On the Stability of Test Particles in Extrasolar Multiple Planet Systems

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ABSTRACT

We present results of extensive numerical studies of the stability of non-interacting particles in the planetary systems of stars $\upsilon$ Andromedae, GJ 876, 47 UMa, and 55 Cancri. We discuss the possibility of the existence of islands of stability and/or instability at different regions in these multi-body systems, and their probable correspondence to certain mean-motion resonances. The results of our study may be applied to questions concerning the stability of terrestrial planets in these systems and also the trapping of particles in resonances with the planets.

Subject headings: celestial mechanics — planetary systems — stars: individual ($\upsilon$ Andromedae, GJ 876, 47 UMa, 55 Cancri)

1. Introduction

The number of extrasolar planetary systems discovered by the precision radial velocity technique has now risen above 100. Except for thirteen of these systems, in all others, the number of detected planets is only one. It has been noted by Fischer et al. (2001) that about 50% of these single-planet systems show trends in the residuals to their radial velocity fits. This strongly suggests that such systems may contain additional companions.

Observations of the present extrasolar multi-planet systems, on the other hand, have revealed some dynamically interesting features of these systems:

i) A few of these systems are in or near mean motion resonances (MMR). For instance, the two planets of the star GJ 876 are in a 2:1 MMR (Marcy et al. 2001; Laughlin & Chambers 2001; Rivera & Lissauer 2001), the 55 Cancri system ($\rho$ Cnc) has two planets near a 3:1 commensurability (Marcy et al. 2002; Zhou et al. 2004), and the two planets in 47 UMa may also be near or in a mean motion resonance (Fischer et al. 2002; Laughlin, Chambers, & Fischer 2002; Rivera & Haghighipour 2003).

ii) Some of these systems resemble our Solar System. As an example, $\rho$ Cnc has a Jovian-mass planet with an orbital semi-major axis analogous to that of Jupiter (Marcy et al. 2002), and the
two planets of 47 UMa were announced to have mass- and period-ratios resembling those of Jupiter and Saturn (Fischer et al. 2002; Rivera & Haghighipour 2003).

iii) It is also possible that these systems may harbor additional low-mass companions. Such bodies are more amenable to detection with other techniques and future space-based missions, such as Darwin, Corot, Kepler, SIM, and TPF. The precision radial velocity technique favors the detection of close-in giant planets and is currently unable to detect terrestrial planets and/or distant gas giants, although continued observations (i.e., extending the time baseline) can help with the latter insensitivity of this technique.

The increasing number of extrasolar multi-planet systems, their diversity, and dynamical complexities call for thorough investigation of their long-term stability, as well as the dynamical evolution of possible small bodies that may exist in these systems. In this paper, we address issues regarding the stability of non-interacting bodies in four extrasolar planetary systems υ Andromedae, GJ 876, 47 UMa, and 55 Cancri. Studies of this kind have been done for these and a few other extrasolar multi-planet systems. However, in this paper, we present results that have been obtained by integrating these systems using initial orbital parameters determined by fitting recent radial velocity data of these stars utilizing a newly developed fitting routine. We present the results of an extensive study of the stability of test particles in these systems, and investigate to what extent these systems can host additional companions. We also carry out studies to look for regions around these stars where low-mass companions, such as terrestrial planets, can have long-term stable orbits. Comparison of the results of our study with those currently available in the literature is also presented.

The outline of this paper is as follows. In §2 we discuss the methodology. The subsequent four sections present the results of our studies for the four systems υ Andromedae, GJ 876, 47 UMa, and 55 Cancri, respectively. In §7 we summarize our results and discuss their applications. It is necessary to mention that any orbital parameter in this study is an osculating element.

2. Methodology

We performed N-body simulations of the planetary systems of stars υ Andromedae, GJ 876, 47 UMa, and 55 Cancri. In order to generate initial conditions for integrating the orbits of the planets, we performed dynamical (also known as N-body or Newtonian) fits to the radial velocity data of the four above-mentioned stars using a modified version of the Levenberg-Marquardt minimization algorithm (Press et al. 1992; Laughlin & Chambers 2001; Rivera & Lissauer 2001).

The Levenberg-Marquardt algorithm is a minimization scheme that can be used to fit data with a model that depends nonlinearly on a set of parameters. We modified the code so that the radial velocity data for any star may be fit. Generally, it is the goodness of the fit, $\chi^2_v$, that is minimized. Close to a minimum, $\chi^2_v$ is assumed to be nearly quadratic in parameters, whereas far from it, the algorithm steps toward a minimum along the gradient of $\chi^2_v$ taken with respect to the
parameters, and smoothly varies between these two extremes (Press et al. 1992).

