Quantum effects and superquintessence
in the new age of precision cosmology

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Abstract

Recent observations of Type Ia supernova at high redshifts establish that the dark energy component of the universe has (a probably constant) ratio between pressure and energy density $w = p/\rho = -1.02^{+0.13}_{-0.19}$. The conventional quintessence models for dark energy are restricted to the range $-1 \leq w < 0$, with the cosmological constant corresponding to $w = -1$. Conformally coupled quintessence models are the simplest ones compatible with the marginally allowed superaccelerated regime ($w < -1$). However, they are known to be plagued with anisotropic singularities.

We argue here that the extension of the classical approach to the semi-classical one, with the inclusion of quantum counterterms necessary to ensure the renormalization, can eliminate the anisotropic singularities preserving the isotropic behavior of conformally coupled superquintessence models. Hence, besides of having other interesting properties, they are consistent candidates to describe the superaccelerated phases of the universe compatible with the present experimental data.
We are entering a new era in cosmology. New generations of experiments are providing us with wealthy sets of experimental data, with a precision simply unbelievable in a very recent past, no more than 20 years ago. New observations of Type Ia Supernova, done from the Hubble Space Telescope (see, for instance, [1]) and from earth telescopes [2] are improving considerably the experimental evidences that our universe is undergoing a phase of accelerated expansion, as first noticed in [3]. Einstein’s equations require a negative pressure cosmological fluid acting as source to an accelerated expansion phase of the universe. Such repulsive fluid must be nonluminous, otherwise it would be detected. This is the “dark” energy component that dominates the evolution of the universe today. Moreover, it corresponds to 70% of the composition of the universe. Steinhardt [4] considers the discovery of dark energy as one of the most surprising and profound of the history of science, and according to him, “we are probably the last generation to have been taught that gravity always attracts”.

The observations of the fluctuations in the cosmic microwave background (CMB), mainly the spectacular results of WMAP [5], have enforced the dark energy hypothesis. Despite the severe degeneracy problems in the CMB parameters analysis, the existence of a dark energy component represented by a small cosmological constant \( \Lambda \) is so favored by the available data that the standard cosmological model is now called the \( \Lambda \) CDM\(^1\) model.

An important feature of the dark energy fluid is the ratio \( w \) between its pressure and energy density. For the cosmological constant, \( w = -1 \). Dark energy can also be described by a field [6], the so-called quintessence: a dynamical field with negative pressure, not necessarily homogeneously distributed, in contrast to the cosmological constant. Typically, quintessence models are constructed from the Einstein-Hilbert action with minimally coupled scalar fields:

\[ 1 \]

\(^1\)CDM stands to cold dark matter, a nonluminous gravitationally attractive form of matter, responsible by approximately 25% of the energy content of the universe.
\[
S = \int d^4x \sqrt{-g} \{ R - \partial_a \phi \partial^a \phi - 2V(\phi) \}. \tag{1}
\]

For such models, the ratio between the pressure and the energy density of a homogeneous scalar field \(\phi\)
\[
w = \frac{p}{\rho} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)} \tag{2}
\]
is bounded from below, \(w \geq -1\). According to the potential \(V(\phi)\), \(w\) can vary between -1 and 0. Typically, \(w\) depends on time in quintessential models.

The recent results of [1], obtained from Type Ia supernova at \(z > 1\), impose strong constraints on rapidly evolving quintessential models, favoring scenarios with constant \(w\).

According to the experimental data, there are some marginal evidences suggesting that \(w < -1\). Indeed, according to [1], \(w = -1.02 \left( ^{+0.13}_{-0.19} \right)\). If one really has \(w < -1\), both cosmological constant and minimally coupled scalar quintessence descriptions for dark energy are ruled out. A regime with \(w < -1\) is called superaccelerated, and a model achieving it, superquintessence [7]. The simplest non-exotic model exhibiting superaccelerated expansion is that of a scalar field conformally coupled with quartic potential [8], corresponding to the action
\[
S = \int d^4x \sqrt{-g} \{ F(\phi) R - \partial_a \phi \partial^a \phi - 2V(\phi) \}, \tag{3}
\]
with \(F(\phi) = 1 - \frac{1}{6} \phi^2\) and \(V(\phi) = \frac{m^2}{2} \phi^2 - \frac{\Omega}{4} \phi^4\). For the homogeneous and isotropic case, all the solutions of (3) are regular ones, and some of them presents other interesting dynamical behaviors besides the superaccelerated regime, as the possible avoidance of big-bang singularities [8].

However, the hypersurface \(F(\phi) = 0\), the frontier between the regions where gravity is effectively attractive \((F > 0)\) and repulsive \((F < 0)\) is known to be problematic. Starobinski was the first to notice it in a particular anisotropic model [9]. The standard perturbation theory for helicity-2 and helicity-0 excitations, derived directly from the action (3) fails on \(F(\phi) = 0\) [10]. One can show, indeed, that the hypersurface \(F(\phi) = 0\) corresponds
generically to an unavoidable anisotropic singularity [11]. Any anisotropic solution crossing it will end in a spacetime singularity, no matter how small is the anisotropy. Such an unstable behavior is, of course, unacceptable for any cosmological model. Unfortunately, all the interesting solutions of the model presented in [8] cross the hypersurface $F(\phi) = 0$.

