Casimir Energy Density at Planck Time: Cosmic Coincidence or Double Solution to the Cosmological Dark Energy Problem?

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Abstract

The Casimir energy density calculated for a spherical shell of radius equal to the size of the Universe projected back to the Planck time is almost equal to the present day critical density. Is it just a coincidence, or is it a solution to the ‘cosmic dark energy’ and the ‘cosmic coincidence’ problems? The correspondence is too close to be ignored as a coincidence, especially since this solution fits the conceptual and numerical ideas about the dark energy, and also answers why this energy is starting to dominate at the present era in the evolution of the Universe.

It is startling to notice that the Casimir energy density of a spherical bounded space with its radius equal to the size of our present Universe scaled back to its size at the Planck time is almost exactly the critical energy density. It is perhaps not reasonable to discard this as a coincidence, since it solves the two important current problems in cosmology with vacuum energy [1], namely the problem of the smallness of the cosmological vacuum energy

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density and the problem of cosmic coincidence of the near equality of the vacuum energy density and the present matter density.

There have been several calculations of the Casimir energy of a conducting spherical shell bounding three dimensional space \[2, 3\]. The result for the electromagnetic vacuum inside a shell of radius \(R\) is

\[
\rho_C = \frac{0.046\hbar c}{(4\pi/3)R^4}
\]  

(1)

For calculations with scalar field one gets a similar expression with a numerical coefficient different by a factor of order unity (A factor of 2 comes from polarization degrees of freedom) \[3\]. An estimate of such a vacuum energy density for the present Universe with the Hubble radius of the order of \(10^{28} \text{ cm}\) gives a value, \(\rho_H\), negligible by a factor \(10^{123}\) compared to the present day critical density of approximately \(\rho_0 = 1.9 \times 10^{-29} \text{ g/cm}^3\). On the other hand, an estimate using Eq. 1 for a Planck size Universe gives a value of the order of the Planck energy density, \(\rho_P\), \(3 \times 10^{92} \text{ g/cm}^3\) – enormous compared to \(\rho_0\). The fact that the number we need to fit the present observations, the critical density itself, is the geometrical mean of \(\rho_H\) and \(\rho_P\) might be a genuine clue, or might just be a coincidence.

However, the Planck length and the Hubble length are not the physically relevant scales for an estimate of the vacuum energy in the Universe. It is well known that a Universe with Planck size at Planck time could not have evolved into the present Universe without large inflationary factors \[4\]. An important and physically relevant scale for the quantum cosmology of an expanding Universe would be its “total size” at Planck time. This is much larger than the Hubble scale at Planck time, since the horizon scale and Hubble scale grows linearly with time whereas the typical scale factor grows as only \(t^{1/2}\) and \(t^{2/3}\) during the radiation dominated and matter dominated eras respectively. Therefore, the horizon scale and the size of the ‘visible Universe’ (part of the Universe that can be in causal contact at some time) were much smaller than the total extent of the Universe at Planck time.

We do not know the total extent of the Universe. It could be infinite, but it could be just very large and finite. All we can say with definiteness now is that it is certainly larger than or equal to about \(10^{28} \text{ cm}\). Extrapolating backwards in evolution to the Planck time from the present Hubble scale of the order of \(10^{28} \text{ cm}\) gives, in the standard big bang picture with a critical evolution, the ‘diameter’ of our presently observable Universe projected to
Planck time \([5]\) as

\[
D_{Pl} \simeq 1.4h^{-1}10^{-3} \, \text{cm}
\]  

With the observed value of the Hubble parameter, \(h \simeq 0.7\), we get \(D_{Pl} \simeq 20\) microns. Therefore the size of the entire Universe at Planck time was larger than 20 microns, and could be a few times larger if the Universe is finite, say \(20 - 100\ \mu m\).