To fit radial velocity data, the Levenberg-Marquardt scheme requires an initial guess for the orbital parameters of each planet orbiting the star. For the N-body fits performed here, the parameters which may be fitted for each planet are the mass, $m_{pl}$, the semi-major axis, $a$, the eccentricity, $e$, the inclination, $i$, the longitude of the ascending node, $\Omega$, the argument of periastron, $\omega$, and the mean anomaly, $M$. In practice, however, we fit for $m_{pl}$, $a$, $h$, $k$, and $M$, where $h = e \sin \varpi$, $k = e \cos \varpi$, and $\varpi = \Omega + \omega$. We take the longitudes of the ascending nodes in all our fits to be $0^\circ$, and for most of our fits we assume a $90^\circ$ inclination with respect to the plane of the sky. That is, the planets are coplanar, and their masses have their minimum values. The best fitted orbital parameters for the planetary systems considered here are given in Tables 1 - 4.

Using the parameters determined from the N-body fitting routine, we numerically integrated the orbits of a few hundred test particles placed in each system. Such simulations enable us to identify regions where low-mass companions can have stable orbits. In order to identify such a region, one has to integrate the orbit of a low-mass object at different distances from the central star. We approximate a low-mass body with a test particle and perform such simulations by placing non-interacting particles at different distances. We then perform one simulation with hundreds of particles at the same time. Such test particle approximations are similar to those in Rivera & Lissauer (2000, 2001), Lissauer & Rivera (2001), and Menou & Tabachnik (2003).

We started all our test particles in the plane of the planets and on circular orbits with respect to the central star. The mean longitudes of all particles were taken to be $0^\circ$. Because different initial eccentricities for test particles have considerable effects on the zones of stability of the system, instead of looking at short term stability for various initial eccentricities, we chose to look at long-term stability for particles initially only on circular orbits. No test particle was placed on an orbit that would initially cross the orbit of a planet.

The simulations were performed with the second-order mixed variable symplectic (MVS) integrator (Wisdom & Holman 1991) in the MERCURY integration package (Chambers 1999). This code was modified to include the relativistic precession of the longitudes of periastron of all bodies in the system (Lissauer & Rivera 2001). In order to investigate if small mutual inclinations would have a significant effect on the dynamics of the extrasolar systems studied here, we also performed fits for fixed mutual inclinations of $1^\circ$ and $5^\circ$ between the planets.

3. \upsilon\ Andromedae

Upsilon Andromedae (HD 9826, HR 458, HIP 7513) is an F8 V star with a mass of 1.2 $M_\odot$ (Fischer 2003, private communication). This star is host to three Jovian-mass planets with periods of about 4.6 days, 242 days, and 3.5 years. Using our N-body fitting routine, we fitted the radial velocity data of 250 observations from JD 2447046.9223 (September 8, 1987) to JD 2452854.9930 (August 3, 2003). Table 1 shows a set of fitted astrocentric orbital elements at the epoch JD
2450000.0 for this system. The $\sqrt{\chi^2}$ and root mean square (RMS) of the differences between the observed and modeled radial velocities for this fit are 1.58 and 12.94 m s$^{-1}$, respectively. Using these orbital parameters, we integrated the orbits of 410 test particles with semi-major axes ranging from 0.06 AU to 10.00 AU. The particles were equally spaced every 0.02 AU except for the regions where the planets made their initial radial excursions. The timestep used in this simulation was 0.23 day.

Figure 1 shows the survival time of a test particle as a function of its initial semi-major axis. The circles represent the planets, and the error bars represent the planets’ radial excursions due to their orbital eccentricities. Note that since the eccentricity of the inner planet is nearly zero, its error bars are smaller than the symbol used for this planet. As shown in this figure, almost all test particles between the outer two planets are lost (i.e., ejected from the system, or collided with the star) within approximately 3000 years. There is a stable region just exterior to the orbit of the inner planet where particles survive for at least 10 Myr. This is in rough agreement with the results by Rivera & Lissauer (2000) and Lissauer & Rivera (2001).

Figure 1 also shows the locations of several MMRs with the outer planet. Except for some long-lived test particles near a few of these MMRs, there is a broad region of instability, outside the orbit of the outer planet. This region extends out to about 7.6 AU, near the location of the 11:2 MMR. As shown in Rivera & Lissauer (2000), the large size of this region is likely due to the presence of the two massive planets on moderately eccentric orbits. This is also in rough agreement with the secular analysis by Malhotra (2004, in preparation).

A detailed analysis of our simulations reveals that most of the stable test particles remain in nearly circular orbits for the duration of the simulation. Of course, there are a few test particles that maintained their orbits and also acquired some eccentricity. For instance, five of the six particles from 0.08 AU to 0.18 AU acquired a maximum eccentricity less than 0.0509 over 10 Myr. The one exception was the particle at 0.12 AU, which became unstable during this time.

Our simulations also show that for stable test particles between 0.20 AU to 0.40 AU, the amplitudes of oscillations in both the semi-major axis and eccentricity rise, and variations in the mean value of the semi-major axis start to appear. This implies that the test particles closest to the middle planet will not survive at longer times.

