CLASSES OF SOURCE PAIRS
IN INTERFERENCE AND DIFFRACTION

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ABSTRACT

A description of interference and diffraction based on the concept of class of source pairs is presented. It is the set of pairs of sources whose contributions to the interference or diffraction patterns exhibit the same phase difference. Each class of source pairs provides a specific cosine-like modulation on the intensity distribution of pattern, in such a way that the set provides an expansion of the intensity distribution of the pattern in an orthogonal basis. From this point of view, the classes of source pairs are the effective elementary sources for those intensity distributions. The characteristics of the classes of source pairs can be accurately determined by Fourier transforming the intensity distribution of the patterns. The central value of this Fourier spectrum is related to the number of individual sources. The remaining values will provide two crucial descriptors of the classes: their positions on the Fourier transform domain will be corresponding to the separation vector of the class of pairs for wave front splitting interferometers (WSI) and diffraction, or the time delay for amplitude splitting interferometers (ASI), and their heights will be proportional to their populations. The concept was also applied to Fraunhofer and Fresnel diffraction, but now the classes of source pairs constitute a continuous set instead of the discrete sets in interference. However, the set of classes can also be determined by Fourier analysis.

1. INTRODUCTION

It is well known that the superposition of coherent light waves produces interference [1]. It results from phase differences due to differences of optical path lengths or time delays or both between the superposed waves. This property is the basis for the development of interferometers of any configuration. The majority of such devices can be classified into two groups, i.e. the wave front splitting interferometers (WSI) and the amplitude splitting interferometers (ASI).

Usually, WSI are based on aperture arrays such as Young’s pairs for two beams interference or gratings for multiple beams interference. ASI are based on cavities for multiple reflections, which can be realised by thin films or a Michelson configuration for two beams interference or plane parallel plates for multiple beams interference [1].

Optical interference is a non-linear phenomenon with respect to the intensity distribution at the observation plane. Indeed, the intensity at each point on this plane can differ from the addition of the intensities provided by the individual sources. However, if \( I_0 \) represents the intensity provided by each aperture in a WSI or the intensity of the incident beam in an ASI, the angular intensity distribution in the far zone (Fraunhofer domain) interference pattern can be expressed as

\[
I(\theta) = \mathcal{B}(N, \Delta) I_0, \tag{1}
\]

where \( \mathcal{B}(N, \Delta) \) is a non-linear function that depends on the number of interfering waves \( N \) and the phase difference \( \Delta = \Delta(\theta) \) between the contributions of two adjacent sources. It determines the structure of interference fringes of the pattern.

By example, \( \Delta(\theta) = a \sin \theta \) for a one-dimensional regular grating of \( N \geq 2 \) point apertures with pitch \( a \), which reduces to \( \Delta(y) = \frac{a y}{z} \) in paraxial approach, with \( y \) the co-ordinate at the
observation plane that is parallel to the axis defined by the point apertures of the grating, and 2 the
distance between the observation and the grating apertures [1]. It is well known that
\[ I(y) = \frac{\text{sen}^2\left(\frac{\pi Na}{\lambda z} y\right)}{\text{sen}^2\left(\frac{\pi a}{\lambda z} y\right)} I_0, \]
with \( \lambda \) the wavelength, for this type of interferometers. Thus,
\[ B(N, y) = \frac{\text{sen}^2\left(\frac{\pi Na}{\lambda z} y\right)}{\text{sen}^2\left(\frac{\pi a}{\lambda z} y\right)}, \] (2)

Eqs.(1) and (2) yield to the intensity distribution of the Young's interference pattern if \( N=2 \).

