

\[
\frac{\mathcal{L}}{\mathcal{L}_0} \approx \left[ 1 + \left( \frac{\theta_c \sigma_z}{2\sigma_x} \right)^2 \right]^{-\frac{1}{2}},
\]

where \( \sigma_z \) is the longitudinal bunch length. Equation (26) holds if at the collision point \( \sigma_x \ll \sigma_z \) and \( \sigma_z \ll \beta_x^* \), which is the case for CLIC. Even small crossing angles are sufficient to reduce considerably the luminosity. The crossing angle must be chosen as a trade-off between the optimization of the multi-bunch kink instabilities, which profit from large angles, and the luminosity performance. The angle \( \theta_c \) should also be large enough for the spent beam not to hit the final doublet of the opposing machine. This constraint imposes a limit on the smallest acceptable crossing angle. For CLIC the value \( \theta_c = 20 \) mrad has been chosen, as explained in detail in [64].

The luminosity reduction due to a crossing angle can be avoided with crab cavities. As first proposed in [46], dedicated cavities are placed upstream of the interaction point and are used to kick the two sides of the bunch in opposite directions. A rotation of \( \theta_c/2 \) is induced at the interaction point without moving the bunch centre. In the following, it is always assumed that crab cavities are used, which provide head-on like collisions. So the effect of the crossing angle will be neglected.

### 3.1.5 Simulations of design luminosity for CLIC

The “nominal” luminosity performance of CLIC is obtained with advanced numerical simulations that take into account the aforementioned effects. Nominal luminosity is the maximum luminosity achievable assuming that (1) all lattice magnets are perfectly aligned to the ideal beam orbit and (2) that the incoming beam in the beam delivery system fulfils the design parameters at the end of the linac. The simulation results discussed in this section are obtained with Merlin [66], which transports the particle beams from the end of the linac to the interaction point. Tracking results have been assessed by comparing in detail several simulation codes worldwide used for particle tracking in beam delivery systems in linear colliders. This is reported published papers [67, 68]. The main results of the code comparison are summarized in Appendix A. The simulations include the synchrotron radiation emission in all lattice dipoles, quadrupoles and sextupoles. The Monte Carlo model used for the photon generation is described in [69]. In order to match the lattice with the actual beam energy, the gradient of the lattice quadrupoles is rescaled according to the beam momentum at the quadrupole location. This avoids a beam-lattice mismatch due to the synchrotron radiation induced energy losses.

Some examples of the transverse particle distributions as obtained at the interaction point after tracking through the collimation section and the final focus system is shown in Fig. 13. The top figure shows the beam transverse profile for an ideal beam with zero energy spread, when the synchrotron radiation is not taken into account. If energy spread is also considered (bottom figure of Fig. 13), beam

\footnote{A compensation of luminosity reduction due to the the crossing angle can also be obtained with a small residual dispersion at the interaction point, \( D^*_x \). \( D^*_x \) must be chosen so that \( D^*_x \sigma_{\delta_k}/\sigma_z = \theta_c/2 \), where \( \sigma_{\delta_k} \) is the correlated component of the RMS relative energy spread [65].}
halos appear at amplitudes much larger than in the ideal case. This is also shown in Fig.14, where the horizontal $x-x'$ distribution is given for a larger $x$ range. Particles are found even at amplitudes larger than $1 \mu m$, while the horizontal beam size expected from the lattice $\beta$-functions is about $40$ nm. According to Merlin calculations, the fraction of particles with amplitudes larger than $3$ sigmas (as calculated from a Gaussian fit) is about $7\%$ in the horizontal direction and $9\%$ in the vertical direction. On the other hand, for an ideal bunch without energy spread, less than $1\%$ of particles are at amplitudes above $3\sigma$, as expected from statistical fluctuations. The large population of the bunch tails is peculiar for this design of the CLIC final focus (results for a previous lattice design can be found in [67]).

In Fig. 15 the transverse beam profile is shown for a realistic case, i.e. for a bunch with energy spread and with synchrotron radiation. The histograms of the horizontal and vertical particle distri-
Figure 15: Transverse scatter plot at the interaction point as calculated for a CLIC bunch with the nominal energy spread. The synchrotron radiation in dipoles, quadrupoles and sextupoles is taken into account. Gaussian fits to the projected particle distributions are also shown.

Distributions are shown with a Gaussian fit. The width of the fitted Gaussian distribution is considerably smaller than the RMS value of the particle positions, which is often used as measure of the beam size\(^6\). For instance, in the vertical plane the standard deviation of the particle distribution is 4.1 nm, but the effective (Gaussian fit) beam size is 0.7 nm.

Dedicated codes are required for the computation of disruption, pinch, beamstrahlung and e\(^+\)/e\(^-\) pair production, which must all be taken into account for a correct estimate of the luminosity performance. The GuineaPig code [70] has been used. It includes all the aforementioned effects and the validity of its results has been shown in [71]. Table 3 summarizes the effect of energy spread and synchrotron radiation on the transverse beam sizes at the interaction point and on the luminosity, with and without beam-beam effects. According to the tracking results of Merlin, including energy spread, synchrotron radiation and beam-beam effect the nominal CLIC luminosity is \(0.7 \times 10^{35} \text{ cm}^{-2}\text{s}^{-1}\).

\(^6\)For Gaussian-like distributions, the RMS of the particle position (standard deviation) and the width of a Gaussian fit are approximately equal. Differences arise for non-Gaussian distributions.
Table 3: CLIC transverse beam sizes at the interaction point and luminosity. Energy spread and synchrotron radiation are progressively taken into account. The particle tracking is performed with Merlin. The luminosity is calculated with GuineaPig for the cases with and without beam-beam effects taking into account hourglass, pinch, beamstrahlung and pair production.

<table>
<thead>
<tr>
<th></th>
<th>$\sigma_x^*$ [nm]</th>
<th>$\sigma_y^*$ [nm]</th>
<th>$\mathcal{L}_{\text{NoBB}}$ $10^{35}$ cm$^{-2}$s$^{-1}$</th>
<th>$\mathcal{L}_{\text{BB}}$ $10^{35}$ cm$^{-2}$s$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple estimate ($\sqrt{\beta^* \epsilon}$)</td>
<td>37.28</td>
<td>0.49</td>
<td>1.188</td>
<td>-</td>
</tr>
<tr>
<td>Tracked, no $\Delta E$, no SR</td>
<td>37.23</td>
<td>0.53</td>
<td>0.94</td>
<td>2.40</td>
</tr>
<tr>
<td>Tracked, $\Delta E$, no SR</td>
<td>48.90</td>
<td>0.59</td>
<td>0.57</td>
<td>1.26</td>
</tr>
<tr>
<td><strong>Tracked, $\Delta E$, SR</strong></td>
<td><strong>57.67</strong></td>
<td><strong>0.69</strong></td>
<td><strong>0.34</strong></td>
<td><strong>0.70</strong></td>
</tr>
</tbody>
</table>

However, the considered lattice has been designed and optimized with another code, MAD [40], to obtain a luminosity of $0.8 \times 10^{35}$ cm$^{-2}$s$^{-1}$. The latter value should then be regarded as the reference design luminosity of CLIC.

### 3.2 Perturbations of the ideal performance

So far, the CLIC luminosity performance has been calculated for a perfect machine assuming that all lattice elements are aligned to the ideal beam trajectory. Unavoidable imperfections reduce the luminosity of an operating collider. Major impact on the achievable luminosity comes from beam-beam effects and emittance blow-up at the interaction point, induced by error along the beam delivery system. This is described in the following sections.

#### 3.2.1 Luminosity reduction due to beam-beam offsets

A good luminosity performance relies on a good overlap between the two opposing beams at the interaction point. Any beam-beam collision offset reduces luminosity. In the limit that two beam misses each other, luminosity is zero. For short Gaussian bunches the luminosity depends exponentially on the relative beam-beam offset $\Delta y$ as

$$\frac{\mathcal{L}}{\mathcal{L}_0} = e^{-\frac{\Delta y^2}{4\sigma_y^*}}, \quad (27)$$

where $\mathcal{L}_0$ is the zero offset luminosity and $\sigma_y^*$ is the vertical spot size at the interaction point $^7$ [72, 73]. Hourglass, pinch and pair production are neglected. A detailed derivation of Eq. (27) is carried out in Appendix B. A similar result is found for horizontal beam offsets. This simple estimate of relative luminosity is shown in Fig. 16 versus beam-beam offset (solid line) and compared with the results of a full beam-beam simulation for the CLIC beam (dots), where the disruption is not taken into account.

7If the colliding beams have different transverse spot sizes, the luminosity reduction due to an offset reads

$$\mathcal{L} = \mathcal{L}_0 \exp \left[ -\frac{1}{2} \frac{\Delta y^2}{\Sigma_y^*} \right],$$

where $\Sigma_y^* = \sigma_{y,+}^2 + \sigma_{y,-}^2$ is the effective vertical beam size. See Appendix B and [72, 73] for details.
Figure 16: Luminosity reduction due to relative beam-beam offsets at the interaction point. The simple estimate of Eq. (27) (solid line) is compared with simulation results obtained for a nominal CLIC beam with and without beam-beam effects and with synchrotron radiation. The case with beam-beam effects and synchrotron radiation (line with diamonds) should be regarded as a reference. Note that for each line the values of $L_0$ and $\sigma_y^*$ are different. Only the relative luminosity reductions are considered.

The agreement is good for $\Delta y / \sigma_y^* \lesssim 2$. For larger offsets the simulations predict a slightly larger luminosity. This might depend on the bunch shape when a tracked bunch in considered instead of the Gaussian distribution assumed in the derivation of Eq. (27).

Equation (27) allows defining tolerances on the relative beam-beam offset, which can by translated in tolerances on the quadrupole displacements. Tolerances are conventionally specified as the beam-beam offset that induces a 2% reduction in luminosity. From Eq. (27) one obtains (in $\sigma_y^*$ units)

$$\Delta y \lesssim 0.28 \sigma_y^* \quad (\text{for} \quad L \gtrsim 0.98 \times L_0).$$

With vertical beam sizes in the range of a few nanometres, sub-nanometre tolerances are thus imposed on the vertical relative beam-beam offset. The horizontal tolerances are typically looser because the beam size is about 100 times larger than vertically.

If the disruption is included, Eq. (27) is no longer adequate. The variation of the effective beam size during collision (Fig. 10) and the attraction from the opposing bunch must be taken into account. Figure 16 shows the relative luminosity reduction versus beam offset when beam-beam effects are taken into account (crossed, dashed line). For $\Delta y / \sigma_y^* \lesssim 1$ ($\Delta y \lesssim 0.7 \text{ nm}$) the luminosity with disruption is more sensitive to beam-beam offsets than without disruption. On the other hand, for larger offsets ($\Delta y / \sigma_y^* \gtrsim 1$) the mutual bunch attraction increases considerably the luminosity for large offsets. Figure 16 shows that at offsets as large as $5\sigma_y$, almost 30% of the luminosity is still attainable.

If the synchrotron radiation is also taken into account, the luminosity reduction versus offset is looser. This beneficial effect is due to the fact that the beam distributions are considerably larger than without synchrotron radiation. The incoherent energy spread induced by radiation losses increases the beam sizes. The beam tails are then more populated than expected from a purely Gaussian distri-
Effect of a displaced quadrupole on the beam motion. The dipole kick experienced by the beam induces a betatron oscillation that propagates to the interaction point and results in a beam offset, $\Delta y^*$, depending on the relative phase between magnet and interaction point.

This is the realistic case to be considered as a reference for the luminosity calculations. It is noted that for small beam-beam offsets, say $\Delta y < 0.5 \sigma_y^*$, the differences between the different considered cases of Fig. 16 are not exceeding a few percent. Hence the analytical estimate of Eq. (27) provides a good approximation that can be used to calculate tolerances on small luminosity reductions.

**Effect of magnetic errors on the beam dynamics**

Errors in the lattice magnets induce perturbations of the beam trajectory with respect to the ideal orbit. These perturbations generate two unwanted effects that reduce the luminosity of a linear collider: relative beam-beam offsets and increase of the transverse spot sizes at the interaction point. The sources of these two effects are discussed.

**Relative beam-beam offsets at the interaction point**

The main source of relative beam-beam offsets at the interaction point is the beam mis-steering due to quadrupole displacements. For a beam passing through the quadrupole magnetic centre, the focusing transverse forces are symmetric and cancel. The beam centroid is not deflected. On the other hand, if the beam enters with an offset it experiences a net dipole field proportional to the distance from the centre.

Let us consider a quadrupole at the location $s_0$ upstream of the interaction point with a vertical offset $\Delta y_{qd}$ with respect to the ideal beam trajectory. The quadrupole kick induces a betatron oscillation of the beam centroid, as schematically drawn in Fig. 17. This oscillation results in a beam offset at the interaction point, $\Delta y^*$, depending on the betatron phase advance:

$$\Delta y^* = R_{3,4}(s_0 \rightarrow \text{IP}) \Delta y' = \sqrt{\beta_y^* \beta_y(s_0)} \sin(\phi_y^* - \phi_y(s_0)) K_{qd} \Delta y_{qd}, \quad (29)$$

where $\beta_y$ and $\phi_y$ are the vertical $\beta$-functions and betatron phases, $R_{3,4}(s_0 \rightarrow \text{IP})$ is the element of the transport matrix from the quadrupole to the interaction point [16] and $\Delta y' = K_{qd} \Delta y_{qd}$ is the deflection due to the quadrupole offset. The quadrupole strength is defined as the integrated field in

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8The increased population of the beam tails should not be confused with the larger effective beam sizes. The latter effect is already accounted for in Fig. 16, as the luminosity is given against the ratio of beam-beam offset over the actual beam size.
units of beam rigidity \((B\rho)\):

\[
K_{qd} = \left( \frac{1}{B\rho} \frac{\partial B_y}{\partial x} \right) l_q = \frac{1}{f_y}
\]  

(30)

where \(l_q\) is the magnet length and \(f_y\) is the vertical focal length.

If each lattice \(N_{qd}\) quadrupole has an offset \(\Delta y_{qd}^{(i)}\) the total offset \(\Delta y^*\) at the interaction point

\[
\Delta y^* = \sum_{i=1}^{N_{qd}} \sqrt{\beta^*_y} \beta_y(s_i) \sin(\phi^*_y - \phi_y(s_i)) K_{qd}^{(i)} \Delta y_{qd}^{(i)},
\]

(31)

where the index \(i\) denotes the quadrupole position. The final doublet is placed at \(\pi/2\) phase advance from the interaction point. The collision point is at the lens focal point and this ensures the incoming “parallel” beam to be focused to the smallest achievable beam size. If \(\beta^*_y \ll 1\) the \(\beta\)-function at the final doublet (FD) is approximately given by \(\beta^*_{FD} \approx f_y^2 / \beta_y^*\), hence Eq. (29) reads:

\[
\Delta y^* \approx \frac{\sqrt{\beta^*_y} \beta_{FD}}{f_y} \Delta y_{FD} \approx \Delta y_{FD}.
\]

(32)

The same equation applies for horizontal offsets.

It is seen that an offset of the final focus quadrupoles is translated one-to-one to a beam offset at the interaction point. It follows that the relative motion between the final doublet quadrupoles at either side of the collision point must be as small as a fraction of the beam size in order not to limit the luminosity performance. The sub-nanometre tolerances on the relative beam size offset are directly translated into sub-nanometre tolerances for the stability of a bulk object such as the final focusing quadrupole.

**Variation of the beam size at the interaction point**

Displaced lattice magnets do not only produce beam offsets at the interaction point but can also induce enlargements of the transverse beam sizes, which result in a reduction of the luminosity performance [75]. A typical source for an increase of the vertical beam size is the relative vertical offset between the beam and a sextupole. The variation of the interaction point beam size induced by a sextupole offset \(\Delta y_{sx}\) can be calculated transporting the sextupole kick to the interaction point with the matrix element \(R_{3,4}(sx \rightarrow IP)\). The vertical deflection is now given by \(\Delta y'(x) = K_{sx,x} \Delta y_{sx}\) [24], where

\[
K_{sx} = \left( \frac{1}{B\rho} \frac{\partial^2 B(x)}{\partial x^2} \right) l_s.
\]

(33)

is the normalized sextupole strength (\(l_s\) being the magnet length). The RMS beam size increase is obtained by integrating over the \(x\) direction:

\[
\Delta \sigma^*_y = \Delta y_{sx} \sigma_x K_{sx} |R_{3,4}(sx \rightarrow IP)|.
\]

(34)

A coupling between the vertical and the horizontal planes is introduced. A similar effect can be caused by a tilted quadrupole magnet, which induces a so-called skew coupling. If the beam has been mis-steered by upstream errors the transport matrices from the quadrupole to the sextupole \((R_{3,4}(qd \rightarrow sx) = R_{3,4}(sx \rightarrow IP) R_{3,4}(sx \rightarrow sx))\). In this case the skew coupling will also contribute to the beam size increase.
(sx)) and from the sextupole to the interaction point \((R_{3,4}(sx \rightarrow IP))\) must both be taken into account to give the spot size increase:

\[
\Delta \sigma_y^* = \left[ K_{dq} \Delta y_{dq} | R_{3,4}(q_{dl} \rightarrow sx)| \right] \sigma_x K_{sx} | R_{3,4}(sx \rightarrow IP)|. \tag{35}
\]

Sources of vertical dispersion are vertical offsets of quadrupoles and rolls of dipoles and quadrupoles. Consider for example a quadrupole displaced by \(\Delta y_{q_{dl}}\). An off-energy particle experiences a kick:

\[
\Delta y' = -\frac{K_{q_{dl}} \Delta y_{q_{dl}}}{1 + \delta} \approx -K_{q_{dl}} \Delta y_{q_{dl}} (1 - \delta), \tag{36}
\]

where \(\delta = \Delta p/p_0\) is the particle energy spread. The particle offset at the interaction point is given by

\[
\Delta y^* = R_{3,4} \Delta y' + R_{3,6} \Delta \delta + T_{3,4,6} \Delta \delta \Delta y' \ldots, \tag{37}
\]

where \(T_{3,4,6}\) is the second-order transport matrix element defined by the equation \(\Delta y^* = R_{3,4} y' + R_{3,6} \Delta \delta + T_{3,4,6} \Delta \delta \ldots\). It describes the second-order contribution to the vertical beam position versus changes of the initial slope or energy. Hence, the induced RMS variation of the vertical beam size is

\[
\Delta \sigma_y^* = \Delta y_{q_{dl}} K_{q_{dl}} \sigma_\delta | R_{3,4} - T_{3,4,6}|, \tag{38}
\]

where \(\sigma_\delta\) is the relative RMS energy spread. There are two contributions to \(\Delta \sigma_y^*\): the chromatic dependence of the deflection (term in \(R_{3,4}\)) and the chromatic dependence of the displaced trajectory.

A waist shift (a longitudinal offset between the focal point and the centre of the two colliding bunches) can be induced by a relative horizontal offset between a sextupole and the beam or by errors in quadrupole gradients. In either case, the additional focusing gradient shifts the transverse position of the interaction point of one beam with respect to the other. This results in an effective increase of the transverse beam size at the collision point:

\[
\frac{\Delta \sigma_{x,y}^*}{\sigma_{x,y}^*} = \Delta K(s_0) \beta_{x,y}(s_0). \tag{39}
\]

Here, \(s_0\) is the longitudinal position of the magnet, \(\beta_i(s_0)\) are the corresponding \(\beta\)-functions and \(\Delta K(s_0)\) is the gradient error. For a horizontally displaced sextupole the error is \(\Delta K(s_0) = K_{sx}(s_0) \Delta x_{sx}\) (\(\Delta x_{sx}\) is the offset) and induces a variation of the particle position given by:

\[
\Delta x^* = -K_{sx} \Delta x_{sx} x^* R_{1,2}^2 \approx -K_{sx} \Delta x_{sx} \beta_x(s_0) \beta_x x^*. \tag{40}
\]

An analogous expression holds for a vertical offset.

3.2.2 Sources of magnet displacements

Ground motion
A source of quadrupole displacements is the ground motion, which is transmitted to magnets via their support structures. Pioneer reports on ground motion studies related to particle accelerator were published by G.E. Fischer [76, 77], who showed that the ground is moving significantly on the scale relevant for the future linear colliders\(^9\). The ground motion is conventionally divided in two frequency

\(^9\)Quoting Fischer: “...this intolerant world, in which steel appears to behave like butter and you dare not to turn the room lights on...” [77].
ranges [78, 79, 80, 81]: “slow” frequencies below \( f_{\text{cut}} \approx f_{\text{rep}}/25 \) and “fast” frequencies above \( f_{\text{cut}} \) (\( f_{\text{rep}} \) is the pulse repetition frequency). As explained later, “slow” vibrations are efficiently corrected with beam-based feedbacks, whereas “fast” vibrations are not. Faster repetition frequencies allow correcting faster motion.

The slow ground vibrations are poorly affected by human activities and are dominated by the tidal motion of the earth surface, mainly due to the gravitational attraction of moon and sun, but is also influenced by temperature variations and atmospheric activities and in general by even far away located vibration sources. Slow motion is not an immediate limitation for the luminosity performance but produces slow misalignments of the lattice magnets, which would induce an emittance growth if not properly corrected with beam-based feedback systems. The so-called ATL law [82, 83] is an empirical diffusive model that describes the ground motion below approximately a tenth of hertz. It states that the relative displacement \( \Delta y \) between two points at a distance \( L \) has an RMS value that increases in time as

\[
\langle \Delta y^2 \rangle = A TL,
\]

where the constant \( A \) varies between \( 10^{-9} \mu m^2/s/m \) and \( 10^{-5} \mu m^2/s/m \) depending on specific conditions and geology of the site \(^{10}\). Detailed experimental verifications of the slow motion are on-going and some theoretical understanding and empirical modelling for simulating its effects on the beam dynamics can be found for instance in [86, 87].

The fast ground motion above approximately 1 Hz consists primarily of elastic waves propagating at velocities in the order of a few km/s. Spectral vibrational amplitudes of fast motion are measured to be decreasing steeply with the fourth power of frequency:

\[
P_{\text{fast}}(f) \approx \frac{1}{f^4}.
\]

The natural behaviour of the ground can be considerably disturbed by the so-called cultural noise, i.e. by all kinds of vibrations related to human activities such as traffic, operating machines, cities, etc. The cultural noise is much larger during the day than during the night [88, 89, 77]. The propagation of the cultural noise and its correlation depend on the geological site properties, but it is typically localized within one wavelength near to the earth surface. Thus, deep underground tunnels profit from the attenuation of the cultural noise, and are much quieter than shallow tunnels (compare, for instance, the LEP [90, 91] and the HERA [92] tunnels). Hard rock sites (high sound velocities) feature better stability and correlation than the sediment rock sites and are considered as a preferable location for a linear collider.

Ground motion is an issue for the next generation of colliders [79] and in particular for future linear colliders, which aim at colliding beams with transverse beam sizes in the nanometre range. On this scale the ground moves considerably, requiring the use of stabilization techniques to damp the ground motion and/or of beam-based feedback correction schemes.

For particle accelerator the noise generated in the tunnel due to the accelerator environment is of particular concern. Water pumps, ventilation systems, air flows, power supply systems, etc. can increase the ground motion by a factor of ten or more. In the LEP tunnel the ground motion was strongly increased when LEP was running. The RMS of the vertical displacement above 4 Hz was increased from 0.2 nm to about 20 nm when all the accelerator equipment was switched on (see [90, 91, 93].

\(^{10}\)The applicability of the ATL model has been experimentally confirmed in the minute to month time scale. On the other hand, the time dependence over years can change as \( T^2 \) [84, 85].
Circulating water used to cool the magnets and resonances of magnet support structures are considered as major source of additional vibrations which can limit the luminosity performance of a linear collider. Other effects such as acoustic noise, air flows, temperature variations, stray field can also perturb the field stability of the focusing quadrupoles. These effects are expected to have a smaller impact on the luminosity performance and are not discussed in detail here.

**Vibrations by cooling water flow**

Circulating water is required to cool normal conducting quadrupoles such as the ones foreseen for the CLIC linac (Section 4.1.3). In order to increase the cooling efficiency, the water flow is kept in the turbulent regime, which induces additional vibrations on the magnet motion. For example, the effect of water induced vibrations was measured at the Stanford Linear Collider [94]. In this case, the vibration level of linac quadrupoles (≈ 60 nm without water) was increased by approximately 200 nm by flowing water. Much of this addition motion was at 59 Hz and was also seen as a jitter of the beam position. This increase of vibration level would not be acceptable for a future linear collider.

A semi-analytical estimate of the water induced vibrations [95, 96] is drawn in Appendix F. According to this theory, circulating water induces additional vibrations above a minimal frequency

\[ f_{\text{min}} = \frac{u}{d}; \quad (43) \]

where \( u \) is the water velocity and \( d \) is the pipe diameter. Measurements on CLIC prototype quadrupoles confirm this prediction within a good accuracy. On the other hand, the estimate of the absolute vibration level is not reliable. A better estimate would require detailed finite-element calculations.

The proposed CLIC final focus quadrupole features a permanent magnet design without a cooling water circuit. The possible use of superconducting final doublets is under investigation for the NLC/JLC and TESLA designs [97]. An experimental study of the vibration level of realistic cryo-quadrupole prototypes has yet to be carried out. Vibration measurements performed on the LHC main dipoles [98] have shown a stability of the order of 1-2 µm at the constant temperature of the super fluid Helium (1.9 K) [99, 100, 101]. However, it is impossible to extrapolate the behaviour of the measured stability of the 15 metre long, 30 ton heavy LHC dipole to the regime of short final doublets of linear colliders.

**Structural resonances of magnet support**

The ground motion can be amplified by structural resonances of the magnet supports, like girders and alignment structures, or of the magnet itself. Every bulk object has several proper oscillation frequencies, at which it behaves like an harmonic oscillator (see also Section 3.3.1). The vibrational motion is amplified around the resonant frequency and damped above. Whether it is better to have a large damping (i.e. a large resonant amplification\(^{11}\)) or a small damping (limited resonant amplification) depends on the value of the proper frequency. A bad support design, not vibration-optimized, can easily amplify the ground vibration by a factor 10 or more, depending on the shape and on the stiffness of the support. Typical resonant frequencies arise above \( \approx 10 \) Hz and hence they are too fast to be adequately cured with beam-based feedbacks in CLIC. Therefore, as a general rule the quadrupole support design has to be optimized for structural resonances to limit amplification of the ground motion.

The calculation of the proper resonances of a complex structure is a typical mechanical engineering problem, which requires finite-element codes and goes beyond the scope of this work. The

\(^{11}\)As discussed in detail in Section 3.3.1, an increase of the damping above resonance can only be obtained with a corresponding increase of the amplification around the resonant frequency.
For each magnet the tolerances on horizontal and vertical displacements are defined as the offset that induces a 2% luminosity reduction. The tolerances on the longitudinal magnet position are of no concern and are not discussed here. The program FFADA [102, 103, 104] allows calculating magnet tolerances based on an analytical expansion of the luminosity as a function of the six-dimensional beam matrix. The results obtained for previous versions of the CLIC final focus lattice can be found in [34, 36, 37]. However, FFADA does not include beam-beam effects at collision nor synchrotron radiation (neither the photon emission in the lattice magnets nor the beamstrahlung during the beam collision). In order to take into account beam-beam and synchrotron radiation effects the tolerances on transverse displacements of quadrupoles and sextupoles have been calculated using particle tracking and beam-beam simulations. Tracking includes the synchrotron radiation emission in all lattice elements and beam-beam simulations take into account hourglass, pinch, beamstrahlung and $e^+e^-$ pair production.

Conventionally the tolerance is calculated for each magnet assuming that all the other lattice elements are perfectly aligned while the magnet under investigation is moving. This approach is also followed here. A perfect CLIC bunch is assumed at the entrance of the beam delivery system. Two different kinds of tolerances are defined, which are related to the two main sources of luminosity reductions: (1) relative beam-beam offsets and (2) increase of the transverse beam sizes at the interaction point. It is useful to separate the two effects in order to understand what is the main source of performance degradation and to define accordingly an *ad hoc* correction strategy.

An example of tolerance calculation is given in Fig. 18. Luminosity and vertical beam position...
Figure 19: Luminosity (left) and vertical beam position and offset (right) versus vertical offset of the lattice sextupole \( SD0 \), which provides the final vertical demagnification of the beam. Shown is the average over 12 sets of seeds for the synchrotron radiation. The statistical errors on size and position are small compared to the given scale and are omitted.

and size are shown as a function of the vertical offset of the \( QD0 \) quadrupole, which is the vertical final focus quadrupole. In this case, the luminosity tolerance is driven only by the beam-beam offset because of the aforementioned one-to-one translation between \( QD0 \) displacements and beam positions at the interaction point. The corresponding tolerance for a 2% luminosity reduction is 0.2 nm. On the other hand, if a sextupole is moved, luminosity, beam offset and beam size vary as shown in Fig. 19. These results are obtained displacing \( SD0 \), i.e. the last sextupole for the vertical chromatic correction, placed right upstream of \( QD0 \). Unlike for \( QD0 \), the luminosity reduction is now entirely driven by the beam size variation, see Eq. (34). The tolerance in this case is much looser than for the previous case: a \( SD0 \) displacement of about 40 nm reduces the luminosity by 2%.

Similarly to the case-studies discussed above, the tolerances on 2% luminosity reduction are calculated for all quadrupoles and sextupoles of the CLIC final focus beam-line. The Gaussian fit provides beam sizes and transverse positions at the interaction point. Therefore, the contribution to the luminosity from spot size variations can be disentangled from the contribution from the relative offsets. Tolerances can be defined separately for these two quantities. According to Eq.(27), the tolerance on the offset-induced luminosity reduction is given by the magnet displacement that induces a \( 0.28 \times \sigma_y^* \), as in Eq.(28). The tolerance on transverse spot sizes is defined as the variation that induces a 2% increase of the luminosity. If only one beam has a beam size variation, the luminosity reduction is half of what is expected if both beams vary. This aspect is discussed in detail in Appendix B.2.

The tolerances for quadrupoles and sextupoles are summarized in Figs. 20-23. Table 4 lists the tolerances for the four last magnets of the final focus lattice. The most severe tolerance is imposed by the motion of the final focus quadrupoles and is driven by beam-beam relative offsets, whereas the effect of the spot size enhancement is of the orders of magnitude smaller (see Figs. 20 and 21).

It is typically found that for a given quadrupole the tolerances on beam size variations are looser than the corresponding tolerances on beam offset, both for the vertical and for the horizontal planes. This feature has important consequences for the correction strategies applied for luminosity optimiza-

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\( ^{13} \)This is not true for the sextupoles, but in this case the tolerance are typically larger than about 50 nm and then require a slow correction anyway. This value corresponds to twice the smaller beam size variation. As explained in Appendix B.2, there is a factor 1/2 between the spot size increase of a single bunch and the induced luminosity reduction.
Figure 20: Vertical tolerances for the lattice quadrupoles of the CLIC final focus system. The black bars give the global tolerance on luminosity reduction, which is inferred from a fit of luminosity versus quadrupole offset. The dashed bars give the tolerance for a 2% increase of the beam spot size. The white bars give the beam-beam offset that induces the 2% according to Eq. (28).

Figure 21: Horizontal tolerances for the lattice quadrupoles of the CLIC final focus system. The black bars give the global tolerance on luminosity reduction, which is inferred from a fit of luminosity versus quadrupole offset. The dashed bars give the tolerance for a 2% increase of the beam spot size. The white bars give the beam-beam offset that induces the 2% according to Eq. (28).
Figure 22: Vertical tolerances for the lattice sextupoles of the CLIC final focus system. The black bars give the global tolerance on luminosity reduction, which is inferred from a fit of luminosity versus quadrupole offset. The dashed bars give the tolerance for a 2\% increase of the beam spot size. The white bars give the beam-beam offset that induces the 2\% according to Eq. (28).

Figure 23: Horizontal tolerances for the lattice sextupoles of the CLIC final focus system. The black bars give the global tolerance on luminosity reduction, which is inferred from a fit of luminosity versus quadrupole offset. The dashed bars give the tolerance for a 2\% increase of the beam spot size. The white bars give the beam-beam offset that induces the 2\% according to Eq. (28).
Tolerance and tuning. An on-line pulse-to-pulse compensation of the beam-beam offset, at frequencies in the Hz range, is mandatory to achieve and keep a high luminosity. On the other hand, re-alignment of the magnets for the spot size tuning are required with much longer time scales. These aspects are discussed in more detail in Section 3.3.

Tolerances on random uncorrelated displacements of all magnets
The above tolerances on magnet offsets have been calculated considering the effect of one single displaced magnet on the luminosity while the others are in their nominal positions. However, it is also interesting to consider the effect of a random misalignment of all the lattice elements, which is a realistic situation for standard machine operation. Simulations were set-up to quantify this effect. It is assumed that the magnets are displaced randomly with respect to their ideal positions and that there is no correlation between the displacements of different magnets. The Gaussian distributions of the magnet offsets are cut at three sigma. The final focus quadrupoles are not displaced from the nominal positions. Their effect on the luminosity has already been quantified in detail: they induce an offset equal to their displacement and hence they would dominate the luminosity reduction if moved like the other magnets.

The simulation results are summarized in Fig. 24. The average luminosity and a second order polynomial fit to the simulated points are shown against the RMS amplitude of the magnet random displacements. Both transverse directions are considered. The vertical and horizontal tolerances for a 2% luminosity reduction are 1.3 nm and 8.3 nm, respectively. These tolerances are mainly induced by displacements of the quadrupoles in the final focus section. The quadrupoles of the collimation section and the sextupoles do not contribute significantly to the luminosity reduction. For instance, if only the sextupoles of the final focus section are moved, the corresponding vertical and horizontal tolerances are 26 nm and 96 nm, i.e. about 20 and 10 times looser than what is obtained when displacing all magnets.

3.2.4 FREQUENCY ANALYSIS OF QUADRUPOLE OFFSETS

Power spectral density and integrated RMS displacement
The finite beam repetition frequency imposes an upper limit for the maximum vibrational frequency that can be corrected with beam-based feedback systems. This aspect is discussed in detail in Section 3.3. It is therefore not only important at what amplitude the lattice elements move, but also at which frequency. In the following, the formalism for frequency analysis of magnet displacements is described. This formalism is used for the analysis of the measurement data.
Figure 24: Average luminosity versus displacement RMS amplitude of the magnets of the CLIC beam delivery system. Vertical (circles) and horizontal (squares) uncorrelated displacements are applied to all sextupoles and quadrupole of the CLIC beam delivery system. The average over 150 random seeds for magnet offsets and synchrotron radiation is shown. The RMS amplitude that induces a 2% luminosity reduction is also given. The solid and dashed lines are second order polynomial fits to the simulation points.

