DarkSUSY: Computing Supersymmetric Dark Matter Properties Numerically

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Abstract

The question of the nature of the dark matter in the Universe remains one of the most outstanding unsolved problems in basic science. One of the best motivated particle physics candidates is the lightest supersymmetric particle, assumed to be the lightest neutralino - a linear combination of the supersymmetric partners of the photon, the Z boson and neutral scalar Higgs particles. Here we describe DarkSUSY, a publicly-available advanced numerical package for neutralino dark matter calculations. In DarkSUSY one can compute the neutralino density in the Universe today using precision methods which include resonances, pair production thresholds and coannihilations. Masses and mixings of supersymmetric particles can be computed within DarkSUSY or with the help of external programs such as FeynHiggs, ISASUGRA and SUSPECT. Accelerator bounds can be checked to identify viable dark matter candidates. DarkSUSY also computes a large variety of astrophysical signals from neutralino dark matter, such as direct detection in low-background counting experiments and indirect detection through antiprotons, antideuterons, gamma-rays and positrons from the Galactic halo or high-energy neutrinos from the center of the Earth or of the Sun. Here we describe the physics behind the package. A detailed manual will be provided with the computer package.

Keywords: Supersymmetry; Dark Matter; Cosmology; Neutrino Telescopes; Gamma-ray Telescopes; Cosmic Antiprotons; Cosmic Antideuterons; Cosmic Positrons; Direct Detection; Indirect Detection; Numerical Code; Galactic Halo
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1 Introduction

During the past few years, remarkable progress has been made in cosmology, both observationally and theoretically. One of the outcomes of these rapid developments is the increased confidence that most of the mass of the observable Universe is of an unusual form, i.e., not made up of ordinary baryonic matter. Recent analyses combining high-redshift supernova luminosity distances, microwave background fluctuations and the dynamics and baryon fraction of galaxy clusters indicate that the present mass density of matter in the Universe \( \Omega_M = \rho_M/\rho_{\text{crit}} \) normalized to the critical density \( \rho_{\text{crit}} = 3H_0^2/(8\pi G N) = h^2 \cdot 1.9 \cdot 10^{-29} \text{ g cm}^{-3} \) is \( 0.1 \leq \Omega_M h^2 \leq 0.2 \), which is considerable higher than the value \( \Omega_B h^2 \leq 0.023 \) allowed by big bang nucleosynthesis \([1]\). Here \( h \approx 0.7 \pm 0.15 \) is the present value of the Hubble constant in units of 100 km s\(^{-1}\) Mpc\(^{-1}\). A recent addition to the wealth of experimental data which support the existence of non-baryonic dark matter is the WMAP microwave background results \([2]\). In a joint analysis of the WMAP data together with other CMBR experiments, large-scale structure data, supernova data and the HST Key Project, the WMAP team report \( \Omega_B h^2 = 0.0224 \pm 0.009 \), \( \Omega_M h^2 = 0.135 \pm 0.009 \), and \( \Omega_\Lambda = 0.73 \pm 0.04 \). Subtracting the baryonic contribution \( \Omega_B h^2 \) from the matter density \( \Omega_M h^2 \) leaves a non-baryonic cold dark matter density \( \Omega_{\text{CDM}} h^2 = 0.113 \pm 0.009 \).

Also from the point of view of structure formation, non-baryonic dark matter seems to be necessary, and the main part of it should consist of particles that were non-relativistic at the time when structure formed (cold dark matter, CDM), thus excluding light neutrinos. Under reasonable assumptions, the WMAP collaboration, using also galaxy survey and Ly-\(\alpha\) forest data, limit the contribution of neutrinos to \( \Omega_\nu h^2 < 0.0076 \) (95\% c.l.).

A well-motivated particle physics candidate which has the required properties is the lightest supersymmetric particle, assumed to be a neutralino \([3, 4, 5]\). (For thorough reviews of supersymmetric dark matter, see \([6, 7]\).) Although supersymmetry is generally accepted as a very promising enlargement of the Standard Model of particle physics (for instance it would solve the so-called hierarchy problem which essentially consists of understanding why the electroweak scale is protected against Planck-scale corrections), little is known about what a realistic supersymmetric model would look like in its details. Therefore, it is a general practice to use the simplest possible model, the minimal supersymmetric enlargement of the Standard Model (the MSSM), usually with some additional simplifying assumptions. Of course, there is no compelling reason why the actual model, if nature is supersymmetric at all, should be of this simplest kind. However, the MSSM serves as a useful template with which to test current ideas about detection, both in particle physics accelerators and in dark matter experiments and contains many features which are expected to be universal for any supersymmetric dark matter model. In fact, the knowledge gained by studying the MSSM in detail may be of even more general use, since it provides one specific example of a WIMP, a weakly interacting massive particle, which is generically what successful particle dark matter models require.\(^7\)

Over several years, we have developed analytical and numerical tools for dealing with the sometimes quite complex calculations necessary to go from given input parameters in the MSSM to actual quantitative predictions of the relic density of the neutralinos in the Universe, and the direct and indirect detection rates. The program package, which we have named DarkSUSY, has now reached a high level of sophistication and maturity, and we have released it publicly for the benefit of the scientific community working with problems related to dark matter. This paper describes the basic structure and the underlying physical and astrophysical formulas contained in DarkSUSY, as well as examples of its use. The version of

\(^7\)A completely different type of particle is the axion \([8]\), which is very light, but was never in thermal equilibrium. Its phenomenology is very different and will not be treated here.
the package described in this paper is **DarkSUSY** 4.1.

For download of the latest version of **DarkSUSY** and for a more technical manual, please visit the official **DarkSUSY** website: [http://www.physto.se/~edsjo/darksusy/](http://www.physto.se/~edsjo/darksusy/)

## 2 Definition of the Supersymmetric model

We work in the framework of the minimal supersymmetric extension of the Standard Model defined by, besides the particle content and gauge couplings required by supersymmetry, the superpotential (the notation used is that of [9] which marked the beginning of the development of **DarkSUSY**, and is similar to [10])

\[
W = \epsilon_{ij} \left( -\bar{e}_R^* Y_E \bar{H}_2^L H_1^i - \bar{d}_R^* Y_D \bar{q}_L^i H_2^i + \bar{u}_R^* Y_U \bar{q}_L^i H_2^i - \mu H_1^i H_2^i \right) \quad (1)
\]

and the soft supersymmetry-breaking potential

\[
V_{\text{soft}} = \epsilon_{ij} \left( -\bar{e}_R^* A_{E} Y_E \bar{H}_2^L H_1^i - \bar{d}_R^* A_D Y_D \bar{q}_L^i H_2^i + \bar{u}_R^* A_U Y_U \bar{q}_L^i H_2^i - B \mu H_1^i H_2^i + \text{h.c.} \right)
\]

\[
+H_1^i m_1^2 H_2^i + H_2^i m_2^2 H_2^i
\]

\[
+ \tilde{q}_L^i M_2^2 \tilde{q}_L^i + \tilde{l}_L^i M_1^2 \tilde{l}_L^i + \tilde{u}_R^i M_1^2 \tilde{d}_R^i + \tilde{d}_R^i M_1^2 \tilde{d}_R^i + \tilde{e}_R^i M_1^2 \tilde{e}_R^i
\]

\[
+ \frac{1}{2} M_1 B \tilde{B} + \frac{1}{2} M_2 \left( \tilde{W}^3 \tilde{W}^3 + 2 \tilde{W}^+ \tilde{W}^- \right) + \frac{1}{2} M_{3} \tilde{g} \tilde{g}. \quad (2)
\]

We give these and the following expressions since they contain our sign conventions. It should be noted that various authors use various sign conventions, and many errors, often difficult to find, can be avoided by keeping careful track of the signs, as we have tried to do consistently in **DarkSUSY**. Here \( i \) and \( j \) are SU(2) indices \((\epsilon_{12} = +1)\). The Yukawa couplings \( Y \), the soft trilinear couplings \( A \) and the soft sfermion masses \( M \) are 3 \times 3 matrices in generation space. \( \tilde{e}, \tilde{l}, \tilde{u}, \tilde{d} \) and \( \tilde{q} \) are the superfields of the leptons and sleptons and of the quarks and squarks. A tilde indicates their respective scalar components. The \( L \) and \( R \) subscripts on the sfermion fields refer to the chirality of their fermionic superpartners. \( B, W^3 \) and \( W^\pm \) are the fermionic superpartners of the U(1) and SU(2) gauge fields and \( \tilde{g} \) is the gluino field. \( \mu \) is the Higgsino mass parameter, \( M_1, M_2 \) and \( M_3 \) are the gaugino mass parameters, \( B \) is a soft bilinear coupling, while \( m_1^2 \) and \( m_2^2 \) are Higgs mass parameters.

These input parameters are contained in common blocks in the program. The full set of input parameters in version 4.1 of **DarkSUSY**, to be given at the weak scale, is \( m_{\tilde{A}}, \tan \beta, \mu, M_1, M_2, M_3, A_{Eaa}, A_{Uaa}, A_{Daa}, M_{Qaa}^2, M_{Laa}^2, M_{Daa}^2, M_{Eaa}^2 \) (with \( a = 1, 2, 3 \)). The user may either provide these parameters directly to **DarkSUSY** or take advantage of the implementation of a MSSM pre-defined through a reduced number of parameters. In this model, the basic set of parameters is \( \mu, M_2, m_{\tilde{A}}, \tan \beta, m_0, A_t, A_b \). Here \( m_{\tilde{A}} \) is the mass of the CP-odd Higgs boson and \( \tan \beta \) denotes the ratio, \( v_2/v_1 \), of the vacuum expectation values of the two neutral components of the SU(2) Higgs doublets. The parameters \( m_0, A_t \) and \( A_b \) are defined through the simplifying Ansatz: \( M_Q = M_U = M_D = M_E = M_L = m_0 \), \( A_U = \text{diag}(0, 0, A_t) \), \( A_D = \text{diag}(0, 0, A_b) \), \( A_E = 0 \).