For distant test particles, one expects small variations to appear in both semi-major axis and eccentricity. One has to note that in the system of $\upsilon$ Andromedae, the planets exert a significant perturbation on the star. From the test particles’ point of view, the star appears to rapidly revolve around the system’s barycenter so that the particles’ astrocentric velocities have relatively large variations. As a result, the astrocentric semi-major axes of long-lived and stable test particles beyond the orbit of the outermost companion librate with amplitudes of order 0.2 AU up to 0.5 AU. We refer the reader to Rivera & Lissauer (2000) and Lissauer & Rivera (2001) for a discussion of a similar effect on the semi-major axes of the planets. In general, these amplitudes increase with increasing distance from the star. For all the test particles whose orbital evolution was examined
here, the libration amplitudes of the barycentric semi-major axes were not more than about half that of the corresponding astrocentric quantities. In a few cases, the amplitudes were reduced by 1-2 orders of magnitude.

The long-lived test particles near MMRs with the outer planet warrant further investigation. Just outside the 1:3 MMR, stable particles experience large oscillations in their orbital eccentricities, reaching up into the range of 0.2 to 0.3. For the duration of the integration, these particles are prevented from close approaches with the outer planet by the \( e-\omega \) mechanism (Milani & Nobili 1984; Gladman 1993; Lissauer & Rivera 2001). In this mechanism, a test particle acquires a low eccentricity when its orbit is anti-aligned with that of the planet.

An analysis of the critical arguments of test particles near the 1:4 MMR suggests that this resonance has a significant effect on the stability of these particles. The eccentricities of these objects either undergo gradual growth, or experience jumps to higher values. For the longest lived test particle (7.25 Myr), such jumps seem to correlate with the libration or circulation of one of its critical arguments, \( \varphi = \lambda_d - 4\lambda' + \varpi_d + 2\varpi' \). In this expression \( \lambda_d \) and \( \varpi_d \) represent the mean longitude and the longitude of periastron of the outer planet, and \( \lambda' \) and \( \varpi' \) correspond to the same quantities for the test particle. Larger eccentricities tend to occur when this critical argument is librating near 0\(^\circ\). Figure 2 shows the variations of the eccentricity and \( \varphi \) for this particle.

The 1:5 MMR in Figure 1 has a considerable effect on the amplitudes of libration of the semi-major axes and eccentricities of test particles in its vicinity. Among the seven stable particles in this region, the two test particles that are in this resonance show the smallest variations in both their semi-major axes and eccentricities. The eccentricities of the particles in this resonance generally remain below 0.06 with very regular oscillations, while those outside this resonance have typical eccentricities of 0.2 to 0.3 with irregular oscillations. Figure 3 shows the eccentricity versus semi-major axis and the only librating critical argument for one of the two particles in this resonance.

Although most of the remaining five particles near the 1:5 MMR have at least one critical argument in non-uniform circulation, the variations in their semi-major axes, and to a lesser extent, their eccentricities, increase with distance from the location of this resonance. These particles also spend a significant amount of time in orbits that are anti-aligned with the orbit of the outer planet. An extension of the simulation to 100 Myr shows that the outermost particles, one interior to and two exterior to the 1:5 MMR, become unstable after 10 Myr.

4. GJ 876

GJ 876 (HIP 113020), an M4 dwarf with a V magnitude of 10.1 (Perryman et al. 1997), has a mass of 0.32 \( M_\odot \) (Marcy et al. 1998). Among currently known planet-hosting stars, GJ 876 is the one with the lowest stellar mass. This star has two Jovian-mass companions with periods of about 30 and 61 days, near a 2:1 MMR. The radial velocity data for GJ 876 was originally fitted with a Levenberg-Marquardt algorithm in which the planets were assumed to be on unperturbed
Keplerian orbits (Marcy et al. 2001). That is, the planet-planet perturbations were not modeled. However, the relatively small stellar mass, the large planetary masses, and their close spacing cause significant perturbations between the planets of this system. That makes the N-body fitting routine, as explained in §2, a more appropriate scheme for fitting the radial velocity data of this star.

To obtain the orbital parameters of the two planets, we carried out three independent fits, corresponding to a total of 106 observations from JD 2450602.0931 (June 2, 1997) to JD 2452851.0567 (July 30, 2003), to the radial velocity data of GJ 876. In the first fit, we assumed that the system was edge-on, as viewed from Earth. For the second fit, we adopted an inclination of 84° with respect to the plane of the sky for both planetary orbits. This is equal to the astrometrically determined inclination of the outer companion as reported by Benedict et al. (2002). In the third fit, we varied the orbital inclinations of both planets by the same amount, keeping their orbits coplanar, and only fitted for the remaining 10 parameters that were listed in §2. We then performed the fitting procedure for various values of this inclination until a minimum in $\sqrt{\chi^2}$ was obtained (see Rivera & Lissauer 2001, for more details on this procedure). Figure 4 shows $\sqrt{\chi^2}$ versus $i$ for this fitting. As shown here, a minimum is reached where $i \sim 39.6°$. It is important to mention that $\sqrt{\chi^2}$ is not sensitive to values of $i > 35°$. Thus, other values of the inclination larger than 35° are nearly as good as the best one.

