On the Nature of Feedback Heating in Cooling Flow Clusters

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ABSTRACT

We study the feedback between heating and cooling of the intra-cluster medium (ICM) in cooling flow (CF) galaxies and clusters. We adopt the popular view that the heating is due to an active galactic nucleus (AGN), i.e. a central black hole accreting mass and launching jets and/or winds. We propose that the feedback occurs with the entire cool inner region ($r \lesssim 5 - 30$ kpc), where the non-linear over-dense blobs of gas with a density contrast $\rho/\rho_a \gtrsim 2$ cool fast and are removed from the ICM before experiencing the next major AGN heating event. We term this scenario cold-feedback. Some of these blobs cool and sink toward the central black hole, while others might form stars and cold molecular clouds.

We derive the conditions under which the dense blobs formed by perturbations might cool to low temperatures ($T < 10^4$ K), and feed the black hole. The main conditions are found to be: (1) An over-dense blob must be prevented from reaching an equilibrium position in the ICM: therefore it has to cool fast, and the density profile of the ambient gas should be shallow; (2) Non-linear perturbations are required: they might have chiefly formed by previous AGN activity; (3) The cooling time of these non-linear perturbations should be short relative to few times the typical interval between successive AGN outbursts. (4) The blobs should be magnetically disconnected from their surroundings, in order not to be evaporated by thermal conduction.
1. Introduction

The observations of Galaxy Clusters with the last generation X-ray satellites Chandra and XMM-Newton have shown the remarkable lack of the large amount of cool gas predicted by the old version (ten years ago) of the cooling flow (CF) model (e.g. Peterson et al. 2003; Tamura et al. 2001b; Molendi & Pizzolato 2001). The most straightforward explanation is that the intra-cluster medium (ICM) in CF clusters must be heated by some mechanism (e.g. Fabian 2003). Probably the most popular heating engine is an active galactic nucleus (AGN) residing at the center of the cluster dominant galaxy, often a cD (see e.g. Binney & Tabor 1995; Tucker & David 1997; Ciotti & Ostriker 2001; Binney 2004; for more references see Peterson, Kahn, Paerels, Kaastra, Tamura, Bleeker, Ferrigno, & Jernigan 2004 and the review by Begelman 2004; see Fujita & Reiprich 2004 and Fabian, Voigt, & Morris 2002 for some problems connected with this scenario). Other heating mechanisms, like supernovae and heat conduction from the cluster outer regions were shown to be problematic for several reasons. It is sufficient to mention that we are looking for a unified mechanism to heat the ICM in CFs, from galaxies to large clusters. Heat conduction cannot work in galactic CFs because there are no large heat reservoirs, and probably there are not enough supernovae in elliptical galaxies to supply a sufficient amount of heat.

The new X-ray observations show that the mass cooling rate to low temperatures is far below predictions by older versions of the cluster CF model (Fabian 1994), but is compatible with low-mass cooling rate models (see Binney 2004, on the expectation for that result). Among them, the moderate CF model (Soker et al. 2001; Soker & David 2003; Soker 2004) is different from many earlier proposed processes whose aim is to prevent CF in clusters of galaxies altogether. In the moderate CF model small but non-negligible quantities of gas are cooling to low ($\lesssim 10^4$ K) temperatures.

Most models of AGN heating agree in that there is some sort of feedback between the heating and the radiative cooling. Many of these models result in intermittent AGN activity. There are two approaches to the feedback between the cooling ICM and the AGN. In the first, the ICM does not cool below X-ray emitting temperatures; the AGN accretion is Bondi-like, and is determined by the ICM properties very close to the central black hole (e.g. Churazov et al. 2002; Nulsen 2004; Omma & Binney 2004). Nulsen (2004) for example, studies the accretion of gas at the virial temperature. We term this type of models hot feedback.
In the second approach, the black hole accretes cold gas, but the mass cooling rate is much below that in old versions of the cooling flow model. In this case the feedback takes place within a region extending to a distance of $\approx 5-30$ kpc from the cluster center. Such is the moderate cooling flow model (Soker et al. 2001; Soker 2004). We term this *cold feedback* model. The two types of feedback models imply other differences.