Below we will give some details to clarify our convention and additional features. Relevant quantities for phenomenological studies, such as the particle masses and mixings, are consistently computed by **DarkSUSY** and available in arrays. The supersymmetry part of the program can thus be used for many applications, in particular for accelerator-based physics studies. Particle decay widths are also available, but currently only the widths of the Higgs bosons are calculated, the other particles having fictitious widths of 1 or 5 GeV (for the sole purpose of regularizing annihilation amplitudes close to poles).
2.1 Neutralino and chargino sectors

The neutralinos $\tilde{\chi}^0_i$ are linear combinations of the superpartners of the neutral gauge bosons, $\tilde{B}$, $\tilde{W}_3$ and of the neutral Higgsinos, $\tilde{H}_0^0$, $\tilde{H}_0^2$. In this basis, their mass matrix is given by

$$M_{\tilde{\chi}^0_{1,2,3,4}} = \begin{pmatrix}
M_1 & 0 & -\frac{g'v_1}{\sqrt{2}} & \frac{g'v_2}{\sqrt{2}} \\
0 & M_2 & \frac{g'v_1}{\sqrt{2}} & -\frac{g'v_2}{\sqrt{2}} \\
-\frac{g'v_1}{\sqrt{2}} & \frac{g'v_2}{\sqrt{2}} & \delta_{33} & -\mu \\
\frac{g'v_2}{\sqrt{2}} & -\frac{g'v_1}{\sqrt{2}} & -\mu & \delta_{44}
\end{pmatrix}$$

where $g$ and $g'$ are the gauge coupling constants of SU(2) and U(1). $\delta_{33}$ and $\delta_{44}$ are the most important one-loop corrections. These can change the neutralino masses by a few GeV up or down and are only important when there is a severe mass degeneracy of the lightest neutralinos and/or charginos. The expressions for $\delta_{33}$ and $\delta_{44}$ used in DarkSUSY are taken from [11, 12] (the tree-level values can optionally be chosen).

The neutralino mass matrix, Eq. (3), is diagonalized analytically and evaluated numerically to give the masses and compositions of four neutral Majorana states,

$$\tilde{\chi}^0_i = N_{i1}\tilde{B} + N_{i2}\tilde{W}_3 + N_{i3}\tilde{H}_1^0 + N_{i4}\tilde{H}_2^0,$$

the lightest of which, $\tilde{\chi}^0_1$ to be called $\chi$ for simplicity, is then the candidate for the particle making up the dark matter in the Universe. The neutralinos are ordered in mass such that $m_{\tilde{\chi}^0_1} < m_{\tilde{\chi}^0_2} < m_{\tilde{\chi}^0_3} < m_{\tilde{\chi}^0_4}$ and the eigenvalues are real with a complex $N$.

The charginos are linear combinations of the charged gauge bosons $\tilde{W}^\pm$ and of the charged Higgsinos $\tilde{H}_1^\pm$, $\tilde{H}_2^\pm$. Their mass terms are given by

$$(\tilde{W}^- \tilde{H}_1^-) M_{\chi^\pm} (\tilde{W}^+ \tilde{H}_2^+) + \text{h.c.}$$

Their mass matrix,

$$M_{\chi^\pm} = \begin{pmatrix}
M_2 & g\nu_2 \\
g\nu_1 & \mu
\end{pmatrix},$$

is diagonalized by the following linear combinations

$$\tilde{\chi}_i^- = U_{i1}\tilde{W}^- + U_{i2}\tilde{H}_1^-,$$

$$\tilde{\chi}_i^+ = V_{i1}\tilde{W}^+ + V_{i2}\tilde{H}_2^+.$$

We choose $\det(U) = 1$ and $U^* M_{\chi^\pm} V^\dagger = \text{diag}(m_{\chi_i^-}, m_{\chi_i^+})$ with non-negative chargino masses $m_{\chi_i^-} \geq 0$. We do not include any one-loop corrections to the chargino masses since they are negligible compared to the corrections $\delta_{33}$ and $\delta_{44}$ introduced above for the neutralino masses [11].

2.2 Sfermion masses and mixings

When discussing the squark mass matrix including mixing, it is convenient to choose a basis where the squarks are rotated in the same way as the corresponding quarks in the Standard Model. We follow the conventions of the Particle Data Group [13] and put the mixing in the left-handed $d$-quark fields, so that the definition of the Cabibbo-Kobayashi-Maskawa matrix is $K = V_1 V_2^\dagger$, where $V_1$ ($V_2$) rotates the interaction left-handed $u$-quark ($d$-quark) fields.
to mass eigenstates. For sleptons we choose an analogous basis, but since in DarkSUSY 4.1
neutrinos are assumed to be massless, no analog of the CKM matrix appears.

The general $6 \times 6$ $u$- and $d$-squark mass matrices are

$$
\mathcal{M}^2_u = \begin{pmatrix}
M^2_U + m_u^u m_u + D_{uu}^{LL} & m_u^u (A_U - \mu^* \cot \beta)
\end{pmatrix},
\mathcal{M}^2_d = \begin{pmatrix}
M^2_D + m_d^d m_d + D_{dd}^{RR} & m_d^d (A_D - \mu^* \tan \beta)
\end{pmatrix},
$$

(9)

(10)

The general sneutrino and charged slepton mass matrices are (for massless neutrinos)

$$
\mathcal{M}_\tilde{\nu}^2 = M^2_{\tilde{\nu}} + D_{LL}^\nu \mathbf{1},
\mathcal{M}_\tilde{e}^2 = \begin{pmatrix}
M^2_L + m_{\tilde{e}}^L m_{\tilde{e}} + D_{LL}^e & m_{\tilde{e}}^L (A_E - \mu^* \tan \beta)
\end{pmatrix},
$$

(11)

(12)

Here

$$
D_{LL}^f = m_Z^2 \cos 2\beta (T_{3f} - e_f \sin^2 \theta_W),
D_{RR}^f = m_Z^2 \cos(2\beta) e_f \sin^2 \theta_W
$$

(13)

(14)

where $T_{3f}$ is the third component of the weak isospin and $e_f$ is the charge in units of the

The slepton and squark mass eigenstates $\tilde{f}_k$ ($\tilde{i}_k$ with $k = 1, 2, 3$ and $\tilde{i}_k, \tilde{u}_k$ and $\tilde{d}_k$ with

$k = 1, \ldots, 6$) diagonalize the previous mass matrices and are related to the current sfermion

eigenstates $\tilde{f}_{La}$ and $\tilde{f}_{Ra}$ ($a = 1, 2, 3$) via

$$
\tilde{f}_{La} = \sum_{k=1}^6 \tilde{f}_k \Gamma_{FL}^{ka},
\tilde{f}_{Ra} = \sum_{k=1}^6 \tilde{f}_k \Gamma_{FR}^{ka}.
$$

(15)

(16)

The squark and charged slepton mixing matrices $\Gamma_{UL}, \Gamma_{UR}, \Gamma_{DL}, \Gamma_{DR}, \Gamma_{EL}, \Gamma_{ER}$ have
dimension $6 \times 3$, while the sneutrino mixing matrix $\Gamma_{\tilde{\nu}L}$ has dimension $3 \times 3$.

The current version of DarkSUSY allows only for diagonal matrices $A_U, A_D, A_E, M_Q,
M_U, M_D, M_E$, and $M_L$. This ansatz, while not being the most general, implies the absence
of tree-level flavor changing neutral currents in all sectors of the model. It also allows the
squark mass matrices to be diagonalized analytically. For example, for the top squark one
has, in terms of the top squark mixing angle $\theta_t$,

$$
\Gamma_{UL}^{ij} = \Gamma_{UR}^{ij} = \cos \theta_t, \quad \Gamma_{UL}^{ij} = -\Gamma_{UR}^{ij} = \sin \theta_t.
$$

(17)

The sfermion masses are obtained with the diagonalization just described.

To facilitate comparisons with the results of other authors, DarkSUSY allows for special
values of the sfermion masses to be set in the program. A common value can be assigned to
all squark masses, and an independent common value to all slepton masses. Alternatively,
to enforce the sfermions to be heavier than the lightest neutralino (which we want to be
the LSP), the squarks and sleptons masses can be set equal to the maximum between the
neutralino mass and a specified value. In the special cases just described, no mixing is
assumed between sfermions. It must be noted that the special choices described in this
paragraph are mathematically inconsistent within the MSSM, but are often made for the
sake of simplicity.
2.3 Higgs sector and interface to FeynHiggs

The Higgs masses receive radiative corrections and DarkSUSY includes several options for calculating these. The default in DarkSUSY is to use FeynHiggsFast \[14\] for the Higgs mass calculations. When higher accuracy is needed, it is possible to instead use the full FeynHiggs \[15, 16\] package.

2.4 Interface to the mSUGRA codes ISASUGRA and SUSPECT

Given the modular structure of DarkSUSY, the user may also run the package using as input for the MSSM definition the output, still at the low energy scale, from an external package. An example of usage under such mode is given in case of the minimal supergravity (mSUGRA) model: in the release, we provide an interface to the output of the ISASUGRA code, as included in ISAJET \[17\] for the current ISASUGRA 7.69 version, as well as an an interface to SUSPECT \[18\] which can be used as an alternative. The interfaces to DarkSUSY are at the level of the full spectrum of masses and mixings, including, for consistency, those for the Higgs sector. Of importance for relic density calculations near a Higgs boson resonance is the possibility of including supersymmetric corrections to the bottom, top, and tau Yukawa couplings as supplied by ISASUGRA.

3 Experimental constraints

Accelerator bounds can be checked by a call to a subroutine. By modifying an option, the user can impose bounds as of different moments in time. The default option in version 4.1 adopts the 2002 limits by the Particle Data Group \[19\] modified as described below. The user is also free to use his or her own routine to check for experimental bounds, in which case there is only need to provide an interface to DarkSUSY.

For the theoretical prediction of the rare decay $b \to s \gamma$ we have implemented the complete next-to-leading order (NLO) Standard Model calculation and the dominant NLO supersymmetric corrections. For the NLO QCD calculation of the Standard Model prediction we have used the expressions of reference \[20\] into which we have inserted the updated so-called “Magic numbers” of \[21\]. Our implementation of the Standard Model calculation gives a branching ratio $BR[B \to X_s \gamma] = 3.72 \times 10^{-4}$ for a photon energy greater than $m_b/20$. This result agrees to within 1% with the result stated in \[21\], but is around 10% larger than the result of previous analyses. The latter is due to the fact that in the reference \[20\] they replaced $m_c/\alpha_b^0$ by $m_c^{MS}(\mu)/m_b^{pole}$ (with $\mu \in [m_c, m_b]$) in the matrix element $\langle X_s \gamma \mid (\bar{s}c)_{V-A}(\bar{c}b)_{V-A} \mid b \rangle$.

The supersymmetric correction to $b \to s \gamma$ has been divided into a contribution from a two Higgs doublet model and a contribution from supersymmetric particles. The expressions for the NLO contributions in the two Higgs doublet model has been taken from \[22\]. The corrections due to supersymmetric particles are calculated under the assumption of minimal flavour violation. The dominant LO contributions which are valid even in the large $\tan \beta$ regime was taken from ref. \[23\], and we also followed their guideline on how the NLO QCD expressions of \[24\] should be expanded to the large $\tan \beta$ regime.