Table 2 shows the fitted astrocentric orbital parameters at epoch JD 2451310.0 for the three fits mentioned above. As shown in this table, the $\sqrt{\chi^2}$ is the same for all three fits implying that these fits are statistically identical. We performed numerical simulations of the orbital stability of test particles for each fit. The timesteps for all three simulations were 1.0 day. For the first fit, we considered 318 test particles placed within the range of 0.1 60 AU to 0.800 AU at equal intervals of 0.002 AU. As discussed in §2, no test particle was placed on an orbit that would initially cross the orbits of the planets. Since the semi-major axes and eccentricities of the planets, obtained from the second fit, are very similar to those of the first fit, we considered an identical set of test particles for the second simulation. Figure 5 shows the results of these simulations. Since the initial conditions for these two simulations are nearly identical, the results are qualitatively the same. These results are also similar to those of Rivera & Lissauer (2001), in which the authors considered a fit with $i = 37.0°$.

As shown in Figure 5, in both simulations, the region between the two planets is cleared out in less than 530 years. This figure also shows that, out to about 0.294 AU from the central star, no test particle survives for more than 530 years. Unlike Rivera & Lissauer (2001), each of these two simulations contains a region extending from 0.294 AU to 0.310 AU, in the vicinity of the 4:7 MMR with the outer planet, where 5 or 6 test particles survive beyond $10^4$ years. All but two of these particles are lost within 250000 years from the start of the simulations. Of these two long-lived particles, one survives for 6.3 Myr.

Figure 5 also shows a region of instability extending from 0.312 AU to 0.336 AU, near the 1:2
MMR with the outer planet. All particles in this region are lost within $10^4$ years. Beyond this region, from 0.336 AU to 0.428 AU, except for one particle at 0.338 AU, which survives for only 2.85 Myr, all other particles are stable for the entire duration of the simulations. The last set of unstable particles is between 0.428 AU and 0.438 AU, around the 1:3 MMR with the outer planet. In this region, for each simulation, all but one test particle at 0.436 AU, are lost in about 1 Myr. Continuations of these simulations beyond 10 Myr show that only two test particles are lost in each simulation; the stable one in the 1:3 MMR region, and one at 0.382 AU, near the location of the 2:5 commensurability with the outer planet.

In these simulations, the region near the 1:3 MMR with the outer planet is probably the most dynamically interesting. The eccentricities of unstable particles in this region undergo sudden jumps. Inspection of the three critical arguments for the 1:3 MMR for all these unstable particles suggests that at least one of their critical arguments repeatedly changes from circulation to large amplitude ($\sim 180^\circ$) libration. This is a characteristic of a particle that repeatedly goes in and out of resonance. Figure 6 shows the time evolution of the eccentricity and the critical argument $\varphi = \lambda_b - 3\lambda' + 2\varpi_b$ of one of these particles. In this formula, $\lambda_b$, and $\varpi_b$ represent the mean longitude and the longitude of periastron of the outer planet, and $\lambda'$ corresponds to the mean longitude of the test particle. The plot of the critical argument $\varphi$ shows that the range $0^\circ$ to $360^\circ$ is not uniformly filled at all times. This indicates that the critical argument is not executing pure circulation.

The most stable test particles in the region near the 1:3 MMR with the outer planet have small eccentricities. The amplitudes of librations of the semi-major axes and eccentricities of these particles are relatively small, and all their three critical arguments circulate. Figures 7 and 8 show the graphs of the eccentricity versus semi-major axis, and the time evolution of all three critical arguments of one such particle, respectively. In both simulations, the continuations of the integrations to longer times indicate that the eccentricity of the single stable particle in that region undergoes a sudden jump, and one of its critical arguments starts librating. Subsequently, this particle is lost.

In the simulations above, one stable particle at the outer edge of the region near the 1:3 MMR with the outer planet shows a different evolution in one of its critical arguments, $\varphi = \lambda_b - 3\lambda' + 2\varpi_b$. While the eccentricity of this particle remains low, its critical argument actually librates about $180^\circ$ with an amplitude of $\sim 100^\circ$. Figure 9 shows the eccentricity of this particle versus its semi-major axis, and its librating critical argument for the simulation with $i = 90^\circ$. This evolution of the critical argument was also observed in the continuations of both simulations to longer times.

Figure 10 shows the survival times of test particles in the simulation based on the third set of fitted planetary parameters (i.e., $i \sim 39.6^\circ$). Since the orbital eccentricities of the planets obtained from the third fit were somewhat larger than in the first two, the set of test particles for the third simulation was slightly different. A total of nine test particles from the first two sets were excluded due to their crossing orbits with the planets. The rather larger masses and eccentricities obtained
from the third fit also have a significant effect on the long-term stability of particles in and near several MMRs with the outer planet. In particular, compared with the previous two fits, the region between the 1:2 and 1:3 MMRs and also the region around the 1:3 MMR are far less stable.

Figure 10 also shows that in this simulation, there is still one test particle in the instability region around the 1:3 MMR which behaves like most stable test particles in the other two simulations. There is also a test particle out as far as the location of the 1:4 MMR with the outer planet that was lost in less than 1 Myr. This result is in agreement with the general finding of Lepage & Duncan (2004) who indicated that the strength of the MMRs increases with the planetary masses and eccentricities (see also Murray & Dermott 1999; Haghighipour 2002). An extension of this simulation to larger times indicates that seven more test particles are lost after 10 Myr. All but one of these particles are at the edges of the three islands of stability closest to the outer planet.