The motion is assumed to be a random stationary process with zero mean, i.e. its characteristics do not depend on the time when they are measured and the average variations of the measurement point are slow compared with the total measuring time. Fourier analysis techniques can be conveniently used for studying the frequency spectral components of a stationary random signal (see, for instance, [105, 106] and [107] for a specific application to ground motion studies). Other phenomena such as fast transients, slow drifts, sudden kicks should be measured and treated differently. They are not dealt with here.

A time-dependent position is measured at the discrete time instants $n\Delta t$, with $n = 1, 2, \ldots, N$. $\Delta t$ is the sampling time and $N$ is the number of measured points. $T = N\Delta t$ is the total observation time. $y(n\Delta t) \equiv y(n)$ is the measured position in the vertical direction. The important parameter to be calculated is the RMS motion in a given frequency range. The overall RMS motion, $\sigma$, is given by:

$$\sigma^2 = \langle y(n)^2 \rangle - \langle y(n) \rangle^2,$$

where $\langle \rangle$ denotes the average. For signals with zero average, the above equation reads:

$$\sigma^2 = \frac{1}{N} \sum_{n=0}^{N} y(n)^2.$$

The discrete Fourier transform of the observation signal, $\tilde{y}(k\Delta f) \equiv \tilde{y}(k)$, is defined as a function of the discrete frequencies $k\Delta f$ as

$$\tilde{y}(k) = \Delta t \sum_{n=1}^{N} y(n)e^{-2\pi i nk}.$$
The frequency resolution $\Delta f = \frac{1}{N \Delta t}$ is the inverse of the total observation time and $k = 1, 2, \ldots N/2$ because only the positive frequency components are considered (see [106] for more details). The maximum observable frequency is fixed by the Nyquist criterion as $f_{\text{max}} = \frac{1}{2 \Delta t}$ and corresponds to $k_{\text{max}} = N/2$.

Random stationary processes by definition last for an infinite time, hence they have an infinite power, i.e. an infinite integral in frequency domain. On the other hand, the average power calculated over a finite time is well-defined and can be used to characterize properly the motion. The power spectral density of the signal, $P_y(k)$, is defined versus frequency as

$$P_y(k) = \frac{2 \Delta t}{N} \left| \sum_{n=1}^{N} y(n) e^{-2\pi i kn} \right|^2 = \frac{2 \Delta t}{N} |\tilde{y}(k)|^2. \quad (47)$$

Here, a factor 2 is introduced to take into account the symmetry of $P_y(k)$ around the null frequency. Depending on the numbering conventions in frequency-domain, null frequency corresponds to $k = 1$ (if $N = 2$) or to $k = N/2$ (if $1 \leq k \leq N$) [105]. In either case, later expressions involving the integral of $P_y(k)$ must include only $N/2$ components.

With Parseval’s theorem

$$\sum_{k=1}^{N} |\tilde{y}(k)|^2 = \frac{1}{N} \sum_{n=1}^{N} |y(n)|^2, \quad (48)$$

the RMS displacement of Eq. (45) can be written as

$$\sigma^2 = \Delta f \sum_{k=1}^{N/2} P_y(k), \quad (49)$$

i.e. the frequency domain integral of the power spectral density gives the total RMS displacement. As only positive frequencies are considered the sum is extended up to $k_{\text{max}} = N/2$. The RMS motion induced by vibrations in a given frequency range $[f_0, f_1] \equiv [k_0 \Delta f, k_1 \Delta f]$ can be calculated summing the spectral components $P_y(k_{\text{min}}), \ldots, P_y(k_1)$ only. The quantity of interest for accelerator physics purposes is the integrated RMS motion above a given frequency, $I_y(f)$, which is defined for all frequencies as

$$I_y(f) \equiv I_y(k \Delta f) = \sqrt{\frac{1}{N \Delta t} \sum_{k'=k}^{N/2} P_y(k')} \quad (50)$$

$I_y(f)$ gives the total RMS motion induced by vibrations at frequencies larger than $f$. Ideally, the sum extends up to infinity. However, in practise the upper limit is fixed by the bandwidth of the devices used for vibration measurements.

Vibrations are measured with geophones that measure velocities versus time. The displacement spectrum is then calculated from the time-dependent velocity signal. The signal has to be integrated in order to obtain the displacement from the velocity. It is convenient to perform the integration in the frequency domain and then calculate the $P_y(k)$ to be used in Eq. (50). Let $v_y(n \Delta t)$ be the time-dependent velocity. Similarly to Eq. (47), the power spectral density of the velocity, $P_{v_y}(k)$, can be defined for the discrete frequencies $f_k = k \Delta f$ as

$$P_{v_y}(k) = \frac{2 \Delta t}{N} \left| \sum_{n=1}^{N} v_y(n) e^{-2\pi i kn} \right|^2 = \frac{2 \Delta t}{N} |\tilde{v}_y(k)|^2, \quad (51)$$

40
is the definition of the (dimension-less) discrete Fourier transform of \(v_y(n)\) used throughout the following text (a factor \(\Delta t\) should be added to have the proper units of (m/s)/Hz). \(\tilde{y}(k)\) is obtained by dividing \(\tilde{v}_y(k)\) with \(2\pi f_k\). With the proper units, \(\tilde{y}(k)\) reads:

\[
\frac{\tilde{y}(k) = \Delta t \tilde{v}_y(k)}{2\pi f_k} = \frac{N(\Delta t)^2 \tilde{v}_y(k)}{2\pi k}.
\]

Therefore, \(P_y(k)\) is given by:

\[
P_y(k) = \left(\frac{N\Delta t}{2\pi k}\right)^2 P_y(k) = \frac{N(\Delta t)^3}{2\pi^2 k^2} |\tilde{v}_y(k)|^2.
\]

This expression has to be used in Eq. (50) to obtain the RMS integrated motion.

**Mutual power spectrum and correlation of motion**

At two points 1 and 2 separated by a distance \(L\) the time-dependent positions \(y_1(t_n)\) and \(y_2(t_n)\) are measured with a sampling time \(\Delta t\) (\(t_n = n\Delta t, n = 1, 2, \ldots N\)). \(P_{y,1}(k)\) and \(P_{y,2}(k)\) are the vibrational spectra at these two locations. The complex mutual power spectral density, \(P_{y,12}(k)\), is defined for the discrete frequencies \(f_k = \frac{k}{N\Delta t}\) as\(^{14}\)

\[
P_{y,12}(f_k; L) = \left(\frac{2\Delta t}{N}\right)^2 \sum_{n=1}^{N} \sum_{m=1}^{N} y_1(n) y_2^*(m) e^{-2\pi i \frac{kn}{N}} e^{2\pi i \frac{km}{N}} = \tilde{y}_1(f_k) \tilde{y}_2^*(f_k).
\]

The normalized mutual power spectrum, \(N_{y,12}(f_k; L)\) is defined as

\[
N_{y,12}(f_k; L) = \frac{P_{y,12}(f_k; L)}{\sqrt{P_{y,1}(f_k)P_{y,2}(f_k)}}
\]

and gives the normalized difference between the two measurement points. The absolute value of \(N_{y,12}(f_k; L)\) is referred to as coherence and its real part is referred to as correlation. The correlation can be calculated from the Fourier transforms of positions\(^{15}\) as

\[
c(f_k; L) = \text{Re} \left[ N_{y,12}(f_k; L) \right] = \text{Re} \left[ \frac{\tilde{y}_1(f_k) \tilde{y}_2^*(f_k)}{|	ilde{y}_1(f_k)| |	ilde{y}_2(f_k)|} \right].
\]

The imaginary part of \(N_{y,12}(f_k; L)\) is given by

\[
2i \text{Im} \left[ N_{y,12}(f_k; L) \right] = P_{y,12}(f_k; L) - P_{y,21}(f_k; L)
\]

\(^{14}\)In order to be consistent with the notation of Eq. (51) a factor \(2^2\) has been included in the definition of \(P_{y,12}(f_k; L)\) to take into account the symmetry of the Fourier transforms \(\tilde{y}_1(f_k)\) and \(\tilde{y}_2(f_k)\). However, this has no practical interest because in the following the normalized mutual spectrum will be considered.

\(^{15}\)Since in the discrete case positions and velocities in frequency-domain differ by a numerical factor, the correlation of positions can be directly calculated from the Fourier transforms of measured velocities.
Since for real signals \( y(t_n) \) the mutual spectrum satisfies the equation \( P_{Y,12}^* = P_{Y,21} \), the imaginary part of the normalized mutual spectrum is null if vibration properties do not depend significantly on the location, which is typically the case unless specific close-by sources of vibrations perturb the motion at a particular location. In the case of real \( N_{Y,12} \), \( c = 1 \) corresponds to a perfect correlation of motion, \( c = 0 \) to a \( \pi \) phase shift (case of larger relative motion between the two points, i.e. \( \sqrt{2} \) times the absolute motion of each point) and \( c = -1 \) to an anti-correlation.

**Two-dimensional power spectral density**

The study of vibrational effects on accelerator performance requires the knowledge of the spatial correlation between beam line elements at different locations. This can be expressed conveniently with the two-dimensional power spectral density, \( P_y(\omega, \kappa) \), which contains also information on the wavelength \( \lambda \) of the ground oscillations[86]. \( \omega = 2\pi f \) is the frequency (in radians) and \( \kappa = \lambda/(2\pi) \) is the wave number. In the discrete case the two-dimensional power spectrum reads

\[
P(\omega_k, \kappa_q) = \frac{2}{N\Delta t} \frac{2}{M\Delta l} \left| \sum_{n=1}^{N} \sum_{m=1}^{M} y(t_n, l_m) e^{-i\frac{\kappa_q l_m}{M}} e^{-i\frac{\kappa q n}{N}} \right|^2,
\]  

(59)

where \( y(t_n, l_m) \equiv y(n, m) \) is the time- and space-dependent ground position in the points \( t_n = 1, 2, \ldots N \) and \( l_n = 1, 2, \ldots M \). Here, \( T = N\Delta t \) like usual and \( L = M\Delta l \).

**Relative motion and integrated difference**

A quantity of interest is the relative RMS displacement between two points versus frequency, \( I_{rel}(k) \). It can be calculated from the measure of the absolute velocities in the two points. Let \( v_{y,1}(n) \) and \( v_{y,2}(n) \) be the vertical velocities as measured in points 1 and 2. At each time \( n\Delta t \) the relative velocity between 1 and 2 is

\[
v_{y,rel}(n) = v_{y,2}(n) - v_{y,1}(n).
\]  

(60)

The power spectral density of the relative motion, \( P_{y,rel}(k) \), is obtained as in Eq. (54):

\[
P_{y,rel}(k) = \frac{N(\Delta t)^3}{2\pi^2k^3} |\tilde{v}_{y,rel}(k)|^2,
\]  

(61)

where \( \tilde{v}_{y,rel}(k) \) is the Fourier transform of \( v_{y,rel}(n) \), as in Eq. (52). The vertical relative RMS displacement is then calculated as

\[
I_{rel}(k) = \sqrt{\frac{1}{N\Delta t} \sum_{k'=k_{\text{min}}}^{N/2} P_{y,rel}(k')}.
\]  

(62)

\( I_{rel}(k) \) will be extensively used in the following.
3.3 Counteracting luminosity degradation

3.3.1 Mechanical stabilization of accelerator magnets

Introduction - need for a stabilization of the magnets

An obvious way to prevent luminosity reduction due to errors of the beam trajectory is to keep all lattice elements aligned to their nominal positions. However, this is not easily achievable because the tolerances on transverse displacements are tight, often in the nanometre range. Only displacements with frequencies above a few Hertz need to be stable since feedback systems can compensate the slower motion. The amplitude of the ground motion decreases with frequency and this helps in stabilizing magnets. If the cultural noise is sufficiently small (e.g. in a site like CERN or in a deep underground tunnel) the ground motion above approximately 50-60 Hz is typically within the tolerances for a linear collider.

It should be noted that to the first order, what has to be stabilized is the relative displacement of the two final doublets at either side of the interaction point. If the two opposing machines move in a correlated way, no reduction of the luminosity performance is induced because the two beams are kept in collision. This can be achieved in different ways. A first approach is to stabilize both doublets in the absolute space. This is the approach that has been mainly pursued for the CLIC study and is discussed in detail here. Techniques for the absolute stabilization of a magnet is also under study at SLAC [108, 109, 110]. Alternatively, it has also been proposed [107] to stabilize the doublet relative to the other with optical system [111].

Absolute stabilization of the magnets

The main source of magnet vibration is the ground motion, which is transmitted to the quadrupole via the support structure. A schematic view of a simple system for the passive isolation from the ground motion is shown in Fig. 25. The object to stabilize sits on a spring with Hooke constant $k$, which provides a force proportional to its elongation $F_{\text{Spring}} = k(y_m - y_g)$, where $y_m \equiv y_m(t)$ and $y_g \equiv y_g(t)$ are the vertical coordinates of load and ground, as defined in Fig. 25. A damper reduces the amplitude of the relative oscillation providing a force proportional to the relative velocity between ground and load: $F_{\text{Damp}} = b(\dot{y}_m - \dot{y}_g)$ ($b$ is the damping constant), with $\dot{y}_m \equiv \frac{dy_m}{dt}$. The equation of

![Figure 25: Scheme with the key elements of a damper system. A pay-load is isolated from the ground by means of a spring and a damper.](image)
motion for this system then reads:

\[ m\ddot{y}_m + b(\dot{y}_m - \dot{y}_g) + k(y_m - y_g) = 0. \]  \hspace{1cm} (63)

If \( Y_m(s) \) and \( Y_g(s) \) are the Laplace transforms of \( y_m \) and \( y_g \) \[112\], Eq. (63) can be written as:

\[ (\omega_0^2 + 2\xi\omega_0 s + s^2)Y_m(s) = (\omega_0^2 + 2\xi\omega_0)Y_g(s), \]  \hspace{1cm} (64)

where the \textit{natural resonance frequency} \( \omega_0 \) without damping \((b = 0)\)

\[ \omega_0 \equiv \sqrt{\frac{k}{m}} = \frac{f_0}{2\pi} \]  \hspace{1cm} (65)

and the \textit{damping ratio} \( \xi \)

\[ \xi \equiv \frac{b}{2m\omega_0} \]  \hspace{1cm} (66)

have been introduced \[16\]. The ground-to-load transfer function versus frequency is referred to as \textit{transmissibility} and is given by

\[ H(f) \equiv \left| \frac{Y_m(s)}{Y_g(s)} \right| = \left| \frac{1 + 2\xi\frac{f}{f_0}i}{1 + 2\xi\frac{f}{f_0} + \left(\frac{f}{f_0}\right)^2} \right| = \sqrt{\frac{1 + \left(2\xi\frac{f}{f_0}\right)^2}{\left(1 - \frac{f^2}{f_0^2}\right)^2 + \left(2\xi\frac{f}{f_0}\right)^2}}, \]  \hspace{1cm} (67)

where the Laplace transform \( Y_m(s) \) and \( Y_g(s) \) have been evaluated at the frequencies \( s = i\omega = 2\pi if \).

The transmissibility \( H(f) \) is shown for three values of \( \xi \) in Fig. 26. The main characteristics of \( H(f) \) are:

\[ \text{Often the equation of motion is parametrized with the quality factor } Q \equiv m\omega_0/b = 1/(2\xi) \text{ instead than with the damping ratio.} \]
Figure 27: Transfer function of a passive and active damping system for different values of $\xi$.

- $H(f) \approx 1$ for frequencies smaller than the natural resonance frequency, i.e. below $f_0$ the load moves almost like the supporting ground.
- At $f \approx f_0$ the load motion is typically amplified with respect to the ground, depending on the value of $\xi$. Large load mass and damper constant $b$ help in keeping the amplification low.
- For $f > f_0$ the isolation goes like $(f/f_0)^2$.
- For $f \gg f_0$ the isolation depends strongly on the damping constant: a larger damping constant induces a smaller isolation.

The parameters of the passive system must be chosen as a trade-off between the damping of the natural resonance frequency and the amplification at high frequencies. A good damping of the natural resonance compromises the isolation above $f_0$. In addition, a pure passive system has also the disadvantage that it is extremely sensitive to even small forces that perturb the load. These drawbacks are partially cured if an active isolator is used, i.e. a system in which a time-dependent force $f_m(t)$ is applied to the load. In this case Eq. (63) becomes

$$\ddot{y}_m + 2\xi\omega_0\dot{y}_m + \omega_0^2 y_m = \frac{\omega_0^2}{k} f_m, \quad (68)$$

where it is assumed that the ground does not move ($y_g(t) \equiv 0$). After a Laplace transformation, the transfer function of this system reads:

$$T(f) \equiv \left| \frac{Y_m(2\pi i f)}{F_m(2\pi i f)} \right| = \frac{1}{k} \sqrt{\frac{1}{\left(1 - \frac{f^2}{f_0^2}\right)^2 + \left(2\xi \frac{f}{f_0}\right)^2}}, \quad (69)$$

where $F_m(s)$ is the Laplace transform of $f_m(t)$. The function $T(f)$ is referred to as the compliance of the system and has units of displacement per unit force. Figure 27 shows $T(f)$ for three values of the damping ratio $\xi$. In this case the natural resonant frequency can be damped while keeping a good response at high frequencies. Note that for active systems $T(f) \approx 1/k$ with $f \ll f_0$, so the curves of Fig. 27 do not necessarily tend to 1 at low frequencies.
Figure 28: Schematic view of the beam-beam deflection at the interaction point generated by a relative beam offset $\Delta y^*$. The transverse kick due to mutual electromagnetic attraction bends the trajectories of both beams. The deflection angle $\theta_y$ can be detected with a beam position monitor (BPM) at a distance $\Delta L_{BPM}$ downstream of the interaction point. If $\Delta y_{BPM}$ is the beam centroid offset at the BPM location, then $\theta$ is approximately given by $\theta_y \approx \Delta y_{BPM}/\Delta L_{BPM}$. This technique allows measuring precisely the beam-beam offset provided that the deflection angle curves are known with sufficient precision (see Fig. 29 later in this section).

The system described by the transfer function of Eq. (68) provides a better isolation from the ground than the passive system because a good damping is achieved in the full frequency range of interest. The high frequency amplification for larger $\xi$ has been suppressed. An active system requires a device for measuring the load vibrations (position, velocity or acceleration can all be used for this purpose) and an actuator which applies a time-dependent force according to the measured motion. Different systems have been conceived for this purpose. In particular, in Section 4.3 two different stabilization devices utilized in the framework of this Thesis work are described in detail. Note that an active isolator is characterized by its natural resonant frequency, which becomes the main design parameter. Below $f_0$ the system does not provide damping with respect to the ground motion and thus $f_0$ is typically chosen as small as possible. This aspect is particularly important for the isolator system to be used for linear colliders. It will be shown in the next section that beam-based corrections of the particle trajectory are effective only up to a few Hz because of the finite pulse repetition frequency. Since it is important to provide mechanical stability in the frequency range which cannot be corrected with the beam, linear collider builders are typically interested in isolators which feature the lowest natural frequency.

3.3.2 FEEDBACK CORRECTION OF THE BEAM-BEAM OFFSET

Need for an interaction point feedback system

It has been shown that the most stringent limitation for the luminosity performance of a linear collider comes from the vibrations of the final focus quadrupoles. Their tolerance for vertical uncorrelated motion above 4 Hz is 0.2 nm instead of 1.3 nm for the linac quadrupoles, i.e. 6.5 times smaller. Relative displacements of the two doublets at either side of the interaction point induce a beam-beam offset that reduces the luminosity. If the offset were not be somehow corrected, any kind of mechanical stabilization technique would not be sufficient to recover the desired luminosity. The low-frequency
content of the magnet vibrations is indeed in the micrometre regime, i.e. about 1000 times larger than the typical tolerances for the displacement of the final doublets.

The strong beam-beam interaction in the vicinity of the interaction point provides a powerful tool for the compensation of the relative beam offsets. If the two opposing bunches collide with a finite offset, the mutual electromagnetic attraction results in a net transverse kick that bends the trajectories of the two beam centroids, as schematically depicted in Fig.28. This beam-beam deflection angle has typical amplitudes of up to 150 micro-radians, big enough to be precisely measured with a beam position monitor a few metres downstream of the interaction point. From the angle measurement the relative beam-beam offset can be inferred, as discussed later in this section. This provides an input for a feedback system that brings the beams into collision (if the initial offset is large) and keeps them around the collision point.

The beam-beam deflection in linear colliders was first observed at the SLC in 1989 [113]. As foreseen a few years earlier [114], this effect became over the years a powerful tool for the beam tuning at the SLC [115]. Not only the setup of an efficient interaction point feedback system [115, 116] but also precise measurements of the beam spot sizes [117] have been made possible. Nowadays, all the designs of future linear colliders heavily rely on a beam-beam based feedback for the orbit correction at the interaction point. In addition, more recent studies foresee the utilization of the deflection angle for more challenging correction schemes, such as the fast intra-train correction of the position of consecutive bunches within the same train [118, 119]. The first experimental results are encouraging [120]. Beam-beam deflection has also been observed in storage rings like LEP [121, 122] and CESR [123], where they were used for luminosity optimization.

In this section the basic features of the interaction point feedback and its implementation for the time-dependent simulations of CLIC are discussed. A classical approach was followed. A detailed study of the modern control design of feedback systems goes beyond the scope of this report.

**Beam-beam deflection curve and angle measurement**

The beam-beam deflection angle can be calculated analytically for Gaussian particle beams starting from the expression of the electric field in Eq. (15) [113]. The trajectory of a test particle passing through the opposing bunch is deflected by an angle $\theta_{x,y}$ given by

$$\theta_{x,y} = \tan^{-1}\left(\frac{p_{x,y}}{p_0}\right) \approx \frac{1}{p_0} \int_0^{\Delta t} eE_{x,y}dt,$$

where $p_{x,y}$ is the horizontal/vertical transverse momentum transmitted to the test particle, $p_0$ is its longitudinal momentum and $\Delta t$ the collision time interval. Let $\Delta x,y$ be the relative beam-beam offset at the interaction point. In the general case of colliding bunches with different transverse spot sizes, the average deflection angle, $\langle \theta_{x,y} \rangle$ is given by

$$\langle \theta_{x,y} \rangle = \frac{2r_e N_b \Delta x,y}{\gamma} \int_0^{\infty} e^{-\left(\frac{\Delta^2}{t+2\Sigma_x^2} + \frac{\Delta^2}{t+2\Sigma_y^2}\right)} dt$$

where $\Sigma_x = \sqrt{\sigma_{x,+}^2 + \sigma_{x,-}^2}$ and $\Sigma_y = \sqrt{\sigma_{y,+}^2 + \sigma_{y,-}^2}$ are the effective beam sizes (the indexes + and - refer to the electron and positron bunches). For small offsets, where the electric field is linear, the

---

**Footnote:** For legibility, the index * denoting the beam parameters at the interaction point is omitted.
Figure 29: Average beam-beam deflection angle versus interaction point beam offset. The particle
distribution at the collision point has a strong influence on the deflection curve shape and on the
maximum deflection angle but the slope at zero offset does not change much. Shown are the cases of
an ideal bunch without energy spread (solid line), a bunch with energy spread only (dashed line) and
also a bunch with synchrotron radiation emission (dotted line).

The kick increases linearly as [121]

$$\theta_y = -\frac{2r_e N_b}{\gamma \sigma_y (\sigma_x + \sigma_y)} \Delta y$$

and the largest deflection angle is approximately given by

$$\theta_{y,\text{max}} \approx \frac{\sqrt{2\pi N_b r_e}}{\gamma \sigma_x}.$$ 

Similar equations hold for the horizontal direction. Note that the slope at zero offset depends only
on the transverse beam sizes. The measurement of deflection angle has been used for measuring the
beam size at the SLC [117].

Figure 29 shows the vertical beam-beam deflection angle versus beam separation for CLIC. The
three lines refer to different kinds of bunches. The ideal case without energy spread and no syn-
chrotron radiation is compared with the cases obtained when both effects are taken into account. The
maximum deflection angle depends on the particle distribution whereas the slope at zero offset is
similar for the different cases. Figure 28 shows schematically the mutual beam deflection and the
quantities required for the angle measurement. The vertical deflection angle is well approximated by

$$\theta_y \approx \frac{\Delta y_{\text{BPM}}}{\Delta L_{\text{BPM}}},$$

where $\Delta y_{\text{BPM}}$ is the difference between the beam position and the ideal trajectory measured without
beam-beam effect (see [113] for the details of the experimental procedure required for the angle
measurement). Note that even if the spent beam has large energy spread and divergence, the measure
of its centroid can be precise enough to ensure the required precision on the angle. The effect of the
precision of the angle measurement on the feedback performance is discussed later in this section.
In order to calculate the interaction point offset the deflection angle curve of Fig. 29 must be inverted. A linear approximation is good for small amplitudes but more complicated fitting algorithms must be used for a precise estimate of the offset versus angle. In any case, the curves of Fig. 29 are not monotonic and therefore the determination of the beam-beam offset from the deflection angle has not unique solution.

Corrector dipole for the feedback

The correction of the beam position at the interaction point is applied with a bending dipole corrector placed upstream of the interaction point. Following Eq. (29), a dipole bending field $B$ over a length $l_B$ induces the following effects on the particle position and divergence at the interaction point:

$$
\Delta y^* = \frac{Bl_B}{B\rho} \sqrt{\beta_B} \beta^* \sin(\phi_B - \phi^*)
$$

$$
\Delta y'^* = \frac{Bl_B}{B\rho} \sqrt{\beta_B} \beta^* \cos(\phi_B - \phi^*),
$$

where $\beta_B$ and $\phi_B$ are the Twiss functions at the corrector location and $Bl_B / B\rho$ is the dipole kick. It is most effective to place the corrector at a $\phi_B - \phi^* = \pi/2$ betatron phase advance upstream of the interaction point because in this case the dipole kick induces a maximum offset. In order to prevent additional aberration effects from the betatron oscillation induced by the kick, the corrector should be put as close as possible to the interaction point. Hence, the offset correctors for both the transverse directions are typically placed close to the corresponding focusing quadrupoles, which are at the right phase from, and close to, the interaction point.

Implementation of the feedback system

A schematic view of the feedback system is shown in Fig. 30, including all elements required for its operation. A beam position monitor provides the angle measurement used for the beam-beam offset determination. This information is then fed back into the corrector, which serves as actuator for the beam-beam offset correction. The device for the angle measurement must be as close as possible to the actuator to reduce the time delay between the offset measurement and its correction. Therefore, the measure of one beam (the positron beam in the example of Fig. 30) is used for steering the other beam. This scheme was adopted at the SLC [124] and is also foreseen in the design of future linear colliders. A diagram with the main ingredients of the feedback implementation into time-dependent luminosity simulations is given in Fig. 31.

The 100 Hz bunch repetition frequency is assumed to be slow enough to allow for a pulse-to-pulse correction, i.e. with a one-pulse delay between the offset measurement and the corrector action. The measured offset of one bunch train is directly fed back for the correction of the next train and hence only the first pair of trains will be uncorrected. Let $\Delta y(n)$ be the interaction-point relative offset between the $n$-th pair of colliding trains. At each time the corrector field, $B(n)$, is chosen to compensate a given fraction of the measured offset. Different correction algorithms can be used. The standard PID control algorithm [125, 126, 127], is considered here. The corrector field for the $(n + 1)$-th bunch train is chosen as:

$$
B(n + 1) = g_1 \times B(n) - g_P \times \frac{\Delta y(n)}{C_{Cal}} - g_D \times \frac{\Delta y(n) - \Delta y(n - 1)}{C_{Cal}},
$$

(77)
Figure 30: Scheme of the implementation of the interaction point feedback for the correction of the relative beam-beam offset. A beam position monitor (BPM) downstream of the interaction point measures the offset of the spent beam with respect to the ideal orbit and allows calculating the deflection angle. This information is fed into the corrector, that deflects the opposite beam so to minimize the RMS offset. The corrector is placed very close to the final doublet (FB), at a $\pi/2$ betatron phase from the collision point.

Figure 31: Diagram with a schematic view of the beam-beam feedback functioning principle. In simulations, tracking begins with a perfectly aligned machine and then the magnets are misaligned in time according to measured vibration spectra and to some correlation models.
where $C_{\text{Cal}}$ is the corrector calibration constant in meter per Tesla and can be inferred from Eq. (75)\(^\text{18}\). The first term of the right hand side of Eq. (77) is the so-called \textit{integral} term (I) and defines the set point. The second, \textit{proportional} term (P) contains a correction which is proportional to the measured beam offset and the last term is a \textit{derivative} contribution (D), which corrects proportionally to the offset variation. This latter term is meant to emphasize the correction of faster displacements. The PID algorithm has three independent gains, one per term, that can be independently used to tune the feedback performance. More details on this algorithm can be found for instance in [125] or in text books like [126, 127]. An example of feedback applications in particle accelerators, with a comparison between the different algorithms, can be found in [128].

As an example, the time evolution of the interaction point offset is shown in Fig. 32 for a step-like quadrupole displacement. Different values of the three PID gains are considered. The integral gain $g_I$ sets the position of the collision point and is normally chosen to 1. Detailed simulations have shown that the value $g_I = 1$ also minimizes the RMS of the beam-beam offset. The comparison between the pure proportional (PI) and the derivative (PID) correction shows that the PID response is slower for a constant offset. However, there exist cases where a correction proportional to the offset velocity can be important [125].

\textit{Feedback response function}

The feedback efficiency depends strongly on the characteristic frequency of the displacement to be corrected. Higher frequencies are corrected less efficiently. This depends on the finite pulse repetition frequency, that limits the time response of the feedback and imposes a finite delay between the measurement of the offset to be compensated and the action of the correctors used to steer the beam.

The frequency dependence of the feedback correction can be explained semi-qualitatively. Assume that the final doublet oscillates vertically at a given frequency $f_0$. Correspondingly, also the

\(^{18}\)Dedicated simulations have been performed to verify that the calibration constant calculated analytically with Eq. (75) is in agreement with the result obtained with the particle tracking.
beam position at the collision point varies like a sine with an amplitude $A_0$ that depends on the amplitude of the quadrupole oscillation and on the lattice transport properties. The interaction point feedback is used for correcting this sine perturbation. The offset to be corrected is sampled for every bunch train, i.e. with a sampling time $\Delta t = 1/f_{\text{rep}}$, and reads:

$$\Delta y^* = A_0 \sin(2\pi f_0 t_n),$$

where $t_n = n\Delta t$ ($n = 1, 2, ...$) are the discrete measurement points. For CLIC $\Delta t = 0.01$ s. Since the correction is effective only for the next pulse, at the time $t_{n+1}$, the maximum expected offset left uncorrected will be of the order of the maximum velocity times $\Delta t$. The velocity of the offset variation is $2\pi f_0 A_0 \cos(2\pi f_0 t)$ and then the maximum residual oscillation amplitude, $A$, will read:

$$\frac{A}{A_0} \approx 2\pi \frac{f_0}{f_{\text{rep}}}. $$

Therefore, for a given $f_0/f_{\text{rep}}$ ratio the achievable closed-loop efficiency is limited. Or, equivalently, the sampling time must be increased to improve the feedback efficiency at a given frequency. For instance, assuming a gain $g_P = 1$, the repetition frequency must be about 125 times larger than the perturbation frequency in order to reduce the initial amplitude to the 5% of its initial values.

As an example, Fig. 33 shows the close-loop response to oscillations of the final doublet at 1 Hz (left) and 3 Hz (right), as obtained in numerical simulations. The uncorrected oscillations (cross lines) are compared with the closed-loop feedback lines (circle lines). If $f_0 = 1$ Hz, $A/A_0 = 0.06$, whereas if $f_0 = 3$ Hz $A/A_0 = 0.18$. The qualitative scaling law of Eq. (79) is nicely verified. Note that the residual oscillation of the beam position is shifted in phase by $\pi/4$ with respect to the uncorrected oscillation.

Typically, it is suggested [129] to sample the signal at least 30 times faster than the frequency to be corrected. This is unfortunately not possible for a linear collider, since the relative offset cannot be

Figure 33: Time variation of the interaction point offset for a sine perturbation of the final doublet vertical position. Two perturbation frequencies are shown: 1 Hz (left) and 3 Hz (right). The crossed lines are the uncorrected oscillation whereas the circle lines are the closed-loop feedback oscillations. An integral-proportional feedback with $g_I = 1$ and $g_P = 0.8$ is applied. The higher the frequency, the larger is the amplitude of the residual oscillation after feedback correction.
Figure 34: Feedback response to a δ-perturbation of the beam position for two values of the proportional gain \( (g_1 = 1 \text{ and } g_D = 0) \). The final doublet is shifted downwards by 5 nm for one pulse only. This induces a step perturbation of the beam centroid at the interaction point, which is corrected by the feedback.

sampled faster than the pulse repetition frequency. Based on the SLC experience, it is commonly expected that the interaction point feedback will be effective only up to \( 1/25 \) of the repetition frequency. This means up to about 4 Hz for the CLIC case. This value fixes a limit between the “slow” motion, which can be compensated with beam-based feedback system, and the “fast” motion, which has to be mechanically stabilized in order to prevent luminosity reductions.

A feedback system is fully characterized by its closed-loop response function versus frequency, \( R(f) \). This function gives the amplification of the uncorrected to corrected motion at a given frequency. For the examples of Fig. 33, the response function can be calculated as corrected to uncorrected ratio of the shown sine oscillations of the beam offset. In principle this procedure can be repeated for all the frequencies of interest so as to give the full feedback response function. A more elegant way to calculate the response function is to move the final doublet with a δ-perturbation, i.e. displacing the magnet for one pulse only and put it back afterwards. The induced oscillation contains contribution at all frequencies and hence can be used to calculate \( R(f) \) as

\[
R(f) = \sqrt{\frac{P_{\Delta y}^{\text{Cor}}(f)}{P_{\Delta y}^{\text{Unc}}(f)}}
\]  

(80)

According to the definition of Eq. (47), \( P_{\Delta y}^{}(f) \) is the power spectral density of the interaction point offset. The indexes “Cor” and “Unc” stand for “corrected” and “uncorrected”, respectively. An example of beam offset after a δ-perturbation is given in Fig. 34 for two values of the proportional gain.