For the current experimental bound on $b \to s \gamma$ we take the value stated by the Particle Data Group 2002 \[19\]. This is an average between the CLEO and the Belle measurements and amounts to $BR[B \to X_s \gamma] = (3.3 \pm 0.4) \times 10^{-4}$. To this we add a theoretical uncertainty which we set to $\pm 0.5 \times 10^{-4}$. The final constraint on the branching ratio then becomes $2.0 \times 10^{-4} \leq BR[B \to X_s \gamma] \leq 4.6 \times 10^{-4}$.
Recently, much interest has been given to the possible contribution of supersymmetry to \((g-2)_\mu\). Although the discrepancy with the Standard Model result is now below 3\(\sigma\), we include for convenience a calculation of the anomalous moment of the muon \((g-2)_\mu\) in DarkSUSY.

4 Calculation of relic density

The WMAP microwave background experiment [2], combined with other sets of data, gives a quite precise determination of the cold dark matter density \(\Omega_{\text{CDM}} h^2 = 0.113 \pm 0.009\). We would like DarkSUSY to compute the relic density of neutralinos to at least the same precision.

We use in DarkSUSY the full cross section and solve the Boltzmann equation numerically with the method given in [25, 26]. In this way we automatically take care of thresholds and resonances.

When any other supersymmetric particles are close in mass to the lightest neutralino they will also be present at the time when the neutralino freezes out in the early Universe. When this happens so-called coannihilations can take place between all these supersymmetric particles present at freeze-out. Coannihilations were first pointed out by Binetruy, Girardi, and Salati [27] in a non-supersymmetric model with several Higgs bosons. Griest and Seckel [28] investigated them in the MSSM for the case where squarks are of about the same mass as the lightest neutralino. Later, coannihilations between the lightest neutralino and the lightest chargino were investigated in [29, 30, 11]. Several authors have also included coannihilations with sfermions [31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 11, 41, 42, 43, 44, 45, 46]. In DarkSUSY we have implemented all coannihilations between the neutralinos, charginos and sfermions as calculated in [46].

Compared to other recent calculations, we believe ours is the most precise calculation available at present. The standard lore so far has been to calculate the thermal average of the annihilation cross section by expanding to first power in temperature over mass and implementing an approximate solution to the evolution equation which estimates the freeze out temperature without fully solving the equation (see, e.g., Kolb and Turner [47]). Sometimes this is refined by including resonances and threshold corrections [28]. Among recent studies, this approach is taken in e.g. Refs. [36, 37]. Other refinements include, e.g., solving the density evolution equation numerically but still using an approximation to thermal effects in the cross section [51, 52, 53, 54, 55, 56], or calculating the thermal average precisely but using an approximate solution to the density equation [58, 59, 60]. At the same time, only in a few of the quoted papers the full set of initial states has been included. As already mentioned, the present calculation includes all initial states, performs an accurate thermal average and gives a very precise solution to the evolution equation. Though the inclusion of initial state sfermions in the DarkSUSY package is a new feature introduced in the present work, other groups [41, 42, 43] have earlier introduced some sfermion coannihilations in an interface with the old DarkSUSY version.

To gain calculational speed we only include the particles with masses below \(f_{\text{co}} m_\chi\). The mass fraction parameter \(f_{\text{co}}\) is by default set to 2.1 or 1.4 depending on how the relic density routines are called (very high precision or fast calculation), but can be set to any value by the user.

4.1 The Boltzmann equation

We will here outline the procedure developed in [26] which is used in DarkSUSY. For more details, see [26].
Consider annihilation of $N$ supersymmetric particles with masses $m_i$ and internal degrees of freedom $g_i$. Order them such that $m_1 \leq m_2 \leq \cdots \leq m_{N-1} \leq m_N$. For the lightest neutralino, the notation $m_1$ and $m_\chi$ will be used interchangeably.

Since we assume that $R$-parity holds, all supersymmetric particles will eventually decay to the LSP and we thus only have to consider the total number density of supersymmetric particles $n = \sum_{i=1}^{N} n_i$. Furthermore, the scattering rate of supersymmetric particles off particles in the thermal background is much faster than their annihilation rate, because the scattering cross sections $\sigma'_{Xij}$ are of the same order of magnitude as the annihilation cross sections $\sigma_{ij}$ but the background particle density $n_X$ is much larger than each of the supersymmetric particle densities $n_i$ when the former are relativistic and the latter are non-relativistic, and so suppressed by a Boltzmann factor \cite{48}. Hence, the $\chi_i$ distributions remain in kinetic thermal equilibrium during their freeze-out. Combining these effects, we arrive at the following Boltzmann equation for the summed number density of supersymmetric particles

$$\frac{dn}{dt} = -3Hn - \langle \sigma_{\text{eff}} v \rangle \left( n^2 - n_{\text{eq}}^2 \right)$$  \hspace{1cm} (18)

where

$$\langle \sigma_{\text{eff}} v \rangle = \sum_{ij} \langle \sigma_{ij} v_{ij} \rangle \frac{n_{\text{eq}}^{ig} n_{\text{eq}}^{gj}}{n_{\text{eq}}^{ig} n_{\text{eq}}^{gj}}.$$  \hspace{1cm} (19)

with

$$v_{ij} = \frac{\sqrt{(p_i \cdot p_j)^2 - m_i^2 m_j^2}}{E_i E_j}.$$  \hspace{1cm} (20)

### 4.2 Thermal averaging

Using the Maxwell-Boltzmann approximation for the velocity distributions one can derive the following expression for the thermally averaged annihilation cross section \cite{26}

$$\langle \sigma_{\text{eff}} v \rangle = \frac{\int_0^\infty dp_{\text{eff}} v^2_{\text{eff}} W_{\text{eff}} K_i \left( \frac{v_{\text{eff}}}{T} \right)}{m_1^4 T \left( \sum g_i g_j m_i^2 K_2 \left( \frac{m_i}{m_\chi} \right) \right)^2}.$$  \hspace{1cm} (21)

where $K_1$ ($K_2$) is the modified Bessel function of the second kind of order 1 (2), $T$ is the temperature, $s$ is one of the Mandelstam variables and

$$W_{\text{eff}} = \sum_{ij} \frac{g_i g_j g^2}{p_{\text{eff}} g_1^2} W_{ij} = \sum_{ij} \sqrt{\frac{(s - (m_i - m_j)^2)(s - (m_i + m_j)^2)}{s(s - 4m_1^2)}} \frac{g_i g_j}{g_1^2} \frac{W_{ij}}{g_1^2}.$$  \hspace{1cm} (22)

In this equation, we have defined the momentum

$$p_{ij} = \frac{[s - (m_i + m_j)^2]^{1/2} [s - (m_i - m_j)^2]^{1/2}}{2\sqrt{s}},$$  \hspace{1cm} (23)

the invariant annihilation rate

$$W_{ij} = 4p_{ij} \sqrt{s} \sigma_{ij} = 4\sigma_{ij} \sqrt{(p_i \cdot p_j)^2 - m_i^2 m_j^2} = 4E_i E_j \sigma_{ij} v_{ij}.$$  \hspace{1cm} (24)
and the effective momentum
\[ p_{\text{eff}} = p_{11} = \frac{1}{2} \sqrt{s - 4m_i^2}. \] (25)

Since \( W_{ij}(s) = 0 \) for \( s \leq (m_i + m_j)^2 \), the radicand in Eq. (22) is never negative. For a
two-body final state, \( W_{ij} \) is given by
\[ W_{ij}^{2\text{-body}} = \frac{|k|}{16\pi^2 g_i g_j S_f \sqrt{s}} \sum_{\text{internal d.o.f.}} \int |M|^2 d\Omega, \] (26)
where \( k \) is the final center-of-mass momentum, \( S_f \) is a symmetry factor equal to 2 for identical
final particles, and the integration is over all possible outgoing directions of one of the final
particles. As usual, an average over initial internal degrees of freedom is performed.

4.3 Annihilation cross sections

In DarkSUSY, all two-body final state cross sections at tree level are computed for all coannihilations
between neutralinos, charginos and sfermions. A complete list is given in table 3 in Appendix A.

The calculation of the relic density is, due to its complexity, the most time-consuming
task of DarkSUSY. For the neutralino-neutralino, chargino-neutralino and chargino-chargino
initial states, to achieve efficient numerical computation, contributing diagrams have been
classified according to their topology (s-, t- or u-channel) and to the spin of the particles
involved. The helicity amplitudes for each type of diagram have been computed analytically
with Reduce\cite{49} using general expressions for the vertex couplings. The Reduce output has
been automatically converted to FORTRAN subroutines called by DarkSUSY.

The strength of the helicity amplitude method is that the analytical calculation of a given
diagram only has to be performed once and the summing of the contributing diagrams for
each given set of initial and final states can be done numerically afterwards.

The Feynman diagrams are summed numerically for each annihilation channel \( ij \rightarrow kl \). We then sum the squares of the helicity amplitudes and sum the contributions of all annihilation
channels. Explicitly, DarkSUSY computes
\[ \frac{dW_{\text{eff}}}{d\cos \theta} = \sum_{ijkl} \frac{p_{ij} p_{kl}}{32\pi S_{kl} \sqrt{s}} \sum_{\text{helicities}} \sum_{\text{diagrams}} |M(ij \rightarrow kl)|^2 \] (27)
where \( \theta \) is the angle between particles \( k \) and \( i \). Finally, a numerical integration over \( \cos \theta \) is
performed by means of an adaptive method\cite{40}.

In rare cases, there can be resonances in the \( t \)- or \( u \)-channels. This can happen when
the masses of the initial state particles lie between the masses of the final state particles.
At certain values of \( \cos \theta \), the momentum transfer is time-like and matches the mass of the
exchanged particle. We regulate the divergence by assigning a small width of 5 GeV to the
neutralinos and charginos and 1 GeV to the sfermions. The results are not sensitive to the
exact choice of this width.

For the coannihilation diagrams with sfermions, the calculations are done with Form\cite{51}
and automatically converted into FORTRAN subroutines.

In the relic density routines, the calculation of the effective invariant rate \( W_{\text{eff}} \) is the most
time-consuming part. Fortunately, thanks to the remarkable feature of Eq. (21), \( W_{\text{eff}}(p_{\text{eff}}) \)
does not depend on the temperature \( T \), and it can be tabulated once for each model.

To perform the thermal average in Eq. (21), we integrate over \( p_{\text{eff}} \) by means of adaptive
gaussian integration, using a spline routine to interpolate in the \( (p_{\text{eff}}, W_{\text{eff}}) \) table. To avoid
numerical problems in the integration routine or in the spline routine, we split the integration interval at each sharp threshold. We also explicitly check for each MSSM model that the spline routine behaves well close to thresholds and resonances.