5. 47 UMa

The star 47 UMa (HD 95128, HR 4277, HIP 53721) has a spectral type of G0 V. The mass of this star is 1.03 $M_\odot$ and it is host to two planets. Initial fits to the radial velocity data of this star by Fischer et al. (2002) indicated minimum masses of 2.54 $M_{\text{Jup}}$ and 0.76 $M_{\text{Jup}}$, and periods of 1089 days and 2594 days, for the two planets of this system, respectively. It was also shown by Fischer et al. (2002) that there are large uncertainties in the orbital parameters of the outer planet, particularly, in its eccentricity. This can be attributed to the short baseline of observations (13 years at that time) compared to the orbital period of the outer planet. Because of this short baseline, the N-body fitting routine discussed in §2 is incapable of fitting the parameters for this system properly\(^1\). Several sets of parameters produce equally good fits. To overcome this difficulty, we iteratively fit for the parameters of one planet while keeping the parameters of the other planet constant. We utilized this iterative fitting procedure for 128 observations from JD 2446959.7372 (June 13, 1987) to JD 2452834.6980 (July 14, 2003), which includes thirty-seven new observations in addition to those of Fischer et al. (2002). Our results indicate that the most stable system with the lowest $\sqrt{\chi^2}$, is obtained by holding the parameters of the inner planet constant and fitting for those of the outer planet. Table 3 shows the fitted astrocentric orbital parameters of this system at epoch JD 2449900.0.

We numerically integrated the orbits of 469 test particles in the 47 UMa planetary system. The timestep for this simulation was 10.0 days, and particles were placed at equal intervals of 0.02 AU, with semi-major axes ranging from 0.44 AU to 10.00 AU. Figure 11 shows the survival times of these test particles as a function of their initial semi-major axes. The resonances marked inside the orbit of the inner planet are internal MMRs with the inner planet, and those outside the orbit

\(^1\)See Rivera & Lissauer 2001 for a presentation of some problems with N-body fitting routines and how one can try to address these problems.
of the outer planet are external MMRs with the outer planet. As shown here, the region from the 5:3 MMR with the inner planet (∼1.48 AU) to the 2:3 MMR with the outer planet (∼5.85 AU) is cleared out in approximately $10^5$ years. An exception to this is observed for several test particles that are temporarily trapped in a 1:1 MMR with the outer planet. This is not unexpected since the orbit of the outer planet is nearly circular (Danby 1992).

Figure 11 also shows that near the locations of several other MMRs, test particles can survive for extended times. Our results indicate that 219 particles are lost by 10 Myr from the beginning of the simulation. An extension of the integration to 100 Myr shows that a significant number of the remaining particles, including those around resonances shown in Figure 11, are lost in less than 50 Myr. Among the particles that are lost after 10 Myr are the ones in the vicinity of the 1:3 MMR with the outer planet. Similar to the unstable test particles near the 1:3 MMR with the outer planet in the GJ 876 system, the eccentricities of these particles undergo sudden jumps indicating transitions from circulation to large amplitude libration for at least one of their critical arguments. Also, only two new instability regions appear around 7.94 AU and 8.67 AU as three test particles are lost from these areas in 43 Myr.

An interesting outcome of the extended simulation is the stability of test particles in the habitable region of 47 UMa. Similar to the results reported by Noble, Musielak & Cuntz (2002), our simulation shows that test particles are stable in this region for 100 Myr. Also, as opposed to Rivera & Haghighipour (2003), who have shown that a test particle at 1 AU, near the 3:1 MMR with the inner planet, could not be in a stable orbit beyond $3 \times 10^5$ years, our simulation indicates that such a particle would be stable for at least 100 Myr. The differences arise from the different initial conditions used in this study. Figure 12 shows the three critical arguments of the test particle at 1 AU, near the 3:1 MMR, over 100 Myr. Since all three critical arguments of this particle circulate (although not uniformly), this particle appears to be stable for this length of time. It is, however, necessary to note that a small change in the position of the inner planet or the test particle could change this situation.

Figure 13 shows the graphs of eccentricities versus semi-major axes for particles in the region of 0.9 AU to 1.5 AU in the 47 Uma planetary system. As shown here, this region is populated by resonances. Most of these resonances are located beyond 1 AU. This dynamically complex region has also been discussed by Laughlin, Chambers, & Fischer (2002), Goździewski (2002), and Asghari et al. (2004), who studied both mean motion and secular resonances. However, significant differences exist between our results and the results of the above-mentioned studies. These differences are most likely due to the different planetary orbital parameters used here. Since the semi-major axis of the outer planet used in this study is very different from that used by these authors, the positions of secular resonances in our simulation are entirely different. This has a significant effect on test particle stability (Lepage & Duncan 2004).