The calculation of the response function is crucial for understanding and optimizing the performance of a feedback system. Figures 35 and 36 show the response function for different values of the proportional and derivative gain of the interaction point feedback as implemented for the CLIC simulations. The integral gain defines the set point of the beam offset and is always set to 1 unless

\[^19\text{The spectrum } P_{\Delta y}^{\text{Unc}}(f) \text{ is flat and in Eq. (80) is used only as a normalization.}\]
Figure 35: Closed-loop feedback response function for the CLIC interaction point feedback for several values of the proportional gain ($g_P$). The integral gain is set to 1 and the derivative gain is set to 0. The simulation time is 3 s (300 pulses), which provides a frequency resolution of 0.033 Hz.

explicitly stated\textsuperscript{20}. The simulations assume that the beam-beam offset is known without error and do not use the deflection angle.

The low-frequency response benefits from larger proportional gains but the high-frequency response becomes worse accordingly. The motion is amplified ($R(f) > 1$) for frequencies larger than a given cutoff frequency $f_{\text{cut}}$. For the $g_P = 1$ it is found that $f_{\text{cut}} = 16.7$ Hz and $R(50 \text{ Hz}) \approx 2$. For increasing gains $f_{\text{cut}}$ shifts slightly towards higher values but the amplification of the fast motion increases. For $g_P \approx 1.5$ the motion becomes unstable. Similar instabilities arise for a derivative gain larger than about 0.6 (the value $g_D = 0.4$ induces a significant amplification of the motion above 40 Hz, as shown in Fig. 36).

Because of the aliasing imposed by the finite bunch repetition frequency, only frequencies from 0 Hz up to $f_{\text{rep}}/2 = 50$ Hz can be considered. Note that the information for all other frequencies is contained in this window (which can be extended to very low frequencies increasing the simulated time). Indeed, since

\[
\sin(2\pi f_0 t_n) = \sin \left(2\pi \frac{f_0}{f_{\text{rep}}} n \right) = \sin \left(2\pi n - 2\pi \frac{f_{\text{rep}} - f_0}{f_{\text{rep}}} n \right) = \sin(2\pi (f_{\text{rep}} - f_0)t_n),
\]

the following relation holds:

\[
R(f_0) = R(f_{\text{rep}} - f_0), \quad \frac{f_{\text{rep}}}{2} < f_0 < f_{\text{rep}}.
\] (82)

Therefore, the response function for frequencies between 50 Hz and 100 Hz, can be extrapolated from the results of Figs. 35 and 36 simply by reflecting the lines with respect to 50 Hz.

\textsuperscript{20}Detailed simulations have shown that the best performance in terms of RMS variation of the beam-beam offset is obtained for $g_I = 1$. Therefore, this value has always been used.
Figure 36: Closed-loop feedback response function for the CLIC interaction point feedback for several values of the derivative gain ($g_D$). For all lines ($g_P = 1$), as for the dashed line of Fig. 35. The simulation time is 3 s (300 pulses), which provides a frequency resolution of 0.033 Hz.

Effect of fitting algorithm and of BPM noise on the feedback efficiency

The calculations of the closed-loop response function discussed so far assume that the relative beam-beam offset at the interaction point is known with no error. Indeed the angle measurement gives a very precise estimate of the beam-beam offset, provided that the beam position monitor is far enough downstream of the interaction point to measure reliably the angle. For instance, angle resolutions in the few micron range were achieved in the measurements at the SLC [113] and also in LEP [121]. Nevertheless, sources of errors can arise and limit the resolution of the offset measurement, which influences the efficiency of the feedback system. Discussed in the following two aspects are: the effect of the offset-to-angle curve fit and the resolution of the beam position monitor used for the angle measure.

In Figure 37 the relative beam-beam offset is shown versus the deflection angle, as obtained from the curves of Fig. 29 (the case of bunches with energy spread and without synchrotron radiation is considered). Three approximations of the data are shown in Fig. 37: (1) a linear approximation with slope tangent to the zero offset angle; (2) a linear fit calculated taking into account beam-beam offsets up to 4 nm; (3) a fifth order polynomial fit up to the maximum deflection angle. The corresponding closed-loop response functions are given in Fig. 38 and compared with the ideal case of perfect beam-beam offset determination (dots).

The linear approximation (1) always underestimates the real offset and this makes the response function at low frequencies worse. More time is required for the correction as an offset smaller than the actual value is calculated. On the other hand, the linear approximation is very accurate for small amplitudes, where it is important that the feedback works well. This approximation was used at the SLC [130]. Note that since the offset is underestimated, gains larger than the values considered so far must be used for obtaining a decent low-frequency response. The approximation (2) fits better larger offsets (so its low-frequency response is better) but overestimates the small offsets, and this can induce feedback instabilities around the collision point. The fifth order fit (3) has the best low-frequency response but its reliability at small offsets may not be good depending on the precision in the experimental measurement of the deflection curve.

55
Figure 37: Vertical beam-beam offset at the interaction point versus deflection angle, $\Delta y(\theta)$, as simulated with GuineaPig (see Fig. 29). Three approximations of the curve are shown: (1) a linear approximation with slope tangent to the zero offset angle (solid line); (2) a linear fit taking into account offsets up to 4 nm (dashed line); (3) a fifth order polynomial fit up to the maximum deflection angle (dashed-dotted line).

In the simulations the different fitting algorithms have all been considered. It is typically found that results obtained with the tangent to the zero angle slope, (1), are not considerably improved if more precise fits to the angle curve are used. This is in agreement with the simulations performed at SLAC for NLC [124].

Note that since the deflection curves versus offset, shown in Fig. 29, are not monotonic, the calculation of the beam-beam offset from the angle can be used only for beam-beam offsets smaller than the one corresponding to the maximum deflection, which for CLIC is between 15 nm and 20 nm. However, even for very large offsets, the two beams “see” each other and the angle measure provides at least the good direction for the corrector to bring the beams closer and back into collision. Once the beams are close enough for the deflection angle curve to be efficiently used, the feedback system works well in keeping the beams in collision.

The angle measurement can be affected by the resolution of the beam position monitor (BPM) placed downstream of the interaction point to measure the position of the spent beam. If $\delta_{BPM}$ is the BPM resolution, the resolution on the angle, $\delta_\theta$, is given by (see Eq. (74))

$$\delta_\theta \approx \frac{\delta_{BPM}}{L_{BPM}},$$

where $L_{BPM}$ is the distance between the BPM and the interaction point, according to Fig. 28. The effect of this angle error is studied assuming a Gaussian distribution for $\delta_{BPM}$. The final doublet is moved according to a measured time-dependent displacement of a CLIC prototype quadrupole and the luminosity is simulated for 3 s of CLIC run, with offset feedback switched on. The maximum average luminosity achieved versus BPM resolution is shown in Fig. 39. A 2% luminosity reduction is obtained for a BPM resolution of about 50 $\mu$m, an easily achievable performance target for modern beam position monitors.
Figure 38: Closed-loop response function of the interaction point PI feedback versus frequency, $R(f)$, when the measure of the deflection angle is used to infer the relative beam-beam offset at the interaction. The three different fits of Fig. 37 to the deflection curve are considered. $R(f)$ is calculated with a $\delta$-perturbation of the quadrupole that corresponds to a beam-beam offset of about 7 nm. The ideal case of perfect offset determination (Fig. 35) is also given (dots).

Figure 39: Achieved relative luminosity, $\langle L \rangle / L_0$, versus resolution of the beam position monitor (BPM) used for the beam-beam deflection measurement. Gaussian resolution is assumed. Sextupoles and quadrupoles of the CLIC beam delivery system are displaced vertically and horizontally according to the vibration measurements performed on prototype magnets (see Chapters 5 and 6). The mean luminosity over 100 pulses averaged over 20 seeds for magnets displacements is shown (error bars, small with respect to graph axes, are omitted).
3.3.3 Knobs for Spot Size Tuning

Fast vibrations limit the luminosity performance via relative beam-beam offsets and require a fast pulse-to-pulse correction. On the other hand, slow vibrations are not an immediate limitation of the machine performance but progressively increase the beam size at the interaction point, which also reduces the luminosity performance but on a longer time scale. A re-alignment of all lattice elements based on the beam measurements has to be periodically performed in order to re-optimize the luminosity [131]. This was routinely done for the SLAC Linear Collider (SLC) [132] and for the Final Focus Test Beam (FFTB), where a beam size of approximately 70 nm was achieved [133].

The spot size tuning is a crucial issue for future linear colliders but is not treated in this Thesis. Detailed studies on beam-based alignment can be found for instance in [131] and [133]. Some studies on linear and non linear tuning knobs for the optimization of beta waist shifts, horizontal and vertical dispersion and transverse coupling for the compact final focus design can be found in [134]. It has been shown that variations of normal and skew quadrupole strengths and horizontal and vertical displacements of sextupoles can be used successfully to tune and optimize linear and second-order aberrations at the interaction point. Similar studies for the tuning of the CLIC performance are under investigations.
4 Experimental set-up and stabilization techniques

The experimental set-up for vibration measurements and magnet stabilization studies is presented. The latest equipment for vibration measurements, realistic prototypes of CLIC quadrupoles and state-of-the-art stabilization technology are brought together with the final goal of investigating the achievable magnet stability.

4.1 Overview of the laboratory and equipment

4.1.1 The CLIC vibration test stand at CERN

The CLIC experimental test stand for vibration measurements and magnet stabilization studies has been set-up since the beginning of 2001 [135, 136]. The location of the laboratory has been chosen on purpose, close to streets, working areas, operating accelerators, workshops and regular office spaces. The typical vibration level above a few Hertz was surveyed and found to be in the 5 nm to 12 nm (RMS) range depending on the environmental conditions. This vibration level is not too noisy and hence ensures good conditions to test stabilization equipments. On the other hand, it is not too quiet either, being 25 to 60 times larger than the aimed stability goal for RMS vertical vibrations of the final focus quadrupoles, i.e. 0.2 nm above 4 Hz. Therefore, the chosen area provides an ideal place for the CLIC stability studies. Much quieter places are available in the CERN area, such as the LEP-LHC tunnel (see [90], where the ground motion is already good enough to meet the CLIC final doublet tolerances. The aim of this work is to show the feasibility of the aimed tolerances in a realistic accelerator environment. The good vibration property of a given deep underground location cannot be guaranteed over the 35 km of the full machine nor can it be ensured in presence of operating accelerator equipment (compressors, ventilation systems, cooling water etc.). For instance, during the LEP operation the ground motion was increased by the accelerator equipment from a fraction of nanometre to 20 nm [90]. Therefore, the existence of one place sufficiently quiet does not provide feasibility demonstration of the CLIC stability requirements.

A scheme with the overall equipment used for vibration measurements and magnet stabilization studies and a photograph of the laboratory area are given in Figs. 40 and Fig. 41, respectively. The main components of the CLIC test stand are:

1) a honeycomb structure (table) to support the objects to stabilize, e.g. the CLIC prototype quadrupoles;
2) some CLIC prototype quadrupoles;
3) two different stabilization devices to isolate the ground motion and possibly to damp also vibrations of the support structure or of the quadrupoles themselves;
4) several sensors to measure the vibrations of ground, table and quadrupole prototypes and the alignment of the whole system.

As shown in Fig. 40, a stabilization system is used to support the honeycomb structure, on top of which quadrupole prototypes are mounted. Several sensors allow monitoring simultaneously the vibration level of ground, table and quadrupoles. Quadrupoles can possibly be connected to incoming and

21 It is noted that the vibrations in the tens of nanometre range were not an issue for the LEP operation and no particular care was taken for the design of the various accelerator equipments. This will not be the case for future linear colliders: all components such as ventilation systems, compressors, powering devices, etc. must be carefully designed and isolated from the ground, in order to reduce as much as possible the additional vibration sources.
Figure 40: Scheme with the overall set-up of the CLIC test stand for vibration measurements and magnet stabilization. A honeycomb support structure (table, see Section 4.1.2) provides a flat and stiff surface to test measurement devices and to install objects to stabilize, e.g. CLIC prototype quadrupoles (Section 4.1.3). Two different stabilization systems (Section 4.3) can be used to damp the table vibrations. Several geophones and a stretched-wire system are used to monitor the vibrations and the alignment of the quadrupoles with respect to ground. Ground vibrations are always monitored as a reference. Quadrupole prototypes can possibly be connected to cooling water channels. The coordinate system is also shown.

Figure 41: Photograph of the vibration test stand.
outgoing channel for the cooling water. The sensors for vibration and alignment measurements and the stabilization systems require a detailed description, carried out in Section 4.2 and 4.3, respectively. The items (1) and (2) are discussed below in this section.

4.1.2 The honeycomb support structure

A flat and quiet working surface is required to test the vibration sensors and to perform a number of vibration measurements. It is also required as a support for mounting the objects to stabilize with the isolation systems. The support platform used for these purposes is referred to as optical table or simply as table. It should be big enough to house quadrupole prototypes and several sensors at the same time. In order not to perturb the measurement, the platform body must also be sufficiently stiff so as not to have relevant structural resonances in the frequency range of interest.

The ideal solution for the CLIC purpose was found to be a honeycomb closed-cell support, whose internal structure is given in Fig. 42. This type of optical table is also used in other domains like holography, interferometry, ultra-fast-optics and spectroscopy. The main advantages of the honeycomb structure are the stiffness and the good damping of vibrations, induced for example by sound waves or by air currents [137]. The technical specifications of the used table are listed in Table 5. Note in particular the high minimum resonant frequency of 230 Hz, above the typical frequencies where the ground motion and the structural resonances of magnets and supports induce their largest contribution. Another constraint for the choice of the table parameters is given by the minimal load required for a proper functioning of the isolation systems (see Section 4.3), which imposed a weight larger than 700 kg for the overall support structure. The table is long enough to house three or four prototype quadrupoles, mounted for example as expected on a linac girder, as in Fig. 41.

4.1.3 The CLIC prototype quadrupoles

Prototype of the CLIC linac quadrupole

Prototypes of the CLIC linac quadrupoles are used for stabilization tests and measurements of mechanical resonances. The quadrupole of the CLIC Test Facility II (CTF2) accelerator [138, 139] is thought to be a representative prototype for the CLIC linac quadrupole, which should have the same
transverse cross-section but a different length. Depending on the location in the linac, the CLIC quadrupoles can be up to 2 m long (the longest magnets are at the linac end, at large beam energies). The main parameters of the CLIC linac and final focus quadrupoles (see next section) are given in Table 6.

The CTF2 quadrupole is a resistive magnet made of four copper coils and an iron yoke and is cooled down with pumped demineralized water. The magnet transverse cross-section is given in Fig. 43. A photograph with a quadrupole doublet mounted on the honeycomb support structure is given in Fig. 44. Each quadrupole is 80 mm long, weighs 6.7 kg and has four copper coils made of six rectangular conductors, with a 3 mm diameter hole for the cooling water to circulate. Two or three quadrupoles sit on a common steel plate and form doublets and triplets. Each magnet can be independently connected to the water cooling circuit, even though quadrupoles of the same doublet or triplet were connected in series in the CTF2 accelerator.

The CTF2 support for quadrupole alignment was used for studies of structural resonances and for stabilization tests. A photograph of this structure fixed on the table is shown in Fig. 45. This support was designed and successfully tested for the micro-metric alignment of CTF2 lattice doublets and triplets [142] but was not optimized against structural resonances. Namely, it features a peaked resonance in the 35 Hz to 40 Hz frequency range (Section 5.5). This can be particularly dangerous for a linear collider because beam-based feedbacks cannot compensate vibrations above a few Hertz. A proper optimization of the quadrupole alignment support against structural resonance is envisaged.

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22No compressor for the demineralized water was available in the laboratory of the CLIC test stand. Hence tests of water induced quadrupole vibrations were performed with the regular tap water of the Geneva network, at a constant pressure of about 4 bar.
Table 6: Main parameters of the CLIC linac and final focus quadrupoles. The tolerances are set for uncorrelated motion above 4 Hz. The tolerances for the final focus quadrupoles are discussed Section 3.2.3, whereas for the linac quadrupoles they have been calculated in the framework of the CLIC Stability Study as in [136]. See also [140].

<table>
<thead>
<tr>
<th></th>
<th>Linac quadrupoles</th>
<th>Final quadrupoles</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>1300 per linac</td>
<td>2 per FF system</td>
</tr>
<tr>
<td>Field</td>
<td>200 T/m</td>
<td>388 T/m</td>
</tr>
<tr>
<td>Transverse size</td>
<td>0.15 x 0.11 m (width x height)</td>
<td>4.3 cm (outer radius)</td>
</tr>
<tr>
<td>Length</td>
<td>0.46 m to 2.08 m</td>
<td>3.5 m</td>
</tr>
<tr>
<td>Weight</td>
<td>69 kg to 312 kg</td>
<td>250 kg</td>
</tr>
<tr>
<td>Field stability</td>
<td>≈ 10^{-3}</td>
<td>&lt; 10^{-5}</td>
</tr>
<tr>
<td>Tolerance, $f_{min} &gt; 4$ Hz</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical</td>
<td>1.3 nm</td>
<td>0.2 nm</td>
</tr>
<tr>
<td>Horizontal</td>
<td>14.0 nm</td>
<td>7.8 nm</td>
</tr>
</tbody>
</table>

Figure 43: Cross-section of the quadrupole of the CLIC Test Facility 2 (CTF2) linac with its transverse dimensions [141]. The transverse x-y coordinate system is also shown. Each quadrupole is 8 cm long and can be mounted on dedicated alignment support structures as shown in Fig. 45. The CLIC linac quadrupoles have the same cross-section but are up to 2 m long, depending upon their location in the machine.
Figure 44: A CLIC prototype doublet used for stabilization tests. Two CTF2 quadrupoles sit on a common steel plate that is directly screwed on the honeycomb table top. On top of the doublet, a geophone is installed for the vibration measurements. Quadrupole triplets are installed similarly.

In the framework of the CLIC stability study, design work for the final focus quadrupoles has been carried out, as reported in [143] (see also [39, 144, 145]). The most promising performance are obtained with a permanent magnet design. It has the main advantages of being small, for an easy installation into the detector region, as safe from additional vibration sources from the refrigeration system. Prototypes of this design are not available yet and have not been used for vibration studies.

**Measurements of water-induced quadrupole vibrations**

Detailed vibration measurements have been performed to quantify the effect of the cooling water on CLIC quadrupole vibrations. The experimental set-up for the water-induced vibration measurements is shown in Figs 44 and 45. A prototype quadrupole doublet is mounted on the top of the honeycomb support structure and is connected to the regular Geneva tap water system, at a constant pressure of about 4 bars. It has not been possible to connect the quadrupole to water pumps using demineralized water, as is expected in operational conditions. Nevertheless, in-situ measurements of CTF2 quadrupole vibrations have been performed [146].

Figure 46 shows schematically how the quadrupole doublet is connected to the tap (left part) and a photograph of the doublet installation on the table top. The water volume rate is measured at the tap with a precision of approximately 1%. The maximum water velocity in the cooling pipes of quadrupoles is about 3 m s$^{-1}$, i.e. larger than the nominal value of the CTF2 quadrupoles of 1.2 m s$^{-1}$ [141]). Three different pipes are required to bring the water from the tap to the magnets. The water is first brought from the tap to a 4 channel manifold and then to the quadrupoles with tubes of different diameters (see Fig 46). All the pipes have to be taken into account in the estimation of the water
Figure 45: The CLIC prototype doublet of Fig. 44 is now installed on its CTF2 support structure conceived for a micrometre alignment of the magnet. Five stepping motors (black and grey cylinders) move the doublet in five degrees of freedom according to the position measured by a stretched-wire system (not installed on the prototypes used for stabilization tests). The longitudinal direction is left uncorrected because it is not critical and does not require a fine on-line adjustment. However, this support is not optimized against structural resonances (see section 5.5) and shows a larger vibration level than the installation of Fig. 44.

Figure 46: Scheme with the set-up for the measurements of water induced quadrupole vibrations (left) with a corresponding photograph (right). Pipes of different lengths and diameters are required to connect the magnet to the regular Geneva tap water system. They have all to be taken into account because vibrations generated upstream of the doublet can be transmitted to the quadrupole via the water itself.
Table 7: Diameter of the different pipes on interest, flow required for the turbulent onset (assumed to arise for Reynold’s number $Re \approx 2000$) and minimal vibration frequency according to Eq. (120).

<table>
<thead>
<tr>
<th>Pipe</th>
<th>$Re$</th>
<th>$d$ [m]</th>
<th>Flow [l/h]</th>
<th>$f_{\text{min}}$ [Hz]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tap→Manifold</td>
<td>2000</td>
<td>0.013</td>
<td>16.4</td>
<td>10.5</td>
</tr>
<tr>
<td>Manifold→Quad</td>
<td>2000</td>
<td>0.008</td>
<td>40.3</td>
<td>27.9</td>
</tr>
<tr>
<td>Quadrupole</td>
<td>2000</td>
<td>0.003</td>
<td>15.1</td>
<td>198</td>
</tr>
</tbody>
</table>

effects on quadrupole vibrations because even vibrations generated upstream of the quadrupole can be transmitted to the magnet via the water itself. A simple theory on the water induced vibrations is developed in Appendix F. It allows estimating the expected vibration frequencies and amplitudes due to the turbulent water. In Table 7, the minimal flows required for turbulence onset and the corresponding minimal vibration frequencies of Eq. (120) are listed for the three different types of pipe of the cooling system. The maximum turbulent frequency, according to Eq. (121), is given by $Re^{3/4} f_{\text{min}}$, where $Re$ is Reynold’s numbers and $f_{\text{min}}$ is given in Table 7, and thus, is well above the maximal frequency of the geophones used for the vibration measurements (315 Hz). The validity of these estimates will be discussed in the next chapter.

4.2 Sensor for vibration measurements

Ideally, one would like to know the absolute vibration amplitudes of the prototype quadrupoles at all frequencies. In practise this is not possible due to limitations of measurements devices, which have a finite measuring bandwidth. The frequency range most relevant for the CLIC stability studies goes from a few Hertz to a few hundreds of Hertz. Slower frequencies are efficiently corrected by beam based feedback systems whereas larger frequencies do not contribute significantly to the total motion due to the natural reduction of vibration amplitudes with frequency, see Eq. (42). The frequency range of interest is covered by the geophones, which provide a measurement of vibration velocities versus time. This type of sensors, mainly used for seismometric studies, are extensively used for vibration studies in particle accelerators [107]. In the following, the used geophones are presented. Other devices for vibration measurements, used to enhance the measurable frequency range of geophones, are also presented.

4.2.1 Overview of the sensors available at CERN

A complete list of the vibration sensors that were available for the CLIC stability studies is given in Table 8\textsuperscript{23}. The geophones GSV-320 by GeoSig [147] should be regarded as reference sensors because they have been used for most of the vibration measurements. They have been characterized and calibrated in detail, as discussed in Section 4.2.2 and in Appendix C. The reference geophones are used to measure vibration in the $\approx 2$ Hz to 315 Hz frequency range, which is the relevant range for the CLIC stabilization studies. Due to the steep decrease of vibration amplitudes with the fourth power of frequency, see Eq. (42), frequencies above a few hundreds of Hertz do not contribute significantly

\textsuperscript{23}For completeness, the build-in sensors of stabilization devices (Section 4.3) are also included in the list of Table 8, even if they have not been used in stand-alone mode.
Table 8: Overview of the sensors for measurements of vibrations and displacements used in the framework of the CLIC stability study. The geophones GSV-320 by GeoSig are the reference sensors which have been used for most of the vibration measurements. Note that the last three sensor types are built-in sensors of the stabilization systems described in Section 4.3.

<table>
<thead>
<tr>
<th>Sensor type</th>
<th>Model</th>
<th>Num.</th>
<th>Measure</th>
<th>Freq. range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geophone</td>
<td>GSV-320 by GeoSig</td>
<td>4</td>
<td>Velocity (3 axis)</td>
<td>≈ 1 Hz to 315 Hz</td>
</tr>
<tr>
<td>Geophone</td>
<td>CMG-40T by Guralp</td>
<td>1</td>
<td>Velocity (3 axis)</td>
<td>0.03 Hz to 50 Hz</td>
</tr>
<tr>
<td>Stretched-wire system</td>
<td>WPS2-D by Nanotech</td>
<td>3</td>
<td>Wire position</td>
<td>&gt; 0 Hz</td>
</tr>
<tr>
<td>Capacitive distance-meter</td>
<td>MCC0.5 by Nanotech</td>
<td>1</td>
<td>Distance</td>
<td>Up to 10 kHz</td>
</tr>
<tr>
<td>Accelerometer</td>
<td>JA5L by Sensorex</td>
<td>2</td>
<td>Accelerat. (tilt angle)</td>
<td>Up to 100 Hz</td>
</tr>
<tr>
<td>Capacitive distance-meter</td>
<td>EI-3015 NAMUR, built-in PEPS</td>
<td>3</td>
<td>Distance</td>
<td>Up to 5 Hz</td>
</tr>
<tr>
<td>Geophone</td>
<td>TMC-VSV1, built-in PEPS-VX</td>
<td>3</td>
<td>Velocity (1 axis)</td>
<td>&lt; 7 Hz</td>
</tr>
<tr>
<td>Geophone</td>
<td>Built-in stacis2000</td>
<td>3</td>
<td>Velocity (1 axis)</td>
<td>&lt; 5 Hz</td>
</tr>
</tbody>
</table>

to the total RMS motion. Therefore, the 315 Hz upper limit of the reference geophones is not a significant limitation for the purposes of the CLIC study.

Much slower frequencies can be measured with a stretched-wire system [148], originally used in the CTF2 accelerator for the on-line alignment of the lattice elements. This measurement device allows measuring slow drifts of the table position with respect to external references where the wire is fixed (see Fig. 40). The system has a wide bandwidth, but the upper measurable frequency is actually limited by the sensor resolution. In some controlled conditions (e.g. short wire) the resolution can be pushed down to a few tens of nanometres$^{24}$ (see Section 4.2.3). The CERN area is known to be particularly quiet and to sensor resolution limit is typically attained at frequencies < 0.5 Hz and hence the wire system cannot be used for the measurements of fast vibrations. In addition, it measures relative displacement with respect to the wire fixed point, which are typically well correlated with the ground motion. The ground-to-table transfer function tend to 1 for decreasing frequencies and hence the system cannot be used to measure low vibrations of the table unless they are very large, as it was found for instance in the case on a soft stabilization system (see Section 4.3.1).

In order to measure absolute vibrations below a few Hz, a CMG-40T geophone by Guralp [150] is used. It allows measuring down to 0.033 Hz with resolution good enough to see the ground motion. Together with the reference sensor, the low frequency geophone allow measuring vibrations over a range of four orders of magnitude, from 0.033 Hz to 315 Hz. For budget reasons it has not been possible to buy more than one Guralp sensor. This prevents performing correlation measurements in the low frequency range, which are of great interest for the accelerator-related ground motion studies.

$^{24}$ The resolution of the wire system was cross-checked with laser interferometer techniques and found to be in the order of 30 nm to 50 nm, depending on the measurement conditions [149].
Two accelerometers JA5L by Sensorex [151] are also mounted on the table top. They are prototypes originally conceived for static measurements of tilt angle in previous CLIC test facilities. In the framework of the CLIC stability study the accelerometers have been used for long term measurements of the table stability against pitch and roll tilts. In addition, they were interfaced with a visual data acquisition system for on-line monitoring of the table vibrations. The accelerometers have a bandwidth from 0 Hz (static angle measurements) to 100 Hz and a resolution good enough to reveal vibrations in the typical range of structural resonances of quadrupoles and supports. However, they have not been accurately calibrated for measuring absolute displacements.

A high-resolution capacitive distance meter MCC0.5 was bought for a dedicated experiment aimed at calibrating the reference geophones. The results are described in Appendix E. A number of other sensors (last three lines of Table 8) are included in the stabilization equipment described later in this chapter. These built-in sensors could, in principle, be used in stand-alone mode.

### 4.2.2 Seismometric Geophones

**Functioning principle of geophones**

Geophones are inertial sensors that measure absolute vibration velocities as a function of time. Geophones are the only devices that can provide with sub-nanometre resolution over a wide frequency range the absolute measurements of support and magnet positions.

The main components of a geophone are (1) a reference mass, (2) a spring, (3) a damper, (4) a case, (5) a magnet and (6) a pickup coil (see, for instance, [152]). These components are connected to each other as in the diagram of Fig. 47. The basic idea for a geophone to measure vibrations is that displacements of the field source fixed to the ground induce a voltage in a pick-up coil fixed to the reference mass, which is supposed to be at rest. This provides an absolute measure of the ground motion. The reference mass can be considered as an inertial frame for vibration frequencies larger than the resonant frequency of the spring/mass system, \( f_0 = \sqrt{k/m/(2\pi)} \). This constraint limits the low frequency response of a geophone.

According to Faraday’s induction law the voltage induced in the pickup coil, \( U_{\text{Coil}} \), is proportional to the (vertical) vibration velocity \( v_y \):

\[
U_{\text{Coil}} = -G v_y, \tag{84}
\]

\( G \) is the so-called sensitivity of the geophone and its value depends on the actual geometry and shape of the components. \( v_y \) is the relative velocity between reference mass and ground (here, it is assumed that the case is rigidly fixed to the ground). Following the naming defined in Fig. 47, \( v_y \) given by \( \dot{y}_m - \dot{y}_g \). Conversely, if a current \( i_{\text{Coil}} \) flows into the pickup coil, the reference mass is subjected to a force \( f_m(t) \) given by Ampère’s law [8]:

\[
f_m(t) = i_{\text{Coil}} \left| \int dl \times \mathbf{B} \right| = G i_{\text{Coil}}, \tag{85}
\]

where the integral has to be performed along the coil loops inserted in the magnetic field, \( \mathbf{B} \).
Figure 47: Diagram with the main components of an inertial geophone. The case and the magnet are rigidly fixed to the ground. The reference mass is supposed to be at rest, which is a good assumption for frequencies above the resonant frequency $f_0 = \sqrt{k/m}/(2\pi)$. The relative motion between magnet and mass induces a time-dependent voltage in the pickup coil proportional to the ground velocity.

is the coil resistance $^{27}$, using Eq. (84) the reference mass equation of motion reads:

$$m\ddot{y}_m + (b + \frac{G^2}{R_{\text{Coil}}})\dot{y}_m + k y_m = m\ddot{y}_g + \frac{G^2}{R_{\text{Coil}}} \dot{y}_g,$$

(86)

where $y_m$ and $y_g$ are the positions of reference mass and ground $^{28}$ and $b$ and $k$ are the damper and spring constants (see Fig. 47). Using the Laplace’s transform, Eq. (85) reads:

$$\frac{U_{\text{Coil}}(\omega)}{V_g(\omega)} = \frac{i\frac{\omega}{\omega_0} m\omega_0 R_{\text{Coil}} - 1}{1 - \frac{\omega^2}{\omega_0^2} + 2i\xi \frac{\omega}{\omega_0}},$$

(87)

where $V_g(\omega)$ and $U_{\text{Coil}}(\omega) = GsY_m(\omega)$ are the Laplace’s transform of ground velocity and coil voltage.

There are several possible error sources for the output signal of a geophone $^{[107, 152]}$:

1) distortion, i.e. additional harmonics found in the output signal, which are not present in the input signal;

2) Brownian motion of the reference mass, i.e. random motion of the mass due to its finite temperature;

3) Johnson noise in the electrical circuits, i.e. a white-noise associated to each circuit with a resistance;

4) noise in the electronics for the data acquisition (sensor amplifier, least significant bit of the analog-to-digital converter, etc.).

$^{27}$Typically a shunt resistance $R_s$ is connected in parallel to the coil resistance $R_C$ to adjust the sensor response and hence the resistance that enters in Eq. (85) is given by $R_{\text{Coil}}^{-1} = R_s^{-1} + R_C^{-1}$

$^{28}$It is assumed that the case moves like the ground because they are rigidly fixed to each other.
The distortion is normally specified by the manufacturer as unwanted to wanted signal ratio versus frequency. A typical value for the distortion at around 10 Hz in a fraction of a percent. The reference geophone used in the CLIC Stability Test Stand, for instance, features a distortion smaller than 0.3% at 12 Hz for a relative mass-to-case velocity of 1.8 cm/s. This value is compatible with the purposes of the CLIC Stability Study. The effect of Brownian motion and Johnson noise are typically found to be on the orders of magnitude below the ground motion [107] and hence can be neglected. The largest contribution to the measurement noise comes from the various sources of electrical disturbances that affect the data acquisition. As described in the following section, this contribution has been empirically measured for the reference geophones.

Details of the reference geophones

The reference geophone used for most of the vibration measurements of the CLIC Stability Test Stand is the model GSV–320 by GeoSig and its heart is the geophone SM–6 by INPUT/OUTPUT, INC. [153]. A schematic view of the sensor is given in Fig. 48. This geophone is designed such that the case is a cylindrical shell that also serves as permanent magnet. The pick-up coil is wound around a cylindrical reference mass, inserted inside the case and supported by a spiral spring (not shown in Fig. 48). The axes of the two cylinders are oriented like the measurement direction. Relative displacements of the case with respect to the reference mass induce a voltage in the coil, \( U_{\text{Coil}} \), approximately given by:

\[
U_{\text{Coil}}(t) \approx -n_c(2\pi r_c B)v(t),
\]

where \( v(t) \) is the instant vibration velocity, \( n_c \) is the number of coil turns, \( r_c \) is the radius of the coil and \( B \) is the magnetic field. The right hand side of Eq. (88) must be multiplied by a constant that takes into account the electronic amplification of the read-out signal. Equation (88) gives the absolute value of the vibration velocity. Full information on the phase is given by Eq. (87).

The used GSV–320 geophone is a tri-axial sensor that contains three SM–6 geophones, one for the vertical direction and two for the horizontal ones. Its technical specifications are given in Table 9. One of the GSV–320 sensors, as mounted on top of a CLIC prototype quadrupole, was shown in Figs. 44 and 45. Note that vibration amplitudes below the geophone resonant frequency \( f_0 = 4.5 \) Hz can be calculated using the sensor response function provided by the manufacturer [153]. The sensitivity drops rapidly below \( f_0 \) and this can be taken into account by correcting off-line the acquisition data\(^{29}\). Vibration amplitudes at frequencies down to approximately 2 Hz can then be obtained\(^{30}\). The comparison with other geophones for low frequency vibrations shows indeed that this off-line correction allows obtaining good amplitudes of vibration spectra, as discussed later in this Section and in Appendix D (see, for instance, Fig. 119).

The upper limit of the reference geophone bandwidth is 315 Hz. Due to the steep decrease of ground vibration spectrum as \( f^{-4} \), vibrations above the sensor cut-off frequencies are negligible for the calculation of the total RMS motion above a few Hertz, which is the quantity of interest for the CLIC purposes. Therefore, in the calculation of the total RMS motion (see Eq. (50)) the choice of

\(^{29}\)The geophone model GSV–310 from GeoSig includes an amplifier to increase the signal from 1 Hz to \( f_0 = 4.5 \) Hz, so as to obtain a constant sensor response in the full frequency range. A comparison between the two sensor types showed that the one with amplifier features a slightly larger electrical noise and hence it was decided to use the sensor without amplifier and to apply the data correction off-line.