As a second example, let us consider a parallel plane plate of refraction index \( n_f \) located in a
medium of refraction index \( n \). Now we have \( \Delta(\theta) = 2 d \sqrt{n_f^2 - n^2} \text{sen}^2 \theta \). Therefore,
\[ \frac{\Delta(\theta)}{c} = \tau_0 \psi(\theta), \]
with \( \psi(\theta) = \sqrt{1 - \left(\frac{n}{n_f}\right)^2} \text{sen}^2 \theta \) and \( \tau_0 = \frac{2 n_f d}{c} \) denotes the time delay
introduced by one internal reflection inside the plate [1]. Complementary interference patterns are
produced by the plate, one of them by reflection and the other by transmission, so that
\[ B_r(N, \psi) = \frac{1}{1 + \left(\frac{2r}{1-r^2}\right)^2 \text{sen}^2\left(\frac{\omega \tau_0 \psi}{2}\right)}, \] (3)
for the transmitted (refracted) pattern and
\[ B_r(N, \psi) = \frac{\left(\frac{2r}{1-r^2}\right)^2 \text{sen}^2\left(\frac{\omega \tau_0 \psi}{2}\right)}{1 + \left(\frac{2r}{1-r^2}\right)^2 \text{sen}^2\left(\frac{\omega \tau_0 \psi}{2}\right)}, \] (4)
for the reflected pattern, with \( r \) the reflection coefficient of the plate surfaces. The value of \( N \)
depends implicitly on the value of the reflection coefficient. Furthermore,
\[ B_r(N, \psi) + B_r(N, \psi) = 1. \] Equation (3) is usually called the Airy function [1].

The cavity of an etalon in a Fabry-Perot interferometer is a parallel plane plate with high reflective
surfaces. Denoting their reflectivity and transmissivity by \( R \) and \( T \) respectively, and taking into
account that \( R + T + A = 1 \) with \( A \) the fraction of light absorbed by the metal coatings of the surfaces,
the Airy function of the etalon will be
Following the usual mathematical description of the behaviour of the WSI and the ASI [1,2], we can express equation (1) as

\[ B_r(N, \psi) = \left(1 - \frac{A}{1 - R}\right)^2 \frac{1}{1 + \frac{4R}{1 - R^2} \sin^2 \left(\frac{\omega \tau_0 \psi}{2}\right)} \]  

(5)

\[ \Psi \]

\section{The Expansion in Classes of Source Pairs in Interference}

Following the usual mathematical description of the behaviour of the WSI and the ASI [1,2], we can express equation (1) as

\[ I(\psi) = \sum_{m=0}^{N-1} \sum_{l=0}^{N-1} e^{-i \frac{2\pi}{\lambda z} (m-l) a} I_0 \]  

(6)

for WSI and

\[ I(\psi) = \left( t_{1f} t_{f2} \right)^2 \sum_{m=0}^{N-1} \sum_{l=0}^{N-1} \left( t_{1f} r_{f2} \right)^{m+l} e^{-i \omega (m-l) r_0 \psi} I_0 \]  

(7)

for ASI, by assuming that \( n_1 \) and \( n_2 \) are the refractive indices of the media before and after the cavity respectively. So, \( t_{1f} \), \( r_{f2} \) denote the transmission and reflection coefficients at the interface with the first medium, and \( t_{f2} \), \( r_{f2} \) the same coefficients at the interface with the second medium.

On applying the Euler's formula, i.e. \( e^{i\phi} + e^{-i\phi} = 2 \cos \phi \), equations (6) and (7) can be written as

\[ I(\psi) = \left[ N + \sum_{p=1}^{N-1} (N - p) 2 \cos \left( \frac{2\pi p a}{\lambda z} \right) \right] I_0 \]  

(8)

and

\[ I(\psi) = \left[ \left( t_{1f} t_{f2} \right)^2 \sum_{m=0}^{N-1} \left( t_{1f} r_{f2} \right)^{2m} \right. \]  

\[ + \sum_{p=1}^{N-1} \left. \left( t_{1f} t_{f2} \right)^2 \sum_{m=p}^{N-1} \left( t_{1f} r_{f2} \right)^{2m-p} \right] 2 \cos \left( \omega p \tau_0 \psi \right) I_0 \]  

(9)

respectively. The both expansions were obtained in the following way:

- The condition \( m=1 \) yields the first term, which is determined by the contributions from the \( N \) individual sources. All the sources provide the same intensity in WSI, but the intensity provided by each individual source in ASI will depend on the number of internal reflections in the cavity.
- The condition \( m \neq 1 \) (\( p=m-1 \geq 1 \)) yields the second term. It contains the cosine-like modulations due to the interference between the contributions of sets of source pairs with the same phase
difference. The amplitude of the modulation provided by a specific set of source pairs is proportional to the number of pairs of the set. Indeed, for \( p = p_0 \) the set contains \( N - p_0 \) pairs of sources, which provide a modulation of amplitude \( N - p_0 \) in WSI, and

\[
\left( t_{1f} t_{2f} \right) \sum_{m=0}^{N-1} \sum_{p=p_0}^{m-p_0} \cos \left( p \Lambda + 2 \pi \right) \text{ in ASI.}
\]

Accordingly we introduce the concept of class of source pairs as the set of pairs of sources whose contributions exhibit the same phase difference. Each class of source pairs provides a specific cosine-like modulation on the intensity distribution of the Fraunhofer interference pattern. In other words, each term of the second summation in equations (8) and (9) is provided by a specific class of source pairs. The source pairs in WSI are usually called Young's pairs. We propose the term Michelson's pairs for denoting the classes of equally time delayed pairs of beams in ASI.