The most noticeable feature of test particles in this region is that most of those with semi-major axes below 1 AU attain maximum eccentricities no more than 0.05 over 100 Myr. Figure
13 shows the eccentricities of these particles versus their semi-major axes. For a comparison, the widths of the 2:1 and 3:1 MMRs with the inner planet have also been shown (Murray & Dermott 1999; Haghighipour 2002). The small width of the 3:1 resonance for small eccentricities strongly indicates that the extreme sensitivity to initial conditions probably contributes to the differences seen in the behavior of test particles near 1 AU in the 47 UMa system (Laughlin, Chambers, & Fischer 2002; Rivera & Haghighipour 2003). This suggests that for a slightly different initial semi-major axis within the width of the resonance, a test particle might be unstable, which would more closely agree with Laughlin, Chambers, & Fischer (2002) and Goździewski (2002). Another interesting feature of particles in the region from 0.90 AU to 1.50 AU is that 28 out of 31 test particles in this region spend at least 60% of their lifetimes in orbits which are within 90° of being aligned with the orbit of the inner planet.

Figure 14 shows a test particle at 1.32 AU. This particle spends most of its time in the 2:1 MMR with the inner planet. This configuration raises the eccentricity of this particle, which generally results in orbital instability. However, this particle survives for 100 Myr. Figure 14 also shows the eccentricity of this particle versus its critical argument, \( \varphi = \lambda' - 2\lambda_b + \varpi' \). In this formula, \( \lambda_b \) is the mean longitude of the inner planet, and \( \lambda' \) and \( \varpi' \) represent the mean longitude, and the longitude of periastron of the test particle. As shown here, for most of the simulation, this critical argument librates. However, since \( \varphi \) circulates non-uniformly for several nearby stable test particles at smaller semi-major axes, this 2:1 MMR appears to have considerable influence on the orbital evolution of such particles. The critical arguments of these objects spend more time near 0° than at 180°.

6. 55 Cancri

The main sequence star 55 Cancri (HD 75732, HR 3522, HIP 43587) has spectral type G8 with a mass of 0.95 \( M_\odot \). This star is host to three planets with minimum masses of about 0.8 \( M_{\text{Jup}} \), 0.2 \( M_{\text{Jup}} \), and 4 \( M_{\text{Jup}} \), and periods of about 14.65 days, 44 days, and 5360 days, respectively. We performed N-body fits to the radial velocity data of 170 observations of this star carried out from JD 2447578.7300 (February 21, 1989) to JD 2452737.7040 (April 8, 2003). Table 4 shows the astrocentric orbital parameters of the planets at epoch JD 2450165.0. The values of \( \sqrt{\chi^2} \) and the RMS for this fit are 2.34 and 8.79 m s\(^{-1}\), respectively. Using these parameters, we numerically integrated the orbits of 455 test particles placed in this system from 0.14 AU to 12.00 AU, at equal intervals of 0.02 AU. The timestep for this simulation was 0.5 day. Figure 15 shows the survival times of these test particles as a function of their initial semi-major axes. The MMRs marked in this figure are the internal and external MMRs with the outer planet for the ones inside and outside its orbit, respectively. The results here resemble those in Rivera & Haghighipour (2003) except that because the orbital parameters for the outer planet are quite different, a different set of MMRs affect the stability of test particles in the regions of 2 AU to 3 AU, and 10 AU to 12 AU.

As shown in Figure 15, there is a small region, extending from 0.30 AU to 0.60 AU, just outside
the orbit of the middle planet, where most test particles are unstable. The survival time of test particles in this region generally rises with increasing distance from the planet. The last unstable test particle in this region is near the outer 1:4 MMR with the middle planet. The simulation also indicates a broad region of stability from 0.62 AU to 2.36 AU. Such a stable region has also been reported by Marcy et al. (2002) who also showed that a terrestrial-mass planet at a distance of 1 AU from the central star would be stable. Barnes & Raymond (2004) have also found a similar broad stable region in this system. These authors find that the stability of particles is not affected significantly by starting them on orbits with eccentricities up to 0.25.

Mean motion resonances affect the stability of test particles depending on their order and also the locations of the particles. In this simulation, the 7:2 MMR affects the stability of the test particle at 2.38 AU by pumping up its eccentricity until it is lost in 47000 years. Gradual drifts were also observed in the semi-major axes of the two nearest neighbors of this particle. Such drifts may cause these particles to become unstable on timescales beyond 100 Myr. The 3:1 and 8:3 MMRs both destabilize a few test particles in their vicinity. The test particles in this small region that survive at least 10 Myr mark the boundary between the broad stable region and a broad unstable region in this system. The broad unstable region extends out to \( \sim 10.1 \) AU, the location of the 2:5 MMR with the outer planet. The last significant region of unstable test particles is just outside the 3:8 MMR with the outer planet at \( \sim 10.5 \) AU.