1. In the hot feedback models the optical filaments observed in many CF-clusters (Heckman et al. 1989), and the cooler molecular gas detected via CO observations (Edge 2001; also Edge & Frayer 2003; Salomé & Combes 2003; Salomé & Combes 2004) come from stripping gas from cluster galaxies. In the cold feedback models the cold gas may come from the cooling ICM as well.

2. In the cold feedback models some X-ray emission from gas at temperatures $\lesssim 10^7$ K is predicted, but at a level more than an order of magnitude below that in old versions of the cooling flow model (Pizzolato, Behar, & Soker in preparation).

3. The feeding of the central black hole with cold gas in the cold feedback models makes the process similar in some aspects to that of AGN in spiral galaxies.

4. It is possible, although not required, that in the cold feedback model the feedback does not only keep energy balance, but mass balance as well. Namely, a substantial fraction of gas that cooled to low temperature is injected back to the ICM and heats up as it is shocked (Soker & Pizzolato 2005). It is possible that during a fraction of the time, most of the cooling gas forms molecular clouds and stars, rather than being injected back to the ICM at high speeds.

To set the stage for the discussion to follows, it will be constructive to present the temperature profiles of some CF clusters (Figure 1). In the presently proposed model a flat temperature profile in the inner region is expected, at least during some fraction of the duty cycle. It is that flat and low temperature profile which facilitates the formation of cold blobs. This is contrary to some hot feedback models, where the accreted gas comes from the immediate AGN neighborhoods. It is our opinion that some of these models may have problems in accounting for the temperature profiles presented in Figure 1. For example, the repeated heating by Omma & Binney (2004) does not heat the outer region as much as the inner one. It seems as if the outer region will cool to low temperature, unlike the higher temperature of the outer regions in clusters. Ruszkowski & Begelman (2002) take both AGN heating and heat conduction. We find their temperature profile in the inner several kpc region to be too steep (see their Figure 1) compared with real clusters. In the temperature profile obtained by Hoeft & Brüggen (2004, their Figure 5) we see two problems.
First, contrary to their claim, we do not think that in the outer CF region their profile fits that of A2052 (taken from Blanton et al. 2003). Second, after a time of $\approx 1.2 \times 10^{10}$ years, the temperature in the inner $\approx 2$ kpc drops to $\ll 10^7$ K, contrary to the observations. There is a fine tuning problem, in that presently all clusters do not show this drop, but they will in a few $10^9$ years.

The considerations above, among other arguments, motivate us to consider the cold feedback model. In the present paper we examine the two types of feedback heating. By analyzing published models and by comparing them with the observations, we constrain the parameters space of the different models, and make some predictions which can be tested with future observations. Our proposed scenario is presented in § 2. In § 3 we calculate the evolution of a dense blob, assuming the dense blobs are magnetically disconnected from their environment, e.g., as in the magnetic flux loop model of Soker (2004) where heat conduction occurs only within the cold blob. Readers interested only in the basic scenario, results, and predictions, can skip § 3 and go directly from § 2 to our summary in § 4.

2. The Proposed Cold Feedback Scenario

The cold feedback scenario entails a cycle in the cooling/accretion activity. We suggest that this cycle starts with a major AGN outburst, which injects a huge amount of energy into the ICM. This event triggers the formation of a wealth of dense blobs. It is important to realize that these blobs are non-linear perturbations of the ICM, and may be distributed with a wide spectrum of densities. These blobs are denser than the surrounding medium, and fall to the black hole. If these blobs have an initial angular momentum, they do not freely fall, but may form an accretion disc. As we shall demonstrate (§ 3.1), for the expected angular momentum distribution this is not a relevant complication. The dense and cool blobs are the fastest to fall, and therefore are removed first from the ICM. The accretion history may be difficult to predict, since it depends on several factors. For most of the time the small blobs may accrete approximately in a steady-state, but some processes may intervene to modify this. For instance if most of the blobs are formed at the same distance to the AGN with a sharply peaked density spectrum, most of the blobs might accrete simultaneously. Moreover, an accretion disc may form and undergo some kind of instability. In either case, a sudden “catastrophic” accretion episode on the black hole is expected, resulting in a new AGN outburst, which restarts the cycle with a fresh injection of blobs. We examine here below some details of the suggested process.