\(^{30}\)The off-line application of the response function gives a good correction to the vibration amplitudes but information on vibration phase below the sensor resonant frequency is lost. This prevents correlation measurements below approximately 4.5 Hz.
Figure 48: Details of the pick-up sensor of the reference geophones used for the vibration measurements. The reference mass is a cylinder and the coil is wound around it. The case is a permanent magnet shaped has a cylindrical shell and surrounds the reference mass.

\[ f_{\text{max}} = 315 \text{ Hz} \] is practically equivalent to \( f_{\text{max}} = +\infty \).

**Sensor resolution and absolute calibration**

Some technical details of the reference geophones, such as an experimental estimate of the sensor resolution and the frequency calibration, are discussed in detail in Appendix C. Here, the main results are summarized. The estimated sensor resolution versus frequency is shown in Fig. 49. This curve gives the amplitude of a Gaussian noise due to sensor electronics, which is unavoidably included in the measure of the integrated RMS motion. For example, at 4 Hz the sensor noise has an amplitude of 0.28 nm. This noise has to be subtracted in quadrature from the measured raw data of integrated RMS motion. The resolution correction has a visible effect only in case of very low vibrations levels, say smaller than 1 nm RMS, as measured for instance in the underground LHC tunnel or on top of the stabilized table (Section 4.3). This is summarized in Fig. 115.

The estimate of the sensor resolution is important to find the smallest displacement that the vibration sensors can measure. It is also mandatory to verify the absolute calibration of the sensor in order to assess the measurement results and to assign an error on the vibration data. The manufacturer of the used sensors performs a calibration of the full scale output voltage by means of a vibrating surface [154]. The confidence in the measurement results for small vibration amplitudes must rely on the linearity of the sensor sensitivity, as expected from Eq. (84). The geophone calibration at the vibration amplitudes of interest, i.e. in the nanometre and sub-nanometre range, is tested by comparing the measurements by different devices which operate in the same frequency range and have the required resolution. The sensors are placed side-by-side and the data are acquired simultaneously, and possibly with the same data acquisition system \(^{31}\). The reference geophones have been compared with other

\(^{31}\) The comparison of different devices placed side-by-side is probably the best way to assess the low-amplitude calibration because it is nearly impossible to generate controlled vibrations of known amplitude and orientation in the nanometre
Table 9: Technical specification of the geophones GSV–320 by Geosig. Vibration amplitudes below the resonant frequency down to approximately 2 Hz can be calculated using the sensor response function provided by the manufacturer.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geophone type</td>
<td>SM–6 by INPUT/OUTPUT, INC. [153]</td>
</tr>
<tr>
<td>Reference mass</td>
<td>16.1 g</td>
</tr>
<tr>
<td>Number of measurement axes</td>
<td>3</td>
</tr>
<tr>
<td>Instrument type</td>
<td>Digital grade long travel geophones</td>
</tr>
<tr>
<td>Resonant frequency</td>
<td>(4.5 ± 0.5) Hz</td>
</tr>
<tr>
<td>Dynamical range</td>
<td>&gt; 96 dB or better</td>
</tr>
<tr>
<td>Analog to digital converter</td>
<td>16bit</td>
</tr>
<tr>
<td>Resolution</td>
<td>15.3 nm/s</td>
</tr>
<tr>
<td>Calibration</td>
<td>5 mV per 1 μm/s</td>
</tr>
<tr>
<td>Linearity</td>
<td>&lt; 0.3% of full scale</td>
</tr>
<tr>
<td>Cross axis sensitivity</td>
<td>&lt; 0.1% of full scale</td>
</tr>
<tr>
<td>Frequency response</td>
<td>≈ 1 Hz to 315 Hz</td>
</tr>
<tr>
<td>Damping</td>
<td>Standard 0.7</td>
</tr>
<tr>
<td>Full scale output</td>
<td>2.5 V±2.5 V</td>
</tr>
</tbody>
</table>

Figure 49: Resolution of the reference geophone versus frequency as calculated from the difference signal of two geophones placed side-by-side in quiet conditions (LHC tunnel). The shown result gives the resolution on the estimate of integrated RMS displacement. For each frequency \( f_0 \) the resolution is obtained summing all spectra components above \( f_0 \). See Appendix C for more details.
for vibration measurements available at CERN and in other research laboratories:

1) The high resolution capacitive sensor developed MCC0.5 by Fogale [148], which will be described in detail in the next section;

2) A CMG-40T geophone by Guralp for low-frequency vibration used at CERN;

3) Geophones by Guralp for low-frequency vibration used at the European Synchrotron Radiation Facility (ESRF) [176].

Detailed comparisons with all the listed sensors are discussed in Appendices D and E [155]. The comparisons are performed taking simultaneously data with two sensors placed side-by-side. Here, the main results of the comparisons are summarized. In particular, the comparison (1) provides an important cross-check of the reference geophone because it is performed against a sensor that measures vibrations based on a different physical phenomenon, i.e. a capacitive measurement of relative distance (see details in Appendix E).

In Fig. 50 the ratio

\[ R(f) = \frac{I_y^{\text{Other}}(f)}{I_y^{\text{Geo}}(f)} \]  

(89)

is shown. \( I_y^{\text{Other}}(f) \) and \( I_y^{\text{Geo}}(f) \) are the vertical integrated displacements versus frequency, \( f \), as measured simultaneously with one of the sensors listed above and with the geophones by GeoSig, respectively. A deviation of \( R(f) \) from the unity gives relative error between two different sensors used simultaneously to measure the same vibration source, i.e. the ground motion. Note that the measurements have been performed in very different vibration conditions, from the quiet CERN site (motion smaller than 10 nm above 4 Hz) to the noisy ESRF site (motion up to 100 nm). This can explain the considerable differences between the measurements of the two geophones from the same manufacturer (dashed and dotted lines of Fig. 50).

Table 10 summarizes the results of Fig. 50). For several frequencies the spread (standard deviation) between the four lines of Fig. 50) is given. Differences of up to 7% are found. On the base of this comparison, a 10% absolute error is assumed as a (pessimistic) intrinsic uncertainty of the reference sensor calibration.

Statistics in the vibrations measurement

A standard way to reduce the variance on the estimate of the power spectral density, \( P(f) \), [106] is to calculate \( P(f) \) for \( M \) consecutive time sets and then to average the results. This procedure reduces the standard deviation by a factor \( \sqrt{M} \) [156]. The value of \( M \) should be chosen large enough to reduce the statistical error to a negligible level.

If \([f_{\text{min}}, f_{\text{max}}]\) is the frequency range allowed by the measurement device, the acquisition time \( T \) should be longer than \( 1/f_{\text{min}} \) and the sampling time should be larger than \( 2/f_{\text{max}} \) in order to cover all the frequency range of interest. Typically, the acquisition is performed for a total time \( MT \). The power spectral densities calculated for the \( M \) subsets are averaged and then integrated. This is equivalent to averaging the power spectral density calculated for the whole acquisition time over the frequency intervals \( \Delta f = \frac{1}{M \Delta t} \).

Vibration data are typically acquired with the reference sensors for a total measurement time of 3 to 5 minutes and then averaged over sub-sets of 5 seconds. This provides a 0.2 Hz frequency resolution and reduces the statistical uncertainty by a factor 6 to 8.
Figure 50: Function $R(f)$ as defined in Eq. (89) versus frequency, $f$. The CLIC reference geophones are compared with two low-frequency seismometers available at the ESRF (dashed line) and at CERN (dotted line, see Section 4.2.2) and with a capacitive distance-meter (solid line). More details on the sensor comparison are discussed in Appendixes D and E. It is noted that the different comparisons have been carried out in different experimental conditions (the total RMS motion above 4 Hz varies from a few nanometres to $\approx 80 \text{ nm}$).

Table 10: RMS spread (standard deviation) between the measurements of the different sensors as a function of the frequency. Note that the comparisons where carried out in much different background environments, from about 1 nm RMS (capacitive distance meter on the stabilized table) to 80 nm RMS (ESRF site). Data with * do not include the measurements with the capacitive sensors, usable only below approximately 30 Hz.

<table>
<thead>
<tr>
<th>Frequency [Hz]</th>
<th>Spread [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>6.9</td>
</tr>
<tr>
<td>10</td>
<td>6.2</td>
</tr>
<tr>
<td>15</td>
<td>6.0</td>
</tr>
<tr>
<td>20</td>
<td>4.1</td>
</tr>
<tr>
<td>25</td>
<td>3.4</td>
</tr>
<tr>
<td>30*</td>
<td>0.9</td>
</tr>
<tr>
<td>35*</td>
<td>2.0</td>
</tr>
<tr>
<td>40*</td>
<td>2.7</td>
</tr>
</tbody>
</table>
Table 11: Technical specification of the broadband feedback seismometer CMG-40T by Guralp.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of sensors</td>
<td>Broadband feedback seismometer</td>
</tr>
<tr>
<td>Measurement axis</td>
<td>3</td>
</tr>
<tr>
<td>Dynamical range</td>
<td>&gt; 145 dB or better</td>
</tr>
<tr>
<td>Analog to digital converter</td>
<td>24 bit</td>
</tr>
<tr>
<td>Calibration</td>
<td>2 × 800 V/(m/s)</td>
</tr>
<tr>
<td>Frequency response</td>
<td>0.033 Hz to 50 Hz</td>
</tr>
<tr>
<td>Damping</td>
<td>ζ = 0.707</td>
</tr>
<tr>
<td>Full scale output</td>
<td>±10 V</td>
</tr>
<tr>
<td>Lowest parasitic vibration</td>
<td>&gt; 450 Hz</td>
</tr>
<tr>
<td>Reference mass</td>
<td>0.036 kg</td>
</tr>
<tr>
<td>Brownian noise</td>
<td>$10^{-18}$ m/s$^4$/Hz</td>
</tr>
</tbody>
</table>

Low-frequency feedback seismometer

The reference geophone has a frequency bandwidth with a lower limit of a few Hertz. In principle this is sufficiently wide for the purposes of the CLIC stability study since lower frequencies are supposed to be compensated by beam-based feedback system. Nonetheless it is also interesting to measure vibration in the range of a fraction of a Hertz. The broadband feedback seismometer CMG-40T by Guralp was acquired. Similar devices are successfully used for vibration measurements in other laboratories such as DESY (see, for instance, [157, 158]) and ESRF [159, 160]. Its technical specifications are listed in Table 11. Note in particular the 0.033 Hz to 50 Hz frequency range, which allows an overlap between the measurements of the CMG-40T with the reference geophones (see Appendix D).

A feedback geophone relies on a capacitive displacement transducer to stabilize the position of the reference mass. This system is typically required for broadband, low-noise seismometers for measurement of vibrations in the fraction of Hertz range. A detailed model of this kind of geophones goes beyond the scope of this Thesis (see [107] for more details). The CMG-40T sensor contains three orthogonal seismometers, one for vertical measurements and two for horizontal measurements. The free oscillation frequency of each seismometer is around 10 Hz. Three pairs of coil/magnet transducers are used to stabilize the mass and to obtain a linear response in the full frequency range.

4.2.3 Capacitive sensors

Capacitive sensors measure the relative position of metal wires or surfaces with respect to a reference metal surface. The capacitor formed by the reference surface and the wire has an electric capacity from which the surface to wire distance can be inferred. A stretched-wire system was used to monitor the long term stability of the honeycomb support structure and a capacitive distance-meter was compared with the reference geophones to cross-check the sensor calibration. The former is described in the following, whereas the latter is described in Appendix E, where its dedicated experiment is also discussed. The common problem of these two capacitive sensors is that they are only capable of measuring relative positions and are not usable to perform an absolute vibration measurement.

The WPS2-D stretched-wire system is a capacitive sensor developed by Fogale Nanotech [148] for the two-dimensional alignments of objects. The technical specifications are listed in Table 12. The
system uses as reference a metal wire stretched all along the object to measure, i.e. the honeycomb support structure in this case (see Fig. 40). Capacitive, non-contact sensors are used to measure the wire position with a sub-micrometre resolution. Views of the wire installation and the geometry of the capacitive sensor are given in Fig. 51 left and right parts, respectively. The wire position is measured with respect to all four inner metal surfaces, which provide two position measurement per measuring plane. This differential measure in each plane allows an improvement of the measurement precision. The best linearity of the measured distance versus capacity of the wire/plate system is achieved around the centre of the four plates but is quite good over the full measuring range of 1 cm (see Table 12).

The resolution in position measurements over the full bandwidth is 0.3 μm. The measurement noise decreases as the square of the bandwidth [161]. Therefore, a slow acquisition frequency of 1 Hz ensures a nominal resolution in the tens of nanometre range. This resolution was cross-checked with laser interferometric techniques [149].

4.3 Stabilization techniques

In this section two modern devices for the passive and active stabilization, as used in the framework of the CLIC Stability Study, are presented and their performances are discussed. The first tested system is a pneumatic isolator based on pressurized air whereas the second one is a stiff isolator based on rubber and piezoelectric actuators. Both devices make use of the basic principles drawn in Section 3.3.1 for stabilizing a support structure: a passive isolator damps the ground motion and an active isolation reduces the resonant frequency of the system and possibly compensates vibrations induced by the pay-load to stabilize.

4.3.1 Soft air-pressure system

*Principle of functioning and installation*

The soft pneumatic system PEPS-VX by TMC [137] has been used to stabilize the honeycomb support structure as shown in Fig 40. The technical specifications of this system are listed in Table 13 and a scheme of a single isolator “foot” is schematically drawn in Fig 52. Photographs of the foot are given in Fig. 53. The system provides active and passive stabilization of a load against vertical vibrations and against pitch and roll angles and also the capability of positioning the load with micro-metric precision. The passive damping of the ground oscillations is provided by a volume of pressurized air that sustains the pay-load. The load is floating on an air pillow that efficiently damps the ground motion above a few Hz. The pressurized air provides a passive damping, like a very soft spring. An active damping can also be achieved by adjusting the airflow (see later). On top of that, a slow feedback system is used to adjust the table position with micro-metric resolution. This feedback is based on the measures of three micro metric proximity sensors (see the yellow cylinder in Fig. 53, right part) which measure continuously the table position with respect to ground. The nominal precision of this system is about 1 μm.

The purely passive vibration damping provided by the air pressure has a relevant resonant frequency (see Section 3.3.1) in the 1 Hz to 2 Hz frequency range. This resonance would considerably limit the stabilization performance if left uncorrected. An active damping system is therefore used to suppress the proper frequency. It is based on geophones placed on the load top (see Fig. 52) to measure the load vibrations. The measurement signal from the geophones is fed back into the control valve that supplies the air to the isolator and the airflow is adjusted on-line to compensate the measured vibrations. Note that the velocity sensors of the PEPS-VX have a low-pass band limited at 5 Hz.
Table 12: Technical specifications of the WPS2-D stretched-wire system by Fogale Nanotech.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of pick-up sensors</td>
<td>3</td>
</tr>
<tr>
<td>Measurement range (both planes)</td>
<td>±5 mm</td>
</tr>
<tr>
<td>Output voltage</td>
<td>0 V-10 V</td>
</tr>
<tr>
<td>Average sensitivity (both planes)</td>
<td>1 V/mm</td>
</tr>
<tr>
<td>Linearity (full range)</td>
<td>±0.150 mm</td>
</tr>
<tr>
<td>Linearity (±0.5 mm)</td>
<td>≈ ±1.5 × 10⁻³ mm</td>
</tr>
<tr>
<td>Horizontal to vertical coupling (full range)</td>
<td>0.8 mm</td>
</tr>
<tr>
<td>Uncertainty of reference centre</td>
<td>±0.1 mm</td>
</tr>
<tr>
<td>Thermal drift</td>
<td>0.5 μm/°C</td>
</tr>
<tr>
<td>Bandwidth (1st order)</td>
<td>10 Hz</td>
</tr>
<tr>
<td>Measurement noise (full bandwidth, 3 m wire)</td>
<td>0.3 μm</td>
</tr>
</tbody>
</table>

Figure 51: Installation of the stretched wire system to monitor the alignment of the support table with respect to the ground (left) and the scheme of the pick-up capacitive sensor to measure the wire position (right).
Figure 52: Key components of one isolator of the soft pneumatic system PEPS-VX by TMC [137], which provides passive and active stabilization of a load. The passive damping is obtained with a pressurized volume that sustains the load. A micrometre distance meter allows levelling and keeping the desired load position with micro-metric precision. The system resonant frequency is in the $1\text{ Hz}$ to $2\text{ Hz}$ range and is suppressed with an active feedback system. Geophones placed on the load top measure vibrations and the airflow is adjusted accordingly in order to damp the load vibrations.

Figure 53: Photographs of one PEPS-VX isolator by TMC, with the main components described in Fig. 52.
Table 13: Technical specifications of the PEPS-VX isolator system by TMC.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of isolators</td>
<td>4</td>
</tr>
<tr>
<td>Active degrees of freedom</td>
<td>3</td>
</tr>
<tr>
<td>Maximum servo bandwidth</td>
<td>10 Hz</td>
</tr>
<tr>
<td>Range of active damping</td>
<td>0.5 Hz to 7 Hz</td>
</tr>
<tr>
<td>Resonant frequency (active system)</td>
<td>(\approx 0.2) Hz</td>
</tr>
<tr>
<td>Levelling accuracy - height</td>
<td>(\approx 2) (\mu)m</td>
</tr>
<tr>
<td>Levelling accuracy - roll, pitch</td>
<td>(\approx 2) (\mu)rad</td>
</tr>
<tr>
<td>Proximity sensors</td>
<td>M-30 NAMUR eddy current sensors</td>
</tr>
<tr>
<td></td>
<td>15 mm operating height</td>
</tr>
<tr>
<td>Velocity sensors</td>
<td>TMC-VSV1 5 Hz long-throw sensors</td>
</tr>
</tbody>
</table>

and hence it is designed to compensate only the system low-frequency resonance and not sources of faster vibrations generated on the load itself.

Four isolators are used to support the honeycomb table and are installed as shown in Fig. 54. The degrees of freedom for the active alignment and stabilization are only three: vertical height, pitch and roll. Three distance meters and three velocity sensors monitor positions and vibrations at three different table locations in correspondence of three isolators (see Fig. 54). The feet 3 and 4 are connected to the same channel for the airflow supply and use the signal of one geophone/distance meter pair only. Note that both geophones and distance meters measures only vibrations/positions along the vertical direction. The feedback gains for the three active degrees of freedom (height, pitch and roll) have to be independently hand-adjusted by the user to find the working point that better suppresses the system resonant frequency. Larger gains improve the low frequency stability at the expense of the passive stabilization efficiency at high frequencies.

The support structure was found not to be wide enough to ensure a good suppression of table rolls when using the PEPS-VX system. In order to overcome this problem, extension slabs, about 1.5 m long, were used to increase the distance between feet 1-2 and 3-4, as shown in Fig. 54.

**Measured performance**

The performance of the PEPS-VX system is tested by mounting the support table as in Fig. 54. Two geophones sit on the table top and one is on the ground. This allows measuring the ground-to-table transfer function in two different table locations. The geophone data are acquired with a sampling time of 0.001 s for a total time of 3 to 5 minutes and the Fourier analysis results are averaged over subsets of 5 s. This procedure reduces the statistical error to negligible values and hence the error bars are not indicated in the following graphs.

In Figure 55, the power spectral density of the vertical ground and table motion, \(P_y(f)\), is given. As seen from Fig. 55, the passive system provides a vibration damping above approximately 6 Hz (dashed line). Instead, lower frequencies are considerably amplified. By switching on the active feedback system with a proper adjustment of the gains, the low frequency amplification can be suppressed (see dotted line). The price to pay is an amplification at higher frequencies. Namely, a peak at about 20 Hz is induced. However, an improvement of the ground motion in all frequency range can be obtained. This is shown in Fig. 56, where the vertical RMS motion versus frequency, \(I_y(f)\), is given.
Figure 54: Installation of the PEPS-VX isolators under the honeycomb support structure. The system stabilizes actively three degrees of freedom (height, pitch and roll) using as input the position and vibration measurements from three pairs of proximity sensors and velocity sensors. The isolators 3 and 4 use only one distance/velocity sensor pair and are connected in parallel to the active airflow control system. Extension slabs made of steel are used to increase the transverse distance between the feet in order to improve the stability against table pitch angles.

Figure 55: Vertical power spectral density, $P_y(f)$, versus frequency, $f$, as measured on the ground and on top of the table mounted on the air-pressure system. The active feedback system reduces the low-frequency resonance of the passive system of approximately 2 Hz, which amplify the ground motion below $\approx 6$ Hz.
Figure 56: Vertical RMS motion, $I_y(f)$, versus frequency, $f$, integrated above $f$ as measured on the ground and on top of the table mounted on the air pressure system.

Figure 57: Ground-to-table transfer function of vertical vibrations, $\sqrt{P_{Tab}(f)/P_{Grd}(f)}$, versus frequency, $f$, when the air-pressure system is used. It is calculated as square root of the table-to-ground ratio of power spectral densities (see Fig 55). Indexes “Tab” and “Grd” state for “table” and “ground”, respectively.
For instance, the RMS vertical motion above 4 Hz is reduced from 7.40 nm to 1.78 nm by the active system, to be compared with 4.80 nm obtained using the passive damping only.

The ground-to-table transfer function is defined as the square root of the table-to-ground ratio of power spectral densities and gives the amplification of the ground motion due to the support (see Eq. (80)). The PEPS–VX transfer function is given in Fig. 57. It is also interesting to look at the table-to-ground ratio of RMS motion versus frequency, as shown in Fig. 58. A maximum damping of a factor 10 can be obtained with the active system.

In Figure 59, typical transfer functions of the active PEPS–VX system for the two horizontal directions are given. The horizontal stabilization is worse than the vertical one. This is related to the fact that the three pick-up geophones used by the active feedback measure only vertical vibrations. Note that the table is more stable longitudinally (z axis as defined in Fig. 40) than transversally (x axis) because the isolators are more distant and the correction is more efficient.

The vibration levels on the table top is found to depend considerably on the location. This is shown in Fig. 60. The table part laying on the legs 3 and 4 (see the diagram in Fig. 54) is more stable below approximately 10 Hz. The active system works less well on the other table side. The feedback gains are set for three degrees of freedom of the whole support structure (height, pitch and roll) and it is not possible to set the system to prevent the mentioned inhomogeneities at different table locations.

Monitoring of the slow table motion with the stretched-wire system has shown that often the table features a strong, broad resonance at approximately 0.035 Hz. An example of power spectral density of table vertical motion is given in Fig. 61. The wire system provides a measure of the relative motion between table and ground, to which the wire is fixed via its two supporting concrete blocks (see Figs. 40 and 41). Note that the low-frequency resonance is not systematically measured in the table motion. When present, it has typical RMS amplitudes ranging from 2 µm to 15 µm. The source of this vibration has not been identified to date. If this issue remained unsolved, this low-frequency vibration would prevent using the soft stabilization system in a high-energy particle accelerator like CLIC.

4.3.2 STIFF PIEZO-BASED SYSTEM

Principle of functioning and installation
The Stacis2000 stabilization system by TMC [137] consists of three or four independent isolators used to support the object to stabilize, i.e. the honeycomb support structure of Fig. 41. The technical specifications of the system are listed in Table 14. Figures 62 and 63 show a schematic view of the vertical cross-section and a photograph of one Stacis2000 isolator. Each isolator (foot) is an independent stabilization system that provides vibration damping in three degrees of freedom (vertical and two horizontal directions). As recommended by the manufacturer, the full system consists of three or four isolators, so as to provide enough degrees of freedom for a 6-dimensional stabilization of the load.

Each Stacis2000 isolator is designed to obtain both a passive and an active damping of the load vibrations. The passive damping from the ground is provided by stiff rubber that surrounds the foot leg (grey area in Figure 62). The rubber has a vertical resonant frequency of about 18 Hz and a horizontal resonant frequency between 12 Hz and 15 Hz, depending upon the load. In order to compensate these natural resonances and to reduce possible vibrations generated on the load, an active system is used. It is based on a closed-loop feedback between geophones, which measure the load vibrations, and piezoelectric actuators, which counteract the measured motion. As shown in Fig. 62, the geophones are mounted on the upper part of the isolator, fixed to the object to be stabilized, whereas the piezoelectric actuators are placed under and at the side of the foot leg. One vertical and
Figure 58: Table-to-ground ratio of integrated vertical RMS motion, $I_{Tab}(f)/I_{Grd}(f)$, versus frequency, $f$, when the honeycomb table is mounted on the PEPS-VX, with (solid line) and without (dashed line) active damping.

Figure 59: Table-to-ground ratio of integrated RMS motion, $I_{Tab}(f)/I_{Grd}(f)$, versus frequency, $f$, when the honeycomb table is stabilized with the pneumatic system (active damping system on). The three Cartesian directions are shown.
Figure 60: Vertical RMS motion, $I_y(f)$, versus frequency, $f$, integrated above $f$ as measured on the ground and on different table locations with active PEPS–VX system. See Fig. 54 for the feet naming.

Figure 61: Power spectral density of the vertical table position relative to ground, $P_{y,rel}(f)$, versus frequency, $f$, as measured with the stretched-wire system (see Fig. 40) when the table is mounted on the PEPS–VX active system. The wire system provides a relative measurement of the table with respect to the ground. The RMS vertical motion corresponding to the broad peak at about 0.035 Hz varies from 2 µm to 15 µm depending on the experimental conditions. The peak was not always present during the PEPS–VX operation. Its source has not been found to date.
two horizontal geophone/actuator pairs provide the load stabilization in three dimensions 32.

The piezoelectric crystals used to counteract the load vibrations need to be loaded to work properly. The load on the vertical actuators is provided by the weight of the object to stabilize, whereas for the horizontal actuators the load is provided by stiff springs that push the foot leg against the crystal on the opposite side, as shown in Fig. 62. The need for a load on the piezoelectric crystal fixes a minimum load weight to be put onto each foot for the active damping to work efficiently and a maximum load weight for the crystals not to break (see Table 14).

Table 14: Technical specifications of the stiff isolation system Stacis2000 by TMC.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of isolators</td>
<td>3 or 4</td>
</tr>
<tr>
<td>Active degrees of freedom</td>
<td>6</td>
</tr>
<tr>
<td>Active bandwidth</td>
<td>0.3 Hz to 250 Hz</td>
</tr>
<tr>
<td>Resonant frequency (active system)</td>
<td>( \approx 0.2 \text{ Hz} )</td>
</tr>
<tr>
<td>Resonant transmissibility</td>
<td>1.1</td>
</tr>
<tr>
<td>Dynamics range</td>
<td>&gt; 60 dB</td>
</tr>
<tr>
<td>Static load capacity per isolator</td>
<td>182 kg to 1590 kg</td>
</tr>
<tr>
<td>Maximum displacement</td>
<td>12 ( \mu \text{m} ) below 10 Hz (peak-to-peak)</td>
</tr>
</tbody>
</table>

As shown in Table 14, the active system with three or four isolators has a resonant frequency of 0.1 Hz to 0.3 Hz depending on the load, with a moderately small transmissibility of 1.1. This should not be a problem for the use of the system in linear accelerators because the resonant frequency is low enough to be efficiently corrected with beam-based techniques, which for CLIC will be effective to correct the motion up to about 4 Hz. On the other hand, the system bandwidth is limited at 250 Hz. For higher frequencies the motion is amplified by the electrical noise in the feedback loop, as also noticed in the studies related to the gravitational wave detectors [162], which have investigated the Stacis2000 system as a possible device to damp ground vibrations. However, the amplitude of the ground motion at these high frequencies is generally so small that the observed amplification up to a factor of 5 from the isolation system is not relevant for linear collider purposes 33.

Note, that the stiff isolation system uses a stabilization method different from the standard active and passive isolator systems, as discussed in Section 3.3.1. The active compensation of the measured load vibrations makes use of a so-called “intermediate mass”, where the damping rubber is installed. In the diagram of Fig. 62 the intermediate mass corresponds to the structure that includes the damping rubber and the foot leg. The piezoelectric actuators are directly fixed on the floor and act on the intermediate mass, which is itself fixed to the load. This method proves [137] to provide a very good damping of the vibrations in the 0.2 Hz to 20 Hz frequency range, where the active system is very effective. Load frequencies above 20 Hz are instead damped passively by the rubber of the intermediate mass. The performance of this method is discussed later in this section and is compared with what is achieved with the PEPS-VX system, which instead uses the standard stabilization techniques (see Section 3.3.1). A drawback of the Stacis2000 design is the limitation of its operating range by the maximum elongation of the piezoelectric crystal, i.e. 20-25 \( \mu \text{m} \). This prevents a large scale of the

32 Unlike in the scheme of Fig. 62, three piezoelectric crystals are actually used for the vertical stabilization in order to support the weight of the load
33 This is not the case for the studies of the gravitational waves, where a stabilization to the femtometre level (\( 10^{-15} \text{ m} \)) is aimed for [163].
Figure 62: Vertical cross-section of one isolator of the Stacis2000 stabilization system. A passive damping is provided by stiff rubber (grey area that surrounds the foot leg) with a natural resonant of about 15 Hz depending upon the load. This vibration is damped with an active system based on a closed-loop feedback between geophones (boxes with arrows), which measure the load motion, and piezoelectric actuators, which counteract the measured vibrations. The geophones are fixed to the load and hence table vibrations within the active bandwidth (Table 14) can also be damped.

Figure 63: Photograph of one Stacis2000 isolator as used in the CLIC Stability Study test stand. Three or four isolators are used to support and stabilize a 700 kg honeycomb table (see Fig. 40).
Figure 64: Possible installation of the Stacis2000 system under the honeycomb support structure using three of four isolators. Each foot works independently of the others and is directly connected to the active stabilization control system.

load position. The following solution can be applied to the CLIC case: the alignment supports as one in Fig. 45 can be directly mounted onto the table top. As discussed in Chapter 5.6, this provides a very good magnet stability while preserving the alignment functionalities of the support structure designed for CTF2.

Two possible installations of the Stacis2000 feet under the honeycomb support structure are shown in Fig. 64. Either three or four isolators can be used. The three feet configuration proves to provide a better ground-to-table transverse and longitudinal transmission but a slightly worse vertical stability than the installation with four feet. The measurement results are discussed in detail below. Since the vertical tolerances are tighter than the horizontal ones, it seems preferable to adopt the four feet system.

Each isolator works independently of the other. The gains of the three feedback loops that correct the Cartesian directions can be hand-set for each isolator. This is done with an oscilloscope that shows the input-to-output signal ratio which has to be minimized. The optimization must be carried out for each isolator iteratively, until an homogeneous performance at the different table locations is achieved.

**Measured performance**

Similarly to the tests of the pneumatic system, the characterization of the stiff stabilization system is performed by mounting the honeycomb table on three or four isolators (see Fig. 64) and measuring simultaneously the vibrations on the ground and on the table top. The geophone data are acquired with a sampling time of 0.001 s for a total time of 3 to 5 minute and the Fourier analysis results are averaged over subsets of 5 s. This procedure reduces the statistical error to negligible values and hence the error bars are not indicated in the graphs below.

The vertical power spectral density and RMS motion versus frequency, as measured on the ground and on the table top, are shown in Figs. 65 and 66, respectively. Figure 67 shows the ground-to-table transfer function calculated as squared ratio of the power spectral densities of Fig. 65. The stabilization system provides a very efficient damping of the vertical ground motion at least up to 100 Hz. Reductions up to a factor $\approx 15$ are obtained. For instance, the RMS motion above 4 Hz is reduced from 6.20 nm to 0.52 nm. This excellent stability is also obtained when prototype magnets are mounted on the table top. The impact on the CLIC luminosity performance is discussed in Section 6.

The horizontal longitudinal and transverse performance of the stiff system are respectively shown in Figs. 68 and 69, where the RMS motion as measured on the ground and on the table is shown. The
Figure 65: Vertical power spectral density, $P_y(f)$, versus frequency, $f$, as measured on the ground (solid line) and on top of the table, mounted on the stiff stabilization system (dashed line).

The system does not provide a horizontal stabilization of the same excellent quality as the vertical one. In particular, with the setup that provides the best vertical stability there is no improvement with respect to the ground in the table transverse direction. The performance is slightly better in the longitudinal direction. As a general feature, the active stabilization works better in the longitudinal table axis than in the transverse since the active feet are more distant.

The ground-to-table transmission of the stiff stabilization system for the three Cartesian directions is summarized in Fig. 70. The table-to-ground ratio of vertical, longitudinal and transverse RMS motion is given against frequency. Note the motion amplification above approximately 250 Hz, out of the active system bandwidth (see Table 14). As shown in Figs. 66, 68 and 69, in this frequency range the ground motion is so small that the amplification due to the stabilization system is not relevant for linear colliders.

The system with three feet, installed as in Fig. 64 left part, features a better transmissibility in the horizontal directions if compared to the system with four feet. This is shown in Fig. 71 [164]. The reason might be the better match of degrees of freedom (in general three supporting points stabilize a plane surface better because a plane is defined by three points). With four isolators the system might be more unstable against roll and pitch angle perturbations.

Since only one low-frequency geophone is available for the CLIC stability study, it is not possible to measure the transmissibility of the system below 1 Hz with simultaneous measurements on the ground and on the table top. Vibration measurements have instead been performed one after the other, with a delay of a few minutes between the end of one data taking and the start of the next one. The table transmissibility can then be calculated in the assumption that the ground motion and the table performance do not change considerably during the total measuring time. This assumption has been verified with the reference geophone.

34 This is true in particular for low frequencies, which are loosely influenced by the cultural noise. Larger differences are expected above a few Hertz. In order to reduce the effects of cultural noise, measurements were performed after the working hours, with no people nor cars moving in the vicinity of the laboratory.
Figure 66: Vertical RMS motion, $I_y(f)$, versus frequency, $f$, integrated above $f$ as measured on the ground (solid line) and on top of the table, mounted on the stiff stabilization system (dashed line).

Figure 67: Ground-to-table transfer function of vertical motion, $\sqrt{P_{\text{Tab}}(f)/P_{\text{Grd}}(f)}$, versus frequency, $f$, as measured when the honeycomb table is mounted on the stiff stabilization system.
Figure 68: Longitudinal RMS motion, $I_z(f)$ (see Fig. 40), versus frequency, $f$, integrated above $f$ as measured on the ground (solid line) and on the table top (dashed line). The honeycomb support structure is mounted on the stiff isolation system.