We then integrate the density evolution equation, Eq. (13). For numerical reasons, we do not integrate the equation directly, but instead rephrase it as an evolution equation for the abundance, \( Y = n/s \) (with \( s \) being the entropy density) and use \( x = m_\chi/T \) as our integration variable instead of time (see [24] for details). The numerical integration is subtle since the equation is “stiff.” For this purpose, we developed an implicit trapezoidal method with adaptive stepsize. In short, if the equation for \( Y \) is written as \( dY/dx = \lambda(Y^2 - Y_e^2) \), with \( Y_e \) being the equilibrium value at temperature \( T \), the numerical integration is based on the recurrence

\[
Y_{i+1} = \frac{C_i}{1 + \sqrt{1 + h\lambda_i c_i Y_i}} \quad \text{(28)}
\]

where \( h \) is the stepsize and \( C_i = 2Y_i + h[\lambda_{i+1}Y_{e,i+1}^2 + \lambda_i(Y_{e,i+1}^2 - Y_i^2)] \). The stepsize is adapted (reduced) if \( |(Y_{i+1} - y_{i+1})/Y_{i+1}| \) exceeds a given tolerance. Here \( y_{i+1} = (c_i/2)(1 + \sqrt{1 + h\lambda_i c_i})^{-1} \) with \( c_i = 4(Y_i + h\lambda_i Y_{e,i}) \).

There are some loop-induced final states, such as two gluons, two photons or a \( Z^0 \) boson and a photon which could in principle give a non-negligible contribution to the annihilation rate and lower the relic abundance somewhat. The cross sections for these one-loop processes are constant and equal to their zero-momentum expressions. These processes can be excluded from the calculation by setting appropriate parameters in the code.

The relic density routines can be called in a precise way where all integrations are performed to a precision better than 1% and coannihilations are included up to a mass difference of \( f_{co} = 2.1 \). It can also be called in a faster way, where the precision of the integrations are of the order of 1% and coannihilations are included up to \( f_{co} = 1.4 \). Usually, the difference between the precise and fast method is completely negligible, but in rare cases it can be of a few %. The fast option is considerably faster and should be sufficient for most purposes. For advanced users, it is also possible to manually decide exactly which coannihilation processes to include.

For users with less demand of calculational precision, we also provide in DarkSUSY the option of a “quick-and-dirty” method of computing the relic density, essentially according to the textbook treatment in [17]. It should be realised, however, that this may sometimes give a computed relic density which is wrong by orders of magnitude.

### 4.4 A note about degrees of freedom and the annihilation rate

We end this section with a comment on the internal degrees of freedom \( g_i \). A neutralino is a Majorana fermion (it is its own antiparticle) and has two internal degrees of freedom, \( g_{\chi_i} = 2 \). A chargino can be treated either as two separate species \( \chi_i^\pm \) and \( \chi_i^- \), each with internal degrees of freedom \( g_{\chi_i^+} = g_{\chi_i^-} = 2 \), or, more simply, as a single species \( \chi_i^\pm \) with \( g_{\chi_i^\pm} = 4 \) internal degrees of freedom. We choose the latter convention in DarkSUSY and use the analogous conventions for sfermions (see [46] for details).

The counting of states in annihilation processes for Majorana fermions is non-trivial, and has led to a factor-of-two ambiguity in the literature which also propagated into earlier versions of DarkSUSY. The current version of DarkSUSY contains the correct normalization of annihilation rates, namely \( \sigma v \) in the Boltzmann equation and \( \sigma v/2 \) for annihilation in the halo. The clearest way to see the origin of the factor of 1/2 is probably to go back to the Boltzmann equation, [52]. In essence, one can view \( \sigma v \) as the thermal average (averaged over
momentum and angles) of the cross section times velocity in the zero momentum limit; in this average one integrates over all possible angles. For identical particles in the initial state, one includes each possible initial state twice, therefore one needs to compensate by dividing by a factor of 2; the prefactor in the zero-momentum limit becomes then \( \sigma v / 2 \). In the Boltzmann equation describing the time evolution of the neutralino number density the \( 1/2 \) does not appear as it is compensated by the factor of 2 one has to include because 2 neutralinos are depleted per annihilation, but we need to include the factor of 1/2 explicitly in other cases where we need the annihilation rate (like for annihilation in the halo). Another way to view this is to think of \( \sigma \) as to the annihilation cross section for a given pair of particles. Let the number of neutralinos in a given volume be \( N \); the annihilation rate would be given by \( \sigma v \) times the number of pairs, which is \( N(N - 1)/2 \). In the continuum limit this reduces to \( \sigma v n^2/2 \).

5 Halo models

The modelling of the distribution of dark matter particles in the Milky Way dark halo plays a major role in estimates of dark matter detection rates. On this issue, however, there is no well-established framework we can refer to. Available dynamical measurements, such as, e.g., the mapping of the rotation curve, the local field of acceleration of stars in the direction perpendicular to the disk, or the motion of the satellites in the outskirts of the Galaxy, provide some constraints on the dark matter density profile, but lack the precision needed to derive a refined model. The guideline for the future will be N-body simulations of hierarchical clustering in cold dark matter cosmologies, which are starting to resolve the inner structure of individual galactic halos. At present, however, the translation of such results into the detailed model we need for the Milky Way still relies on large extrapolations, and, to some extent, faces the problem of possible discrepancies between some of its properties and observations in real galaxies (for a recent review see, e.g., [53]).

In light of these large uncertainties, the definition of the model for the dark matter halo in DarkSUSY has been kept in a very general format. Two sets of properties need to be implemented:

a) Local properties: To derive counting rates in direct detection and \( \chi \)-induced neutrino fluxes from the center of the Sun and the Earth, the user has to specify the halo mass density \( \rho_0 \) at our galactocentric distance \( R_0 \) and the relative particle velocity distribution \( f(u) \), where \( u \) is the modulus of the velocity \( \vec{u} \) in the rest frame of the Sun (or Earth, respectively) and an average of incident angles has been assumed. Options to set \( f(u) \) include the possibility to implement the expression valid for a (truncated) isothermal sphere, or the numerical interpolation of a table of pairs \( (f_i, u_i) \) provided by the user. We have also implemented routines which compute the function \( f(u) \) in the Earth or the Sun rest frame for a given isotropic velocity distribution function in the Galactic reference frame as needed for direct detection and for neutralino capture rates by the Sun. Finally a numerical table for \( f(u) \) in the Earth frame and including the modelling of the diffusion process of neutralinos through the solar system [54] is available to compute capture rates by the Earth.

b) Global properties: To compute indirect signals from pair annihilations in the halo, the full dark matter mass density profile is needed. Charged cosmic ray fluxes are computed in two dimensional models and a generic axisymmetric profile \( \rho(R, z) \) can be implemented by the user. Line of sight integrals with angular averaging over acceptance angles, needed to compute gamma-ray fluxes, are given for spherically symmetric
profiles $\rho(r)$. The options to specify $\rho$ include several analytic forms, with most classes of profiles proposed in recent years [55, 56, 57, 58], as well as the numerical interpolation of a table of pairs $(\rho_i, r_i)$ provided by the user. It is also possible to switch on an option to compute the signals for annihilations taking place in a population of small, unresolved clumps. In this case the user should specify the probability distribution for the clumps and the average enhancement of the source per unit volume compared to the smooth halo scenario.

An option to rescale $\rho_0$ and $\rho(r)$ by the quantity $\Omega h^2/\Omega_{\text{min}} h^2$, where $\Omega h^2$ is the neutralino relic abundance and $\Omega_{\text{min}} h^2$ a minimum reference value for neutralinos to provide most of the dark matter in our Galaxy, is available for the case where subdominant dark matter candidates are considered. The effect can be introduced a posteriori as well for all detection techniques except for neutrino fluxes, where the response to rescaling is non-linear.

A separate package interfaced to DarkSUSY with self consistent pairs $(\rho(r), f(u))$ in $\Lambda$CDM inspired models which fit available dynamical constraints will be available shortly from one of the authors [59].

6 Detection rates

For the detection rates of neutralino dark matter we have used the rates as calculated in Refs. [52, 54, 60, 61, 62, 63, 64, 65, 66], with some improvements which we report in this Section where the formulas used in DarkSUSY are presented.

6.1 Direct detection

The current version of DarkSUSY provides the neutralino-proton and neutralino-neutron scattering cross sections (spin-independent and spin-dependent), as well as cross sections and form factors for elastic scattering of neutralinos off nuclei.

The rate for direct detection of galactic neutralinos can be written as

$$\frac{dR}{dE} = \sum_i c_i \rho_\chi \sigma_{\chi i} |F_i(q_i)|^2 \int_{v>\sqrt{M_i E/2\mu_{\chi i}}} \frac{f(v,t)}{v} d^3v. \quad (29)$$

The sum is over the nuclear species in the detector, $c_i$ being the detector mass fraction in nuclear species $i$. $M_i$ is the nuclear mass, and $\mu_{\chi i} = m_\chi M_i/(m_\chi + M_i)$ is the reduced neutralino-nucleus mass. Moreover, $\rho_\chi$ is the local neutralino density, $v$ the neutralino velocity relative to the detector, $v = |v|$, and $f(v,t) d^3v$ the neutralino velocity distribution. Finally, $\sigma_{\chi i}$ is the total scattering cross section of a WIMP off a fictitious point-like nucleus, $|F_i(q_i)|^2$ is a nuclear form factor that depends on the momentum transfer $q_i = \sqrt{2M_i E}$ and is normalized to $F_i(0) = 1$. The integration is over all neutralino speeds that can impart a recoil energy $E$ to the nucleus.

The cross section $\sigma_{\chi i}$ scales differently for spin-dependent and spin-independent interactions. For spin-independent interactions with a nucleus with $Z$ protons and $A-Z$ neutrons, one has

$$\sigma_{\chi i}^{\text{SI}} = \frac{\mu_{\chi i}^2}{\pi} |Z G_p^p + (A-Z) G_n^n|^2, \quad (30)$$

where $G_p^p$ and $G_n^n$ are the scalar four-fermion couplings of the WIMP with point-like protons and neutrons, respectively (see, e.g., [67], and below). As default, the spin-independent form factor in DarkSUSY is taken to be of the Helm form

$$|F^{\text{SI}}(q)|^2 = \left( \frac{3j_1(q R_1)}{q R_1} \right)^2 e^{-q^2 s^2}, \quad (31)$$
with \( j_1 \) a spherical Bessel function of first kind, \( R_1 = \sqrt{R^2 - 5s^2} \), \( R = [0.9(M/\text{GeV})^{1/3} + 0.3] \) fm and \( s = 1 \) fm. An exponential form factor is also available as an option.