In this duty cycle some of the gas cools to low temperatures ($\lesssim 10^4$K) before the next major heating, while the rest is heated back to a relatively high temperature. We differ from
many previous models in that in the moderate CF model a substantial fraction of the ICM gas cools to low temperatures. The presence of a detectable amount of gas cooling below X-ray emitting temperatures is a prediction of this model. Indeed, in the CF cluster Abell 2597 both extreme-UV and X-ray observations indicate a mass cooling rate of $\sim 100 \ M_\odot \ yr^{-1}$, which is $\sim 0.2$ of the value quoted in the past based on ROSAT X-ray observations (see the discussion in Morris & Fabian 2005). In the CF cluster Abell 2029, Clarke et al. (2004) find a substantial amount of gas at a temperature of $\approx 10^6$K; a CF model gives a mass cooling rate of $\sim 50 \ M_\odot \ yr^{-1}$.

The AGN outburst interacts in a very complicated fashion with the ICM (e.g. Begelman 2004), e.g. it heats and inflates radio bubbles, which rise buoyantly in the ICM. The ICM itself is displaced and thickened by the rising bubbles, as shown by the enhanced X-ray brightness (e.g. Blanton et al. 2001). A non-homogeneous thickening may result in the formation of a multi-phase gas inside or near the radio lobes, which harbor relatively strong magnetic fields, up to few tens $\mu G$.

The magnetic field inside the radio lobes is related to a fundamental issue for the cold feedback model, namely the efficiency of thermal conduction. A highly efficient conduction would evaporate the cold gas blobs before they can accrete on the AGN, and is therefore incompatible with the cold feedback model.

It is well-known that magnetic fields are able to suppress thermal conduction, and that the degree of suppression strongly depends on their topology (see e.g. the discussion in Narayan & Medvedev 2001).

For our purposes, we assume that there is essentially no heat conduction between the over-dense blobs and their surroundings, i.e., the over-dense blobs are magnetically disconnected from their environment, as in the model of Soker (2004). Effectively, for our blobs of radius $a \sim 10 - 100 \ pc$, we find from Figure 3 of Nipoti & Binney (2004) that the effective heat conduction should be $\lesssim 0.001$ times the Spitzer (1956) value in order for the blobs not to be evaporated.

This suppression factor is somewhat high, but consistent with some recent observations. Molendi (2002), e.g., finds evidence of a gas component cooler than the ambient gas inside the radio lobes of M87. The very existence of these cold pockets led Molendi (2002) to estimate a conduction suppression factor $\lesssim 0.01$ with respect to the nominal Spitzer (1956) value.

The conclusion is that the radio lobes, permeated by magnetic fields, may be “safe corridors” where the blobs may accrete on the AGN without being evaporated by conduction.
The coexistence of gas phases at different temperatures is not exclusive of M87. In their analysis of the group NGC 5044, Buote et al. (2003) find evidence for a moderate multi-phaseness. Their data are well fit by a two temperature model, a relatively cool component with \( T_{\text{cool}} \approx 0.7 \) keV and a hot component at \( T_{\text{hot}} \approx 1.4 \) keV. These temperatures seem to coexist in the inner \( \approx 30 \) kpc, with the cooler component dominating in the inner \( \approx 10 \) kpc. The cool component has a sizeable filling factor: \( f_B \approx 0.5 \) for \( r \lesssim 25 \) kpc and \( f_B \approx 0.1 \) at \( r \gtrsim 30 \) kpc.