It is apparent that the cosine functions are orthogonal and their set is complete, because the corresponding classes contain all the pairs that contribute to the interference pattern. So, the set of classes of source pairs provides an expansion of the intensity distribution of the Fraunhofer interference pattern in an orthogonal basis.

Consequently, the intensity distribution of the pattern produced by multiple beam interference is a function of an \( N \) dimensional space of functions. The dimension of the space is determined by the set of classes of source pairs, understanding the individual radiators as pairs with null phase difference. From this point of view, the classes of source pairs are the effective elementary sources for those intensity distributions.

Accordingly, we can expand the function \( B (N, \Lambda) \) in the canonical form

\[
B (N, \Lambda) = B_{\text{individuals}} (N) + B_{\text{pairs}} (N, \Lambda) = \sum_{m=0}^{N-1} A_m + 2 \sum_{p=1}^{N-1} B_p \cos (p \beta \Lambda),
\]

with \( \beta = \frac{2 \pi \alpha}{\lambda z} \) and \( \Lambda = y \) for WSI and \( \beta = \omega \tau_0 \) and \( \Lambda = \psi \) for ASI.

As an example, let us consider the Fraunhofer interference of the light provided by four equidistant point sources as depicted in Figure 1. For the WSI it can be realized by a line of pinholes and for the ASI by reflections within a plane parallel plate. In both cases, there are three classes of pairs, i.e. one of three pairs characterized by \( \beta \), one of two pairs characterized by \( 2 \beta \) and one of only a pair characterized by \( 3 \beta \).

Table 1 shows the orthogonal basis of the corresponding expansions and Table 2 the normalized profiles of the intensity distribution of the Fraunhofer diffraction patterns. The WSI produces a high contrast interference pattern due to the population of the classes of source pairs, whilst the contrast of the transmitted interference pattern produce by the ASI closely depends on the reflection coefficient of the plate too, i.e. the higher the reflection coefficient the higher the contrast of the interference pattern. However, the high contrast pattern produced by the ASI is very similar to the pattern produced by the WSI.
Fig. 1: Four equidistant point sources depicted by the circles. $\beta$ is corresponding to the separation $a$ in WSI and to the time delay $\tau_0$ in ASI. In both cases, there are three classes of pairs, i.e. one of three pairs characterized by $\beta$, one of two pairs characterized by $2\beta$ and one of only a pair characterized by $3\beta$.

TABLE 1: Orthogonal basis of the expansion in equation (10)

$$\cos(0\beta \Lambda) = 1$$

![Graph showing the cosine function of $0\beta \Lambda$]

$$\cos(\beta \Lambda)$$

![Graph showing the cosine function of $\beta \Lambda$]
TABLE 2: Normalized profiles of the intensity distribution of the Fraunhofer diffraction patterns. The abscise-axis for the WSI has length units and for the ASI $\beta \Lambda = \omega T \psi$ has angle units.

On the other hand, it is possible to identify the classes of source pairs by Fourier transforming the intensity distribution of the interference pattern, i.e.

$$I(u) = I_0 \int B(N, \Lambda) e^{-iu} d\Lambda = B'(N, u) I_0.$$  \hspace{1cm} (11)

From eqs. (10) and (11) we conclude that

$$B'(N, u) = \frac{I(u)}{I_0} = \left[ \sum_{m=0}^{N-1} A_m \right] \delta(u) + \sum_{p=1}^{N-1} B_p \left[ \delta(u - p\beta) + \delta(u + p\beta) \right]. \hspace{1cm} (12)$$
with $δ(ω)$ the Dirac's delta function [3]. It is clear that $\tilde{B}(N,ω)=\tilde{B}^*(N,-ω)$ because $I(θ)$ is a real defined quantity. Therefore, it is sufficient to consider $|\tilde{B}(N,u)|$ for $u ≥ 0$ to identify all the classes of pairs as follows:

- The height of the peak at the origin of co-ordinates is proportional to the total number of interfering sources. It takes the values $N$ or $\left(t_{1f} t_{2f}\right)^2 \sum_{m=0}^{N-1} (t_{1f} r_{2f})^{2m}$ depending on the considered interferometer.