Now I estimate the radius of the bounded space that will generate a Casimir energy density that is equal to 2/3 the critical energy density at present, \((2/3)\rho_0 = 1.3 \times 10^{-29}\ \text{g/cm}^3\), which is the estimated amount of dark energy in the Universe at present. From eq. 1 for the Casimir energy density (and dividing by \(c^2\) to get the mass density),

\[
D = 2R = 2 \left( \frac{0.046h}{(4\pi/3)(2/3)\rho_0 c} \right)^{1/4} \simeq 55\mu m
\]

With a scalar field this number is about 46 \(\mu m\). The Casimir energy of the electromagnetic field is not the relevant energy here, since it does not have the required equation of state \(p = -\rho\). With the scalar field, there is the possibility to get the Casimir energy from its quantum fluctuations with the required equation of state. Therefore, the size of the Universe that contains a Casimir energy density equal to the critical density is in the same range as the present Hubble size of the Universe extrapolated back to the Planck time. This is surprisingly good agreement. If this number was less than 10 \(\mu m\) the hypothesis that the present dark energy could be the Casimir energy generated at Planck time would have been ruled out immediately since the present Hubble size is known well within a factor of 2.

It is possible that this is a mere coincidence (Then, contrary to Einstein’s assertion, G is malicious!). But, here the correspondence is too close to be ignored as a mere coincidence, especially since there is no accepted solution to the various questions raised by the possible presence of a small vacuum energy density comparable to the critical energy density in the Universe. In fact, a more precise estimate of the scalar field Casimir energy density for the Universe, with its size at Planck time taken as the present horizon scale projected to the Planck time, might show that \(\rho_C \approx \rho_0\) even more closely than we have estimated. There is a calculation of the scalar field vacuum energy density for the manifold \(M^4 \times S^3\), product of the 4-d Minkowski space-time and a compact 3-d space, giving a similar estimate, with the energy density becoming comparable to the critical energy density for \(D \simeq 20\) microns \([3]\).
In our scenario, a size of the order of 20 microns is not the size of the compact extra dimension, but it is the size of the entire bounded Universe at Planck time.

It may be noted that the estimated size of the Universe at Planck time is the geometric mean of the Planck length and the present Hubble length. This is a coincidence without any physical significance, since this relation will change as a function of time. Thus we are able to explain why the present critical density is approximately the geometric mean of the Planck energy density and the vacuum energy density calculated using Hubble length as the relevant scale. While this analysis suggests that there is no special physical significance to this fact, the considerations in ref.\[6\] clarify the possible relations between these length scales.

The scenario I described naturally answers the important unresolved question why the dark energy density has started to dominate the matter energy density only recently in the evolution of the Universe. This is the Cosmic Coincidence problem: why is the vacuum energy density comparable to the present day critical density? Due to the special equation of state, \( p = -\rho \), the vacuum energy density remains constant as the Universe expands. During the early evolution of the Universe an energy density of \( 1 \times 10^{-29} \text{ g/cm}^3 \) in the Casimir form is totally insignificant compared to the energy density in radiation or in matter. As the Universe expanded the energy density in radiation and matter dropped as \( R^4 \) and \( R^3 \) respectively. The scalar vacuum energy density remained constant and insignificant till recently when the matter energy density has dropped to about \( 10^{-29} \text{ g/cm}^3 \), and the vacuum energy density has just exceeded the matter density. The magnitude of the Casimir energy at Planck time for a Universe of size of approximately 40-50 microns is such that it will dominate matter density after about 14 billion years, if the evolution of the Universe is as in the standard big bang picture.

Thus we have the double solution cosmologists are seeking – explanation of the present value of the vacuum energy density and an explanation for the question why the vacuum energy density has started dominating now.

This scenario addresses the cosmological constant problem directly; the question why we do not see any effect of the infinite zero point energy of the quantum vacuum. The cosmological constant is small because quantum vacuum has no energy density that can act as a source of gravity, and only the Casimir type vacuum energy density arising from bounding the vacuum in finite sized boundaries has any physical relevance. Free quantum vacuum is truly empty. This is of course restating Schwinger’s view on the vacuum
energy density, and I think that it is an important stand that can solve many problems. At present, such an idea is consistent with all experimental and theoretical facts.

In conclusion, I wish to stress a viewpoint that we are probably faced with several observational evidences that are indicative of a quantum origin of our Universe—evidence we do not take seriously today, but might be compelled to accept tomorrow. Earlier we have pointed out that some of the standard “classical” observations are perhaps evidence for quantum cosmology [6]. In this paper I have pointed out a possible solution to the cosmological dark energy problem in terms of the Casimir energy of the entire bounded Universe at Planck time. The numerical estimates match the present observational parameters well, and it also answers the query why the dark energy has started dominating relatively recently.

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References