An extension of this simulation to 100 Myr resulted in the loss of an additional 12 particles. Among these particles, six were at the edge of an island of stability near 3 MMRs with the outer planet. Our extended simulation also showed that three test particles at the inner boundary of the broad region of stability were lost, and that three new unstable regions appeared at 1.66 AU, 10.82 AU, and 11.40 AU. The latter region corresponds to the 1:3 MMR with the outer planet. The test particles around this resonance show three types of behaviors that are similar to those observed for test particles near the 1:3 MMR with the outer planet of GJ 876. In correlation with an increase in their eccentricities, two stable particles in this region showed an increase in the libration amplitudes of one of their critical arguments. Also, the stable particle at 11.42 AU in this region had two critical arguments turning from circulation to libration, with a corresponding jump in its eccentricity. Based on the results for GJ 876, it is likely that this region is unstable on longer times beyond 100 Myr. This is the same mechanism that makes some resonances (e.g., 3:1) unstable in the asteroid belt.

Between the two outer planets of this system, a stable region exists from 0.7 AU to 1.3 AU, which includes the star’s habitable zone (Menou & Tabachnik 2003). To ensure that our study would also include an analysis of the stability of test particles in this region, we took a closer look at the stability of particles in the region from 0.64 AU to 2.90 AU. Similar to the situation in 47 UMa, this region is populated by resonances with the middle and outer planets. The most easily discerned feature of the test particles in this region is that their maximum eccentricities rise almost monotonically from about 0.03 at 0.76 AU to 0.4 at 2.90 AU. The amplitudes of libration of the semi-major axes of these particles also increase in this range. Figure 16 shows the eccentricity
versus semi-major axis of all test particles with initial semi-major axes between 0.64 AU and 1.48 AU. As shown in this figure, only the particles closest to the middle planet appear to be strongly affected by its presence, although all the test particles in this figure are stable for 100 Myr. Figure 17 shows the same quantities for test particles with initial orbital radii between 1.50 AU and 2.90 AU. In this figure, the indicated MMRs are internal resonances with the outer planet. The width of the 3:1 MMR is also shown here. The MMRs with the outer planet of the form $n : 1$ have a significant effect on the eccentricities of particles at their nominal locations.

7. Summary and Discussion

We have studied the stability of non-interacting particles in the multi-planet extrasolar systems of $\upsilon$ Andromedae, GJ 876, 47 UMa, and 55 Cancri. We identified regions of stability of these systems where test particles maintain their orbits for 100 Myr without being ejected from the system or colliding with the central star. Although examples of secular resonances, in which the lines of apsides remain closely aligned, are also present in a few extrasolar systems (Chiang, Tabachnik, & Tremaine 2001; Chiang & Murray 2002; Malhotra 2002), we restricted ourselves to study of the effects of mean motion resonances on the dynamical evolution of test particles in these planetary systems.

The simulations presented in this study were initially run for 10 Myr and were subsequently extended to 100 Myr. The results indicate that particles near MMRs with planets and at the edges of the zones of stability could be lost very slowly over extended times. Most particles placed in regions between planets became unstable on short timescales ($< 10^5$ years). Three exceptions to this were the region just outside the orbit of the inner planet in $\upsilon$ Andromedae, some captured Trojans in 47 UMa, and also the region between the middle and outer planets in 55 Cancri.

Among all the mean motion resonances studied in this paper, the external 1:3 MMR played a more significant role in the dynamical evolution of test particles. For this resonance, a test particle is stable either when all its three critical arguments circulate for the length of the simulations, or when only one of its critical arguments librates for that duration of time. Instability generally arises when at least one critical argument undergoes a transition between circulation to libration. Sudden jumps in eccentricity accompany such transitions. This behavior has also been observed in simulations of test particles near Jupiter’s internal 3:1 MMR, the location of a Kirkwood gap (Wisdom 1983).

Studies such as the one presented here are the first step in attempting to identify extrasolar planetary systems that may harbor terrestrial planets. Terrestrial planets are on average about 2 orders of magnitude less massive than Jupiter-like planets. Unless two terrestrial planets are close together (e.g., Earth-Venus), or are involved in a secular resonance with giant planets (e.g., Mercury-Venus-Jupiter, cf. Laskar 1994, 1997), the perturbative effects of such bodies on the dynamics of the system is so small that to a good approximation, they can be neglected. This enables one to
consider terrestrial planets as test particles. To search for regions where a terrestrial planet can have a stable orbit, one has to run simulations for different values of the orbital parameters of such a body. In this study we accomplished this by considering a battery of test particles, systematically placed at different semi-major axes, to represent a terrestrial planet at those locations. Such studies have also been done by other authors. However, this study appears to be one of the first in which the planetary orbital parameters were determined by performing N-body fits to radial velocity data.

In general, most of our results agree with previous studies. As Jones & Sleep (2003), Jones, Sleep, & Chambers (2001), and Menou & Tabachnik (2003) have found, it is unlikely that \( \upsilon \) Andromedae and GJ 876 harbor a terrestrial planet in a stable orbit in their respective habitable zones. However, for both 47 UMa and 55 Cancri, the existence of such a terrestrial planet is not impossible. It is necessary to emphasize that, if significant giant planet migration occurs during the formation of the two latter systems, it will be difficult for a terrestrial planet to maintain a stable orbit for the duration of the formation (Laughlin, Chambers, & Fischer 2002; Marcy et al. 2002). As the giant planet migrates, the locations of the mean motion and secular resonances also vary. As a result, either the terrestrial planet will be removed from the system, or the material that needs to go into the making of such a planet will be depleted by the passage of mean motion and secular resonances.