- The locations of the peaks for $p ≥ 1$ will be $u_p = pβ$. They take the values $2π λ z pa$ for WSI. It suggests that the classes of source pairs in a grating of $N$ sources and pitch $a$ can be characterised by the separation of the pairs. So, the grating will contain $N-1$ classes of source pairs with separations $a$, $2a$, $3a$, ..., $(N-1)a$, and populations (number of source pairs) $N-1$, $N-2$, ..., 1 respectively.

- For ASI, the locations of the peaks for $p ≥ 1$ will take the values $ω pτ_0$. Now the classes of source pairs, provided by a plane parallel plate of thick $d$ and refraction index $n_f$, will be characterised by the time delay $τ_0$. If the transmitted beam inside the plate suffers $N$ internal reflections, it will produce $N$ point sources on the plate surface. They determine $N-1$ classes of source pairs with populations $N-1$, $N-2$, ..., 1 and time delays $τ_0 ψ(θ_0)$, $2τ_0 ψ(θ_0)$, $3τ_0 ψ(θ_0)$, ..., $(N-1)τ_0 ψ(θ_0)$ respectively, where $θ_0$ denotes the angle of incidence on the plate.

- From equation (8) it is apparent that $B_p = N - p$ for WSI. It means that the height of each peak for $p ≥ 1$ gives the population of the corresponding class in this case.

- From equations (9) and (12) we have $B_p = \left(t_{1f} t_{2f}\right)^2 \sum_{m=p}^{N-1} (t_{1f} r_{2f})^{2m-p}$ for ASI. The height of the peaks for $p ≥ 1$ will depend not only on the population of the corresponding class but also on both the reflection and refraction coefficients of the interfaces.

Table 3 shows $|\tilde{B}(N,u)|$ corresponding to the intensity distributions in Table 2. The same profile holds for the WSI and the ASI with high reflection coefficient. The peak of height 4 at the origin denotes the four point sources we are considering. The further peaks are equidistant in arbitrary units that are corresponding to both the separations $a$, $2a$, $3a$ that characterise the classes of source pairs of the WSI and the time delays $τ_0$, $2τ_0$, $3τ_0$ that characterise them for the ASI. The populations of such classes of source pairs are given by the corresponding peak heights, i.e. 3, 2, 1 respectively.

The influence of the reflection coefficient of the ASI on the set of classes is apparent in the second diagram in Table 3. A low reflection coefficient causes the suppression of the classes of higher time delays and consequently a low contrast of the interference pattern. For $r^2 = 0.18$ only the class with time delay $τ_0$ and a population of one source pair will contribute. Therefore, the interference pattern will exhibit cosine-like fringes as depicted in the second diagram of Table 2.
TABLE 3: Profiles of $\tilde{B}(N, u)$ for the intensity distributions in Table 2

\[ \tilde{B}(4, u) \]

WSI

ASI with

$\rho^2 = 0.87$

The same behaviour can be verified in the profiles of Table 4.
3. THE EXPANSION IN CLASSES OF SOURCE PAIRS IN FRAUNHOFER DIFFRACTION

Fraunhofer diffraction phenomenon can be considered as interference of multiple beams provided by a continuous set of sources, determined by the aperture shape.

Let us consider a diffracting aperture of area $\mathcal{A}$ at a distance $z$ from the observation plane in the Fraunhofer domain [2], illuminated by a coherent field of complex amplitude $\Psi(\xi) = |\Psi(\xi)| e^{i \alpha(\xi)}$ and wavelength $\lambda$. Then, according to the Kirchhoff-Sommerfeld integral [1-3], the intensity distribution of the diffraction pattern at the observation plane will be given by

$$I(r) = \left( \frac{1}{\lambda z} \right)^2 \left| \int_{\mathcal{A}} \Psi(\xi) e^{-i\frac{k}{\lambda} r \xi} \, d^2 \xi \right|^2,$$