Figure 69: Transverse horizontal RMS motion, $I_x(f)$ (see Fig. 40), versus frequency, $f$, integrated above $f$ as measured on the ground (solid line) and on the table top (dashed line). The honeycomb support structure is mounted on the stiff isolation system.
Figure 70: Table-to-ground ratio of integrated RMS motion, $I_{\text{Tab}}(f)/I_{\text{Grd}}(f)$, versus frequency, $f$, when the honeycomb table is mounted on the stiff stabilization system with four isolators. Vertical (solid line), longitudinal (dashed) and transverse (dotted) directions are shown.

Figure 71: Table-to-ground ratio of the RMS motion, $I_{\text{Tab}}(f)/I_{\text{Grd}}(f)$, versus frequency, $f$, when the honeycomb table is mounted on the stiff stabilization system with three isolators. Vertical (solid line), longitudinal (dashed) and transverse (dotted) directions are shown.
Figure 72: Square root of table-to-ground ratio of vertical power spectral density, $\sqrt{P_{\text{Tab}}(f)/P_{\text{Grd}}(f)}$, versus frequency, $f$, when the honeycomb table is mounted on the stiff isolation system. Measurement are performed after the working hours with one low-frequency geophone with a delay of 15 minutes (the total measurement time is 10 minutes). The ground spectrum is assumed to be the same for the two data sets, which should be a good approximation at low frequencies.

The square root of the table-to-ground ratio of vertical power spectral densities is shown in Fig. 72. The ratios of the RMS motion in the three dimensions are given in Fig. 73. The vertical transmissibility is good below 1 Hz: no significant amplification of the ground motion is observed. On the other hand, longitudinally the motion below approximately 0.1 Hz is amplified. Transversally, a large amplification is observed below 1 Hz.

It is noted that the stiff stabilization system Stacis2000 was found to be very sensitive to electromagnetic waves. For instance, mobile phones working close to the table induced instabilities on the active system and spoilt the stabilization performance. Thus, some kind of screening of the isolators must be foreseen for applications in the particle accelerator domain.
Figure 73: Table-to-ground ratio of the RMS motion, $I_{\text{Tab}}(f)/I_{\text{Grd}}(f)$, versus frequency, $f$, when the honeycomb table is mounted on the stiff isolation system. Vertical (solid line), longitudinal (dashed) and transverse (dotted) directions are shown. Measurements are performed with one low-frequency geophone and not simultaneous. The ground spectrum is assumed to be the same for the two data sets, which should be a good approximation at low frequencies. On the other hand, differences above approximately 1 Hz are more likely because the ground motion in this range is strongly affected by the cultural noise. This might explain the differences with respect to the results in Fig. 70.
5 Measured stability performance of the CLIC quadrupoles

The experimentally achieved stabilizaty of CLIC prototype quadrupoles is discussed. The analysis of the magnet motion is divided in high-frequency range (above $\approx 1$ Hz) and low-frequency range (below $\approx 1$ Hz). “Fast” and “slow” motions have different effects on the beam dynamics and impose different correction techniques. It is shown, that a sub-nanometre RMS motion at the frequencies of interest for CLIC can be achieved with state-of-the-art stabilization equipment. The measurements of other effects, such as the cooling water and the structural resonances of magnet supports, are also discussed. These results are the base for a realistic estimate of the luminosity performance of CLIC.

5.1 Achieved stabilization of CLIC quadrupoles

The best damping of quadrupole fast vibrations ($f \gtrsim 1$ Hz) is obtained by fixing the magnets on top of the honeycomb support structure, as in Fig. 44 and the stiff piezo-based isolators (Section 4.3.2). The measured vertical and horizontal quadrupole stability is shown in Figs. 74 and 75, respectively. The RMS motion, $I(f)$, versus frequency, $f$, as integrated above $f$ is shown (solid lines). Measured quadrupole vibrations are compared with the CLIC linac and final focus tolerances above 4 Hz (dotted lines, see Table 6). The ground vibration level is also shown as a reference (dashed lines). The vertical and horizontal measured RMS displacements above some values of the minimal frequency are listed in Tables 15 and 16, respectively. The results presented in this section are obtained by integrating the measured vibrational spectra up to $f_{\text{max}} = 315$ Hz, as limited by the final bandwidth of the reference geophone. For the purpose of this section, this is equivalent to integrating up to infinity since faster vibrations do not contribute significantly to the total RMS (see Section 4.2.2).

Above 4 Hz the quadrupole doublet was stabilized to ($0.43 \pm 0.04$ nm) with a ground motion of ($6.19 \pm 0.62$ nm). The quadrupole vertical vibration level is within the CLIC linac tolerance ($1.3$ nm) and only a factor 2 larger than the final doublet tolerance ($0.2$ nm). The transverse horizontal RMS motion above 4 Hz was ($0.79 \pm 0.08$ nm) instead of ($3.04 \pm 0.30$ nm) on the ground, i.e. factors 10 and 18 smaller than the linac and final focus tolerances ($14.0$ nm and $7.8$ nm, respectively). The longitudinal horizontal RMS motion above 4 Hz was ($4.29 \pm 0.43$ nm) instead of ($4.32 \pm 0.43$ nm) on the ground. The quadrupole doublet is mounted such that its transverse horizontal axis ($x$) is aligned with the table longitudinal axis (see Fig. 40), which provides the best horizontal performance.

This is not the installation expected in a linear collider tunnel. In order to avoid too wide tunnels (more expensive) it would be preferable to align the table longitudinal axis with the beam path ($z$). Nevertheless, the table does not induce any significant amplification of the horizontal ground motion and measured vibration stays within tolerance.

Real and imaginary parts of the correlation between table and ground vertical motions are shown in Fig. 5.1 left and right parts, respectively. Due to the passive and active isolation (resonant frequency of $\approx 0.2$ Hz) the motion on table top is basically not correlated with the ground above a few Hertz. Correlation measurements in deep underground tunnels such as at LEP/LHC, show that ground motion can show a good correlation up to approximately 10 Hz at distances of tens of metres [82]. This is beneficial for the luminosity performance because the final doublets of the opposing machines are only some meters apart. The use of stabilization equipment can decrease the correlation with respect to the ground properties. This issue is discussed in detail in the next Section.

\[35\] Note that table settings have been found which provide a better horizontal performance than the one discussed here, for instance when it is used with three isolators instead of four (Section 4.3.2). However, the results that provide the best vertical stability are discussed here. This is justified because an even poor horizontal stability performance allows keeping
Figure 74: Vertical RMS motion, $I_y(f)$, versus frequency, $f$, integrated above $f$ as measured on the ground (dashed line) and on a CLIC prototype quadrupole (solid line), mounted on the stiff stabilization system as in Fig. 44. The dotted lines indicate the CLIC linac (1.3 nm) and final focus (0.2 nm) tolerances above 4 Hz (see Table 6). Measurements are performed in the morning of a working day (February 2003).

Figure 75: Horizontal transverse RMS motion, $I_x(f)$, versus frequency, $f$, integrated above $f$ as measured on the ground (dashed line) and on a CLIC prototype quadrupole (solid line), mounted on the stiff stabilization system as in Fig. 44. The dotted lines indicate the CLIC linac (14.0 nm) and final focus (7.8 nm) tolerances above 4 Hz (see Table 6). The horizontal quadrupole axis ($x$) is aligned with table longitudinal axis (see Fig. 40). Measurements are performed in the morning of a working day (February 2003).
Table 15: Vertical RMS motion above different minimal frequencies \( (f) \) as measured on the ground and on a quadrupole doublet stabilized with the stiff isolation system. The quadrupole is directly screwed on top of the honeycomb support structure (see Fig. 44). Measurements are performed in the morning of a working day (February 2003). Measured data are corrected to take into account the measured sensor resolution. According to Section 4.2.2, a 10% error is assigned as a (pessimistic) estimate of the absolute measurement error.

<table>
<thead>
<tr>
<th>Vertical RMS motion [ nm ]</th>
<th>Ground</th>
<th>Quadrupole</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f \geq 4 \text{ Hz} )</td>
<td>6.19 ± 0.62</td>
<td>0.43 ± 0.04</td>
</tr>
<tr>
<td>( f \geq 20 \text{ Hz} )</td>
<td>2.67 ± 0.27</td>
<td>0.36 ± 0.04</td>
</tr>
<tr>
<td>( f \geq 60 \text{ Hz} )</td>
<td>1.01 ± 0.10</td>
<td>0.14 ± 0.01</td>
</tr>
<tr>
<td>( f \geq 100 \text{ Hz} )</td>
<td>0.40 ± 0.04</td>
<td>0.08 ± 0.01</td>
</tr>
</tbody>
</table>

Table 16: Horizontal and longitudinal RMS motion above different minimal frequencies \( (f) \) as measured on the ground and on a quadrupole doublet stabilized with the stiff isolation system. Here, the data set obtained with the vertical stability of Table 15 are listed. Measurements are performed in the morning of a working day (February 2003). Measured data are corrected to take into account the measured sensor resolution. According to Section 4.2.2, a 10% error is assigned as a (pessimistic) estimate of the absolute measurement error.

<table>
<thead>
<tr>
<th>Horizontal RMS motion [ nm ]</th>
<th>Horizontal</th>
<th>Ground</th>
<th>Quadrupole</th>
<th>Longitudinal</th>
<th>Ground</th>
<th>Quadrupole</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f \geq 4 \text{ Hz} )</td>
<td>3.04 ± 0.30</td>
<td>0.79 ± 0.08</td>
<td>4.29 ± 0.43</td>
<td>4.32 ± 0.43</td>
<td>4.29 ± 0.43</td>
<td></td>
</tr>
<tr>
<td>( f \geq 20 \text{ Hz} )</td>
<td>0.50 ± 0.05</td>
<td>0.49 ± 0.05</td>
<td>0.63 ± 0.06</td>
<td>0.63 ± 0.06</td>
<td>0.63 ± 0.06</td>
<td></td>
</tr>
<tr>
<td>( f \geq 60 \text{ Hz} )</td>
<td>0.20 ± 0.02</td>
<td>0.12 ± 0.01</td>
<td>0.28 ± 0.03</td>
<td>0.14 ± 0.01</td>
<td>0.14 ± 0.01</td>
<td></td>
</tr>
<tr>
<td>( f \geq 100 \text{ Hz} )</td>
<td>0.08 ± 0.01</td>
<td>0.09 ± 0.01</td>
<td>0.15 ± 0.02</td>
<td>0.08 ± 0.01</td>
<td>0.08 ± 0.01</td>
<td></td>
</tr>
</tbody>
</table>
The phases of vertical quadrupole motion, $\phi_y(f)$, are shown versus frequency in the left part of Fig. 5.1. The right part of Fig. 5.1 shows the phase distribution. These measurements show that there is no correlation between the phases of different spectral components. Similar results are found for the vibrations on the ground. These results will be used for the generation of time-dependent magnet displacements starting from the measured vibrational spectra.

Figure 77: Phase of vertical quadrupole vibrations, $\phi_y(f)$, versus frequency, $f$, as measured when a doublet is mounted on the stiff stabilization system (left graph) and the corresponding distribution (right graph).

the quadrupole vibrations within tolerances.
5.1.1 Comparison with previously achieved magnet stabilities

In Figure 78, the experimentally achieved stability of magnet-like objects, as obtained in experimental test stands at DESY [157] and SLAC [108, 109, 165], is compared with the results of Fig. 15. Shown, is the vertical RMS motion above 4 Hz of stabilized magnets (diamonds) and of the supporting ground (triangles) as measured at DESY, SLAC and CERN test stands. The DESY data refer to the achieved vertical stabilization of a quadrupole magnet by means of a hand-made feedback system used to damp ground motion [157]. At SLAC, a 40 kg aluminium block is supported by six springs and stabilized by means of closed-loop feedback systems between inertial sensors and piezoelectric actuators (the system has six active degrees of freedom) [108].

Figure 78 shows that in the framework of the CLIC stability study the absolute magnet stability was advanced by approximately a factor 20 with respect what was previously achieved, i.e. from the $\approx 10$ nm level to the $\approx 0.5$ nm level (vertical stability). It is noted that the various stabilization experiments of Fig. 78 were carried out in different environment conditions. The ground motion at the CERN site is of several nanometres (RMS), i.e. considerably smaller than at the SLAC site ($\approx 20$ nm) and the DESY site ($\approx 40$ nm). Nevertheless, at the CLIC test stand the vertical ground motion was reduced by a factor $\approx 15$, whereas in the other experimental test stands the improvement was of a factor 4 (DESY) and 2 (SLAC).

![Figure 78: Vertical RMS motion above 4 Hz as measured on the ground (triangles) and on stabilized prototype quadrupoles (diamonds) in different stabilization experiments at DESY [157], at SLAC [108, 109, 165] and at CERN in the framework of the CLIC stabilization study. The SLAC experiment is not actually stabilizing a magnet prototype but a block of aluminium.](image)

5.2 Measured long-term stability

5.2.1 Long-term stability of fast vibration damping

Quadrupole stability has continuously been monitored for a total period of several days. Every 1.5 hours two geophones measured simultaneously the vibrations on the ground and on a quadrupole
doublet mounted on the table top. Results are shown in Fig. 79, where the vertical RMS motion above 4 Hz is measured on the ground (solid line) and on the stabilized doublet (line with circles) is shown versus time. Measurements started in the evening of Wednesday 10th, September, and finished on the Friday of the following week. The cultural noise has a relevant influence on vibrations above 4 Hz, increasing the RMS ground motion by 3 nm to 5 nm during working hours. Note that the first Thursday was a local Geneva bank holiday: the cultural noise for this day was considerably smaller than for the other working days. A stabilization of the quadrupole doublet below the linac tolerance (1.3 nm, dotted line of Fig. 79) is achieved throughout the measurement time. The quadrupole vertical RMS motion above 4 Hz varies between 0.59 nm and 1.0 nm with a ground motion between 5.90 nm and 11.22 nm (see Table 17).

Figure 80 shows the transverse horizontal and longitudinal RMS motion of the quadrupole above 4 Hz. The ground horizontal vibrations are less affected by the cultural noise than the vertical ones. In Fig. 80, top graph, the day-night variation is barely visible. For instance, the day-night variation is barely visible. Both transversally and longitudinally, the quadrupole stability is kept well below the horizontal linac tolerance of 14 nm for all the measuring period. Figure 80 shows that the tolerance for final focus quadrupoles are also basically met, even in the quadrupole longitudinal direction where the table performance is less good.

The achieved quadrupole stabilities above 4 Hz in all directions are summarized in Table 17 and
Table 17: Long-term stability of a quadrupole doublet, mounted on the honeycomb support structure and stabilized with the stiff isolators. Maximum and minimum achieved RMS motion above 4 Hz as is given obtained over a total period of approximately nine days. Ground motion is also listed as a reference.

<table>
<thead>
<tr>
<th>RMS motion above 4 Hz [ nm ]</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground</td>
<td>5.90 ± 0.59</td>
<td>11.22 ± 1.12</td>
</tr>
<tr>
<td>Quadrupole</td>
<td>0.59 ± 0.06</td>
<td>1.00 ± 0.10</td>
</tr>
<tr>
<td>Vertical</td>
<td>5.36 ± 0.54</td>
<td>9.62 ± 0.10</td>
</tr>
<tr>
<td>Horizontal</td>
<td>0.75 ± 0.08</td>
<td>1.47 ± 0.15</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>4.37 ± 0.44</td>
<td>9.19 ± 0.92</td>
</tr>
<tr>
<td></td>
<td>4.46 ± 0.45</td>
<td>9.07 ± 0.91</td>
</tr>
</tbody>
</table>

Table 18: Long-term stability of a quadrupole doublet, mounted on the honeycomb support structure and stabilized with the stiff isolators. The average achieved RMS motions above 4 Hz during the days (from 7 a.m. to 7 p.m.) and during the nights (from 7 p.m. to 7 a.m.) are given with their spread (standard deviation). Ground motion is also listed as a reference. Data are measured over a total period of approximately nine days.

<table>
<thead>
<tr>
<th>RMS motion above 4 Hz [ nm ]</th>
<th>Day</th>
<th>Night</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ground</td>
<td>Avg</td>
<td>StDev</td>
</tr>
<tr>
<td>Quadrupole</td>
<td>Avg</td>
<td>StDev</td>
</tr>
<tr>
<td>Vertical</td>
<td>7.83</td>
<td>1.12</td>
</tr>
<tr>
<td>Horizontal</td>
<td>7.70</td>
<td>0.78</td>
</tr>
<tr>
<td>Longitudinal</td>
<td>6.94</td>
<td>0.84</td>
</tr>
</tbody>
</table>

compared with the ground vibration level. The average RMS vibration levels above 4 Hz as measured during the days (from 7 a.m. to 7 p.m.) and during the nights are given with its spread (standard deviation) in Table 18. An average day-night variation of the vertical ground motion of approximately 1 nm RMS is damped to a 0.1 nm variation of the stabilized magnet. The spread is also reduced by a factor 10, which proves that the active system is efficient in reducing the cultural noise.

5.2.2 Alignment stability of the support structure

The long-term alignment of the honeycomb support structure mounted on the stiff isolation system was monitored with the stretched-wire system, which measures table drifts with respect to ground. An example of measured vertical and horizontal table positions versus time is given in the two top graphs of Fig. 81. The total measurement time was approximately eight days. Data are recorded every second. Ambient temperature versus time as measured in the vicinities of the table is also given (bottom graph). The table position depends on the temperature. A maximum variation of approximately 40 μm has been measured for a temperature variation of ≈ 3 °C. Variations of horizontal position are three to four times smaller. A temperature decrease induces a lowering of table height, probably due to the shrinking of the rubber in each isolator.

It has been verified that the measured displacements are not artifact of the wire system from temperature variation. No significant effect is expected for the short length (≤ 3 m) of the wire
Figure 80: Transverse ($I_x(4\text{ Hz})$, top graph) and longitudinal ($I_z(4\text{ Hz})$, bottom graph) horizontal RMS motions above 4 Hz versus time as measured on the ground (solid line) and on a quadrupole doublet (circles) versus time for a total period of approximately nine days. The magnet is mounted on the honeycomb table and stabilized with the stiff system. The CLIC final focus tolerance is also shown (dotted line). The error bars from statistical uncertainty on doublet data are small compared to the axis and are omitted.
Figure 81: Vertical ($\Delta y(t)$, top graph) and horizontal ($\Delta x(t)$, middle graph) table positions versus time, $t$, as measured with the stretched-wire system over a total period of approximately eight days. The room temperature, $T(t)$, as measured in the vicinity of the honeycomb table is also given (bottom graph). Data are acquired with an acquisition frequency of 1 Hz. The measurement resolution is approximately 30 nm.

[166]. Temperature variations have very small influence of the measurement because the wire sagitta is entirely determined by the weight which stretches the wire.

The effect of temperature variations on the table alignment is shown in more detail in Fig. 82. The table vertical position (top graph) and the ambient temperature (bottom graph) are shown versus time. A sudden temperature variation of approximately 1.5 °C induces a slow drift of the table vertical position, which reaches its maximum of $\approx 20 \mu m$ after 5 hours. This corresponds to a variation of 1 nm per second. Beam-beam offsets at corresponding frequencies can be compensated by the beam-based feedback at the interaction point. The tunnel ambient temperature should nevertheless be kept constant during the machine operation.

5.3 Low-frequency quadrupole stability

Low-frequency vibrations of the quadrupole doublet are measured with the Guralp seismometer placed on the table, at the magnet side. The size of the geophone is comparable with the magnet size and therefore it cannot be easily mounted on the magnet top like the smaller reference geophones. As discussed in Section 5.5, in the low frequency range quadrupole structural resonances are negligible and hence the table vibrations close to quadrupoles provide a good estimate of its stability. Data from the reference geophone, mounted as in Fig. 44, are also acquired simultaneously. This enables measuring quadrupole vibrations from 0.033 Hz to 315 Hz, i.e. over four orders of magnitude in frequency. This is shown for in Fig. 83, where the spectra of vertical vibrations as measured with the reference geophone (solid line) and the low-frequency geophone (dashed line) are put together to
Figure 82: Vertical table position, $\Delta y(t)$, as measured with the stretched-wire system (top) and variation of ambient temperature ($T(t)$, bottom) versus time, $t$. A sudden temperature variation of approximately $1.5^\circ C$ induces a slow table drift which lasts for several hours and reaches a maximum value of $\approx 20 \mu m$.

cover the full frequency range. The corresponding integrated RMS motion is given in Fig. 84, solid line. The ground motion as measured with one reference geophone is also shown (dashed line). The transverse and longitudinal horizontal motion of the magnet is given in Fig. 85. A large component in the low-frequency range is given by the tidal peak at approximately $0.2 \text{ Hz}$, mainly visible in the vertical direction (see Fig. 83). The induced RMS motion above $0.1 \text{ Hz}$ is approximately $250 \text{ nm}$. Horizontally the quadrupole moves more and reaches RMS values above $0.2 \text{ Hz}$ as large as $1.5 \mu m$.

A measurement of the correlation of motion between table and ground has been obtained in collaboration with DESY. A measurement campaign was set-up in the CLIC test stand using two low-frequency Guralp seismometers used at DESY for vibration measurements [157]. One was placed on the ground and the other on the table top. The measurement conditions were not ideal because time was limited and the sensors were left settling only for some hours before the measurements. This prevented measuring very low frequencies. Reliable measurements can be extracted from $0.1 \text{ Hz}$ to $10 \text{ Hz}$.

The measured correlation between ground and table vertical vibrations is shown in Fig. 86. Above a few tenths of Hertz, where the passive damping provided by the stiff rubber is effective, the correlation goes to zero. For decreasing frequencies the correlation increases and reaches its maximum of approximately 1 at $\approx 0.2 \text{ Hz}$, where the motion is dominated by the tidal peak. Measurements performed in the LEP/LHC tunnel with the low-frequency seismometers from DESY also show a steep correlation loss below $\approx 0.1 - 0.2 \text{ Hz}$. Hence the correlation losses at very low frequencies, as in Fig. 86, do not seem to be induced by the isolation system.

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36 A campaign of vibration measurements has been performed on the CERN site with W. Bialwons and H. Ehrlichmann on February 2003.

37 The low-frequency limitation probably comes from measurement noise whereas the high-frequency limitations comes from the software for the data acquisition.
Figure 83: Power spectral density of vertical vibrations, $P_y(f)$, versus frequency, $f$, as measured on top of a CLIC prototype quadrupole, mounted on the honeycomb support structure (Fig. 44) and stabilized with the stiff system. Measurements data taken with the low-frequency seismometer by Guralp (dashed line) are put together with the data taken with the reference geophone (solid line), resulting in a total measurement frequency range of 4 orders of magnitude (0.033 Hz to 315 Hz). The dotted lines show the low-frequency limit of the reference geophone and the high-frequency limit of the Guralp sensor. Differences in width and height of the spectrum peaks are induced by the different frequency resolutions of the two kinds of geophones. In the comparable frequency range, the RMS vibrational amplitudes for the two curves agree within a few percent.

The correlation in motion between stabilized table and ground can, in principle, be calculated combining the measure of absolute table vibration, provided by the geophones, with the measure of relative table-to-ground displacements, provided by the stretched-wire system. It can be shown that the power spectral density of the relative (vertical) motion between two points at a distance $L$, $P_{y, \text{Rel}}(f; L)$, is given by [86]

$$P_{y, \text{Rel}}(f; L) = 2P_y(f)[1 - c(f; L)], \quad (90)$$

where $P_y(f)$ is the spectrum of absolute motion. It is assumed here that the absolute spectra in the two measurement points (ground and table) have the same amplitudes. This seems a good approximation in the frequency range of interest, i.e. below $\approx 0.2$ Hz, as confirmed by the measurements of low-frequency ground-to-table transfer function, see Fig 72. The explicit $L$ is not relevant for this discussion and will be dropped in the following. If the motion is perfectly correlated ($c(f) = 1$), the relative motion is zero. If there is no correlation ($c(f) = 0$), the relative motion is $\sqrt{2}$ times larger than the absolute motion, as expected for random noises. However, the finite resolutions of the measurement devices limit the frequency range, where Eq. (90) can be used to extract the correlation from the measurements of $P_{y, \text{Rel}}(f)$ and $P_y(f)$.

In Figure 87 the spectra of vertical table vibrations as measured with the geophone (solid line) and with the stretched-wire system (line with circles) are given. The geophone measures absolute table vibrations whereas the capacitive system measures relative table displacements with respect to
Figure 84: Vertical RMS motion, $I_y(f)$, versus frequency, $f$, integrated above $f$ as measured on a quadrupole doublet stabilized with the stiff isolator system (solid line). Time is shown on the upper axis of the graph. Measurement data from the low-frequency seismometer by Guralp and from the reference geophone are put together. The ground vibration as measured simultaneously with another reference geophone (at frequencies $\geq 1$ Hz only) is also shown (dashed line).

Figure 85: Longitudinal (dotted line) and horizontal (solid line) RMS motion, $I(f)$, versus frequency, $f$, integrated above $f$ as measured on a quadrupole doublet stabilized with the stiff isolator system. Time is shown on the upper axis of the graph. Measurement data from the low-frequency seismometer by Guralp and from the reference geophone are put together.
Figure 86: Correlation of vertical motion, \( c(f) \), versus frequency, \( f \), as measured on the ground and on top of the honeycomb support structure, stabilized with the stiff isolators.

Figure 87: Absolute power spectral density of vertical table motion, \( P_y(f) \), versus frequency, \( f \), as measured with the low-frequency seismometer (solid line) and power spectral density of vertical relative table displacements from ground, \( P_{y,\text{Rel}}(f) \), versus frequency, as measured with the capacitive stretched-wire system (line with circles). In the low-frequency range \( f \lesssim 1 \text{ Hz} \), relative displacements are smaller than the stretched-wire system resolution and hence the system records a Gaussian noise. The noise RMS amplitude, as extrapolated form measured data (dotted line), is \( \approx 35 \text{ nm} \), i.e. \( P_{\text{res}} = 1.23 \text{ nm}^2/\text{Hz} \), in agreement with the expected sensors resolution (see Table 12).
Figure 88: Correlation between table and ground motion, \( c(f) \), versus frequency, \( f \), as extrapolated from the measurements of Fig. 87 and Eq. (90). The resolution of the stretched-wire system only allows finding a lower limit for the actual correlation, which must be above the solid line. The bold dashed line shows the extrapolation of \( c(f) \) at frequencies below the geophone bandwidth.

The ground, to which the wire is fixed. In the shown frequency range the wire system is limited by the sensor resolution of approximately 35 nm, as extrapolated from the measured data (the dotted line shows a fit to the flat part of the measured spectrum). Similar results are obtained for the horizontal motion. According to Eq. (90), this measure allows defining a lower limit for the table-to-ground correlation:

\[
c(f) \gtrsim 1 - \frac{P\text{wire}(f)}{2P\text{Geo}(f)}.
\]  

(91)

This limit is shown in Fig. 88. Below approximately 0.2 Hz the correlation is always larger than 0.8. This is a very encouraging result that shows that the isolation system would not perturb the spatial characteristics of the low-frequency correlation versus distance of the ground motion (see [82]), which is of great importance for a linear collider to operate.

In Figure 89 the measured correlation above 0.1 Hz (solid line) and the extrapolated correlation from the combined measurements of low-frequency geophone and stretched-wire system (dashed line) are put together on the same graph. The two curves merge well at frequencies around 0.2 Hz.

5.4 Water induced quadrupole vibrations

Detailed measurement campaigns have been performed to quantify the effect of cooling water on the vibrations of CLIC linac quadrupoles. Magnet prototypes are connected to the regular Geneva tap water network and stabilized with the different isolation equipments. This is reported in Appendix F (see also [96, 167]). Here, the main results are summarized.

The best achieved quadrupole stability with nominal water flow (30 l/h) is given in Figs. 90 and 91, which respectively show vertical and horizontal RMS motion as measured on the stabilized table with (solid lines) and without (dotted lines) cooling water. Ground motion (dashed lines) is also shown as a reference. The latter did not change considerably during the measurement time. Figures 90 and 91 show that the CLIC linac tolerances can basically be met with circulating water. Vertically
the quadrupole has been stabilized to \((1.38 \pm 0.14)\) nm with a motion of the supporting ground of approximately 6 nm. The horizontal motion is \((2.12 \pm 0.21)\) nm, i.e. 3.7 times smaller than the linac tolerance of 1.3 nm (Table 6).

The pure contribution of circulating water to magnet vibrations has been quantified for different values of water flow. It can be calculated as the squared difference between the measurements with and without water [96]. The water contribution above different values of minimal frequencies is shown as a function of the flow in Fig. 92. Data refer to vertical vibrations and are not obtained with the same experimental set-up of Fig. 90. Depending on the conditions, increases of RMS vibrations above 4 Hz up to approximately 2 nm are measured. Interestingly, the water contribution to vibrations does not increase monotonically with flow but shows some peaks, as if different water velocities excite different resonant vibrations.

The simple theory of water induced vibrations (Appendix F) provides an accurate estimate of the expected vibration frequencies from the magnet cooling system. In particular, it has been proven that most of the increase in motion in the frequency range of interest comes from the feeding pipes upstream of the quadrupole pipes themselves. The small quadrupole pipe induces high frequency vibrations \((f \gtrsim 200\) Hz), in a range where displacement amplitudes are typically below tolerance. On the other hand, the larger feeding pipes induce vibrations at frequencies of a few tens of Hertz. These vibrations are transmitted to the quadrupole via water and pipes and amplify the motion up to \(\approx 50\) Hz. This is particularly dangerous for the luminosity performance of linear colliders because in this frequency range the motion has typically the same order of magnitude as the tolerances and is not properly corrected by beam-based feedbacks. Water vibrations can also amplify resonances of quadrupoles and their supports. However, it seems that this kind of problem can be avoided with a proper design of the water circuit, optimizing the effect of water on magnet vibrations. Increasing the diameter of the main feeding pipes, for instance, could reduce the frequency of induced vibrations to a value which can be compensated efficiently with beam-based feedbacks.
Figure 90: Vertical RMS motion, $I_y(f)$, versus frequency, $f$, integrated above $f$ as measured on the ground (dashed line) and on a prototype of the CLIC linac quadrupole doublet, without circulating water (solid line) and with the nominal water flow (dotted line). The magnet is stabilized with the stiff isolator. The water is not pumped as expected in machine operation but comes from the regular tap network ($\approx 4$ bar).

Figure 91: Transverse horizontal RMS motion, $I_x(f)$, versus frequency, $f$, integrated above $f$ as measured on the ground (dashed line) and on a prototype of the CLIC linac quadrupole doublet, without circulating water (solid line) and with the nominal water flow (dotted line). The magnet is stabilized with the stiff isolator. The water is not pumped as expected in machine operation but comes from the regular tap network ($\approx 4$ bar).
Figure 92: Additional RMS contribution to quadrupole vertical vibrations from circulating water versus water flow. Different lines show the integrated contributions above different minimal frequencies. The water contribution is calculated as the difference in quadrature between the measured RMS vibration amplitudes with and without water.

5.5 Transmission of quadrupole supports

Studies of structural resonances for the quadrupole doublet and its support structure were performed on the stabilized table, measuring simultaneously the vibration on the table and on the magnet. Measurements were performed using the stiff isolation system to stabilize the supporting table because it provides quieter conditions. The square root ratio of the corresponding power spectral densities, $\sqrt{P_{y,\text{Quad}}(f)/P_{y,\text{Tab}}(f)}$, shows the resonant frequencies: if the ratio is larger than 1, the quadrupole motion amplifies the table motion.

The vertical resonances of the quadrupole doublet are shown in Fig. 93. A significant amplification with respect to the table motion is found only above approximately 150 Hz. Notably, the largest resonances are found at 170 Hz and 240 Hz, where the doublet motion is up to three times larger than on the table. However, at these frequencies the natural motion is below the 0.1 nm level and hence the doublet amplification is not dangerous. The quadrupole doublet has a main horizontal resonance at approximately 93 Hz, which amplifies the motion up to 15 times with respect to the table. The corresponding RMS motion is in the fraction of nanometre range. It is also not a problem, because the tolerances on horizontal displacements are less tight than the vertical ones (see Section 3.2.3).

The resonance diagram of a quadrupole doublet mounted on the CTF2 alignment support is given in Fig. 94. Similarly to the previous case, the support is fixed on the honeycomb support and vibrations are measured simultaneously on doublet and table. (see Fig. 45). The main structure resonance arise at 44 Hz (sharp peak in Fig. 45) and amplifies the table motion by 10 to 15 times depending on the experimental conditions. A broader resonance arises at approximately 145 Hz, where the motion is

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38 A small peak is noticeable at 8.8 Hz in Fig. 93. It is not thought to be a doublet structural resonance but rather an inhomogeneity induced because the doublet and the geophone are not exactly at the same table location. Similar peaks have also been found on the table surface without doublet. Nevertheless, the difference in the corresponding integrated motion is negligible ($\approx 0.06$ nm for the given case).
Figure 93: Square root of quadrupole (Quad) to table (Tab) ratio of vertical power spectral densities, \( \sqrt{P_y^{\text{Quad}}(f)/P_y^{\text{Tab}}(f)} \), versus frequency, \( f \), as measured on top of the honeycomb table stabilized with stiff isolation system.

Figure 94: Square root quadrupole (Quad) to table (Tab) ratio of vertical power spectral densities, \( \sqrt{P_y^{\text{Quad}}(f)/P_y^{\text{Tab}}(f)} \), versus frequency, \( f \), as measured on top of the honeycomb table stabilized with. The quadrupole doublet is now mounted on its CTF2 alignment support structure, as in Fig. 45.
Figure 95: Squared ratio of the spectral amplitudes, \( \sqrt{P_{y_{\text{Quad}}}(f_0)/P_{y_{\text{Tab}}}(f_0)} \), versus frequency, \( f_0 \), as measured on the table (\text{Tab}) and on the quadrupole mounted (\text{Quad}) on its alignment support structure. The table is stabilized with the stiff isolation system. The vibrations of table and quadrupole are excited with a loudspeaker, placed about 30 cm apart from the table, which induces sound waves of fixed amplitudes but at different perturbing frequencies \( f_0 \). The two graphs show resonance diagrams in different frequency ranges.

normally smaller. Unlike the case of a single quadrupole doublet, which shows resonances in the high-frequency range (above \( \approx 150 \text{ Hz} \), where vibration amplitudes are small), the resonances of the support can be harmful for a linear collider. Indeed, they typically have frequencies of some tens of Hertz and in this frequency range the vibration level is larger. Support resonances can possibly amplify other sources of vibration, such as the water induced vibrations (see Appendix F). The main horizontal resonances of the quadrupole support also arise at 44 Hz and amplify the motion by 20 to 25 times. It should be noted that the alignment support used in the CTF2 accelerator was not optimized against structural resonances. A much more careful design should be envisaged for the CLIC quadrupoles.