Our simulations also indicate that the extrasolar multi-planet systems studied here could harbor more planets in one of the following four regions: 1) close-in orbits, even if the system already has a Jovian-mass planet very close to the star as in \( \upsilon \) Andromedae, 2) distant orbits, 3) orbits between widely spaced planets, and 4) mean motion and secular resonance-protected orbits, where a small planet could be protected from close encounters with its Jovian neighbors.

Systems like 47 UMa and 55 Cancri, which contain significant regions of stability inside the orbit of the outermost planet, could also harbor asteroids in addition to, or instead of additional planets. The asteroid belt in our Solar System has two properties that are also apparent in the zones of stability in the extrasolar planetary systems studied here; there are gaps at the locations of MMRs with Jupiter, and also the belt is very slowly being eroded (Nesvorny & Morbidelli 1998). This may also happen in extrasolar asteroid belts. Similar to the Solar System, collisions among these extrasolar asteroids would produce dust. This asteroidal dust could be detected by ground-based interferometry and also by TPF and Darwin. Resolving structures in this dusty environment, over scales of a few AU from the star, would help in constraining the parameters for the Jovian and possibly other nearby planets.

It is important to emphasize that the work presented here is based on recent radial velocity data. Also, as mentioned before, some of the orbital parameters of the planetary systems studied here are not strongly constrained. For instance, a range of values for a planet’s orbital inclination can result in equally good fitted orbital parameters. As the data acquisition techniques are modified, instruments are improved, and more observational data are gathered, it is likely that the best fitted orbital parameters will change. How significant these changes will be, and how they will help
in constraining the orbital parameters depend partly on the length of time for which a system has been observed. With the future data, it will be vitally important to conduct studies similar to the one presented here to refine the orbital parameters of planetary systems. For instance, if future data indicate that the masses and eccentricities of planets in some systems are larger than the current best values, based on this work and similar studies, the regions around MMRs would possibly become less stable. For now, however, studies such as the one presented here can be used to predict where possible additional planets may exist in extrasolar planetary systems, based on all the recent data.

We are grateful to Alan Boss and John Chambers for critically reading the original manuscript, and for their helpful comments and suggestions. We thank Debra Fischer, Geoff Marcy, and Paul Butler for providing the radial velocity observations and fits for the systems studied in this work. We also appreciate Alycia Weinberger for insightful discussions. This research is supported by the NASA Astrobiology Institute under Cooperative Agreement NCC2-1056 for E. J. R. and N. H., and also by the NASA Origins of Solar Systems Program under Grant NAG5-11569 for N. H.

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Table 1. Astrocentric orbital parameters for the three planets orbiting υ Andromedae.

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<th>Parameter</th>
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<td>$m_{pl}$ (M$_{Jup}$)</td>
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<td>119.4</td>
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Table 2. Astrocenric orbital parameters for the two planets orbiting GJ 876.

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<td>$m_{pl} (M_{Jup})$</td>
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<td>$M$ (deg)</td>
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$\sqrt{\chi^2} = 1.60$, RMS = 7.77 m s$^{-1}$

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<td>$\omega$ (deg)</td>
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<td>$M$ (deg)</td>
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<td>271.7</td>
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$\sqrt{\chi^2} = 1.60$, RMS = 7.82 m s$^{-1}$

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<td>$a$ (AU)</td>
<td>0.130</td>
<td>0.209</td>
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<td>$e$</td>
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<td>$\omega$ (deg)</td>
<td>329.6</td>
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<td>$M$ (deg)</td>
<td>165.2</td>
<td>282.1</td>
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Table 3. Astrocentric orbital parameters for the two planets orbiting 47 UMa. ($\sqrt{\chi^2} = 1.55$, RMS = 10.01 m s$^{-1}$)

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<td>$a$ (AU)</td>
<td>2.09</td>
<td>4.47</td>
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<tr>
<td>$e$</td>
<td>0.0466</td>
<td>$5.97 \times 10^{-6}$</td>
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<tr>
<td>$\omega$ (deg)</td>
<td>108.2</td>
<td>328.6</td>
</tr>
<tr>
<td>$M$ (deg)</td>
<td>270.6</td>
<td>330.7</td>
</tr>
</tbody>
</table>
Table 4. Astrocentric orbital parameters for the three planets orbiting 55 Cancri.

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<td>$m_{pl}$ (MJup)</td>
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<tr>
<td>a (AU)</td>
<td>0.115</td>
<td>0.240</td>
<td>5.49</td>
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<tr>
<td>$e$</td>
<td>0.0657</td>
<td>0.201</td>
<td>0.244</td>
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<tr>
<td>$\omega$ (deg)</td>
<td>122.4</td>
<td>33.8</td>
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<tr>
<td>$M$ (deg)</td>
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<td>29.7</td>
<td>187.8</td>
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