with $r$ and $\xi$ denoting the position vectors at the aperture and the observation planes respectively, and $k = \frac{2\pi}{\lambda}$. By assuming illumination with a plane wave with null phase, i.e. $\Psi(\xi_1) = \Psi(\xi_2) = \Psi_0$ a real constant, equation (13) can also be expressed as

$$I(r) = I_0 \left( \frac{1}{\lambda z} \right)^2 \int_{\mathcal{A}} \int_{\mathcal{A}} e^{-i\frac{k}{\lambda} (r - \xi_1) \xi_2} \, d^2 \xi_1 \, d^2 \xi_2,$$

with $I_0 = \Psi_0^2$ the intensity across the aperture. Equation (14) can be considered as the continuous version of equation (6).

**Fig 2.** The swept condition for solving equation (14).

Now, we can apply a similar procedure to that for interference in the following way. At the first, we introduce the dimensionless function $\lambda z \delta(\xi_1 - \xi_2) + [1 - \lambda z \delta(\xi_1 - \xi_2)]$ in the integrand of equation (14), with $\delta(\xi_1 - \xi_2)$ the Dirac’s delta function [3]. Because of the properties of this
function, we can separate the contributions provided by individual sources within the aperture \((\xi_1 = \xi_2 = \xi)\) from the contribution provided by source pairs \((\xi_1 \neq \xi_2)\).

Then, we apply the Euler's formula on the integral for source pairs. Consequently, the integration region should be changed taking into account the swept condition \(|\xi_1| < |\xi_2|\) if \(|\xi_1| \neq |\xi_2|\) and \(\theta_1 < \theta_2\) if \(|\xi_1| = |\xi_2|\), with \(\theta_1\) and \(\theta_2\) the angles subtended by the position vectors \(\xi_1\) and \(\xi_2\) to the positive x-axis respectively (Figure 2).

After this algorithm, equation (14) becomes

\[
I(r) = I_0 \left( \frac{1}{\lambda z} \right)^2 \left\{ \mathcal{A} + 2 \int_{\xi_1, \xi_2} \cos \left[ \frac{k}{z} (\xi_1 - \xi_2) \cdot r \right] d^2\xi_1 d^2\xi_2 \right\}, \tag{15}
\]

The first term of equation (15) \((\xi_1 = \xi_2 = \xi)\) represents the contributions of individual sources within the diffracting aperture. The second one \((\xi_1 \neq \xi_2)\) denotes the modulations due to the interference of the contributions provided by source pairs. The mathematical form of this expression is similar to a continuous version of equation (8), which describes the intensity distribution of the interference pattern at the observation plane of a WSI.

Thus, the source pairs within the aperture behave as Young pairs of point sources, and we can use the before defined concept of class of source pairs for describing Fraunhofer diffraction too. Indeed, the intensity distribution of the Fraunhofer diffraction pattern will result from the superposition of the Young interference patterns provided by the classes of source pairs inside the aperture. The fringes of each pattern will be orthogonal to the separation vector of the class and its period will be \(\Delta r = \frac{\lambda z}{|\xi_1 - \xi_2|}\).

On comparing equations (1) and (15) we conclude that

\[
\mathcal{B}(\mathcal{A}, \Delta) = \left( \frac{1}{\lambda z} \right)^2 \left\{ \mathcal{A} + 2 \int_{\xi_1, \xi_2} \cos \left[ \Delta(r; \xi_1, \xi_2) \right] d^2\xi_1 d^2\xi_2 \right\}, \tag{16}
\]

with

\[
\Delta(r; \xi_1, \xi_2) = \frac{k}{z} (\xi_1 - \xi_2) \cdot r. \tag{17}
\]

The mathematical form of equation (16) is the continuous version of the canonical equation (10), which confirms the initial statement of this section. The modulation provided by a specific class will have the form \(2 \cos \cos [\Delta(r; \xi_1, \xi_2)]\), with \(\xi_1 - \xi_2\) the separation vector that characterises the class.

As in interference, this continuous but bounded set of cosine functions is orthogonal and complete, because the corresponding classes contain all the pairs that contribute to the diffraction pattern. So,
the power spectrum of the diffraction pattern will be a function of a $\infty$ dimensional space of functions. The bound of the basis is corresponding to the integration region of equation (16), which is defined by the aperture shape.