The resonances of Fig. 94 are calculated as unperturbed square root ratio of the power spectral densities. In order to measure the motion amplification from the doublet support in presence of external perturbations, sound waves were excited with a loudspeaker of tunable frequency and amplitude, placed some tens of centimetres apart from the table. A frequency scan around the two main resonances of 44 Hz and 145 Hz was performed and the results are summarized in Fig. 95. The loudspeaker induces a sharp peak in the power spectral densities of the quadrupole and table motions. Figure 95 gives the square root ratio of the two peak amplitudes versus frequency. In presence of a perturbing source the amplification of the doublet support structure is much larger than for the unperturbed case. For instance, the support resonance amplified the vertical vibrations by approximately 160 times if the loudspeaker excites sound waves at 44 Hz.
5.6 Honeycomb support used as a girder

It is known that a potential drawback of the stiff stabilization system, which could prevent its use in a linear collider, is that the isolators do not allow aligning vertically and horizontally the table position with respect to some external reference\(^{39}\). The capability of aligning the lattice elements in the micro-metre range is a crucial ingredient for the operation of a linear collider. The day-one pre-alignment of the machine, for instance, is mandatory to enable transporting of pilot bunches, which will then provide the required information for beam-based alignment to be effective. The pre-alignment tolerance on the transverse position of CLIC girders, quadrupoles and beam position monitors is 10 \(\mu\)m over 200 m [142]. Constant adjustments of the lattice elements will be required at any time during the machine operation.

The alignment problem can be accounted by mounting the quadrupoles together with their CTF2 alignment support structure on the table top (see Fig. 45). The alignment support has been designed for a micro-metric alignment of quadrupole doublets and triplets in five degrees of freedom (only the longitudinal position is left uncorrected since its positioning tolerances are orders of magnitude looser). Two doublets and one triplet, each with one reference geophone on top, have been installed on the honeycomb support structure (see Fig. 41) mounted on stiff isolation system. Another geophone monitors ground vibrations as a reference.

The alignment problem can be accounted by mounting the quadrupoles together with their CTF2 alignment support structure on the table top (see Fig. 45). The alignment support has been designed for a micro-metric alignment of quadrupole doublets and triplets in five degrees of freedom (only the longitudinal position is left uncorrected since its positioning tolerances are orders of magnitude looser). Two doublets and one triplet, each with one reference geophone on top, have been installed on the honeycomb support structure (see Fig. 41) mounted on stiff isolation system. Another geophone monitors ground vibrations as a reference.

The vertical RMS motion versus frequency as measured on one doublet is given in Fig. 96 (solid line). The ground motion (dashed line) and the RMS difference between the vertical motion of two doublets (dotted line) are also shown. All quadrupoles have a vertical stability below the tolerances for the CLIC linac quadrupoles (1.3 nm RMS above 4 Hz), which is a very encouraging result. Differences between different doublets are also small. The horizontal stability performance of the isolators

\[^{39}\text{As suggested by T. Mattison, a piezo-based feedback for the low-frequency variation of the table height could be designed and integrated rather easily into the stiff isolation system for an alignment at the micron level. This approach has not been pursued for the moment.}\]

Figure 96: Vertical RMS motion, \(I_y(f)\), versus frequency, \(f\), integrated above \(f\) as measured on a quadrupole doublet (solid line) and on the ground (dashed line). Three magnets together with their alignment support structures are mounted together on the honeycomb table, as on a girder (see Fig. 41). The table is stabilized with the stiff isolator system. The integrated difference between the two quadrupoles (dotted line) and the CLIC linac tolerance (dots) are also shown.
Figure 97: Vertical RMS motion above 4 Hz, $I_y(4 \text{ Hz})$, versus time as measured on the ground (solid line) and on a quadrupole doublet (line with circles) versus time for a total period of approximately four days, from Friday to Tuesday. CLIC linac tolerance is also shown (dotted line). The error bars on the doublet from statistical uncertainty are small compared to the vertical scale and are omitted.

is less good than the vertical one. Nevertheless, transverse quadrupole vibrations are also within tolerances (14 nm RMS above 4 Hz).

Figure 97 shows the long-term stability of the quadrupoles as measured continuously for several days, from a Friday to the following Tuesday. The RMS motion above 4 Hz as measured on the ground (solid line) and on one doublet (line with circles) is shown against time. The vibration level of the quadrupole doublets and triplet is kept below the CLIC linac tolerances (dotted line) independently of the ground motion for a total period of more than four days. Similar results are obtained for the transverse horizontal direction.

It should be mentioned that the use of the stiff stabilization system all along the CLIC linac would require further feasibility studies. In particular, issues like the radiation hardness of the main isolator components (e.g., rubber, electronics cards) and the influence of background electromagnetic fields on the damping performance remain to be addressed.
6 Time-dependent luminosity performance of CLIC

The time-dependent CLIC luminosity can be predicted based on the measured magnet stability. Fast vibrations above 1 Hz impose an immediate limitation for the CLIC performance. Beam-based feedbacks at the interaction point cannot compensate properly fast beam jitters above a few Hertz because they are effective only at frequencies larger than approximately \( f_{\text{rep}}/25 = 4 \) Hz. Therefore, the fast and uncorrelated frequency regime is the range where mechanical stabilization of magnet is crucial for the performance of a linear collider. Several studies presented here aim at characterizing this important regime. Some studies are also presented on slower and partially correlated magnet vibrations.

6.1 Overview of predicted luminosity performance and assessment of CLIC feasibility

Before discussing in detail the luminosity performance predicted for CLIC under various conditions, an overview of the main achieved results is given. The time-dependent CLIC luminosity is summarized in Table 19 as predicted based on vibrational spectra measured in different experimental conditions. All magnets of the CLIC beam delivery system are displaced in time according to the 2-dimensional measured spectra. Vertical and horizontal feedback systems for the correction of beam-beam offsets at the interaction point are included in the simulation. Fast vibrations above 1 Hz are considered, which allows simulating 1 second of machine operation. This is the frequency range where the mechanical stabilization of magnet is crucial to produce luminosity. Slower vibrations are corrected efficiently by beam-based feedback systems and are not considered here (see Section 6.3). No correlation between the motion of different magnets is assumed.

If state-of-the-art stabilization technology is used to damp the ground motion, up to 70 % of the design CLIC luminosity (i.e., \( 0.6 \times 10^{35} \text{ cm}^{-2} \text{ s}^{-1} \)) can be achieved in a normal working area on the CERN site, like for example the CLIC test stand. The luminosity obtained with stabilization is approximately ten times larger than without stabilization and is similar to what is achievable in a deep underground tunnel without technical noise and without stabilization. Typical noisy sites (like ESRF) but even rather quiet sites like the ground CERN area would provide no significant luminosity without stabilization techniques. Detailed studies on the effects of perturbing effects such as water flows and magnet support (CTF2 design, not optimized against magnet vibrations) can be kept under control.

Even though further improvements and optimizations of the tested stabilization techniques are required to recover the full design luminosity, the results of Table 19 demonstrate the principle feasibility of colliding high energy nanobeams for CLIC.

6.2 Effect of uncorrelated quadrupole vibrations

6.2.1 2D MODEL FOR THE TIME-DEPENDENT LUMINOSITY SIMULATIONS

Time-dependent simulations of the CLIC luminosity performance are carried out with the assumption that the mechanical motion of each magnet of the beam delivery system may be stabilized with an isolation system as described in Section 4. Each magnet is assumed to be stabilized with an isolator system independent of the ones used for the other magnets. First, vibrations above 1 Hz are considered. This allows predicting the total machine operation for 1 second. It has been shown in Section 5.1 that in this frequency range vibrations of stabilized magnet are uncorrelated with respect to ground, both horizontally and vertically. Therefore, even if a correlation over distances of tens of metres is expected on the ground [90], all stabilized lattice elements move in an uncorrelated way with respect to each other. Notably, the final focus quadrupoles of the two opposing machines, which are only a
Table 19: Summary of the predicted CLIC luminosity as expected in various experimental conditions. The average luminosity calculated over one second of CLIC operation is given with its error. All magnets of the beam delivery systems move in time according to the 2D vibrational spectra as measured in different conditions (first column). No correlation between the motion of different magnets is assumed. Simulations include horizontal and vertical feedback systems for the correction of beam-beam offsets at the interaction point.

<table>
<thead>
<tr>
<th>Input spectra</th>
<th>$\langle \mathcal{L} \rangle / \mathcal{L}_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLIC test stand</td>
<td></td>
</tr>
<tr>
<td>Ground, no stabilization</td>
<td>$(6.07 \pm 0.50)%$</td>
</tr>
<tr>
<td><strong>Stiff stabilization system</strong></td>
<td>$(68.97 \pm 0.72)%$</td>
</tr>
<tr>
<td>Soft stabilization system</td>
<td>$(50.08 \pm 1.52)%$</td>
</tr>
<tr>
<td>LHC tunnel (quiet site)</td>
<td>$(64.86 \pm 1.42)%$</td>
</tr>
<tr>
<td>ESRF site (noisy site)</td>
<td>$(0.49 \pm 0.10)%$</td>
</tr>
<tr>
<td>Cooling water, with stabilization</td>
<td>$(68.01 \pm 0.83)%$</td>
</tr>
<tr>
<td>Alignment support (not optimized), with stabil.</td>
<td>$(50.26 \pm 0.66)%$</td>
</tr>
</tbody>
</table>

few metres apart, do not show any correlation above $\approx 1$ Hz when actively and passively stabilized, even if the supporting ground might.

Vibrational spectra as measured on prototype quadrupoles are used to generate time-dependent offsets of the lattice elements. The spectrum amplitude as a function of frequency, $P_y(f)$, gives the amplitude of each vibration harmonics in frequency domain as:

$$|\hat{y}(f)| = \sqrt{P_y(f)},$$

(92)

where $\hat{y}(f)$ is the Fourier transform of vertical magnet position in units of nm/$\sqrt{\text{Hz}}$. The time-dependent magnet position, $y(t)$, is generated assuming no correlation between the phases of different frequency components of $\hat{y}(f)$. The vibrational phases, $\phi(f)$, are given by a squared, uniform random distributed between $0$ and $2\pi$. This assumption was confirmed by experimental results (see Fig. 5.1) and is commonly used for generating time-dependent displacements from the vibrational spectra [86]. Thus $y(t)$ can be written as:

$$y(t) = F^{-1}\left\{ \sqrt{P_y(f)} \times e^{i\phi(f)} \right\},$$

(93)

where $F^{-1}$ denotes the inverse Fourier transform. Only the real part of the Fourier transform has to be considered. A Fast Fourier Transform (FFT) algorithm is actually used for a faster calculation. The horizontal motion is generated similarly. It is assumed that there is no coupling between the motion of the two planes.

An example of Fourier amplitudes of vertical vibrations as measured on a stabilized quadrupole doublet is given in Fig. 98. The two corresponding time-dependent displacements, used to move the two final quadrupoles in the simulations discussed later, are given in Fig. 99. The motion of other lattice magnets is generated similarly, using the same input spectrum. Uncorrelated motion of different magnets is obtained using sets of random phases independent of each other. No sextupole
prototypes were available for vibration measurements. Sextupoles are thus assumed to move like quadrupoles.

Vertical and horizontal misalignments are applied simultaneously to quadrupoles and sextupoles of the beam delivery system, resulting in a full 2-dimensional model of the magnet dynamics. Longitudinal motion of magnets is neglected. The achieved magnet stability at the frequencies of interest \((f \gtrsim 4 \text{ Hz})\) is in the few nanometre range along both horizontal magnet axes, whereas the longitudinal tolerances are some orders of magnitude looser than the transverse ones. Therefore, the achieved longitudinal vibration level of quadrupoles has no significant effect on the luminosity performance of CLIC.

Horizontal and vertical feedback systems for the correction of the beam-beam offset at the interaction point are used to steer the opposing beams into collision. The feedbacks are based on the measurements of beam-beam deflection angles, from which the beam-beam offset to be corrected is inferred (Section 3.3.2). Vertical and horizontal beam-beam scans at the collision point have been performed to provide the beam-beam deflection curves in both planes. Linear fits with slope tangent to the origin (i.e., slope at zero offset, see Fig. 37) are used to calculate the beam-beam offsets from the measured deflection angles. In the case of fast motion \((f > 1 \text{ Hz})\), no significant improvement is obtained if more complex fitting algorithms are used. Higher order polynomial fits typically improve the feedback low-frequency efficiency (see Section 3.3.2) which is not relevant for the purpose of this section.

The beam tracking through the CLIC beam delivery system is simplified by not taking into account synchrotron radiation. The main effect of fast vibrations is a reduction of the luminosity via production of beam-beam offsets. Emittance dilution from photon emission enlarges the beam transverse spot size at the interaction point but does not modify the beam-beam offset. To first order, the luminosity performance is determined by the efficiency of beam-based feedbacks and by the achieved magnet stability. According to simulation results of Section 3.1.5, the achieved luminosity per bunch train with synchrotron radiation is decreased by approximately \(0.5 \times 10^{35} \text{ cm}^{-2}\text{s}^{-1}\). This is assumed to apply also for the results of time-dependent simulations. Photon emission is instead taken into account in beam-beam simulations at the interaction point, which provide achieved luminosity and beam-beam angle.
Figure 99: Examples of vertical time-dependent magnet positions, $y(t)$, generated according to the spectrum of Fig. 98. Uncorrelated random phases between different harmonics in frequency-domain are assumed (see Fig. 5.1). The absolute displacement of the opposing final quadrupoles (solid and dashed lines) as used in the simulations discussed later (see Figs. 100 and 101) and their relative displacement (line with dots) are given as a function of time. In first approximation, the difference between the final quadrupoles, multiplied by the $R_{3,4}$ transport element to the interaction point, gives the vertical offset of colliding beams.

Examples of time-dependent luminosity performance simulated for CLIC when all lattice magnets move like a stabilized magnet are shown in Figs. 100 and 101. Relative luminosity (top graphs of the two figures), vertical beam-beam offset (middle graphs) and relative variations of the two beam vertical spot sizes (bottom graphs) are shown versus time. The luminosity relative to the zero-offset luminosity ($\mathcal{L}/\mathcal{L}_0$) is given. 2-dimensional vibrations are applied to all lattice magnets according to the best magnet stability achieved with the stiff isolator system (Section 5.1). Simulations start with a perfectly aligned machine. In order to consider steady operation conditions, 1.5 s of CLIC operation are simulated and the first 50 pulses are neglected in the calculation of the quantities of interest, such as the average luminosity and the RMS beam-beam offset.

Figures 100 and 101 show the effect of the vertical interaction point feedback while the settings of horizontal feedback remain unchanged. If there is no vertical correction of the relative beam-beam offset (Fig. 100), the magnet stabilization is not sufficient to obtain a decent luminosity. The low-frequency uncorrected motion of the two beams (up to tens of nanometres in the given example) induces beam-beam offsets much larger than the vertical beam size (0.7 nm). In steady operation, the corresponding luminosity is basically zero. It is interesting to note that even if all lattice elements are moving according to the same measured spectrum, the offset at the interaction point is dominated by the displacements of the final doublets (compare middle graph of Fig. 100 with Fig. 99).

When the interaction point feedback is switched on (Fig. 101), the slow frequency drifts of colliding beams are efficiently corrected. This enables recovering a considerable fraction of the nominal luminosity (approximately 70% in the given example). Nonetheless, the correction is not perfect. A pulse-to-pulse jitter is left uncorrected, which induces a jitter of the luminosity performance. This effect, also experienced at the SLC, is unavoidable. It depends on the limited efficiency of the feedback.
Figure 100: Relative luminosity \( (\mathcal{L}(t)/\mathcal{L}_0, \text{top graph}) \), vertical beam-beam offset \( (\Delta y_{BB}(t), \text{middle graph}) \) and relative variations of colliding beam vertical sizes \( (\sigma^*_y(t)/\sigma^*_{y,0}, \text{bottom graph}) \) versus time during 1.5 second of CLIC operation. 2-dimensional vibrations are applied to quadrupoles and sextupoles of the CLIC beam delivery system using as input horizontal and vertical vibrational spectra of a stabilized quadrupole doublet (stiff isolator system is used). The horizontal feedback for beam-beam offsets is active whereas the vertical one is off. Fast vibrations \( (f > 1 \text{ Hz}) \) are only considered. The luminosity is therefore severely reduced. Simulations start with a perfect machine. The first 50 pulses are neglected and the average luminosity in calculated over 1 second of steady operation.

which cannot correct vibrations faster than approximately \( f_{\text{rep}}/25 = 4 \text{ Hz} \). Note that to a first approximation the luminosity performance is dominated by the relative beam-beam offset whilst beam size variations are less important. The use of beam-based feedbacks is mandatory to compensate the slow frequency \( (f \lesssim 4 \text{ Hz}) \) content of motion, which otherwise would prevent the production of any significant luminosity independent of the good damping of fast magnet vibrations. The motion at around 1 Hz has typically an RMS amplitudes of up to tens of nanometres, corresponding to peak-to-peak oscillation amplitudes \( 2\sqrt{2} \) times larger. For uncorrelated motion of the final doublets of the opposing machines, the expected beam-beam offset is a factor \( \sqrt{2} \) larger (see Fig. 99). An additional factor 1.36 has to be taken into account for the amplification on vertical doublet offsets due to the \( R_{3,4} \) transport element. Therefore the expected beam-beam offset, if no feedback correction is applied, results in zero luminosity independent of the good damping of magnet vibrations.

Figure 102 shows instead the effect of the horizontal interaction point feedback. The settings of the vertical feedback are now left unchanged while the horizontal feedback is switched off and on (top and bottom figures of Fig. 102, respectively). Since horizontal tolerances on beam-beam offsets are looser than vertical ones, horizontal vibrations are less critical. However, also in this case the feedback system is mandatory to produce a significant luminosity because uncorrected horizontal vibrations (up to \( \approx 150 \text{ nm} \) in the example) are sufficient to severely reduce the luminosity (see Fig. 102, top graph).
Figure 101: Relative luminosity ($\mathcal{L}(t)/\mathcal{L}_0$, top graph), vertical beam-beam offset ($\Delta y_{BB}^v(t)$, middle graph) and relative variations of colliding beam vertical sizes ($\sigma_y^v(t)/\sigma_{y,0}^v$, bottom graph) versus time during 1.5 second of CLIC operation. 2-dimensional vibrations are applied to quadrupoles and sextupoles of the CLIC beam delivery system using as input horizontal and vertical vibrational spectra of a stabilized quadrupole doublet (stiff isolator system is used). Both horizontal and vertical feedbacks for beam-beam offsets are active. Fast vibrations ($f > 1$ Hz) are only considered. Simulations start with a perfect machine. The first 50 pulses are neglected in the calculation of average luminosity (1 second of steady CLIC operation is considered).

In order to optimize the achieved luminosity performance, the settings of interaction point feedbacks must be optimized to reduce the horizontal and vertical RMS beam-beam offsets at collision. An example of a 2-dimensional gain optimization, carried out when lattice magnets move like a stabilized doublet, is shown in Fig. 103. The optimal gain settings are chosen on the base of the largest average luminosity achieved over 1 second of CLIC steady operation. Note that feedback gains larger than 1 are required because the linear fit of the beam-beam deflection slope at zero-offset (Fig. 37) always underestimates the actual beam-beam offset. Since horizontal tolerances are looser, the luminosity performance depends less on the choice of the horizontal feedback gain. Note that since the main contribution of the beam-beam offset at the interaction point comes from the final focus quadrupoles, the gain optimization can be carried out considering only displacements of the final doublets. This is shown for instance in Fig. 104, where the average achieved luminosity in a noisy and quiet place is shown versus vertical feedback gain, moving only the final quadrupoles of the opposing machines. This example shows that both magnet stability and beam-based feedbacks are mandatory for a linear collider to produce luminosity: even in a quiet site, uncorrected low-frequency vibrations prevent keeping the beams in collisions. Conversely, the interaction point feedback alone cannot compensate efficiently high-frequency vibrations of magnets, which must be mechanically stabilized.
Figure 102: Relative luminosity ($L(t)/L_0$, top graph of each figure), horizontal beam-beam offset ($\Delta x_{BB}(t)$, middle graph) and relative variations of colliding beam horizontal sizes ($\sigma_x^*(t)/\sigma_{x,0}^*$, bottom graph) versus time during 1.5 second of CLIC operation when the horizontal interaction point feedback is switched off (top graphs) and on (bottom graphs). The vertical feedback is always on. 2-dimensional vibrations are applied to quadrupoles and sextupoles of the CLIC beam delivery system using as input horizontal and vertical vibrational spectra of a stabilized quadrupole doublet (stiff isolator system is used). Simulations start with a perfect machine. The first 50 pulses are neglected in the calculation of average luminosity (1 second of steady operation is considered).
Figure 103: Average luminosity over 1 second of CLIC operation versus gain of the horizontal and vertical feedbacks for the beam-beam offset correction at the interaction point. The average relative luminosity with respect to the zero-offset luminosity ($\langle \mathcal{L} \rangle / L_0$) is given. 2-dimensional gain optimization is carried out for the case of all lattice magnets moving like a prototype doublet stabilized with the stiff isolation system. Similar results are found if only the final doublets move.

Figure 104: Average luminosity, $\langle \mathcal{L} \rangle / L_0$, over 1 second of CLIC operation versus gain of the vertical interaction point feedback. The final focus quadrupoles at either side of the collision point move vertically according to typical vibrational spectra as measured in a quiet site (deep underground LHC tunnel, solid line), in a noisy site (ESRF, dashed line) and on the ground of the CLIC test stand (dotted line). Other lattice magnets do not move. Error bars are calculated as error on the average luminosity.
The average CLIC luminosity achieved obtained in various conditions is given in Table 20. The first line of Table 20 gives the case of all lattice magnets are moved in time according to the best measured quadrupole stability (Section 5.1). The data of Figures 74 and 75, obtained when a quadrupole doublet is mounted on the stiff stabilization system, are used as input for the simulations. The average luminosity was calculated if the lattice magnets moved like the ground (1) of the CLIC test stand, (2) of a typical quiet place (LHC tunnel) and (3) of a known noisy place (ESRF site). Simulations show that the tested stabilization technology would allow achieving approximately the 70% of the CLIC design luminosity in normal surface working area on the CERN site, i.e. approximately $0.6 \times 10^{35}$ cm$^{-2}$ s$^{-1}$. This result is slightly better that what would be expected if the magnets move like the LHC deep underground tunnel. Noisy locations like the ESRF site and even quiet sites like the surface CERN area would not allow achieving the goal luminosity without a proper damping of ground motion.

The simulations discussed so far assume that lattice magnets move horizontally according to the spectra measured along the table longitudinal direction, which provides a slightly better vibration damping above 4 Hz than horizontally. An average luminosity of $(67.64 \pm 0.74)$%, i.e. smaller by 2%, is obtained if the horizontal motion of the transverse table direction is applied. The achieved luminosity performance is still very good. Larger differences are expected if the low-frequency motion is included, since below approximately 1 Hz the amplification of ground motion from transverse table transmission is more significant (see Section 6.3).

The luminosity performance for the case that only the final doublets at either side of the interaction point are moved is given in Table 21. The comparison with Table 20 shows that most of the luminosity reduction with respect to design value comes from the vibrations of the final doublets. The relative contribution to total luminosity from the last quadrupoles actually depends on the specific features of the considered vibrational spectrum. The difference is of the order of 1% if the spectra as measured on the stabilized magnets are considered, whereas differences up to 10% arise if the motion of the LHC tunnel is used. It should be noted that simulations only include a feedback correction for the beam-beam offset at the interaction point and no correction of the beam position along the beam.

Table 20: Average relative luminosity with respect to its zero-offset value ($\mathcal{L}_0$) as calculated over one second of CLIC operation when different time-dependent vibrations are applied to all magnets of the beam delivery system. The vibration measurements performed on the LHC tunnel (quiet), on the ESRF site (noisy), on the CLIC test stand and on top of a doublet stabilized with the stiff stabilization system are used to model the 2-dimensional magnet vibrations. The measured vertical RMS motions above 2 Hz and 4 Hz are listed for all cases. Vibrations above 1 Hz are simulated. All magnets have the same absolute motion but move without correlation with respect to the others. Simulations start with a perfect machine and the first 50 pulses are neglected for the calculations of average luminosity. The average of 20 seeds for the generation of magnet displacements and its error are given.

<table>
<thead>
<tr>
<th>Input spectra</th>
<th>$I_y(2\text{ Hz})$ [nm]</th>
<th>$I_y(4\text{ Hz})$ [nm]</th>
<th>$\langle \mathcal{L} \rangle / \mathcal{L}_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLIC test stand, with stabil.</td>
<td>0.59 $\pm$ 0.06</td>
<td>0.46 $\pm$ 0.05</td>
<td>$(68.97 \pm 0.72)%$</td>
</tr>
<tr>
<td>CLIC test stand, ground</td>
<td>9.23 $\pm$ 0.92</td>
<td>7.12 $\pm$ 0.71</td>
<td>$(6.07 \pm 0.50)%$</td>
</tr>
<tr>
<td>LHC tunnel (quiet site), no stabil.</td>
<td>0.77 $\pm$ 0.08</td>
<td>0.24 $\pm$ 0.02</td>
<td>$(64.86 \pm 1.42)%$</td>
</tr>
<tr>
<td>ESRF site (noisy site), no stabil.</td>
<td>20.94 $\pm$ 2.09</td>
<td>14.02 $\pm$ 1.40</td>
<td>$(0.49 \pm 0.10)%$</td>
</tr>
</tbody>
</table>
Table 21: Average relative luminosity with respect to its zero-offset value ($L_0$) as calculated over one second of CLIC operation when different time-dependent vibrations are applied to all the final doublets only. Each quadrupole of the doublets moves horizontally and vertical according to the vibrational spectra as measured in the different locations (see also Table 20). Vibrations above 1 Hz are considered. Simulations start with a perfect machine and the first 50 pulses are neglected for the calculations of average luminosity. The average and error are given for 20 seeds for the generation of magnet displacements.

<table>
<thead>
<tr>
<th>Input spectra</th>
<th>$\langle L \rangle / L_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLIC test stand, with stabilization</td>
<td>(69.95 ± 0.44)%</td>
</tr>
<tr>
<td>CLIC test stand, ground</td>
<td>(10.26 ± 0.98)%</td>
</tr>
<tr>
<td>LHC tunnel, no stabilization</td>
<td>(75.86 ± 0.59)%</td>
</tr>
<tr>
<td>ESRF site, no stabilization</td>
<td>(1.44 ± 0.34)%</td>
</tr>
</tbody>
</table>

delivery system. This makes the collider performance sensitive to the low-frequency content of the vibrations applied to the lattice magnets because their effect is not corrected like in the case of the final doublets.\(^{40}\) A more complete model of the dynamics and correction as achievable in reality all along the lattice is required to estimate the achievable luminosity. However, the results of Table 20 demonstrate that the achieved magnet stability provides good performance even without beam-based corrections along the lattice upstream of the interaction point.

6.2.3 Effect of Water Vibrations on CLIC Luminosity

In order to quantify the effect of water induced magnet vibrations on the CLIC luminosity performance, quadrupoles and sextupoles of the beam delivery system are displaced according to the vibrational spectra as measured on a stabilized doublet with nominal water flow (see Figs. 90 and 91). Vibrations above 1 Hz are included and the motion of different magnets is assumed to be uncorrelated. Since no measurements of water effects on sextupole vibrations are available, it is assumed that the spectrum of sextupole water vibrations is the same as the one measured on quadrupole doublets. In any case, sextupoles have a small impact on the short-term luminosity performance because their tolerances on vertical and horizontal offsets are much looser than for the quadrupoles. The final doublets are not moved with respect to their nominal position. They feature a permanent magnet design without cooling circuit. Their effect on luminosity reduction was discussed and is neglected here.

Simulation results show that, if the water induced vibrations are taken into account, the achieved fraction of nominal luminosity is lower by (0.96 ± 0.41)% than what is expected if stabilized magnet with no water are considered. This result is obtained by averaging the results of forty seeds of magnet motion. This small variation, almost at the limit of the simulation resolution, does not seem to be an issue for CLIC. Nonetheless, the increase of vibration level due to water must be kept under control because water effects depend considerably on water flow (Fig. 92). Quantification of water effects in full scale CLIC magnets, connected to water pumps as expected in the machine operation, remains yet to be carried out.

\(^{40}\)The RMS motion below 4 Hz, larger in the LHC tunnel than on top of the stabilized doublet, can explain the larger differences obtained when applying LHC-like motion to the final doublet or to all the machine.
Figure 105: Average vertical luminosity, $<\mathcal{L}>/L_0$, versus vertical feedback gain when lattice magnets move like a stabilized doublet, with (solid line) or without (dashed) alignment support and like the ground (dotted). Average luminosity over 1 second of CLIC operation is given.

It is noted that water induced vibrations can be a major limitation in the water cooled linac quadrupole of CLIC. Vertical tolerances for these magnets are on the 1.3 nm level (see Table 6), which can easily be attained due to water vibrations if the cooling circuit is not properly designed (see, for example, Fig. 92).

6.2.4 Effect of Alignment Supports

The effect of resonances in quadrupole alignment support (Section 5.5) is simulated by moving the magnet of the beam delivery system according to the spectra measured on top of a stabilized quadrupole mounted on its alignment support (see Fig. 45). An example scan of the vertical feedback gain to find the optimum settings is shown in Fig. 105. For the given example the achieved fraction of nominal luminosity is approximately 10% lower than what is expected if magnets move without alignment support. The performance in case that the magnets move like the supporting ground is also given.

The average maximum luminosity obtained for 20 seed of magnet displacements is $(50.26 \pm 0.66)$ % of the nominal luminosity, i.e. 18.7% smaller than what is achieved if magnets move like a stabilized doublet without alignment support. The main reason for this reduction of luminosity performance is the support resonance at approximately 40 Hz, i.e. well above the maximal frequency that the beam-based feedbacks can correct. However, even if the support design was not optimized against structural resonances (as in the tested CTF2 supports), 50% of nominal luminosity can be achieved with stabilization systems.

6.2.5 Comparison of Different Stabilization Technologies

The simulation results presented in the previous section have been obtained using as input for the time-dependent displacements of magnets measured with the stiff isolation system. Here, the perfor-
Table 22: Average luminosity ($\langle \mathcal{L} \rangle / \mathcal{L}_0$) over one second of CLIC operation as obtained when lattice magnets are moved. The stability performance of the soft isolation system (Section 4.3.1) is used as input. Displacements are either applied to all lattice magnets (second column) or to the final doublets only (third column) using the longitudinal (first row, $z$ direction of Fig. 40) or horizontal (second row, $x$ direction) table motion.

<table>
<thead>
<tr>
<th>Applied motion</th>
<th>All magnets</th>
<th>Final doublets only</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soft system, longitudinal</td>
<td>(50.08 ± 1.52)%</td>
<td>(55.38 ± 1.49)%</td>
</tr>
<tr>
<td>Soft system, horizontal</td>
<td>(25.06 ± 4.25)%</td>
<td>(47.33 ± 3.97)%</td>
</tr>
</tbody>
</table>

Performance of the soft pneumatic system is considered for comparison. In Table 22, the maximum average luminosity is given over one second of CLIC operation, obtained with the soft isolation system. The cases if displacements are applied to all magnets (second column) and to the final doublets only (third column) are both considered.

It is seen that 50% of nominal luminosity is achieved with the soft stabilization system if the best transverse stabilization (provided along the table longitudinal axis) is relied on. This number is halved if the transverse direction is considered instead. Differences between the two cases are smaller if only the final doublets move. Since no correction of the beam position is applied along the beam delivery system, the uncorrected low-frequency magnet vibrations reduce the luminosity more than if the final doublets only are moved.

The soft stabilization system is performing less than the stiff system but still provides quite good luminosity performance. However, the measured low-frequency instabilities of this system (see Fig. 61) could prevent using it for accelerator purposes. The low-frequency and high-amplitude vibrations that this system introduces are uncorrelated with respect to the ground and destroy the good correlation over long distances between the lattice elements, a crucial ingredient for a linear collider.

6.3 Effect of slow vibrations

So far, vibrations above 1 Hz have been considered to estimate the pulse-to-pulse stability of luminosity performance over 1 second of CLIC operation. Experimental results show that above 1 Hz the motion of actively and passively stabilized magnets is uncorrelated with respect to ground. Based on these measurements, each magnet was moved independently of the others, according to the same measured absolute spectrum. This means that the relative motion between any two magnets (notably, the two final quadrupoles) is $\sqrt{2}$ times larger than the absolute motion of each of the two. In order to simulate the machine performance for longer times, magnet motion below 1 Hz is taken into account, which requires detailed knowledge of the spatial correlation of motion between different lattice elements.

Due to, for instance, tidal effects, the amplitude of ground motion below a fraction of a Hertz increases steeply for decreasing frequencies (see Fig. 84). For example, the total RMS motion above 0.1 Hz is up to several hundreds of nanometres. However, correlated displacements of the accelerator as a whole are not a limiting factor for the luminosity performance. What matters are the relative displacements of magnets: if there were no correlation at all between the low-frequency motion of the lattice elements, significant luminosity would be produced. If, for instance, the final focus doublets
Figure 106: Correlation of horizontal (left) and vertical (right) ground vibrations as measured in the underground LEP tunnel at a distance of 200 m. Even at this large distance between probes, a good correlation of motion exists below approximately 1 Hz. The graphs are from [90].

are offset by a micron with respect to the upstream machine, aberration effects annihilate the luminosity by increasing the beam sizes even if the colliding beams are re-centred by the interaction point feedback.

The correlation of the low-frequency, large amplitude motion is therefore a key ingredient for the operation of a long particle accelerator. A full modelling of the 2-dimensional magnet dynamics in the low-frequency range, including spatial correlation of motion along the machine, is required to fully predict the long-term luminosity performance (simulated times larger than a few seconds). This goes beyond the scope of this thesis. Instead, a simplified but relevant model is used to study this regime.