For spin-dependent interactions, one has instead, at zero momentum transfer,

\[
\sigma_{\chi_i}^{SD} = \frac{4\mu_{\chi_i}^2}{\pi} \frac{J + 1}{J} \left| \langle S_p \rangle G_p^n + \langle S_n \rangle G_n^n \right|^2,
\]

where \( J \) is the nuclear spin, \( \langle S_p \rangle \) and \( \langle S_n \rangle \) are the expectation values of the spin of the protons and neutrons in the nucleus, respectively, and \( G_p^n \) and \( G_n^n \) are the axial four-fermion couplings of the WIMP with point-like protons and neutrons \([67, 68]\). At finite momentum transfer, the spin-dependent cross section times the form factor reads

\[
\sigma_{\chi_i}^{SD} |F_i^{SD}(q)|^2 = \frac{4\mu_{\chi_i}^2}{2J + 1} \left[ (G_p^n)^2 S_{pp}(q) + (G_n^n)^2 S_{nn}(q) + G_p^n G_n^n S_{pn}(q) \right],
\]

where \( S_{pp}(q) = S_{00}(q) + S_{11}(q), S_{nn}(q) = S_{00}(q) + S_{11}(q) - S_{01}(q), \) and \( S_{pn}(q) = 2[S_{00}(q) - S_{11}(q)] \). The spin structure functions \( S_{00}(q), S_{11}(q), \) and \( S_{01}(q) \) are defined in \([69]\) and are given in the literature.

For protons and neutrons, the previous expressions reduce to

\[
\sigma_{\chi_i}^{SI} = \frac{\mu_{\chi_i}^2}{\pi} |G_p^n|^2, \quad \sigma_{\chi_i}^{SI} = \frac{\mu_{\chi_i}^2}{\pi} |G_n^n|^2, \quad \sigma_{\chi_i}^{SD} = \frac{3\mu_{\chi_i}^2}{\pi} |G_p^n|^2, \quad \sigma_{\chi_i}^{SD} = \frac{3\mu_{\chi_i}^2}{\pi} |G_n^n|^2.
\]

For the neutralino, the scalar and axial four-fermion couplings with the proton and neutron arise from squark, Higgs and Z boson exchange. In DarkSUSY, the default expressions for a nucleon \( N = p, n \) are

\[
G_s^N = \sum_{q=u,d,s,c,b,t} \langle N|\bar{q}q|N \rangle \left( \frac{1}{2} \sum_{k=1}^{6} \frac{g_{Lq_i\chi_q}g_{Rq_i\chi_q}}{m_{q_i}^2} - \sum_{H_1, H_2} \frac{g_{h\chi\chi}g_{hqq}}{m_h^2} \right), \tag{35}
\]

\[
G_a^N = \sum_{q=u,d,s} \langle \Delta q \rangle_N \left( \frac{g_{h\chi\chi}g_{hqq}}{m_Z^2} + \sum_{k=1}^{6} \frac{g_{Lq_i\chi_q}^2 + g_{Rq_i\chi_q}^2}{m_{q_i}^2} \right), \tag{36}
\]

where \( g_{abc} \) are the coupling constants in the vertex involving particles \( abc \) (see \([2]\) and \([65]\) for explicit expressions). The more complicated expressions of Drees and Nojiri \([70]\) are also available as an option.

Default values of the parameters in DarkSUSY are \([71, 72]\) (with \( \langle N|\bar{q}q|N \rangle = f_{\bar{T}q}^N M_N/m_q \))

\[
f_{\bar{T}u}^p = 0.023, \quad f_{\bar{T}d}^p = 0.034, \quad f_{\bar{T}s}^p = 0.14, \quad f_{\bar{T}c}^p = f_{\bar{T}b}^p = f_{\bar{T}t}^p = 0.0595, \tag{37}
\]

\[
f_{\bar{T}u}^n = 0.019, \quad f_{\bar{T}d}^n = 0.041, \quad f_{\bar{T}s}^n = 0.14, \quad f_{\bar{T}c}^n = f_{\bar{T}b}^n = f_{\bar{T}t}^n = 0.0592, \tag{38}
\]

\[
(\Delta u)_p = (\Delta d)_n = 0.77, \quad (\Delta d)_p = (\Delta u)_n = -0.40, \quad (\Delta s)_p = (\Delta s)_n = -0.12. \tag{39}
\]

Other sets of values for these parameters are available. These values can also be overridden by the user.

### 6.2 Indirect detection

There are several ways of detecting dark matter particles indirectly. Pair annihilations of dark matter particles in the Galactic halo may give an exotic component in positron, antiproton or antideuteron cosmic-rays and gamma-rays. There may also be annihilation in astrophysical environments where the density of neutralinos may be enhanced, such as annihilation in the center of the Earth or Sun (detected in neutrino telescopes through a high-energy neutrino flux) or near the central black hole of the Galaxy.
6.2.1 Monte Carlo simulations

In several of the indirect detection processes below we need to evaluate the yield of different particles per neutralino annihilation. The hadronization and/or decay of the annihilation products are simulated with Pythia \[73\] in essentially the same way (with a few differences) for all these processes and we here describe how the simulations are done. We can divide the simulations into two groups: a) annihilation in the Earth and the Sun and b) annihilation in the halo. In both cases the simulations are done for a set of 18 neutralino masses, \(m_\chi = 10, 25, 50, 80.3, 91.2, 100, 150, 176, 200, 250, 350, 500, 750, 1000, 1500, 2000, 3000\) and 5000 GeV. We tabulate the yields and then interpolate these tables in DarkSUSY.

These two groups of simulations differ slightly in other aspects, namely

a) Annihilation in the Earth and the Sun. In this case we are mainly interested in the flux of high energy muon neutrinos and neutrino-induced muons at a neutrino telescope. We simulate 6 ‘fundamental’ annihilation channels, \(c\bar{c}, b\bar{b}, t\bar{t}, \tau^+\tau^-, W^+W^-\) and \(Z^0Z^0\) (if kinematically allowed) for each mass listed above. The lighter leptons and quarks do not contribute significantly and can safely be neglected. Pions and kaons get stopped before they decay and are thus made stable in the Pythia simulations so that they do not produce any neutrinos. For annihilation channels containing Higgs bosons, we can calculate the yield from these fundamental channels by letting the Higgs bosons decay in flight (see below). We also take into account the energy losses of \(B\)-mesons in the Sun and the Earth by following the approximate treatment of \[74\] but with updated \(B\)-meson interaction cross sections as given in \[65\]. Neutrino-interactions on the way out of the Sun are simulated with Pythia including neutral current interactions and charged-current interactions as a neutrino-loss. The neutrino-nucleon charged current interactions close to the detector are also simulated with Pythia and finally the multiple Coulomb scattering of the muon on its way to the detector is calculated using distributions from \[13\]. For more details on these simulations see \[75, 76\].

For each annihilation channel and mass we simulate \(1.25 \times 10^6\) annihilations and tabulate the final results as a neutrino-yield, neutrino-to-muon conversion rate and a muon yield differential in energy and angle from the center of the Sun/Earth. We also tabulate the integrated yield above a given threshold and below an open-angle \(\theta\). We assumed throughout that the surrounding medium is water with a density of 1.0 g/cm\(^3\). Hence, the neutrino-to-muon conversion rates have to be multiplied by the density of the medium. In the muon fluxes, the density cancels out (to within a few percent). For the neutrino-nucleon cross sections, we have used the parameterizations in \[65\].

b) Annihilation in the halo. The simulations are here simpler since we do not have a surrounding medium that can stop the annihilation products. We here simulate for 8 ‘fundamental’ annihilation channels \(c\bar{c}, b\bar{b}, t\bar{t}, \tau^+\tau^-, W^+W^-, Z^0Z^0, \mu^+\mu^-\). Compared to the simulations in the Earth and the Sun, we now let pions and kaons decay and we also let antineutrons decay to antiprotons. For each mass we simulate \(2.5 \times 10^6\) annihilations and tabulate the yield of antiprotons, positrons, gamma rays (not the gamma lines), muon neutrinos and neutrino-to-muon conversion rates and the neutrino-induced muon yield, where in the last two cases the neutrino-nucleon interactions has been simulated with Pythia as outlined above.

With these simulations, we can calculate the yield for any of the above mentioned particles for a given MSSM model. For the Higgs bosons, which decay in flight, an integration over the angle of the decay products with respect to the direction of the Higgs boson is performed. Given the branching ratios for different annihilation channels it is then straightforward to
compute the muon flux above any given energy threshold and within any angular region around the Sun or the center of the Earth.

6.3 Neutrinos from the Sun and Earth

One of the most promising indirect detection methods \[77\] relies on the fact that scattering of halo neutralinos by the Sun and the planets, in particular the Earth, during the several billion years that the Solar system has existed, will have trapped these neutralinos within these astrophysical bodies. Being trapped within the Solar or terrestrial material, they will sink towards the center, where a considerable enrichment and corresponding increase of annihilation rate will occur.

Searches for neutralino annihilation into neutrinos will be subject to extensive experimental investigations in view of the new neutrino telescopes (AMANDA, IceCube, Baikal, NESTOR, ANTARES) planned or under construction \[78\]. A high-energy neutrino signal in the direction of the center of the Sun or Earth is an excellent experimental signature which may stand up against the background of neutrinos generated by cosmic-ray interactions in the Earth’s atmosphere.

The detector energy threshold for “small” neutrino telescopes like Baksan, MACRO and Super-Kamiokande is around 1 GeV. Large area neutrino telescopes in the ocean or in Antarctic ice typically have thresholds of the order of tens of GeV, which makes them sensitive mainly to heavy neutralinos (above 100 GeV) \[79\]. In DarkSUSY, the low energy cut-off can be set.

Final states which give a hard neutrino spectrum (such as heavy quarks, \(\tau\) leptons and \(W\) or \(Z\) bosons) are usually more important than the soft spectrum arising from light quarks and gluons.

Neutralinos are steadily being trapped in the Sun or Earth by scattering, whereas annihilations take them away. The balance between capture and annihilation has the solution for the annihilation rate implemented in DarkSUSY

\[
\Gamma_A = \frac{C_c}{2} \tanh^2 \left( \frac{t}{\tau} \right),
\]

where the equilibrium time scale \(\tau = 1/\sqrt{C_cC_a}\), with \(C_c\) being the capture rate and \(C_a\) being related to the annihilation efficiency. In most cases for the Sun, \(\tau\) is much smaller than a few billion years, and therefore equilibrium is often a good approximation \((\dot{N}(t) = 0)\). This means that it is the capture rate which is the important quantity that determines the neutrino flux. For the Earth, \(\tau\) is, on the other hand, usually of the same order as, or much larger than, the age of the solar system, and equilibrium has often not occurred. In either case, in the program we keep the exact formula \[40\] (except for the modifications needed for a Damour-Krauss population of WIMPs, discussed below).