Now, let us calculate the Fourier transform of the intensity distribution of the diffraction pattern i.e. $\tilde{I}(\eta) = \int I(r) e^{i \frac{2\pi}{\lambda} \eta r} d^2 r$. To perform it, it is more efficient to start from equation (13) instead of equation (15), because we can directly apply the Wiener-Khintchine theorem [3]. According to it, $\tilde{I}(\eta)$ will be proportional to the autocorrelation of $\Psi(\xi)$. Thus, the aperture shape plays a definitive role in this calculation.

By example, for mathematical simplicity and without lack of generality, let us consider a one-dimensional slit of width $a$, which diffracts a plane wave of phase null in the Fraunhofer domain.

So, we have $\Psi(\xi) \rightarrow \Psi_0 \text{rect} \left( \frac{\xi}{a/2} \right)$, with $\text{rect} \left( \frac{\xi}{a/2} \right) = \begin{cases} 1 & \text{if} \ |\xi| \leq \frac{a}{2} \ \text{otherwise} \\ 0 & \text{otherwise} \end{cases}$. (Figure 3). It is well known that the autocorrelation of a rect function is a tri function, which is defined as [3]

$$\text{tri} \left( \frac{\eta}{a} \right) = \int_{-\infty}^{\infty} \text{rect} \left( \frac{\xi}{a/2} \right) \text{rect} \left( \frac{\xi - \eta}{a/2} \right) d\xi = \begin{cases} \frac{|\eta|}{a} + 1 & \text{if} \ |\eta| \leq a \\ 0 & \text{otherwise} \end{cases}.$$ Thereby, we conclude that

$$\tilde{I}(\eta) = I_0' \text{tri} \left( \frac{\eta}{a} \right), \quad \text{(18)}$$

with $I_0'$ the total intensity that emerges from the slit and $\eta = \xi_2 - \xi_1$. As in the case of interference, this result allows as to identify the classes of source pairs within the aperture. Note that $\tilde{I}(\eta) = \tilde{I}^*(-\eta)$ because $I(r)$ is a real definite quantity, so that the portion $\tilde{I}(\eta)$ for $\eta \geq 0$ provides complete information about the classes.
Fig. 3: a) Plane wavefront that emerges from a slit of width $a$. b) Normalised intensity distribution of the Fraunhofer diffraction pattern it produces. c) Normalized Fourier transform of the intensity distribution in b).

In contrast to the multiple-beam interference, the set of classes by diffraction is continuous but bounded by a support of width $a$. The value $\tilde{I}(\eta = 0) = \tilde{I}(\xi_1 = \xi_2) = I_0'$ is an estimative of the total number of secondary point sources inside the slit. It is corresponding to the first term of the Fourier transform of equation (18).

Each value of $\tilde{I}(\eta)$ for $\eta > 0$, i.e. $\xi_1 > \xi_2$, characterises the corresponding class with separation $\eta = \xi_1 - \xi_2$ in the sense that it is proportional to the density of source pairs with this separation. It is related to the value of the second term of the Fourier transform of equation (18) for $\eta = \xi_1 - \xi_2$. Note that the greater the separation the smaller $\tilde{I}(\eta)$, in such a way that there are no classes with separation $\eta > a$, as expected.

Most two-dimensional symmetric apertures, which diffract plane waves in the Fraunhofer domain, exhibit a similar behaviour, i.e. $\tilde{I}(\eta)$ will have a central maximum for $\eta = 0$ ($\xi_1 = \xi_2$) and decays
to null when $\eta$ increases ($\xi_1 \neq \xi_2$) up to the aperture dimensions. So, the above interpretation of $\tilde{T}(\eta)$ as a map of the continuous set of classes remains valid.

The value $\tilde{T}(\eta = 0)$ will be an estimative of the total number of secondary point sources within the aperture and each value $\tilde{T}(\eta \neq 0)$ will be proportional to the density of source pairs with separation vector $\eta$. Furthermore, $\tilde{T}(\eta) = 0$ for all $|\eta|$ greater than the dimensions of the aperture. It means that there are no classes of pairs with such separations.