We therefore consider the inner region, \( r \lesssim 5 - 30 \) kpc of cooling-flow clusters to posses an ICM with non-linear perturbations, i.e. dense blobs spread within it. The fate of a blob depends on the relative magnitudes of its cooling time \( t_{\text{cool}} \), the time interval to the next AGN heating event, and on the time \( t_{\text{fall}} \) the blob takes to accrete on the central black hole. The time scale \( t_{\text{fall}} \) depends on several factors. The first is the blob’s over-density with respect to the ambient gas. The second is the blob’s angular momentum: a high angular momentum prevents the blob from approaching the black hole altogether. We shall discuss the issue of angular momentum more fully in § 3.1: for the time being we assume that the content of angular momentum of a blob is small. Under this hypothesis, the relation between the time scales \( t_{\text{cool}} \) and \( t_{\text{fall}} \) mainly depends on the blob’s density \( \rho \) relative to that of the ambient medium \( \rho_a \). A sinking blob has \( t_{\text{cool}} \lesssim t_{\text{fall}} \), with the extreme case of \( t_{\text{cool}} \ll t_{\text{fall}} \), where the blob significantly cools down on a very short time scale, and almost free-falls to the center. If \( t_{\text{fall}} < t_{\text{cool}} \) the blob would not cool much, and will reach a position where it is as dense as the ambient gas; if \( t_{\text{cool}} \approx t_{\text{fall}} \) the blob sinks fairly slowly, owing to its small over-density.

In this way, the ICM efficiently disposes of the cooler phases, i.e., the highly non-linear over-dense perturbations.

If a blob cools isobarically in pressure equilibrium with its surrounding, its cooling time scales as \( t_{\text{cool}} \propto \rho^{-2} \). Later on, when the blob temperature drops below \( T \approx 0.1 \) keV, the sound waves become too slow to keep up with the outer pressure, and cooling occurs isochorically: \( t_{\text{cool}} \propto \rho^{-1} \) (e.g. Burkert & Lin 2000). In either case the denser — and the cooler — is a perturbation, the faster it cools, and the more efficiently it is removed from the ICM.

The discussion above leads us to the proposed scenario. Non-linear over-dense blobs of gas, \( \delta \rho/\rho_a \gtrsim 2 \), i.e. \( \rho/\rho_a \gtrsim 3 \), cool on short time scales such that they are removed from the ICM before the next major AGN heating event. Some of these blobs cool and sink toward the central black hole. Other non-linear perturbations may form stars, as is inferred in some CF clusters, e.g., in the CF cluster A1068 the cooling rate within \( r \approx 30 \) kpc is about equal to the star formation rate there (Wise et al. 2004; McNamara et al. 2004). The dense blobs
that sink to the center feed the AGN. The feedback is with the entire cool inner region, and not only with the gas close to the black hole. Any over-cooling taking place in the inner region, where the temperature profile is flat, will lead to many small and dense blobs, which feed the AGN.

We assume that the cold blobs are magnetically disconnected from the surrounding, so that most of them can survive long enough to be delivered to the central AGN. Indeed, while some blobs might certainly be evaporated, yet the inability of thermal conduction to re-heat all the cold gas is testified by the presence of sizeable amounts of molecular gas in the central few kpc of some clusters (NGC 1275/Perseus: Inoue et al. 1996). In a recent paper Wilman et al. (2005) resolved a ring of molecular gas with radius of 50 pc from the center of NGC 1275 at the center of the Perseus cluster. Adding the presence of molecular gas at distances of up to \( \sim 10 \) kpc, their finding shows that cold gas originating at large distance in the cluster, but still within the low temperature region, can feed the central black hole.

This ends our simple demonstration that highly over-dense blobs can be accreted to the central black hole before the next major AGN heating event. More detailed calculations are presented in the next Section; readers not interested in them may skip directly to the last Section.