For the actual capture rate calculations we have several expressions implemented in DarkSUSY. As a default, we use the full expressions given in appendix A of \[80\] where we numerically integrate over the velocity distribution. To speed-up the calculations, it is possible to perform this integration only once and use a saved tabulated version for subsequent calls. In the capture rate calculations we also need the density profile of the Earth and the Sun and the chemical composition as a function of radius. For the Sun we use the solar model BP2000 \[81\], complemented with the estimates of the mass fractions of the heavier elements from \[82\]. For the Earth, we use the density profile of \[83\] with the compositions given in \[84\] (see \[54\] for a table of these). For comparison, the approximate capture rate expressions in \[6\] are also available.
The capture rate induced by scalar (spin-independent) interactions between the neutralinos and the nuclei in the interior of the Earth or Sun is the most difficult one to compute, since it depends sensitively on the Higgs mass, form factors, and other poorly known quantities. However, this spin-independent capture rate calculation is the same as for direct detection treated in Section 6.1. Therefore, there is a strong correlation between the neutrino flux expected from the Earth (which is mainly composed of spin-less nuclei) and the signal predicted in direct detection experiments [79, 85].

Due to the low counting rates for the spin-dependent interactions in terrestrial detectors, high-energy neutrinos from the Sun constitute a competitive and complementary neutralino dark matter search. Of course, even if a neutralino is found through direct detection, it will be extremely important to confirm its identity and investigate its properties through indirect detection. In particular, the mass can be determined with reasonable accuracy by looking at the angular distribution of the detected muons [86, 87].

The capture rate in the Earth is dominated by scalar interactions, where there may be kinematic and other enhancements, in particular if the mass of the neutralino almost matches one of the heavy elements in the Earth. As shown by Gould [88], the Earth does not dominantly capture WIMPs from the halo directly. Instead, the Earth captures WIMPs that, due to gravitational interactions, have diffused around and become bound to the solar system. However, solar depletion of these bound WIMPs could be an important effect [89], and as a default, DarkSUSY uses a new estimate of the velocity distribution in the solar system, where these solar depletion effects have been included [54]. It is possible to change to a standard Gaussian distribution if the user prefers.

There is also a possibility that there exists a special population of WIMPs, the Damour-Krauss population [90], arising from WIMPs that have just skimmed the Sun’s surface. As an optional choice, this population can be included in the calculation [66]. The enhancement caused by the new population is only important for a neutralino mass less than 150 - 170 GeV (the exact number depending on details about the angular momentum distribution [66]). The total capture rate is computed according to the formulas in [66], which take into account that the annihilation rates from the Earth will in general depend on time in a different way than the simple result in Eq. [86].

### 6.4 Indirect searches through antimatter signals

Pair annihilation of neutralinos in the Galactic halo produces the same amounts of matter and antimatter. As antimatter seems to be scarce in the Universe, apparently with no standard primary sources, there is a chance that by measuring antimatter in cosmic ray fluxes one may find evidence of the existence of dark matter particles (see, e.g., [91, 92, 93, 94, 95, 96, 97, 98, 99]). In the current release of the code we consider neutralino induced fluxes of antiprotons, positrons and antideuterons. To produce estimates of such fluxes, there are several steps one needs to follow: i) Evaluate for each dark matter candidate the probability for a pair annihilation to take place locally in space, i.e. compute $1/2 (\rho(x)/m_\chi)^2 \sigma_{\text{ann}} v^8$; ii) Estimate the production rate of a given species by folding together, for each model, the branching ratios for the annihilation into a given two-body final state with the Monte Carlo simulation of the hadronization and/or decay of that state, as described in Section 6.2.1 (except for $\bar{D}$ sources where we have implemented the prescription suggested in Ref. [98] to convert from the $\bar{p}$ yield); iii) Propagate these sources through the Galactic magnetic fields to make predictions for the induced cosmic ray fluxes at the Sun’s location in the Galaxy; iv) Include the effect of solar modulation to propagate the fluxes from the interstellar medium to our location inside the solar system. The implementation in DarkSUSY is written in a

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8The origin of the factor of 1/2, missing in earlier versions of DarkSUSY, is explained in Sect. [49].
modular format which allows the user to eventually modify and/or replace each of the four steps above independently.

At tree level, the relevant final states of neutralino pair annihilations for $\bar{p}$ and $\bar{D}$ production are $q\bar{q}$, $W^+W^-$, $Z^0Z^0$, $W^+H^-$, $ZH^0$, $ZH^0$, $H^0H^0$, and $H^0H^0$; among the $q\bar{q}$ states we have included in DarkSUSY all the heavier quarks ($c$, $b$ and $t$). In addition, we have included the $Z\gamma$ and the 2 gluon final states which occur at one loop-level. The same list of final states is implemented for positrons, with the addition of $e^+e^-$. The $e^+e^-$ final state gives rise to a positron monochromatic source (however this is negligible in most particle physics setups, and tends to be smeared out by propagation).

To model the propagation we have considered a semi-empirical diffusion equation, in the steady state approximation, applied to a two-dimensional model with cylindrical symmetry and with free escape boundary conditions. The parameters in such a model should be fixed in agreement with values inferred from available cosmic ray data in analogous propagation models (see, e.g., the GALPROP package). For simplicity, rather than considering the most general setup, we refrain to cases in which, still including all relevant effects, the Green function of the transport equation can be derived analytically, so that we can avoid the CPU time-consuming problem of having to solve a partial differential equation for each dark matter candidate. Therefore, we do not include reacceleration effects but mimic them through a diffusion coefficient which has a broken power law in rigidity as functional form. Also, for antiprotons and antideuterons we neglect energy losses (whenever a scattering with a nucleus takes place the particle is removed), while for positrons we consider an average over space of the energy loss effect due to inverse Compton on starlight and the cosmic microwave background, and in addition the synchrotron radiation from the effects of the galactic magnetic field. For comparison, we allow also the option to treat the propagation of antiprotons according to the propagation models by Chardonnet et al. and Bottino et al., while for the positron we have implemented the option to use the leaky-box treatment given by Kamionkowski and Turner or the numerical Green functions derived by Moskalenko and Strong with the GALPROP code (the latter two cases however cannot be interfaced to a generic axisymmetric dark matter density profile, as for our default propagation model).

Given a set of parameters for the propagation model, and a given neutralino number density profile or a given probability distribution of small clumps, the effect of propagation, in the cases we have considered, can be factorized out into effective energy-dependent “diffusion times”, $\tau_{\bar{p}}(T)$, $\tau_{\bar{D}}(T)$ and $\tau_{e^+}(T)$, which are independent of the parameters defining the particle physics setup for the dark matter candidate. In some cases, such as when considering very large samples of neutralino candidates or when implementing singular halo profiles for which the computation of the diffusion times can be very CPU time-consuming, it is advantageous to exploit the option provided by DarkSUSY to tabulate the diffusion times over a given energy range (optionally saving the tabulation to disk for later use) and use an interpolation between tabulated values, rather than linking directly to the computation for each dark matter candidate.

Finally, regarding solar modulation, we implement the one parameter model based on the analytical force-field approximation by Gleeson & Axford for a spherically symmetric model. This approach is expected to be slightly less accurate than, e.g., the full numerical solution of the propagation equation of the spherically symmetric model, but again it is much less CPU time-consuming. DarkSUSY allows an output with both solar modulated and local interstellar fluxes, and the latter can eventually be solar modulated by the user with more sophisticated methods. For the positrons we allow for the option to reduce the effects of solar modulation by considering the positron fraction, $e^+/(e^++e^-)$, rather than the absolute positron fluxes. In this case an estimate of the background is needed: in DarkSUSY
we provide background $e^+$ and $e^-$ fluxes taken from [106].

### 6.5 Gamma rays

Gamma-rays have a low enough cross section on gas and dust that the Galaxy is essentially transparent to them (except perhaps in the innermost part, very close to the region where a massive black hole is inferred); absorption by starlight and infrared background becomes efficient only for very far away sources and high-energy photons.

The bulk of the gamma-rays from neutralino annihilations arise in the decay of neutral pions produced in the fragmentation processes initiated by tree level final states [109, 60, 110] (analogously to the other halo signals, in DarkSUSY we include all tree level final states and make use of a Monte Carlo simulation for fragmentation and decay processes, see Section 6.2.1). However, $\pi^0$ production is common also to other astrophysical processes, and this may turn out to be a limiting factor to disentangle dark matter sources. At the same time, however, a relevant gamma-ray contribution may arise directly (at one-loop level) in two body final states; although such photons are much fewer than those from $\pi^0$ decays they have a much better signature: neutralinos annihilating in the galactic halos move with a velocity of the order $v/c \sim 10^{-3}$, hence these outgoing photons (as any particle in any of the allowed two body final states) will then be nearly monochromatic, with energy of the order of the neutralino mass [111, 112, 113, 102, 100, 60]. There is no other known astrophysical source with such a signature; the detection of a line signal out of a spectrally smooth gamma-ray background would be a spectacular confirmation of the existence of dark matter in form of exotic massive particles.