Two usual examples are the Cartesian symmetrical aperture (i.e. the rectangular slit) and the rotation symmetrical aperture (i.e. the circular hole). For the first case we have

$$\tilde{T}(\eta_x, \eta_y) = I_0 \text{tr}_a \left( \frac{\eta_x}{a} \right) \text{tr}_b \left( \frac{\eta_y}{b} \right),$$

with $a$ and $b$ the sides of the rectangle, and for the second one

$$\tilde{T}(\eta) = \begin{cases} \frac{2}{\pi} \arccos \left( \frac{\eta}{|\eta_0|} \right) & \text{for } |\eta| \leq |\eta_0|, \\ 0 & \text{otherwise} \end{cases}$$

with $|\eta_0| = \frac{a}{2\lambda z}$ and $a$ the radius of the hole [5]. Because of the redundancy due to the symmetry of $\tilde{T}(\eta)$ in both cases, the classes of source pairs can be analysed by considering its values on only two quadrants of its support.

4. EFFECTS OF THE FRESNEL’S ZONES ON THE EXPANSION IN CLASSES OF SOURCE PAIRS

Fresnel diffraction takes place when the geometry of the diffractive set-up induces a quadratic phase distribution on the wavefront that illuminates the aperture. As a consequence, the intensity distribution of the diffraction pattern will be given by

$$I(r) = \left( \frac{1}{\lambda z} \right)^2 \left| \int_\rho \Psi(\xi) e^{\frac{k}{2z} r^2} e^{-i k \frac{r \cdot \xi}{r}} \, d^2 \xi \right|^2,$$  \hspace{1cm} (19)$$

instead of equation (13). By applying the same algorithm as for Fraunhofer diffraction, equations (19) becomes

$$I(r) = I_0 \left( \frac{1}{\lambda z} \right)^2 \left\{ A + 2 \int_{\xi_1, \xi_2} \cos \left[ \frac{k}{z} (\xi_1 - \xi_2) \cdot r - \frac{k}{2z} \left( |\xi_1|^2 - |\xi_2|^2 \right) \right] d^2 \xi_1 d^2 \xi_2 \right\}.$$  \hspace{1cm} (20)$$

Thus, the canonical form of $\mathcal{B}(\delta, \Delta)$ for diffraction remains with the only difference that
\[ \Delta(r : \xi_1, \xi_2) = \frac{k}{z} (\xi_1 - \xi_2) \cdot r - \frac{k}{2z} |\xi_1|^2 - |\xi_2|^2. \]

The term \( \frac{k}{2z} (|\xi_1|^2 - |\xi_2|^2) \) in equation (21) shifts laterally the cosine functions at the observation plane as a consequence of the Fresnel's zones [4]. This lateral shifting can be interpreted as a time delay, as follows.

Figure 4: Time delay \( \tau = \frac{d}{c} \) in Fresnel diffraction

Figure 4 shows that \( |\xi|^2 = z^2 - (z-d)^2 = 2zd \). Thus, \( \frac{k}{2z} |\xi|^2 = kd = \omega \tau(\xi) \), with \( \omega \) the frequency of the wave and \( \tau(\xi) = \frac{d}{c} \) the time delay between the contributions provided by the sources at \( O \) and \( Q \) respectively. Then, equation (20) becomes

\[ I(r) = I_0 \left( \frac{1}{\lambda z} \right)^2 \left\{ A + 2 \int_{\xi_1, \xi_2} \int \cos \left[ \frac{k}{z} (\xi_1 - \xi_2) \cdot r - \omega [\tau(\xi_1) - \tau(\xi_2)] \right] d^2 \xi_1 d^2 \xi_2 \right\}. \] (22)

On comparing equations (9) and (22) we conclude that the source pairs within the aperture does not only behave as Young's pairs but also as Michelson's pairs (as in ASI, particularly in a Newton's interferometer [1]) by Fresnel diffraction.

More precisely, the time delay between the contributions from pairs of sources located within the same Fresnel's zone can be neglected, and the modulation they produces will be similar to that in Fraunhofer diffraction. But if the sources are located inside different Fresnel's zones, the time delay between their contributions will be appreciable and should be included in their phase difference.

Therefore, distinguishing between Fraunhofer and Fresnel diffraction [4] means to determine if the effective elementary sources of the power spectrum of the diffracted field are only Young's pairs or if they also behaves as Michelson's pairs.
Obviously, the behaviour of the classes will be affected by the quadratic phase distribution in the last case, because $\tilde{I}(\eta)$ will be now proportional to the autocorrelation of $\Psi'(\xi)e^{k/2zd|\eta|^2}$. Not only the aperture shape but also the Fresnel number and the match of the Fresnel’s zones to the aperture shape will be crucial for this calculation.