Based on measurements performed in the LEP underground tunnel [90], a low-frequency correlation up to distances of hundreds of metres is expected in hard-rock sites. Figure 106 shows, for example, the measured horizontal (left graph) and vertical (right graph) correlation between two probes placed along the LEP tunnel, 200 m apart. In both directions, the correlation of motion is very close to 1 below approximately 1 Hz. Measurements performed at SLAC [107] show that a similar correlation of motion is also measured in sedimentary sites at ground level. An example is given in Fig. 107, where the correlation of vertical motion as measured at a distance of 100 m is shown. Ideally, the magnet support should be designed such that the magnet motion at low frequencies is correlated with the ground, in order to maintain between lattice magnets the good spatial correlation measured on the supporting ground. This condition is difficult to achieve because supports are not perfectly rigid and show structural resonances. In addition, passive and active stabilization breaks the ground-to-magnet on purpose.

In order to prove that an isolation device can be used for stabilizing magnets of an accelerator, it must be verified that passive and active damping do not decouple the low-frequency vibrations of a magnet from the ground. This can be a concern because the resonant frequencies of stabilization devices are typically below 1 Hz. No luminosity would be produced if the magnets moved without correlation in the low-frequency range, independently of the good achievable stability above a few Hertz. Based on the ground-to-table correlation measurements performed with the stiff stabilization system, simulations have been set up with a simple model for the spatial correlation. The aim is to verify that this device is suitable for the CLIC purposes (see below).
Figure 107: Correlation of vertical motion (solid line) as measured on the ground at the SLAC site, with probes at a distance of 100 m. The graph is from [107].

A model for the implementation of the spatial correlation of motion has been developed in [86]. A 2-dimensional power spectrum of ground vibrations, see Eq. (59), is used to generate time- and spatial-dependent misalignments of all lattice elements based on some assumptions on the propagation of sound waves in the medium. Namely, waves are assumed to have a cylindrical symmetry and to propagate with known velocity, possibly frequency-dependent (see also [107] for details on measured frequency dependence of sound velocity). However, preliminary simulations have shown that this model needs to be modified to study the time-dependent performance of an accelerator like CLIC, if active and passive stabilization devices are used to damp the ground motion (as it is assumed here). Indeed, the model describes the correlation properties of ground motion, entirely determined by the used sound velocity (the only free parameter of the model). It is implicitly assumed that the good correlation versus distance as measured on the ground can be applied directly to all lattice magnets.

For instance, the model would predict a correlation up to several Hertz between the two final doublets at either side of the interaction point because they are only a few meters apart. This overestimates the achievable luminosity because the correlation above 1 Hz is zero even for close-by magnets if passive/active stabilization techniques are applied. For the CLIC study it is preferable to use ad hoc correlation curves which can reproduce the measured magnet-to-ground correlation (see Fig. 86).

In order to model the measured ground-to-table correlation of the stiff stabilization system, it is assumed that the two opposing machines move together with their final doublets. Each of the two machines is then displaced in time as a whole according to the horizontal and vertical vibration spectra as measured on a stabilized doublet from 0.03 Hz to 315 Hz (e.g., Fig. 83). This assumption is justified by the fact that displacements of the final focus quadrupoles have the largest impact on the luminosity performance: in the previous section it has been shown that their contribution to the luminosity reduction is comparable or larger than the contribution of all other magnets together.

The time-dependent displacements of the final doublets at either side of the interaction point are generated such as to reproduce the measured correlation between stabilized magnets and ground. The motion of the supporting ground is assumed to be correlated at least below 1 Hz at the short distance

---

41Work on the modification of the ground motion model of [86] is on-going at SLAC. Recently some effects such as local sources of “cultural” noise have been added to the model [168].
between the two final doublets, which is fully in agreement with measurement results of Figs. 106 and 107. As a first approximation, the correlation between magnet and ground is assumed to drop linearly from 1 at 0.2 Hz to 0 at 1 Hz. In Figure 109 this approximation is compared with the measured ground-to-table correlation (see Section 5.3, Fig. 86). The agreement between the two curves is good in the considered frequency range. The same magnet-to-ground correlation curve is assumed for horizontal and vertical vibrations.

The motion of one doublet, \( y_1(t) \), is generated from the measured spectrum, as discussed in Section 6.2.1, using a given set of random phases \( \phi_1(f) \) for different vibration harmonics. If \( \phi_2(f) \) is the phase of motion of the other magnet, \( y_2(t) \), from the definition of correlation, Eq. (57), it is found that

\[
\mathcal{C}(f) = \text{Re} \left\{ e^{i(\phi_2(f) - \phi_1(f))} \right\}.
\]

The phases \( \phi_2(f) \) induce the desired linear decrease of \( \mathcal{C}(f) \) and are calculated inverting the above expression. They are then used to generate the motion of the other doublet according to Eq. (93). A Gaussian error with sigma of 0.1 is assumed on the correlation curve.

An example of time-dependent vertical displacements of the final doublets is shown in Fig. 108 (solid and dashed lines). The difference between the two magnets is also given (bold line). The obtained correlation versus frequency, given in Fig. 110, matches well a linear decrease from 0.2 Hz to 1 Hz (dotted line of Fig. 110). Above 1 Hz the noisy variation of \( \mathcal{C}(f) \) around the null value, induced by the small frequency resolution (\( \Delta f = 0.033 \) Hz), denotes the uncorrelated motion. The absolute and relative integrated RMS amplitudes versus frequency are given in Fig. 111 as calculated from the results of Fig. 108. The integrated relative motion above 0.1 Hz is approximately one order of magnitude smaller than the absolute one. Above 1 Hz, where correlation is zero, the relative RMS motion is instead \( \sqrt{2} \) times larger.

The time-dependent luminosity performance of CLIC is estimated assuming that the two opposing machines move horizontally and vertically like the final quadrupoles at either side of the interaction point. A simulation result is given in Fig. 112, where relative luminosity \( (\langle \mathcal{L} \rangle / \mathcal{L}_0) \), top graph) and
Figure 109: The correlation versus frequency, $c(f)$, as measured between table and ground (dashed line) is compared with the model for the correlation between the final doublets at either side of the interaction point (solid bold line) used in the simulations of long-term luminosity performance. Note that the correlation is directly measured down to approximately 0.1 Hz, whereas below this value it is inferred by comparing the measurements of low-frequency geophones and stretched-wire system, as discussed in Chapter 5.

Figure 110: Correlation of vertical motion of final quadrupoles as calculated from solid and dashed lines of Fig. 108. The dashed line shows the aimed linear (note the logarithmic scale) decrease from $c(f) = 1$ at 0.2 Hz to $c(f) = 0$ at 1 Hz. It is noted that $c(f)$ is very noisy above $\approx 1$ Hz. This is because only one seed is shown (the curve corresponds to the time-dependent displacements of Fig. 108). Measured correlation curves are usually averaged over several sets of data and hence look smoother.
Figure 111: Absolute (solid line) and relative (dashed line) integrated RMS motion versus frequency calculated from the time-dependent quadrupole positions of Fig. 108. Note that the relative integrated RMS motion is flat below 0.2 Hz where doublet displacements are perfectly correlated.

vertical (middle) and horizontal (bottom) beam-beam offset versus time are shown. The average luminosity achieved over 20 seed for the magnet displacements is (70.87 ± 0.15)%. This is comparable to what is obtained considering only fast vibrations above 1 Hz. Therefore, the interaction point feedback is capable of compensating the uncorrelated motion below 1 Hz introduced by the passive/active stabilization device. The tested system does not modify in an intolerable way the spatial property of ground motion and therefore is suitable to be used for the stabilization of operating particle accelerators.

The model used for the 2-dimensional displacements of magnets takes into account only the effects of relative misalignments of the final doublets at either side of the interaction point. All along the machine, the beam passes through the magnet centres. The model does not include uncorrelated relative displacement of other lattice elements. This prevents using it to estimate degradation of luminosity performance due to beam size increases.
Figure 112: Relative luminosity ($L(t)/L_0$, top graph), and vertical (middle) and horizontal (bottom) beam-beam offsets at interaction point versus time when each machine moves as a whole like the final doublet (see Fig. 108). Horizontal and vertical feedback systems are switched on.
7 Summary

The feasibility of colliding high-energy nanobeams in CLIC has been assessed from detailed experimental and numerical studies. A test stand has been set up and equipped with advanced measurement equipment, prototypes of accelerator magnets and state-of-the-art stabilization technology. Vibration measurements above 4 Hz were established with a resolution of 0.28 nm. The error on the vibration measurements was evaluated to be below 10%. This is based on experimental studies performed in addition to the manufacturer’s calibration. Spectral vibration measurements were established from 0.03 Hz up to 300 Hz, covering four orders of magnitude in frequency.

The measurement tools were used to characterize the mechanical vibration of CLIC magnets under various conditions. In particular, the magnet motion could be decoupled successfully from the vibration of the supporting ground, using modern stabilization equipment. The stabilized magnets then had up to 15 times less vibration than the ground. Under optimal conditions a magnet was stabilized in the vertical direction to 0.43 nm ± 0.04 nm. Over a period of nine days, the RMS motion was kept below the 1 nm level. These measurements advance the achieved stability of accelerator magnets by more than a factor of 20 with respect to studies carried out in other accelerator laboratories. This is the first time that a magnet was routinely stabilized to the sub-nanometre level in a normal working environment. The horizontal directions were characterized as well, finding RMS vibration levels of 0.79 nm ± 0.08 nm (perpendicular to the beam path) and 4.32 nm ± 0.43 nm (longitudinally to the beam path). Full 3D vibrational spectra were established for further use in numerical studies of luminosity. In addition to the studies under optimal conditions, the impact of various disturbing effects, such as water flow and support resonances was investigated and described in detail.

A complete numerical model for the time-dependent luminosity simulation was set up, taking into account the beam transport through a vibrating beam line, the beam-beam interaction, and a beam-based feedback circuit at the interaction point. Important effects (beam-beam deflection, synchrotron radiation, QED effects) were all included in the simulation set up. In particular a full 2D vibration modeling was implemented for all magnets and interfaced to the various measurements, including all uncorrelated and some correlated effects.

The numerical model was used to predict the achievable luminosity in CLIC under various conditions. It was shown that a luminosity of about $6 \times 10^{35} \text{cm}^{-2} \text{s}^{-1}$ can be achieved, if state-of-the-art stabilization equipment is used and if standard “slow” beam-beam feedbacks are employed. This represents 70% of the CLIC nominal luminosity. A similar luminosity performance can be obtained in a deep underground tunnel without technical noise and without stabilization. Based on the ground at the CLIC stability test stand, only 6% of nominal luminosity could be achieved, if no stabilization were applied. The stabilization technology allows in a normal technical environment achieving magnet stability as good as in a deep underground tunnel without technical installations. Based on these results it is concluded that the CLIC parameters with collision of nanometre size beams are indeed feasible, if modern stabilization equipment is relied on.

Future work will focus on developing particular hardware solutions that are targeted at the accelerator-specific environment and needs. It is expected that the performance and cost can be optimized significantly. In addition, a more complete model, which takes into account for example updated magnet designs, stabilization equipment, correlated ground motion, the experimental detector, “slow” and “fast” beam-beam feedbacks and the extraction lines of the colliders, should be developed. The luminosity reduction due to beam size variations and emittance drifts and its correction should also be investigated.
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9 Bibliography

References


[38] http://www-conf.slac.stanford.edu/1c02/


[40] http://mad.home.cern.ch/mad/


J. Irwin and F. Zimmermann, “Impact of final-focus ground motion on NLC luminosity,” SLAC-PUB-7142 Talk given at 5th European Particle Accelerator Conference (EPAC 96), Sitges, Spain, 10-14 Jun 1996


R. Pitthan, “Re-alignment: It is the tunnel floor which moves, isn’t it?,” SLAC-PUB-7043, Invited talk at 4th International Workshop on Accelerator Alignment (IWAA95), Tsukuba, Japan (1995).


sensor development for active vibration stabilization,” SLAC-PUB-9843, Presented at Particle Accelerator
Conference (PAC 03), Portland, Oregon (2003).

[111] M. N. Obergfell, “Construction And Performance Of A Permanent Earth Anchor (Tieback) System For
The Stanford Linear Collider,” SLAC-PUB-4211, Presented at Int. Symp. on Prediction and Performance


2949.

[114] P. Bambade and R. Erickson, “Beam-Beam Deflections as an Interaction Point Diagnostic for the SLC,”
SLAC-PUB-3979 Contributed to Stanford Linear Accelerator Conference (LINAC86), Stanford, CA
(1986).


the interaction point of the Stanford linear collider,” SLAC-PUB-5512 (1991), presented at IEEE Particle

[117] P. Raimondi and F. J. Decker, “Flat beam spot sizes measurement in the SLC final focus,” SLAC-PUB-
6806, Presented at 16th IEEE Particle Accelerator Conference (PAC 95) and International Conference on
High Energy Accelerators (IUPAP), Dallas, Texas (1995).

proceedings of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001) ed.

Nanobeam2002, 26th Advanced ICFA Beam Dynamics Workshop on Nanometer Size Colliding Beams,
Lausanne, Switzerland (2002).

Timescales for Linear Colliders: Results from the First Beam Tests at the NLC Test Accelerator at SLAC,”
Presented at IEEE Particle Accelerator Conference (PAC2003), Portland, OR (2003).

beam-beam deflections at LEP,” CERN-SL-96-25-OP, Talk given at 5th European Particle Accelerator
Conference (EPAC 96), Sitges, Spain (1996).


In CESR,” prepared for 16th IEEE Particle Accelerator Conference (PAC 95) and International Conference on
High-energy Accelerators (IUPAP), Dallas, Texas (1995).


(1994).
[137] See the WWW site http://www.techmfg.com/. In particular, many details are available in the paper “Technical Background”.


[153] See the website of INPUT/OUTPUT, INC., http://www.i-o.com/


[175] http://www.dt.insu.cnrs.fr/garchy/etalonnages.html#moyens

[176] S. Redaelli, L. Zhang et al., to be published.


A Assessment of tracking results

In order to assess the tracking results, several codes worldwide used for the particle tracking in beam delivery systems of linear colliders have been systematically compared. The CLIC beam delivery system has been used as a case-study to carry out this comparison. Detailed reports on this work have been published in [67] and [68] (see also [169]). Here, the main results are summarized. Four codes have been compared: MAD8 [40], DIMAD [170, 171], Merlin [66], Placet [172, 173].

- **MAD8** is a general all purpose simulation code developed at CERN. Tracking is performed using the *Transport formalism* [174].
- **DIMAD** is a SLAC code that tracks trajectories of the particles according to a second order matrix formalism similar to *Transport*. Release 2.8 has been used.
- **Merlin** is a class library developed at DESY for performing charged particle accelerator simulations using first- and second-order transport matrices.
- **Placet** is a tracking program originally conceived for linac simulations. It can handle both a ray and a macroparticle description of the beam. [173].

The code comparison has been performed by tracking the same input bunches from the entrance of the beam delivery system to the interaction point. Five sets of 6-dimensional particle distributions are generated according to the nominal beam parameters listed in Table 2. The assessment of tracking results is based on the comparison of the predicted horizontal and vertical sizes of colliding beams, as calculated with a Gaussian fit to the tracked particle distributions. The simulation results for different kinds of input bunches are summarized in Table 23 and are graphically depicted in Fig. 113. The bunch with energy spread and synchrotron radiation is the relevant case for a realistic prediction of luminosity. With the exception of MAD, different codes agree within a few percent.

The codes are also compared on the base of the predict luminosity performance. The tracked bunches are used as input for the beam-beam calculations of GuineaPig. Results are given in Table 24. The codes use different models for synchrotron radiation [68]. Placet can simulate all models and is correspondingly compared with the other codes. Discrepancies up to $0.11 \times 10^{35}$ cm$^{-2}$ s$^{-1}$ (i.e., up to 16%) are found between different models. On the other hand, if the same model is used differences are within a few percent, not relevant for the CLIC study purposes. Note that the lattice design was done with MAD and its prediction of $0.82 \times 10^{35}$ cm$^{-2}$ s$^{-1}$ should be regarded as CLIC nominal value.
Table 23: Horizontal (top) and vertical (bottom) beam sizes at the interaction point after tracking as calculated with four tracking codes. MAD, DIMAD, Merlin and Placet. Beam sizes are calculated as width of Gaussian fits to the tracked particle distributions. The average of five seeds for the bunch generation is given with its error.

<table>
<thead>
<tr>
<th></th>
<th>No $\Delta E$ - No SR</th>
<th>$\Delta E/E=1$ - No SR</th>
<th>$\Delta E/E=1$ - SR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal beam sizes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAD</td>
<td>38.35 nm±0.14 nm</td>
<td>42.83 nm±0.08 nm</td>
<td>55.39 nm±0.07 nm</td>
</tr>
<tr>
<td>DIMAD</td>
<td>37.45 nm±0.12 nm</td>
<td>44.67 nm±0.08 nm</td>
<td>54.59 nm±0.17 nm</td>
</tr>
<tr>
<td>Merlin</td>
<td>37.38 nm±0.13 nm</td>
<td>44.48 nm±0.07 nm</td>
<td>57.49 nm±0.13 nm</td>
</tr>
<tr>
<td>Placet</td>
<td>36.96 nm±0.12 nm</td>
<td>43.99 nm±0.08 nm</td>
<td>54.12 nm±0.17 nm</td>
</tr>
<tr>
<td>Vertical beam sizes</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MAD</td>
<td>0.793 nm±0.001 nm</td>
<td>0.758 nm±0.003 nm</td>
<td>0.680 nm±0.001 nm</td>
</tr>
<tr>
<td>DIMAD</td>
<td>0.524 nm±0.001 nm</td>
<td>0.590 nm±0.001 nm</td>
<td>0.800 nm±0.002 nm</td>
</tr>
<tr>
<td>Merlin</td>
<td>0.527 nm±0.001 nm</td>
<td>0.601 nm±0.001 nm</td>
<td>0.688 nm±0.002 nm</td>
</tr>
<tr>
<td>Placet</td>
<td>0.523 nm±0.001 nm</td>
<td>0.606 nm±0.001 nm</td>
<td>0.775 nm±0.002 nm</td>
</tr>
</tbody>
</table>

Figure 113: Horizontal (left) and vertical (right) beam sizes as simulated with the different codes (listed on the graph horizontal axes), see Table 23. Spot sizes of an ideal bunch with no energy spread nor synchrotron radiation are given by the stars. Energy spread (crosses) and synchrotron radiation (diamonds) are also considered. Error bars are small and are omitted.

Table 24: Average luminosity (5 seeds for bunches to track) as calculated with different codes for CLIC bunches with energy spread and synchrotron radiation. MAD, DIMAD, Merlin use different models for photon emission [68], Placet can simulate all models and thus its results (last column) are correspondingly compared with the other codes.

<table>
<thead>
<tr>
<th>Average CLIC luminosity [$10^{35}$ cm$^{-2}$ s$^{-1}$]</th>
<th>Placet</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAD</td>
<td>0.817±0.003</td>
</tr>
<tr>
<td>DIMAD</td>
<td>0.747±0.005</td>
</tr>
<tr>
<td>Merlin</td>
<td>0.704±0.002</td>
</tr>
</tbody>
</table>
B  Luminosity reduction from offsets and beam size

B.1  Luminosity reduction from beam-beam offsets

Consider two short bunches with normalized transverse distribution given by the Gaussian:

$$\psi_i(x, y; \sigma_x^i, \sigma_y^i) = \frac{1}{2\pi\sigma_x^i\sigma_y^i} \exp\left[-\frac{x^2}{2\sigma_x^i} - \frac{y^2}{2\sigma_y^i}\right].$$  \hspace{1cm} (95)

The index $i \equiv +, -$ refers to the positron and electron beams, which have charge $N_b^+\text{ and } N_b^-$, respectively. Neglecting the disruption effects, the luminosity is given by the geometric superimposition of the particle distributions and reads:

$$\mathcal{L}_{\text{Geo}} = f_{\text{rep}} n_b N_b^+ N_b^- \int dx dy \psi_+(x, y) \psi_-(x, y),$$  \hspace{1cm} (96)

where $f_{\text{rep}}$ is the bunch train repetition frequency and $n_b$ the number of bunches per train. It is assumed to have a head-on collision between short bunches, so that the integration in the longitudinal and time dimensions can be disregarded. In the case of perfectly aligned beams, the integral of Eq. (96) reads

$$\mathcal{L}_{\text{Geo}} = \frac{f_{\text{rep}} n_b N_b^+ N_b^-}{(2\pi)^2\sigma_x^+\sigma_x^-\sigma_y^+\sigma_y^-} \times \int_{-\infty}^{+\infty} dx dy \exp\left[-\frac{x^2}{2\sigma_x^+\sigma_x^-}\sigma_x^2 x^2 - \frac{y^2}{2\sigma_y^+\sigma_y^-}\sigma_y^2 y^2\right].$$  \hspace{1cm} (97)

The integration of the Gaussian functions gives

$$\mathcal{L}_{\text{Geo}} = \frac{f_{\text{rep}} n_b N_b^+ N_b^-}{2\pi \Sigma_x^+ \Sigma_y^+}$$  \hspace{1cm} (98)

where

$$\Sigma_x^2 = \sigma_x^+ + \sigma_x^-$$  \hspace{1cm} (99)
$$\Sigma_y^2 = \sigma_y^+ + \sigma_y^-$$  \hspace{1cm} (100)

are the effective horizontal and vertical beam sizes at the interaction point. For identical colliding beams, the geometric luminosity of Eq. (98) reduces to Eq. (11), which also include the disruption coefficient $\mathcal{H}_D$.

Let us assume that the electron and positron bunches centroids have a relative vertical offset $\Delta y$. The case of horizontal offsets is analogous. According to Eq. (96), the luminosity can be calculated considering the shifted particle distribution $\psi_-(x, y - \Delta y)$ (it is assumed here that the positron bunch is centred in the nominal collision point whereas the electron bunch is displaced). The geometric
luminosity in this case, \( L_{\text{Geo}}(\Delta y) \), is then given by\(^{42}\):

\[
L_{\text{Geo}}(\Delta y) = \frac{f_{\text{rep}} n_b N_b^+ N_b^-}{(2\pi)^2 \sigma_x^+ \sigma_x^- \sigma_y^+ \sigma_y^-} \times \int_{-\infty}^{+\infty} dx dy \exp \left[ -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right] \exp \left[ -\frac{x^2}{2\sigma_x^2} - \frac{(y - \Delta y)^2}{2\sigma_y^2} \right]
\]

\[
= \frac{f_{\text{rep}} n_b N_b^+ N_b^-}{(2\pi)^2 \sigma_x^+ \sigma_x^- \sigma_y^+ \sigma_y^-} \exp \left[ \frac{(\Delta y)^2}{2\sigma_y^2} \right] \times \int_{-\infty}^{+\infty} dx dy \left\{ \frac{1}{2} \frac{\sigma_y^+ + \sigma_y^-}{\sigma_y^+ \sigma_y^-} x^2 + \frac{1}{2} \frac{\sigma_y^+ - \sigma_y^-}{\sigma_y^+ \sigma_y^-} y^2 - 2\frac{y \Delta y}{\sigma_y^2} \right\}. \quad (101)
\]

The integration can be performed, for example, by completing the square in \( y \) in the last exponent term (expression in squared brackets). This results in an additional factor \( \exp\left\{ (\sigma_y^+)^2/[2\sigma_y^2 (\sigma_y^+ + \sigma_y^-)]\Delta y^2 \right\} \) in front of the integral. Thus, the final result is

\[
\left[ \frac{L(\Delta y)}{L_0} \right]_{\text{Geo}} = \exp \left[ -\frac{\Delta y^2}{2\Sigma_y^+ \Sigma_y^-} \right], \quad (102)
\]

where \( L_0 \) is the zero-offset geometric luminosity of Eq. (98). Again the index * denotes beam parameters of the interaction point. Similar results are obviously found if the positron beam is moved instead or if a horizontal offset is considered because what matters is the relative offset between the colliding bunches. For the case of identical colliding beams, discussed in Section 3.2, with \( \sigma_y^+ = \sigma_y^- \equiv \sigma_y^* \), Eq. (102) simplifies as

\[
\left[ \frac{L(\Delta y)}{L_0} \right]_{\text{Geo}} = \exp \left[ -\frac{\Delta y^2}{4\sigma_y^2} \right]. \quad (103)
\]

**B.2 Luminosity reduction from beam size variations**

In order to calculate the effect of beam size variation on the luminosity, the following transformations

\[
\sigma_y^+ \rightarrow \sigma_y^+ + \delta^+ \\
\sigma_y^- \rightarrow \sigma_y^- + \delta^-
\]

are plugged into Eq. (98). \( \delta^+ \) and \( \delta^- \) are the variations of the positron and electron vertical spot sizes and are assumed to be different for the moment. The vertical direction only is considered but the results obtained will also hold for the horizontal plane. Note that for legibility, the index * denoting interaction point values is omitted. The geometric luminosity thus reads:

\[
L_{\text{Geo}} \sim \frac{1}{\sqrt{\sigma_y^+ + \sigma_y^-}^2} \rightarrow \frac{1}{\sqrt{(\sigma_y^+ + \delta^+)^2 + (\sigma_y^- + \delta^-)^2}}
\]

\[
= \frac{1}{\sqrt{\sigma_y^+ + \sigma_y^-}^2} \times \frac{1}{\sqrt{1 + \frac{2(\sigma_y^+ \delta^+ + \sigma_y^- \delta^-)}{\sigma_y^+ + \sigma_y^-} + \frac{\delta^+ \delta^-}{\sigma_y^+ + \sigma_y^-}}}.
\quad (105)
\]

\(^{42}\)For legibility, the index * denoting beam parameters at the interaction point is omitted in Eq. (101)
where the constants have been omitted.

Assuming that the beam size variations $\delta^+$ and $\delta^-$ are small compared to $\sigma_y^+$ and $\sigma_y^-$, the second-order terms can be neglected and the second square root term of Eq. (105) can be developed so to give:

$$L_{\text{Geo}} \approx \frac{1}{\sqrt{\sigma_y^{+2} + \sigma_y^{-2}}} - \frac{\sigma_y^+ \delta^+ + \sigma_y^- \delta^-}{(\sigma_y^{+2} + \sigma_y^{-2})^{3/2}}.$$  \hspace{1cm} (106)

Therefore, the relative luminosity reduction is given by:

$$\left[ \frac{\Delta L}{L_0} \right]_{\text{Geo}} = \frac{\sigma_y^+ \delta^+ + \sigma_y^- \delta^-}{\sigma_y^{+2} + \sigma_y^{-2}},$$  \hspace{1cm} (107)

where $\Delta L = L_0 - L$ is the luminosity reduction with respect to its nominal value. Only the geometric contribution to the luminosity is considered and the disruption is neglected.

If two colliding beams with the same transverse spot size are considered, $\sigma_y^+ = \sigma_y^- \equiv \sigma_y^*$, then the relative luminosity reduction is given by:

$$\left[ \frac{\Delta L}{L_0} \right]_{\text{Geo}} = \frac{\delta^+ + \delta^-}{2 \sigma_y^*} \text{ (same size).}$$  \hspace{1cm} (108)

In particular, if also the two spot size variations are identical, $\delta^+ = \delta^- \equiv \Delta \sigma_y^*$

$$\left[ \frac{\Delta L}{L_0} \right]_{\text{Geo}} = \frac{\Delta \sigma_y^*}{\sigma_y^*} \text{ (same size and variation).}$$  \hspace{1cm} (109)

So, if the size perturbations are identical for the two beams and the beams have the same unperturbed beam size, then the relative luminosity reduction corresponds to the relative beam size increase. This can be directly seen from Eq. (11). On the other hand, if only one bunch changes its vertical spot size, e.g. $\delta^+ \equiv \Delta \sigma_y^*$ and $\delta^- = 0$, then Eq. (108) reads:

$$\left[ \frac{\Delta L}{L_0} \right]_{\text{Geo}} = \frac{1}{2} \frac{\Delta \sigma_y^*}{\sigma_y^*} \text{ (only one bunch changes).}$$  \hspace{1cm} (110)

This is indeed the case discussed in Section 3.2.3. If the luminosity reduction is mainly induced by beam size variations rather than by beam-beam offsets, the total luminosity variation is half of what is expected from the beam size increase of a single bunch. It is noted that the factor $1/2$ on the right hand side of Eq. (110) explains why the tolerances for 2% beam size increase of one bunch are typically twice as small as the global tolerances on the 2% luminosity increase for those magnets that mainly induce beam size variation (see Figs.20 - 23).

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\(^{43}\)This is obviously the case for the tolerance calculations, when spot size variations as small as a few percent are considered.

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C Details of the reference geophones

In this appendix an empirical estimate of the sensor resolution and a frequency calibration of the reference geophones are discussed.

C.1 Measure of sensor resolution

On the assumption that two sensors have the same absolute calibration\(^44\), an upper estimate of the sensor resolution can be found as the difference signal between two sensors placed side-by-side. The two output signals from close-by sensors should be identical but differences within the sensor resolution can arise. Here, the effect of the finite sensor resolution is assumed to be a frequency independent Gaussian noise with amplitude \( \sigma_{\text{Res}} \), to be added to the real signal \( S_{\text{Real}} \). The validity of this assumption will be discussed later. The measured signal, \( S_{\text{Meas}} \), is then given by the sum in quadrature of the two above contributions: \( S_{\text{Meas}}^2 = S_{\text{Real}}^2 + \sigma_{\text{Res}}^2 \). In the case of a velocity measurement from a geophone the measured signal is the time dependent velocity. The frequency independent resolution on the velocity corresponds to a resolution on the displacement that decreases as \( 1/f \), since in frequency domain the displacement \( \hat{y}(f) \) depends on the velocity \( \hat{v}(f) \) as \( \hat{y}(f) \approx \frac{\hat{v}(f)}{2\pi f} \).

The Fourier techniques discussed in Section 3.2.4 is used to analyze the geophone measurement data. Data are acquired with an acquisition frequency of 1 kHz for a total time of 300 s and averaged over subsets of 5 s to reduce the statistical uncertainty. Measurements are performed in the LHC tunnel, which ensures very quiet environmental conditions suitable to measure the resolution limit of the sensors. Measurements performed on the stabilized support table (see Section 4.3) are in agreement with the underground measurements. The vertical direction is considered in the following. The electronics the same for all geophone pick-up coils and hence the results discussed below apply also to the horizontal directions.

If two sensors are placed side-by-side and measure the same \( S_{\text{Real}} \), the integrated difference of Eq.(62) gives a measure of the resolution according to:

\[
I_{\text{rel}}(k) = \sqrt{2} \hat{\sigma}_{\text{Res}}(k).
\]  
\[
(111)
\]
\(\hat{\sigma}_{\text{Res}}(k)\) is the Fourier transform of the resolution signal. For a Gaussian noise the amplitudes in time and frequency-domain are the same. The factor \( \sqrt{2} \) arises because the resolution error of the two sensors must be summed in quadrature. The quantity of interest is the resolution of the integrated RMS displacement versus frequency, \( I_{\text{Res}}(k) \). It can be obtained by measuring the power spectral density of the difference signal from two sensors placed side-by-side, \( P_{\text{Res}}(k) \). \( P_{\text{Res}}(k) \) is actually obtained as a fit of the measured spectrum, as shown in Fig. 114. The linear variation of resolution on the displacement spectrum shows that the assumed Gaussian, frequency independent resolution of velocity is a good approximation. Since data vary over several orders of magnitude, it is more convenient to fit the logarithm of \( P_{\text{Res}}(k) \) versus the logarithm of the frequency. In order to have dimension-less quantities in the logarithm arguments, the parameters \( P_0 = 1 \text{ nm}^2/\text{Hz}, A_0 = 1 \text{ nm} \) and \( f_0 = 1 \text{ Hz} \) are introduced so that the function to fit reads:

\[
\frac{P_{\text{Res}}(k)}{P_0} = \frac{A}{A_0} \left( \frac{f}{f_0} \right)^B,
\]
\[\text{(112)}\]
\(^{44}\) This assumption has been experimentally verified by checking that the average of the difference signal from close-by sensors is zero within the measurement uncertainty.
Figure 114: Measured power spectral density of the difference signal ($P_{\text{Res}}(k)$) as measured by two geophones placed side-by-side (dots) and its linear fit according to Eq. (114) (dashed line). The resolution of the two sensors must be added in quadrature when calculating the power spectral density of the difference signal. Therefore, the fit must be performed for the measured $P_{\text{Res}}(k)$ divided by 2.

or:

$$\log \left[ \frac{P_{\text{Res}}(k)}{P_0} \right] = \log \left[ \frac{A}{A_0} \right] + B \left[ \log \frac{f}{f_0} \right],$$

(113)

with $P_0 = A_0 f_0^B$. $A$ and $B$ are the fitting parameters. $P_{\text{Res}}(k)$ is obtained as

$$P_{\text{Res}}^{\text{Fit}}(k) = P_0 \times 10^{\frac{A}{45}} \times \left( \frac{f}{f_0} \right)^B.$$

(114)

The best fit to the measured data is obtained for $A = 0.7551 \pm 0.0201 \, \mu m$ and $B = 2.1820 \pm 0.0060$. The quantity of interest is the frequency-dependent resolution on the RMS displacement, $I_{\text{Res}}(k)$. It can be calculated integrating $P_{\text{Res}}^{\text{Fit}}(k)$ as in Eq. (50)\(^{46}\). The vertical sensor resolution versus frequency is given in Section 4.2.2, Fig. 49. For instance, the resolution on the RMS motion above 4 Hz is 0.28 nm.

The resolution curve of Fig. 49 must be subtracted in quadrature from the measured integrated RMS motion versus frequency in order to obtain the actual RMS motion. Figure 115 shows an example of this resolution correction to the vibration RMS motion as measured in the LHC underground tunnel and in the CLIC test stand laboratory.

\(^{45}\)The linear fit of Eq. (113) gives the ratio $A/A_0 = 10^a$ with $a = -6.1220 \pm 0.0115$. The corresponding error on $A$, $\sigma_A$, must be calculated with the formula of the error propagation as $\sigma_A = \ln(10) \times 10^{-6.122} \times 0.0115$ (see, for instance, [156]).

\(^{46}\)Note that the fitted data of Fig. 114 are not corrected with the sensor response function as it is normally done for the measurement data. The fit without actual response function is much simpler because $P_{\text{Res}}(k)$ is linear in all the frequency range (see Fig. 114). The response function is then applied to the fitted $P_{\text{Res}}^{\text{Fit}}(k)$ before performing the integration.
C.2 Frequency calibration

The reference sensors are typically used in connection with the data acquisition system provided by the manufacturer (CR4 box), to be connected to a dedicated PC software for the data taking. It was found that the acquired signal features a frequency distortion: the measured frequencies are systematically lower than the actual one. Even though the difference is almost negligible for the purposes of the CLIC study, measurements have been performed to calibrate the frequency response of the reference geophones. Note that the measured frequencies are correct if the analog signal is taken out of the geophones and integrated with an independent analog to digital converted (ADC).