#### 6.5.1 Sources and fluxes

Following the discussion in [60], the monochromatic gamma-ray flux measured in a detector with angular acceptance $\Delta \Omega$ is:

$$\Phi_{\gamma}(\psi, \Delta \Omega) = 0.94 \cdot 10^{-11} \left( \frac{N_\gamma}{10^{-29} \text{cm}^3 \text{s}^{-1}} \right) \left( \frac{10 \text{ GeV}}{M_\chi} \right)^2 \langle J(\psi) \rangle \Delta \Omega \times \Delta \Omega \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1},$$

(41)

where $\psi$ is an angle to label the direction of observation and where $N_\gamma = 2$ for $\chi \chi \to \gamma \gamma$, $N_\gamma = 1$ for $\chi \chi \to Z \gamma$. Here the dimensionless line-of-sight dependent function is

$$J(\psi) = \frac{1}{8.5 \text{ kpc}} \cdot \left( \frac{1}{0.3 \text{ GeV/cm}^2} \right)^2 \int_{\text{line of sight}} \rho_{\chi}^2(l) \, dl(\psi),$$

(42)

and its angular average over the resolution solid angle $\Delta \Omega$ is

$$\langle J(\psi) \rangle_{\Delta \Omega} = \frac{1}{\Delta \Omega} \int_{\Delta \Omega} d\Omega' J(\psi'),$$

(43)

Analogously, the gamma-ray flux with continuum energy spectrum is obtained by replacing $N_\gamma$, $v \sigma_{\chi \chi \gamma}$ with $\sum_f dN_f^\gamma/dE \, v \sigma_f$, where the sum is over all tree level final states. Finally, the formalism we introduced can be used also to estimate the flux in the simple case of a single source which, for a given detector, can be approximated as point-like. If such a source is in the direction $\psi$ at a distance $d$, Eq. (43) becomes:

$$\langle J(\psi) \rangle_{\Delta \Omega} = \frac{1}{8.5 \text{ kpc}} \cdot \left( \frac{1}{0.3 \text{ GeV/cm}^2} \right)^2 \cdot \frac{1}{d^2} \cdot \frac{1}{\Delta \Omega} \int d^3r \, \rho_{\chi}^2(\vec{r})$$

(44)

where the integral is over the extension of the source (much smaller than $d$).
Several targets have been discussed as sources of gamma-rays from the annihilation of
dark matter particles. An obvious source is the dark halo of our own galaxy [114] and in
particular the Galactic center, as the dark matter density profile is expected, in most models,
to be peaked towards it, possibly with huge enhancements close to the central black hole. The
Galactic center is an ideal target for both ground- and space-based gamma-ray telescopes.
As satellite experiments have much wider field of views and will provide a full sky coverage,
they will test the hypothesis of gamma-rays emitted in clumps of dark matter which may
be present in the halo [115, 110, 116, 117]. Another possibility which has been considered is
the case of gamma-ray fluxes from external nearby galaxies [118]. Furthermore, it has been
proposed to search for an extragalactic flux originated by all cosmological annihilations of
dark matter particles [119, 120].

DarkSUSY is suitable to compute the gamma-ray flux from all these (and possibly other)
sources. Two cases are fully included in the package: assuming that neutralinos are smoothly
distributed in the Galactic halo with \( \rho_\chi \) equal to the dark matter density profile, in DarkSUSY
Eq. (43) is computed for a specified halo profile and any given \( \psi \) and \( \Delta \Omega \) [60]. The second
option deals with the case of a portion of dark matter being in the form of clumps, each of
which is treated as a non-resolvable source in the detector, distributed in the Galaxy according
to a probability distribution which can be specified by the user; in DarkSUSY the default
probability distribution stems from the assumption that it follows the dark matter density
profile (see [116] for details). It is straightforward to extend this to all other astrophysical
sources; in case of cosmological sources one has just to pay attention to include redshift
effects and absorption on starlight and infrared background, see [120].

The case of the possible enhancement, a “spike” in the vicinity of the galactic center [121]
should be kept in mind. However, since there is no consensus in the literature [122, 123, 124]
about important quantities for the calculation such as the magnetic field radial profile and
the optical depth for synchrotron self-absorption, we have chosen not to include routines for
the effects of this possible enhancement of gamma rays and neutrinos. And the very existence
of a spike is dependent on fine details, still unknown for the Milky Way [125, 126].

6.5.2 Gamma rays with continuum energy spectrum

The gamma-ray flux produced in neutralino annihilations through \( \pi^0 \) decays can be large but
in general lacks distinctive features. The photon spectrum in the process of a pion decaying
into \( 2\gamma \) is, independent of the pion energy, peaked at half of the \( \pi^0 \) mass, about 70 MeV,
and symmetric with respect to this peak if plotted in logarithmic variables (i.e. \( dN_\gamma/dE \) vs.
\( \log E \)). Of course, this is true both for pions produced in neutralino annihilations and, e.g.,
for those generated by cosmic ray protons interacting with the interstellar medium.

When considered together with the cosmic ray induced Galactic gamma-ray background,
the neutralino induced signal looks like a component analogous to the secondary flux due to
nucleon nucleon interactions: it is dwarfed by the bremsstrahlung component at low energy,
while it may be the dominant contribution at energies above 1 GeV or so. In DarkSUSY
the continuum gamma flux from all annihilation channels is computed and may be easily
obtained for a given energy or energy threshold.

6.5.3 \( \chi\chi \to \gamma\gamma \)

At the one-loop level, it is possible to get two-body final states containing one or two photons,
with a distinctive signature which may provide a “smoking gun” for dark matter. The
drawback is of course that the processes are loop-suppressed, so one probably needs a halo
with a large central concentration, or small-scale structure (“clumps” of dark matter) to
detect a signal.
In DarkSUSY the full expression for the annihilation cross section of the process

$$\tilde{\chi}_1^0 + \tilde{\chi}_1^0 \rightarrow \gamma + \gamma$$

(45)
is computed at full one loop level, in the limit of vanishing relative velocity of the neutralino pair, i.e. the case of interest for neutralinos in galactic halos; the outgoing photons have an energy equal to the mass of $\chi_1^0$:

$$E_\gamma = m_{\chi}.$$  

(46)

The total amplitude is implemented in DarkSUSY as the sum of the contributions obtained from four different classes of diagrams:

$$\tilde{A} = \tilde{A}_{ff} + \tilde{A}_{H} + \tilde{A}_{W} + \tilde{A}_{G},$$

where the indices label the particles in the internal loops, i.e., respectively, fermions and sfermions, charged Higgs and charginos, W-bosons and charginos, and, in the gauge we chose, charginos and Goldstone bosons. For every $\tilde{A}$ term, real and imaginary parts are separately computed; the full set of analytic formulas are given in [102], following the notation of [113], where some of the contributions were first computed. They are rather lengthy expressions with non trivial dependences on various combinations of parameters in the MSSM.

The branching ratio for neutralino annihilations into $2\gamma$ is typically not larger than 1%, with the largest values of $v\sigma_{2\gamma}$, for neutralinos with a cosmologically significant relic abundance, in the range $10^{-29} - 10^{-28}$ cm$^3$s$^{-1}$. Such values may be large enough for the discovery of this signal in upcoming measurements; at the same time it should be kept in mind that very low values for the cross section are possible as well.

In the very high mass range above a TeV, it has been suggested that the line rates may be very much larger due to binding effects and resonant conversion between neutralinos and charginos [127]. In the present version of DarkSUSY we have not included these effects, however.

### 6.5.4 $\chi\chi \rightarrow Z\gamma$

The process of neutralino annihilation into a photon and a $Z^0$ boson[100]

$$\tilde{\chi}_1^0 + \tilde{\chi}_1^0 \rightarrow \gamma + Z^0$$

(47)

also gives a nearly monochromatic line (with a small smearing caused by the finite width of the $Z^0$ boson, in any case unlikely to be important for current or proposed gamma ray experiments), with energy

$$E_\gamma = M_\chi - \frac{m_Z^2}{4M_\chi}.$$  

(48)

The steps followed in DarkSUSY to compute the cross section are essentially the same as those described for the $2\gamma$ case. Again the total amplitude is obtained by summing the contribution from four classes of diagrams and by splitting for each of them real and imaginary parts. The analytic formulas were derived in [100], and are much less compact than those obtained for the process of neutralino annihilation into two photons.

The maximum value of $v\sigma_{Z\gamma}$, for neutralinos with a cosmologically significant relic abundance, is around $2 \cdot 10^{-28}$ cm$^3$s$^{-1}$ and occurs for nearly pure higgsinos. In the heavy mass range, the value of $v\sigma_{Z\gamma}$ reaches a plateau of around $0.6 \cdot 10^{-28}$ cm$^3$s$^{-1}$. This interesting effect of a non-diminishing cross section with Higgsino mass (which is due to a contribution to the real part of the amplitude) is also valid for the $2\gamma$ final state in the corresponding limit, with a value of $1 \cdot 10^{-28}$ cm$^3$s$^{-1}$ [102]. Since the gamma-ray background drops rapidly
with increasing photon energy, these processes may be interesting for detecting dark matter neutralinos near the upper range of allowed neutralino masses.

Whenever the lightest neutralino contains a significant Wino or Higgsino component the value of $v\sigma Z\gamma$ may be as large as, or even larger than, twice the value of $v\sigma 2\gamma$. It is therefore usually not a good approximation to neglect the $Z\gamma$ state compared to $2\gamma$.

6.6 Neutrinos from the halo

Usually, the flux of neutrinos from annihilation of neutralinos in the Milky Way halo is too small to be detectable, but for some clumpy or cuspy models, or for annihilation in a possible spike around the central black hole, it might be detectable. The calculation of the neutrino-flux follows closely the calculation of the continuous gamma ray flux, with the main addition that neutrino interactions close to the detector are also included. Hence, both the neutrino flux and the neutrino-induced muon flux can be obtained. The neutrino to muon conversion rate in the Earth can also be obtained. The neutrino rate from other sources than the interior of the Earth or the Sun is generally negligible. If there would exist a spike at the galactic center [121], there may in some cases be a detectable flux. These neutrino rates are constrained by existing limits on the gamma-ray flux [121, 128].

7 Examples of results obtained with DarkSUSY

We will here go through a set of benchmark models as examples of the performance of DarkSUSY. We will consider two popular setups. We will start with a set of mSUGRA models and then turn to more general MSSM models. We will here use the default DarkSUSY setup, in particular an isothermal sphere with a Maxwell-Boltzmann velocity distribution for the halo model.

7.1 Benchmark models in mSUGRA

We will consider here some of the benchmark models from Battaglia et al. [129]. In table 1 we list the properties of the selected models, as derived by DarkSUSY using the ISASUGRA 7.69 package to describe the renormalization group running of the theory from the grand unification scale to the low energy scale. The models we are focusing on are those with a top mass of 175 GeV and that are still viable with ISASUGRA 7.69. As the table in [129] was produced with ISASUGRA 7.67, some differences occur due to different versions of the codes, e.g. model M is no longer physical in ISASUGRA 7.69, and thus not included here.