It is quite easy to understand the origin of the anisotropic singularities studied in [11] for the models (3) with general $F(\phi)$. As the kinetic term for the metric $g_{ab}$ is $F(\phi)R$, we have no guarantee that the Cauchy problem is well posed on the hypersurface $F(\phi) = 0$. Indeed, for the anisotropic case, it is not, meaning that one cannot assign any value of $\dot{\phi}$, $g_{ab}$ and $\dot{g}_{ab}$ when $F(\phi) = 0$. On the contrary, the Cauchy problem is well posed everywhere in the isotropic case. This is the reason why all the homogeneous solutions of (3) studied in [8] are regular.

The relevant question arising here is if one should give up of conformal coupling to construct superquintessence models due to their anisotropic instabilities or not. Conformally coupled superquintessence models are the simplest ones that we can construct with homogeneous and isotropic solutions compatible with superaccelerated phases of the universe. They also have other particular predictions that could be checked experimentally [7]. Our question is obviously related to another one: is it possible to modify the model (3) in order to avoid the anisotropic singularities and, at the same time, preserve its isotropic solutions? The answer is, fortunately, affirmative. Moreover, the desired modification is achieved by the inclusion of a new term coming from semiclassical analysis.\(^2\)

The idea of incorporating vacuum semiclassical effects into gravity has a long history, and a good set of references is presented in [12,13]. Zeldovich was the first to propose [14], in 1967, that a cosmological constant term could arise from quantum considerations of matter. Yet in the sixties, in a set of seminal works, Parker considered [15] the effect of the creation of particles in a expanding universe, and discussed the possible backreaction, opening the

\(^2\)By semiclassical here one means that $\phi$ is quantized on a classical gravitational background.
discussion of anisotropy damping and avoidance of the initial singularity due to quantum corrections [16]. A semiclassical treatment of the model described by (3) with $F(\phi) = 1 - \xi \phi^2$ and $V(\phi) = \frac{m^2}{2} \phi^2 - \frac{\Omega^4}{4} \phi^4$, requires the inclusion of higher orders counterterms to ensure the renormalization of the theory. These terms are [12,13]

$$S_{\text{vac}} = \int d^4 x \sqrt{-g} \left( \alpha_1 R^2 + \alpha_2 R_{ab} R^{ab} + \alpha_3 R_{abcd} R^{abcd} + \alpha_4 \Box R \right).$$  (4)

The quantum divergences of the semiclassical theory can be removed by the renormalization of the constants $\alpha_{1,2,3,4}$ and the Newtonian constant $G$. In fact, the full set of quantities affected by the renormalization includes the scalar field, its mass $m$ and selfcoupling constant $\Omega$, the non-minimal coupling constant $\xi$ and yet a cosmological constant. Note that the last counterterm in (4) does not contribute to the classical dynamics, since it is merely a total divergence.

The Weyl tensor $C_{abcd}$ vanishes identically for homogeneous and isotropic spacetimes, and hence, if included in the dynamics, it would affect only the anisotropic case by construction, preserving all isotropic solutions. In four dimensions, we have

$$C_{abcd} C^{abcd} = R_{abcd} R^{abcd} - 2 R_{ab} R^{ab} + \frac{1}{3} R^2.$$  (5)

Hence, it is possible, in principle, to combine $\alpha_1$, $\alpha_2$ and $\alpha_3$ in order to have the counterterm that preserves, by construction, the isotropic solutions. The resulting action is

$$S + S_{\text{vac}} = \int d^4 x \sqrt{-g} \left\{ \left( 1 - \frac{1}{6} \phi^2 \right) R + \alpha C_{abcd} C^{abcd} - \partial_a \phi \partial^a \phi - 2 V(\phi) \right\},$$  (6)

where $V(\phi) = \frac{m^2}{2} \phi^2 - \frac{\Omega^4}{4} \phi^4$ and $\alpha$ is a parameter typically small when compared to $1/G$. The action (6) has, by construction, the same homogeneous isotropic solutions considered in [8], since the term $C_{abcd} C^{abcd}$ and its contributions to the dynamics vanish in this case. As to the anisotropic solutions, they are now free of singularities. The induced counterterm acts a high order kinetic term for the metric $g_{ab}$, and the Cauchy problem is well posed everywhere. We can show [17] that, provided that $\alpha$ is small, the new term will act only in the region close to the $F(\phi) = 0$ hypersurface. Far from there, the solutions do not differ much from the ones of (3).
A relevant issue is the naturalness of the necessary adjust in the constants $\alpha_1$, $\alpha_2$ and $\alpha_3$ in order to have the conformal counterterm. As this constants may arise from quantum corrections, the only reasonable hypothesis about them is that they must be small if compared with $1/G$. This point is now under investigation [17], and preliminary results show that, under the only hypothesis of small $\alpha_1$, $\alpha_2$ and $\alpha_3$, the singularities on $F(\phi) = 0$ can be eliminated preserving almost all of the isotropic behavior.

Concluding, in models like (3), the passage through the frontier between $F(\phi)$ positive and negative, which is essential to the novel interesting cosmological histories described in [8], is classically forbidden as it is unstable at $F = 0$ with respect to anisotropic classical fluctuations. Nevertheless, precisely these fluctuations, when considered semiclassically together their feedback response, render this passage physically meaningful, enriching dramatically the whole problem and helping us to elucidate the Physics behind the fantastic cosmological data available today.

ACKNOWLEDGMENTS

The authors acknowledge the financial support from the EEC (project HPHA-CT-2000-00015) and FAPESP (Brazil).
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