A geophone is placed on top of the honeycomb support structure stabilized with the Stacis2000 system (see Section 4.3.2. This provides a very quiet environment ideal for the sensor calibration. Sound vibrations are induced with a loudspeaker placed a few centimetre apart from the table. The vibration amplitude, i.e. the loudspeaker voltage, is kept constant while changing the frequency, even if it should not have any relevance for the frequency calibration. The vibration generator has a frequency error negligible for the calibration purpose. No distortions are expected from possible structural resonances of the table.

The geophone signal is acquired with a 0.001 s sampling time for a total acquisition time of 300 s and the vibration power spectral density is calculated as average of 10 s long subsets. The input frequency from the loudspeaker appears as a sharp peak in the power spectral density. The error on the peak position is taken as half the frequency resolution, i.e. $0.5 \times \frac{1}{10} = 0.05$ Hz. The frequency error of the geophone acquisition system versus measured frequency if shown in Fig. 116. A linear fit to the

$$f = 0.01057 f_{\text{meas}} - 0.0070,$$

A frequency calibration was also performed in the French CNSR laboratory [175] for seismometric calibrations. There, vibrations were induced with an oscillating surface suitable for geophone calibration. The calibration results are in agreement within less than 1% with the ones performed at CERN with loudspeaker induced vibrations. The fitted frequency error is $\Delta f = 0.01057 f_{\text{meas}} - 0.0070$, to be compared with the value specified of Fig. 116.
Figure 116: Difference between actual ($f_{\text{actual}}$) and measured ($f_{\text{meas}}$) frequencies versus measured frequency. Vibration of different $f_{\text{actual}}$ is induced on the support table with a loudspeaker and measured with geophone placed on the table top. The error on each point is calculated as half of the frequency resolution, i.e. 0.05 Hz. A measured point gives a frequency error of about $+1.1\%$ (the measured frequency is systematically larger than the actual one). All the measurement results presented in this thesis take into account this small distortion.
D  Comparison of geophones for vibration measurements

In order to assess the absolute calibration of the reference sensor, it has been systematically compared with some low-frequency seismometers for vibration measurements, available at ESRF and at CERN. The comparisons are performed in different experimental conditions, with ground motion between a few nanometres (CERN area) up to tens of nanometres (ESRF area).

D.1 Low frequency geophones used at ESRF

In collaboration with European Synchrotron Light Facility (ESRF), the reference geophones have been compared [176] with low-frequency seismometers by Guralp, similar to the ones described in Section 4.2.2 (see Table 11), used for vibration measurements at the ESRF. The ESRF sensors measures vibration velocities in 0.033 Hz to 50 Hz frequency range. The experimental installation setup for on the ESRF site for the sensor comparison is shown in Fig. 117. Three ESRF geophones are placed side-by-side with three CLIC reference sensors. A couple of sensors lays on the concrete floor of the ESRF building and two other couples sit on a granite table stabilized with the stiff stabilization system (see Section 4.3.2). The experimental set-up of Fig. 117 is located along an ESRF beam line. In order to synchronize the data from the two kinds of geophones, the sensor analog outputs are integrated with the same analog to digital converter. This also prevents possible inhomogeneities induced by the different data acquisition systems. Data are recorded at an acquisition frequency of 1280 Hz for a total time of 120 s. Power spectral densities and integrated RMS displacements are calculated over subsets of 5 s and then averaged, which reduces the statistical uncertainty on the measurement results by a factor $\sqrt{120/5} = 4.9$. The results of the measurement discussed here are taken in the late afternoon, after working hours. The room that houses the experimental set-up of Fig. 117 has a controlled temperature. The sensors had settled for about one day at the same location before the data recording.

Figure 117: Experimental set-up for the sensor comparison at ESRF. The sensors where installed in an ESRF beam line at controlled temperature and had settled for about one day before the data taking.

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48 Two measurement campaigns have been set up at CERN (July 2002) and at ESRF (March 2003) in collaboration with L. Zhang. M. Lesourd (ESRF) helped in the data acquisition. In this appendix, the results of the measurement performed on the ESRF site are discussed.
The power spectral density of vertical vibrations, $P_y(f)$, as measured on the ground of the ESRF site with the two kinds of geophones is shown versus frequency $f$, in Fig. 118. The ESRF geophones are referred to as Guralp (Gur) whereas the CLIC reference sensors are labelled as GeoSig (Geo). The agreement between the two sensors in the comparable frequency range (i.e., from a few Hertz to 50 Hz) is good. In Figure 119 the low-frequency part of the spectra in Fig. 118 is zoomed out. Here, the dashed line shows the raw data as measured by the reference geophone, without taking into account the off-line correction due to the actual sensor response function (Section 4.2.2). It is noted that the off-line application of the response function allows obtaining spectral amplitudes in agreement with the ones from the Guralp sensor down to approximately 1.5 Hz even if the resonance of the geophone reference mass is $\approx 4.5$ Hz.

The integrals of the vibration spectra of Fig. 118 are given in Fig. 120. The integration is performed up to the maximum frequency $f_{\text{max}} = 45$ Hz. A good agreement between the different sensors is found over almost two orders of magnitude in displacements (from about 1 nm to about 100 nm). As discussed in Section 4.2.2, the comparison of different devices for vibration measurements is based on the integrated RMS displacements versus frequency as measured by the two sensors. The ratio

$$R(f) \equiv \sqrt{\frac{P^\text{Gur}_y(f)}{P^\text{Geo}_y(f)}}$$

is given in Fig. 121. $1 - R(f)$ gives the relative difference between the measurements from the two devices. Based on the results of Fig. 119, the comparison can be carried out above approximately 1.5 Hz, below which the low-frequency limit of the reference geophone becomes effective. In the comparable frequency range, i.e. from 1.5 Hz to 45 Hz, the differences are always smaller than 10%. The largest difference arises at 6 Hz, where the ESRF sensor measures an RMS motion approximately 10 % larger than the CLIC reference geophone. The results of Fig. 121 are used in Fig. 50 and Table 10 to estimate the absolute error of the reference geophone.
Figure 119: Power spectral densities of vertical ground vibrations, $P_y(f)$, versus frequency, $f$, as measured with the Guralp seismometer used at ESRF (solid line) and with the CLIC reference sensors by GeoSig (dashed and dotted lines). For the reference geophone, the cases obtained with (dashed) and without (dotted) applying the actual sensor response function are considered. Measurements are performed on the ESRF site. Data are recorded simultaneously with the same data acquisition system.

Figure 120: Vertical RMS ground vibrations, $I_y(f)$, versus frequency, $f$, integrated above $f$ as measured with the Guralp seismometer used at ESRF (solid line) and with the CLIC reference sensors by GeoSig (dotted lines). Measurements are performed on the ESRF site. Data are recorded simultaneously with the same data acquisition system. The power spectral densities of Fig. 118 are integrated up to 45 Hz in order not to take into account an electromagnetic noise that was found to affect the reference geophones.
Figure 121: Ratio of vertical RMS motion, $R(f)$, versus frequency, $f$, as measured with the Guralp seismometer used at ESRF and with the CLIC reference sensors by GeoSig, see Eq. (115). Measurements are performed on the ESRF site. Data are recorded simultaneously with the same data acquisition system.

D.2 Low frequency geophones used at CERN

The reference geophone has also been compared with the low-frequency seismometer by Guralp available for the CLIC stability studies since April 2003 (Section 4.2.2). The comparison is performed by placing the two sensors side-by-side on the table or on the ground. Data from the different data acquisition systems are recorded using the same software, which ensures simultaneity and synchronization. Data are taken at an acquisition frequency of 200 Hz, i.e. a factor 4 larger than the maximum frequency measurable by the low frequency geophone (i.e., 50 Hz).

The vertical power spectral densities as measured by the two devices on the ground of the CLIC test stand are given in Fig. 122. The corresponding integrated RMS displacements are shown in Fig. 123. Only frequencies up to 55 Hz are considered in the calculation of the integrals. The measurements show a good agreement in all the comparable frequency range. The ratio $R(f)$ as defined in Eq. (115) is shown in Fig. 124. The percentage differences of the measures of integrated RMS motions from the two devices, given by $1 - R(F)$, are smaller than 10% at all the compared frequencies. The largest difference is found at approximately 1.5 Hz, i.e. at a frequency which is at the limit of measurable range of the reference geophone (see also Fig. 119). In this low-frequency range, where the amplitudes measured by the reference geophone are inferred with the off-line application of the sensor response function, a difference of 10% only seems reasonably small.

The results of Fig. 124 are used in Fig. 50 and Table 10 to estimate the absolute error of the reference geophone.
Figure 122: Power spectral density of the ground vertical vibrations, $P_y(f)$, versus frequency, $f$, as measured by the Guralp (solid line) and the GeoSig (crossed line) sensors in the CLIC test stand. Data are recorded simultaneously with the same data acquisition system.

Figure 123: Integrated RMS vertical displacements, $I_y(f)$, versus frequency, $f$, integrated above $f$ as measured by the Guralp (solid line) and by the GeoSig (crossed line) sensors in the CLIC test stand.
Figure 124: Ratio of the integrated RMS displacements of Fig. 123, $R(f)$, versus frequency, $f$, as defined in Eq. (115).
E  Comparison between geophones and a capacitive sensor

The comparison between different kinds of geophones, operating in different frequency ranges, has shown a very good agreement (see Appendix D). This provides an assessment of the absolute calibration of the reference sensors. An even stronger validation of the measurement results can be obtained by comparing devices that measure vibrations based on different physical phenomena. In this appendix, the comparison between the reference geophone and a capacitive distance-meter is discussed.

E.1  Detail of the capacitive sensor

A photograph of the capacitive sensor used for the comparison is given in Fig. 125. This is the high-performance MCC0.5 prototype by Fogale [148] which provides a non-contact measure of relative distances between the sensors active area (small round part in Fig. 125) and a target surface. The measurement distance is inferred by a measure of the electric capacity of the capacitor formed by the sensor/target system. Over a range of some centimetres, the capacity is within a good approximation of a linear function sensor-to-target distance. The signal from the MCC0.5 sensor is treated with the acquisition box MC900 by Fogale, which provides 1 V per mm. The technical specifications of the overall system are listed in Table 25. Note the large bandwidth (0 Hz-10 kHz) which enables measuring fast vibrations 49.

The MCC0.5 sensor can be used over a total measuring range of 5 cm with a linearity of 0.4%. In order to perform a comparison with the reference geophone on the nanometre level, the manufacturer was asked to perform a more precise calibration over a shorter range. The most precise calibration

Figure 125: Detailed photograph of the high resolution capacitive distance-meter used for the comparison with the reference geophones. It measures the distance to a metallic surface. The sensitive part of the sensor is the round area of a 2 mm diameter whereas the metal is the sensor support structure adapted for the setup of Fig. 126.

49Reducing the bandwidth upper limit by a factor 100 reduces the signal noise by a factor $\sqrt{100} = 10$ [161].
Table 25: Specification of the capacitive distance-meter MCC0.5 by Fogale.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band width</td>
<td>0 Hz-10 kHz</td>
</tr>
<tr>
<td>Signal noise (full bandwidth)</td>
<td>0.05 %</td>
</tr>
<tr>
<td>Full scale output</td>
<td>0 V-10 V</td>
</tr>
<tr>
<td>Measurement range</td>
<td>0-50 mm</td>
</tr>
<tr>
<td>Calibration (full scale)</td>
<td>10 V per 50 mm</td>
</tr>
<tr>
<td>Linearity (full scale)</td>
<td>±0.1%</td>
</tr>
<tr>
<td>Calibration (0 μm-50 μm)</td>
<td>( d[\mu m] = 0.20 + 4.691V[Volt] +0.0728V^2 - 0.00336V^3 )</td>
</tr>
<tr>
<td>Diameter of active surface</td>
<td>1.2 mm</td>
</tr>
<tr>
<td>Reproducibility (static measurements)</td>
<td>0.01 %</td>
</tr>
<tr>
<td>Operating temperature range</td>
<td>0-50 °C</td>
</tr>
</tbody>
</table>

has been performed over a range of 50 μm and is given by a third order polynomial of the voltage (see Table 25). The sensor comparison was carried out using this configuration.

### E.2 Experimental set-up

The main difference between a geophone and a distance-meter is that the former measures absolute vibration velocities with respect to a reference mass whereas the latter measures relative distances between two surfaces. Therefore, a dedicated measurement set-up has to be conceived in order to compare these measurements. This can be done with a set-up as shown in Fig. 126. A photograph of this installation is given in Fig. 127. A platform is placed on top of the honeycomb table (Section 4.1.2) and isolated from it with three soft rubber feet, which allow a relative motion between platform and table. One geophone is fixed on the platform top and another one is put under the platform, fixed on the table. The simultaneous measurement from the two geophones enables measuring the platform-to-table relative motion (see Section 3.2.4), which is also simultaneously measured with the capacitive distance-meter. The non-contact sensor is fixed on the platform such as to measure the distance from the table. A micrometre palmer is used to adjust the distance between the sensor active area and the table surface. In order to use the non linear calibration of Table 25, this distance should be smaller then 50 μm, (the typical operating value is approximately 30 μm). On the platform top, a loudspeaker is also fixed, which allows generating mechanical vibrations of controlled frequencies and amplitudes. The experimental setup, as shown in Fig. 127, is installed on top of the honeycomb table, stabilized with the stiff isolation system described in Section 4.3.2. This system provides quiet conditions suitable for the sensor comparison on the nanometre and sub-nanometre scale.

A study of the platform resonances was performed with the geophones prior to the measurements for the sensor comparison. Fig. 128 shows the function

\[
A(f) = \sqrt{\frac{P_{Pl}(f)}{P_{Tb}(f)}}, \tag{116}
\]

where \( P_{Pl}(f) \) and \( P_{Tb}(f) \) are the power spectral densities of vertical motion as measured with two geophones on the platform and on the table, respectively. For each frequency \( f \), \( A(f) \) gives the
Figure 126: Scheme of the experimental setup for the comparison between two geophones and a capacitive distance-meter. The top resolution of the distance-meter is obtained over a 50 $\mu$m range, which requires the non-contact sensor to be placed very close to the table top surface. This is done with a micrometric palmer (typical operating distance of about 30 $\mu$m). To make the system more stable against drifts of the soft rubber, the platform was loaded with about 50 kg. A larger weight helped to stabilize the supporting feet.

Figure 127: Photograph of the experimental setup for the comparison between the geophones and the capacitive sensor.
amplification of vibration level due to the platform. \( A(f) \) depends mainly on the kind of rubber used as platform support, on the geometry of the system and on its weight. The maximum amplification occurs at 24 Hz, where the motion on the platform is 6.5 times larger than on the table.

**E.3 Measurement results**

An example of relative table-to-platform motion as measured by geophones and capacitive distance-meter is given in Fig. 129, as obtained without excitation of the platform with the loudspeaker. The power spectral density of the vertical platform-to-table distance (\( P_{y,\text{Rel}}(f) \), left graph) and the corresponding integrated RMS motion (\( I_{y,\text{Rel}}(f) \), right graph) are given as a function of frequency, \( f \). The distance-meter has a constant noise of \((1.72 \pm 0.39) \times 10^{-8} \mu \text{m}^2/\text{Hz}\) (one sigma is given), corresponding to a noise in the displacement measure of 0.13 nm ± 0.06 nm. This limits the frequency window where sensors can be compared. As shown in Fig. 129, above 35 Hz the displacements are below the noise level of the distance-meter and can only be measured by the geophones, which have a better resolution at high frequencies (the constant velocity resolution corresponds to a displacement resolution that decreases with frequency as \( \frac{1}{2\pi f} \), see Section 4.2.2). In addition, the distance-meter also features a relevant electromagnetic noise at 50 Hz and its multiples. Therefore, the integrated motion given in the right part of Fig. 129 is calculated taking into account only frequencies up to \( f_{\text{max}} = 35 \) Hz. A comparison at larger frequencies can also be performed by means of the loudspeaker, which excites vibrations of amplitudes that rise above the noise level of the distance-meter (see below).

The results of Fig. 129 show that the two different measurement devices are in good agreement. A systematic comparison is performed considering the ratio \( R(f) \), defined as:

\[
R(f) = \frac{I_{y,\text{Rel}}^{\text{Cap}}(f)}{I_{y,\text{Rel}}^{\text{Geo}}(f)},
\]

where the labels Geo and Cap refer to the geophone and the capacitive sensor, respectively. The ratio \( R(f) \) is shown in Fig. 130. The function \([1 - R(f)]\) gives the relative difference between the two
Figure 129: Vertical power spectral density of relative platform-to-table motion, $P_{y,\text{Rel}}(f)$ (left graph), and corresponding integrated RMS motion, $I_{y,\text{Rel}}(f)$ (right graph), versus frequency, $f$, as measured with the capacitive sensor (solid lines) and with the reference geophones (dashed lines). The integral is performed up to $f_{\text{max}} = 35\,\text{Hz}$ in order to avoid the contribution of the noise of the capacitive sensor.

Figure 130: Ratio $R(f)$ of Eq. (117) versus frequency, $f$. The results of Fig. 129 (right graph), which are obtained by integrating the vibration spectra up to 35 Hz, are used. The arrows indicate the low-frequency limitation of the reference geophones and the range where the noise of the capacitive sensor becomes relevant.
Figure 131: Geophone measurement of the relative platform-to-table motion, $I_{y, \text{Rel}}^{\text{Geo}}(f)$, versus measurement of the capacitive sensor, $I_{y, \text{Rel}}^{\text{Cap}}(f)$. The results of Fig. 129 (right graph) are used. A first-order polynomial fit of data below 2 nm is also shown (dashed line).

Slope 1 with zero constant term would correspond to a perfect agreement. A 2.6% difference is found between the measurements from the two devices. This is a very good result that strongly assesses the absolute calibration of the reference geophone. The 0.1 nm shift can be explained with the larger noise of the distance-meter.

**Measurements with loudspeaker**

A comparison at frequencies larger than 35 Hz is performed with the loudspeaker by exciting platform vibrations whose amplitude rises above the noise level of the distance-meter (see Fig. 129). The sensor comparison has been performed for different values of exciting frequencies above the platform resonance (24 Hz). An example is shown in Fig. 132 for data referring to an exciting oscillation of 67 Hz. Some measurement results are summarized in Table E.3. The RMS amplitude of the oscillation induced by the loudspeaker, as measured with the geophones and with the distance-meter, is given for different values of exciting voltages. The differences are always smaller than 7%.

---

50 The data used for the comparison with the loudspeaker excitations have been taken when both devices were connected to the same analog to digital converter (ADC).
Figure 132: Power spectral density of the relative platform-to-table motion, $P_{\text{Rel}}(f)$, as measured by the distance-meter and with the geophones when a 67 Hz oscillation is induced with the loudspeaker on the platform top (see Fig. 127). The peak-to-peak exciting voltage is of 1 Volt.

Table 26: RMS amplitudes of the relative oscillations between platform and table induced by the loudspeaker (see Fig. 126) at $f_0 = 62$ Hz. Three exciting voltages are given. The RMS amplitudes as measured with the capacitive sensor ($I_{\text{Cap}}(f_0)$, second column), with the geophones ($I_{\text{Geo}}(f_0)$, third column) and their ratio ($R(f_0)$, last column) are given.

<table>
<thead>
<tr>
<th>Voltage [Volt]</th>
<th>$I_{\text{Cap}}(f_0)$ [nm]</th>
<th>$I_{\text{Geo}}(f_0)$ [nm]</th>
<th>$I_{\text{Cap}}(f_0)/I_{\text{Geo}}(f_0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.711 ± 0.007</td>
<td>0.696 ± 0.012</td>
<td>1.022</td>
</tr>
<tr>
<td>1.0</td>
<td>1.531 ± 0.010</td>
<td>1.643 ± 0.012</td>
<td>0.932</td>
</tr>
<tr>
<td>2.0</td>
<td>3.123 ± 0.007</td>
<td>3.222 ± 0.010</td>
<td>0.969</td>
</tr>
</tbody>
</table>

In conclusion, the comparison between geophone for the measurement of absolute vibration velocity and distance-meter for the measurement of relative distances have shown that the two different devices are in very good agreement. The measured integrated displacements agree within a few percent to a wide range of frequencies (5 Hz-70 Hz) and displacements (0.1 nm-3 nm).
F Effect of cooling water on magnet vibrations

The quadrupoles of the CLIC linac are resistive magnets cooled down with pumped water, which will increase the magnet vibration level. This is an important issue for CLIC because the stability requirements for all linac quadrupoles are very tight (vertical tolerances of 1.3 nm above 4 Hz, see Table 6). Measurements of water-induced magnet vibrations [94, 177] have shown that this effect can be of orders of magnitudes larger than the CLIC requirements. In this appendix, the present theoretical understanding of the phenomenon is reviewed and results of vibration measurements on the CLIC prototype quadrupoles are reported. Similar studies have also been performed at SLAC in order to quantify the contribution of circulating water to the vibrations of the RF cavities [178].

F.1 Theory of water induced vibrations

Consider a straight pipe with diameter $d$, with water circulating at a velocity $u$. For laminar motion, no vibration should be generated since the water velocity at the internal pipe wall is zero. Turbulence in the water motion will instead transmit additional vibration to the pipe. Turbulence onset depends on the values of the Reynold’s number, defined as

$$Re = \frac{ud\rho}{\eta},$$  \hspace{1cm} (119)

where $u$ is the water velocity, $d$ the pipe diameter, $\rho=10^3 \text{ kg m}^{-3}$ and $\eta=0.89 \times 10^{-3} \text{ kg m}^{-1} \text{ s}^{-1}$ the water density and dynamic viscosity. For given pipe geometry and cooling fluid, the Reynold’s number depends only on the fluid velocity $u$. Turbulence occurs when the Reynold’s number reaches a critical value $Re_{cr}$, which depends for instance on the roughness of the pipe surface, on the pipe shape and on the status of the water upstream of the pipe. A typical value is $Re_{cr} \approx 2000$. In the following, the water motion will be assumed to be laminar upstream and downstream of the pipe under consideration, i.e. other water induced vibrations than the ones induced by the pipe itself are neglected.

In the turbulent regime, an eddy-like motion is superimposed to the drift of the water. Domains of coherent eddy motion appear above turbulence and drift with velocity $u$. These domains induce a modulation of the water motion with a frequency that depends on their size. This is shown in Fig. 133. The local rotation velocity of the eddies in a frame moving with the water ($v_\lambda$) depends on the eddy size ($\lambda$) as $v_\lambda \sim \Delta u (\lambda/d)^{3/4}$, where $\Delta u$ is the velocity of the largest eddies and can be of the order of the water velocity $u$ (see [179] for more details). The largest domains have a typical size of the order of the pipe radius i.e. $d/2$. For too small values of $v_\lambda$, locally the motion can go below the turbulence limit. This occurs for local velocities corresponding to Reynold’s number smaller then the critical

![Diagram of water flow in a pipe](image)

Figure 133: In the turbulent regime, an eddy-like motion is superimposed to the drift of the water. Coherence motion domains drift with velocity $u$. They induce a modulation to the water motion whose wave length is roughly twice the eddy diameter.
value of Eq. (119), i.e. \( u \lambda \sim ud \). This condition determines the smallest size of the eddies, which can be estimated from the above scaling law as \( d/Re^{3/4} \).

The largest domains fix a lower value of the turbulence induced vibration frequency as

\[
    f_{\text{min}} = \frac{u}{d},
\]

which is a frequency associated to coherence domains of length equal to the tube radius. On the other hand, the smallest eddies fix the largest vibration frequency as

\[
    f_{\text{max}} = \frac{u}{d} Re^{3/4}.
\]

This value is intended to give only the order of magnitude of the expected frequency range of water induced vibration.

Following the approach of [95] it is possible to estimate the amplitude of the turbulence induced pipe vibrations. In the turbulent regime, the pressure drop along the pipe depends on \( u^2 \) as

\[
    \Delta p = \frac{\rho u^2}{2d} \lambda,
\]

where \( l \) is the pipe length and \( \lambda \approx 0.316 \text{Re}^{-1/4} = 0.04 \) is an empirical parameter (see, for instance, [180]). In turbulence, the hydrostatic energy density is converted into kinetic energy density. If the pump power is completely converted in irretrievable kinetic energy, i.e. \( \frac{\partial \mathbf{v}}{\partial t} \Delta p = \frac{\partial \mathbf{v}}{\partial t} \rho \overline{v^2} \), then the pressure drop \( \Delta p \) is given by:

\[
    \Delta p = \frac{1}{2} \rho \overline{v^2}.
\]

Here, \( \overline{v^2} \) is the mean square velocity of turbulence seen from a frame moving with velocity of the water, \( u \).

The fluid can be divided in coherence cells of mean velocity \( \overline{v^2} \). In the approximation of isotropy of the turbulence, i.e. \( \overline{v^2} = 3 \overline{v^2} / 3 \) (\( y \) denotes the, say, vertical direction), Eqs. (122) and (123) give the following expression for the RMS vertical vibration velocity of one coherence cell:

\[
    \overline{v_{y}} = \frac{u \sqrt{\lambda}}{3d}.
\]

The local momentum density of one cell is \( \rho_{y} \overline{v_{y}} \). In order to calculate the total kinetic energy released to the pipe, some assumptions on the volume over which the energy is released have to be made. It seems reasonable (and somehow pessimistic) to assume the kinetic energy to be concentrated in cells of the same size as the coherence domains, i.e. \( d/2 \). The contributions from the different cells are assumed to be uncorrelated and must be added in quadrature. The total momentum transferred to the water is then

\[
    P_{y, \text{Tot}}^{\text{RMS}} = m u \sqrt{\frac{\lambda}{6}},
\]

where \( m \) is the mass of the water in the pipe. The coefficient \( \sqrt{2l/d} \) is the square root of the cell number and comes from the sum in quadrature. \( \pi d^3 \rho/2 \) is the mass of the water in one cell. If the
pipe itself does not have internal resonances and can be considered as a rigid body of total mass $M$, i.e. $P_{y,\text{Tot}}^{\text{RMS}} = M v_{y,\text{Tot}}^{\text{RMS}}$, its velocity is

$$v_{y,\text{Tot}}^{\text{RMS}} = \frac{m}{M} u \sqrt{\frac{\lambda}{6}}. \quad (126)$$

For a pure harmonic oscillation at the frequency $f_0$, the RMS amplitude corresponding to the RMS velocity of Eq. (126) would be $v_{y,\text{Tot}}^{\text{RMS}}/(2\pi f_0)$. In the pessimistic assumption that all the energy is concentrated at the minimal vibration frequency $f_{\text{min}} = u/d$, the vertical RMS vibration amplitude of the pipe becomes

$$y_{\text{Tot}}^{\text{RMS}} = \frac{1}{2\pi} m \frac{d}{M} u \sqrt{\frac{\lambda}{6}}. \quad (127)$$

Surprisingly, the vibration amplitude does not depend directly on the water velocity, there is only a small dependence as $u^{-1/8}$ due to the $\lambda$ dependence on $Re$. The agreement of this result with the experimental data will be discussed in the next sections.

### F.2 Measurement results

Measurements of quadrupole vibrations versus water flow were performed as described in Section 4.1.3. Both the stiff\(^{51}\) and the soft stabilization systems were used to stabilize the honeycomb table supporting the quadrupole (see Fig. 46). The measurement results are summarized in the next two sections, respectively.

#### F.2.1 Measurements on the stiff stabilization system

In Fig. 134 the power spectral densities of the vertical quadrupole displacements are given for different values of water flow. The peaks of the zero flow line are mostly induced by floor motion, damped by a factor between 10 and 100 by the stabilizing support (Section 4.3.2). Fig. 134 shows that the turbulence is a threshold phenomenon. Effects of water induced vibrations are found only for flows larger than 15 l/h\(^{52}\). In fact, the 12.5 l/h flow line in Fig. 134 is completely superimposed to the zero flow line. This is in good agreement with what is to be expected from Table 7. The pipe from the tap to the manifold and the pipe of the quadrupoles themselves are the possible candidates for the turbulence to occur. The pipes from the manifold to the quadrupole is expected to have an effect for flows above 40 l/h.

Typical power spectral densities of the doublet vertical displacements for different water flows are given in Figs. 135 and 136. The squared ratio of the power spectral densities with and without circulating water are given in Fig. 137\(^{53}\). Arbitrarily, data have been divided in two different frequency ranges. Above the turbulence threshold, two main effects induced by the circulating water are observed. (1) The released energy increases the overall noise level of the quadrupole vibrations. The

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\(^{51}\)The measurements of water induced vibrations presented in this appendix have been performed with the stiff isolation system in the three-feet configuration.

\(^{52}\)The specified values of the water flow refer to the water flowing into the quadrupole pipe. The flow in the larger pipe from the tap to the manifold (see Fig. 46) is four times as large.

\(^{53}\)The graphs of Fig. 137 are only meant to show qualitatively the frequencies where the water contribution is more effective. However, the ratio of the power spectral densities is not the proper quantity to describe the water induced vibration because the contribution of the water adds on quadrature to the unperturbed spectrum, according to the present theoretical understanding (Appendix F.)
existing peaks of the power spectral density become considerably amplified. This is, for instance, the case of the noise below 10 Hz and for the peak at around 170 Hz (see also Fig. 134). (2) A number of new peaks arise, which are not present without turbulence. This is the case for a strong peak at 15 Hz (appearing above 45 l/h) and for broad peaks in the 25–45 Hz frequency range (see Fig. 135). In the higher frequency range these two features are even more remarkable: amplifications of the zero flow vibration level of up to 1000 and more are clearly shown in Fig. 136. Three new peaks appear at \( \approx 90 \text{ Hz}, \approx 180 \text{ Hz} \) and \( \approx 270 \text{ Hz} \), both in vertical and horizontal directions, whose amplitudes increase for increasing water flows. They remind three harmonics of the same oscillation. In some cases the largest amplification with respect to the vibration level without water is obtained for the central peak (see Figure 137) but this is not a systematic feature. Interestingly enough, the middle peak arise exactly at the resonant frequency of the quadrupole pipes, as foreseen by the theory of Section F.1 (see Table 7). Nevertheless, the peak frequencies do not increase for increasing water velocities, as in Eq. (120). However, it should be noted that the vibrations above 60 Hz, even though of academic interest, contribute less than 0.2 nm to the total integrated motion (see below).

Fig. 135 shows that the new peaks appearing above turbulence shift with increasing water flows. This is a nice result in qualitative agreement with the theory of Section F.1, which foresees that the spectrum of turbulence induced vibrations increases with the water velocity (see Eqs. (120) and (121). On the other hand, this feature is not found for the peaks of Fig. 136, which only increase their amplitude for larger water flows.

Regardless of the theory prediction, the water induced motion is strongly dependent on the water flow. This is shown in Section 5.4, Fig. 92 where the RMS vibration amplitudes are shown versus water flow for different minimal frequencies. A peak on the vibration level curves is found for a flow of around 60 l/h. Interestingly, the vibration levels are lower at even higher water flows. The maximum increase of the quadrupole motion above 4 Hz due to the water is of the order of 3 nm. Note that the contribution of the vibrations above 60 Hz are less than 0.2 nm.

The small motion which is measured above 60 Hz suggests that the water induced vibrations of the doublet are mainly driven by the turbulence in the upstream pipes (that feed the quadrupoles).
Figure 135: Low frequency content of the water induced vibration. The power spectral density of vertical displacements versus frequency as measured on top of the CLIC prototype doublet is shown.

Figure 136: High frequency content of the water induced vibration. The power spectral density of vertical displacements versus frequency as measured on top of the CLIC prototype doublet is shown.
rather than in the magnet pipes themselves. From the theoretical estimates, it seems unlikely that the small quadrupole pipe can induce vibrations at low frequencies (see Table 7). Vibrations are generated in the larger pipe coming from the tap and then transmitted downstream via the pipe or via the water. This feature was also confirmed in measurements done at SLAC [178].

Vibration measurements of a doublet mounted on its CTF2-like alignment support and fixed on the stabilized table have also been performed. The preliminary results show that the horizontal RMS motion above 4 Hz can be amplified by a factor of 2 and more. A support internal resonance at 37 Hz [164] is considerably amplified by turbulence and is the main contribution to the increased motion. The vertical direction is not much affected by the alignment support, as also confirmed by in-situ measurements of the CTF2 quadrupoles.

F.2.2 Measurements on the Soft Stabilization System

Vibration measurements were also taken on a soft stabilization device (Section 4.3.1). The power spectral densities versus frequency for different water flows are shown in Fig. 138. The integrated motion versus water flow is given in Fig. 139. With this stabilization system, much larger displacements are measured. The integrated motion above 4 Hz increases monotonically for increasing water flows. At 60 l/h, an RMS motion of about 65 nm was measured. However, the motion at the nominal water flow (30 l/h) is about 3.3 nm. These larger displacements are mainly induced by the low frequency content of the vibrations, say up to 10 Hz. Above this frequency, the same features as with the stiff systems are found. For instance, Fig. 138 shows one peak at about 25 Hz that moves for increasing water flows and eventually disappears. This behavior in the 20 Hz to 40 Hz frequency range was inducing the non-monotonic curves of Fig 92.

The conclusion of the water induced vibration measurement studies on CLIC prototype quadrupoles is that the CLIC tolerances for the vibration of the linac quadrupoles, with nominal water flow, can basically be met. The considered CTF2 quadrupoles have the same cross section as the ones foreseen
Figure 138: Power spectral density of vertical displacement as measured on the doublet mounted on the air pressure stabilization system.

Figure 139: Vertical RMS motion of the doublet when mounted on the air pressure system. Different water flows are shown.
for CLIC, but they are about twenty times shorter. Whether these results can be reproduced for the full scale magnet, is still to be verified. The effect of water pumps also needs to be quantified.

The simplified theory of water induced vibrations gives a reliable rough estimate of the frequency range of the vibrations but is not sufficiently accurate for estimating the RMS amplitude of the motion. Water induced magnet vibrations have been proven to be driven by turbulence in the water motion. The main source of quadrupole vibration does not come from the quadrupole pipes but from the pipes feeding the magnet, as is also confirmed by measurement done at SLAC. This is an important issue since the large pipe feeding the quadrupoles induces vibrations in the frequency range of few tens of hertz. In the same frequency range are the structural resonances of the magnets and of their supports. It seems feasible to reduce this effect by adjusting properly the diameter of the feeding pipes, for instance increasing the diameter to concentrate the motion in a frequency range where beam based feedbacks are effective.

The stability of the quadrupole doublet and the effect of water were found to depend considerably on the stabilization device. A stiff system gives the best performance, whilst a soft air pressure system shows an unacceptable amplification of the low frequency vibrations.