Our results for the relic density and for the SUSY contributions to $B(b \rightarrow s\gamma)$ and to the anomalous magnetic moment of the muon $a^\mu$, agree reasonably well with those in [129]. The expected sensitivities of future neutrino telescopes are of the order of 20 events km$^{-2}$ yr$^{-1}$ for the Earth and 50–1000 events km$^{-2}$ yr$^{-1}$ for the Sun; we see in the table that all benchmark models, unfortunately, produce much lower fluxes. The outreach of future direct detection experiments is expected to be of the order of $10^{-9}$ pb for the spin-independent scattering cross section; we see that some of models we are considering are potentially detectable. The cross section for annihilation into gamma rays (times the number of photons produced) are also given for these models; the detectability depends strongly on the halo profile, but one can in general say that these cross sections are too low to be seen in current data, unless the halo profile is very cuspy towards the galactic center. The $e^+$ fluxes are here given in the energy bin 6.0–8.9 GeV of the HEAT 94+95 [130] experiment. The measured $e^+$ flux is $(7.2 \pm 1.2) \times 10^{-6}$ GeV$^{-1}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$, i.e. the predicted fluxes are much lower than the measured one. For the antiprotons, we choose, as an example, to show the predicted (average) flux in the energy
Table 1: Relic density and various rates for the benchmark models of [129]. There are five free parameters in mSUGRA: $\tan\beta$, sign($\mu$), $m_{1/2}$, $m_0$ and $A_0$. The latter three are the unification values (at the grand unification scale) of the soft supersymmetry breaking fermionic mass parameters, scalar mass parameters and trilinear scalar coupling parameters, respectively. All the benchmark models have $A_0 = 0$. We have here used ISASUGRA 7.69 for the RGE-running (but have not taken the $b$, $t$, and $\tau$ Yukawa couplings from ISASUGRA). The neutrino-induced muon fluxes from the Earth and the Sun are for a threshold of 1 GeV. ‘best’ refers to the suppressed fluxes resulting from the estimate in [54] and ‘gauss’ refers to the usual approximation of a Gaussian velocity distribution of neutrinos for capture in the Earth. The scattering cross sections are given here evaluated both with the standard expressions (labelled ‘std’) and with the Drees and Nojiri expressions [10] (labelled ‘dn’). The continuum $\gamma$’s are given in terms of the number of photons times the cross section, $N_{\gamma}$ cont.($\sigma v$), and refers to $\gamma$’s above 1 GeV. The $e^+$ flux is the average solar modulated flux in the energy range 6.0–8.9 GeV, i.e. in one of the HEAT 94-95’s bins [130]. The $\bar{\nu}$ flux is the average solar modulated flux in the energy range 0.56–0.78 GeV, i.e. in one of the BESS 98 bins [131]. The $D$ flux is the average flux in the energy range 0.1–0.4 GeV, as applicable to e.g. the proposed GAPS probe [132]. In this case, the flux is the average of the solar modulated fluxes at solar minimum and maximum. Parameters other than those mentioned above are set to their default values as described in the text.
range 0.56–0.78 GeV, which is one of the BESS 98 \textsuperscript{131} bins. The measured flux in this
bin is \((1.23^{+0.38}_{-0.33}) \times 10^{-6} \text{ GeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}\), i.e. the measured flux is much higher than
the predicted one in this energy range. For the antideuterons, we show the expected flux
in the energy range 0.1–0.4 GeV, which is reasonable for the proposed GAPS probe \textsuperscript{132}. For
the antideuterons, there is essentially no background and the sensitivity is thus given by
the ability to detect one antideuteron. For GAPS this corresponds to a sensitivity of about
\(2.6 \times 10^{-13} \text{ GeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}\). Hence, two of these benchmark models have high enough
fluxes for being just about detectable in this way. For further examples of rates in mSUGRA,
as calculated with DarkSUSY, see e.g. \textsuperscript{133}.

7.2 Benchmark models in MSSM

We will now turn to models in the MSSM framework as generated by fixing free parameters
at the weak scale. We refer to the setup with seven free parameters, i.e. \(\mu, M_2, m_A, \tan \beta, m_0, A_t \text{ and } A_b\), we described in Section \textsuperscript{2}. We will here show an example of results that
can be obtained with a simple scan over this parameter space. We have generated 5000
models assuming: \(\mu \in [-3000, 3000] \text{ GeV}, M_2 \in [-3000, 3000] \text{ GeV}, m_A \in [100, 1000] \text{ GeV},\)
\(\tan \beta \in [1, 55], m_0 \in [100, 5000] \text{ GeV}, A_t \in [-3.3] m_0 \text{ and } A_b \in [-3.3] m_0\). We have then
selected a few sample models with a relic density in the range \(0.09 \leq \Omega h^2 \leq 0.11\) and with
reasonably high detection rates; these are the first ten models in Table \textsuperscript{2}. In addition to this
scan, we have also made a small scan of 300 models in which we required the mass of the
\(CP\)-odd Higgs boson, \(A\), to be in the range \(m_A \in [90, 150] \text{ GeV}\); model number 11 in Table
\textsuperscript{2} has been chosen from this latter scan to illustrate that higher-rate models are possible to
find with these kind of dedicated scans.

As seen in the table, the rates are typically slightly higher than for the mSUGRA benchmark
models considered above. Please remember though, that this set of MSSM benchmark
models is achieved with a rather small scan over the MSSM parameter space. Models with
even higher rates would be possible to find with more extensive scans of the parameter space.

Many of our models produce fluxes from the Sun that exceed the future sensitivity of
50–1000 events \text{ km}^{-2} \text{ yr}^{-1} and hence would be detectable. For the Earth, on the other hand,
the fluxes in this set of models are typically much lower than the projected future limit of 20
events \text{ km}^{-2} \text{ yr}^{-1}. Most of the selected models are potentially detectable with future direct
detection experiments as they have a spin-independent scattering cross section larger than
\(10^{-9} \text{ pb}\). Note the complementarity between the direct detection signal and the neutrino
flux from the Sun. For example, for model 9, the spin-independent scattering cross section
is very close to the sensitivity of future detectors, whereas the neutrino-flux from the Sun
is clearly above expected future sensitivities of neutrino telescopes like Antares or IceCube.
The gamma-ray yields are in general larger than for the mSUGRA benchmark points, but the
issue regarding detectability is still difficult to address as it is strongly affected by the halo
model dependence. The positron and antiproton fluxes are generally enhanced as well, but
still well below measured fluxes. Finally, there are several models among those we selected
which have an antideuteron flux in the energy range 0.1–0.4 GeV exceeding \(2.6 \times 10^{-13} \text{ GeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1}\), the expected sensitivity of the GAPS detector \textsuperscript{132}.

7.3 Discussion

The potential of DarkSUSY package has been illustrated here, for a few sample models and
in two popular scenarios. The code provides the state of the art calculation of the neutralino
relic abundance, and allows for the estimate of several quantities of interest for dark matter
searches. It has been shown that the code has a very flexible structure in the definition of
the supersymmetric dark matter candidate, as well as a rather broad freedom in the choice
Table 2: Same as Table 1, but for the MSSM benchmark models of our sample scan.
of the relevant astrophysical setups. Outputs are in simple and general formats, which, as we have shown, can be very easily compared to current and future sensitivities. Some trends on predictions can be extracted from benchmark models, as we have done for some of the mSUGRA models proposed in [129], and with sample models selected according to their relic abundance in a more general low energy scan (with the latter being more promising than the former). One should keep in mind however that firmer statements are possible just in light of dedicated and more extensive scans.

8 Conclusions

We have here described the computer package DarkSUSY, that can be used to calculate various quantities of interest for supersymmetric dark matter searches. We have gone through great efforts and used state-of-the-art techniques to obtain a package that can deliver very accurate results in a flexible setting. We also believe that this package can be of great use for the physics community.

In this paper we have described briefly the ingredients of DarkSUSY and shown, with some examples, what one can calculate with it. We encourage the reader to download DarkSUSY and start using it.

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A Included coannihilations

In this Appendix, we tabulate the coannihilation processes computed in DarkSUSY. For more details, see [43].

In the table, we list all $2 \rightarrow 2$ tree-level coannihilation processes with sfermions, charginos and neutralinos. All the processes are included in the DarkSUSY code.

It should be noted that we have not included all flavour-changing charged current diagrams. The DarkSUSY vertex code for the charged current couplings is written in a general form that includes all possible flavour-changing (and flavour-conserving) vertices. The flavour-conserving couplings are much larger than the flavour-changing. For the sfermion coannihilations with charged currents we only take the flavour-conserving contributions, while for the chargino coannihilations we include the flavour-changing contributions as well. In a future version of DarkSUSY, we may as well include the flavour-changing processes for the sfermion coannihilations, even if they are not expected to be important.

We have used the notation $\tilde{f}$ for sfermions and $f$ for fermions. Whenever the isospin of the sfermion/fermion is important, it is indicated by an index $u$ ($T_3 = 1/2$) or $d$ ($T_3 = -1/2$). The sfermions have an additional mass eigenstate index, that can take the values 1 and 2 (except for the sneutrinos which only have one mass eigenstate). A further complication to the notation is when the sfermions and fermions in initial, final and exchange state can belong to different families. Primes will be used to indicate when we have this freedom to choose the flavour. So, e.g. $\tilde{f}_u$ and $f_u$ will belong to the same family while $\tilde{f}_u$ and $f'_u$ can belong to the same or to different families. Note that the colour index of (s)quarks as well as gluons ($g$) and gluinos ($\tilde{g}$) is suppressed.
Besides the sfermions we also have neutralinos and charginos in the initial states. The notation used for these are the following. The neutralinos are denoted by $\chi^0_j$ with the index running from 1 to 4. The charginos are similarly denoted $\chi^\pm_j$ with the index taking the values 1 and 2.

In the table, a common notation is introduced for gauge and Higgs bosons in the final state. We denote these with $B$ with an upper index indicating the electric charge. So $B^0$ means $H^0_1, H^0_2, H^0_3, Z, \gamma$ and $g$ while $B^\pm$ is $H^\pm$ and $W^\pm$. We will use additional lower indices $m$ and $n$ when we have more than one boson in the final state. Thus indicating that the bosons can be either different or identical. Note that the case of two different bosons also includes final states with one gauge boson and one Higgs boson.

The table has been made very general. This means that when a set of initial and final state (s)particles has been specified, the given process might not run through all the exchange channels listed for the generic process. Exceptions occur whenever an exchange (s)particle does not couple to the specific choice of initial and/or final state. As an example we see that since the photon does not couple to neutral (s)particles, none of the exchange channels listed for the generic process $\tilde{f}_i + \chi^0_j \rightarrow B^0 + f$ actually exist for the specific process $\tilde{\nu} + \chi^0 \rightarrow \gamma + \nu$. All these exceptions can be found in the extended tables in Ref. [134]. Also note that the list of processes is not complete with respect to trivial charge conjugation. For each process of non-vanishing total electric charge in the initial state there exists another process which is obtained by charge conjugation.
Table 3: Included coannihilation processes through $s-$, $t-$, $u-$channels and four-point interactions (p). For the $\tilde{f}_d, \tilde{f}_d^{(*)}$ processes the corresponding process for up-type sfermions can be obtained by interchanging the $u$ and $d$ indices.
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