As example, let us consider that the slit in Figure 3a exactly matches two Fresnel zones (as a one-dimensional case). Figure 5 shows the profile of the intensity distribution of the Fresnel diffraction pattern, characterised by the dark central fringe, and its Fourier spectrum.

![Fig. 5: a) Normalised intensity distribution of the Fresnel diffraction pattern produced by a slit that exactly matches two (one-dimensional) Fresnel zones, b) Normalised Fourier transform of the intensity distribution in a).](image)

As by Fraunhofer diffraction, the value $\tilde{I}(\eta=0) = \tilde{I}(\xi_1 = \xi_2) = I_0'$ by Fresnel diffraction is an estimative of the total number of secondary point sources inside the slit, and each value of $\tilde{I}(\eta)$ for $\eta > 0$, i.e. $\xi_1 > \xi_2$, characterises the corresponding class with separation $\eta = \xi_1 - \xi_2$ in the sense that it is proportional to the density of source pairs with this separation.

Furthermore, the decay $\tilde{I}(\eta)$ for $\eta > 0$ is also apparent in Figure 5b, but in contrast to Fraunhofer diffraction, it now oscillates taking negative values. As a consequence:

- The classes corresponding to its zero points, i.e. $\tilde{I}(\eta_0) = 0$, are suppressed.
- The classes corresponding to lobes of $\tilde{I}(\eta)$ with the same sign will have phase differences with the same sign. So, their contribution will interfere constructively.
- The classes corresponding to lobes of $\tilde{I}(\eta)$ with the contrary signs will have phase differences with the contrary signs, and their contribution will interfere destructively. This feature together to the symmetry of the set up produces the behaviour of the aperture pairs as Michelson’s pairs that generate the central dark fringe of the pattern.

For this point of view, to evolve from Fraunhofer diffraction to Fresnel diffraction means to modulate the set of classes of source pairs as depicted in Figure 5b. Note that there are no classes with separation $\eta > \alpha$ because it depends explicitly on the aperture shape. These results suggest a
novel interpretation of optical filtering as a modulation (in amplitude, phase or both) over the set of classes of sources pairs in the aperture of the optical system.

CONCLUSION

We presented a description of interference and diffraction based on the concept of class of source pairs, which is the set of pairs of sources whose contributions to the interference and diffraction patterns exhibit the same phase difference.

Each class of source pairs provides a specific cosine-like modulation on the intensity distribution of pattern, in such a way that the set provides an expansion of the intensity distribution of the pattern in an orthogonal basis. Consequently, the intensity distribution of the pattern produced by multiple beam interference is a function of an $N$ dimensional space of functions, whereas the diffraction patterns will be functions of a bounded but $\infty$-dimensional space.

From this point of view, the classes of source pairs are the effective elementary sources for those intensity distributions.

The characteristics of the classes of source pairs can be accurately determined by Fourier transforming the intensity distribution of the patterns. The central value of this Fourier spectrum is related to the number of individual sources. The remain values will provide two crucial descriptors of the classes: their positions on the Fourier transform domain will be corresponding to the separation vector of the class of pairs for WSI and Fraunhofer diffraction, or the time delay for ASI and Fresnel diffraction, and their heights will be proportional to their populations.

In this way, optical filtering can be understood as a modulation (in amplitude, phase or both) over the set of classes of sources pairs in the aperture of the optical system.

Acknowledgments
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REFERENCES
TABLE 4: Profiles of the intensity distribution of the interference patterns and the moduli of their corresponding Fourier transform for 10 and 30 point sources.

| ASI | \( I(\beta \Lambda) \) | \( |\tilde{b}(N,u)| \) |
|-----|----------------|-----------------|
| \( r^2=0.87 \) | ![Graph for r^2=0.87] | ![Graph for |\tilde{b}(N,u)|] |
| \( N=10 \) | ![Graph for N=10 r^2=0.87] | ![Graph for |\tilde{b}(N,u)|] |
| \( r^2=0.18 \) | ![Graph for r^2=0.18] | ![Graph for |\tilde{b}(N,u)|] |
| \( N=10 \) | ![Graph for N=10 r^2=0.18] | ![Graph for |\tilde{b}(N,u)|] |
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