3. Nonlinear Evolution of the Blobs

In this Section we consider the evolution of a single blob. Let \( V \) and \( S \) be its volume and cross-section, respectively, and \( \rho \) its mass density. The blob is subjected to the overall gravitational acceleration \( g \), to the hydrostatic buoyancy force and to the drag force. Its equation of motion may be written (e.g. Loewenstein 1989; Kaiser 2003)

\[
\rho V \frac{dv}{dt} = g (\rho - \rho_a) V - C S \rho_a v v,
\]

where \( \rho_a \) is the mass density of the ambient gas and \( C \) is the dimensionless drag coefficient. For \( C \) we shall adopt the value \( C \approx 0.75 \) used by Kaiser (2003) and derived from the numerical simulations by Churazov et al. (2001). It will be convenient to express Equation (1) in terms of the blob over-density \( \delta \) with respect to the surrounding medium

\[
\delta \equiv \frac{\rho - \rho_a}{\rho_a}.
\]

Eliminating \( \rho \) from Equation (1), we may rewrite it as

\[
\frac{dv}{dt} = g \frac{\delta}{1 + \delta} - \frac{3C}{8} \frac{v}{1 + \delta} \frac{v}{a}.
\]
where we have calculated $V$ and $S$ assuming that the blob is a sphere of radius $a$. If $a$, $\delta$ and $g$ are constants, the last equation is easily integrated:

$$v = v_t \frac{e^{t/\tau} - 1}{e^{t/\tau} + 1},$$

where

$$\tau = \left( \frac{2a}{3Cg} \right)^{1/2} \frac{1 + \delta}{\delta^{1/2}} \approx 5 \times 10^6 \left( \frac{a}{100 \text{ pc}} \right)^{1/2} \left( \frac{g}{10^{-8} \text{ cm s}^{-2}} \right)^{-1/2} \frac{1 + \delta}{\delta^{1/2}} \text{ yr} \quad (5)$$

is the characteristic time in which the blob attains its terminal velocity

$$v_t = \left( \frac{8}{3C} g a \delta \right)^{1/2} \approx 33 \left( \frac{a}{100 \text{ pc}} \right)^{1/2} \left( \frac{g}{10^{-8} \text{ cm s}^{-2}} \right)^{1/2} \delta^{1/2} \text{ km s}^{-1}. \quad (6)$$

Note that if the gravitational acceleration $g$ is relatively small, then a blob will take more time $\tau \propto g^{-1/2}$ to accelerate to a smaller terminal velocity $v_t \propto g^{1/2}$. This explains why in Figure 2 the terminal-velocity fall time drops below the free-fall time at small radii, where $g$ is small; the blobs starting there take a comparatively long time to accelerate.

In order to calculate the evolution of a blob, we need three further equations. The first equation simply relates the position and the velocity of the blob:

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}. \quad (7)$$

The second equation is the blob mass conservation

$$\rho a^3 = \text{constant}. \quad (8)$$

The last equation provides the blob energy balance. If $E$ is the blob total internal energy and $P$ is its pressure, the combined first and second law of thermodynamics

$$P \, dS = -n_e n_H \Lambda V \, dt \quad (9)$$

yield the variation of the blob entropy

$$S = \frac{3}{2} \ln \left( \frac{P}{n_e^{5/3}} \right) \quad (10)$$

under the radiative loss of the amount of energy $dQ = n_e n_H \Lambda V \, dt$ in the time interval $dt$. This loss depends on the cooling function $\Lambda(T)$ and on the electron and hydrogen densities $n_e$ and $n_H$ of the gas inside the blob. We now suppose that the blob is instant by instant in pressure equilibrium with the ambient gas, whose pressure is $P_a$:

$$P \equiv P_a. \quad (11)$$
This assumption holds until the blob sound crossing time is short in comparison to the blob cooling time, and therefore the acoustic waves can efficiently pressurize the blob (see e.g. Burkert & Lin 2000). We shall have to check a posteriori the validity of this hypothesis. We further assume that the ambient gas is in hydrostatic equilibrium. With these hypotheses, the pressure variation $dP$ experienced by the blob is only due to its drift across the cluster gravitational potential $\Phi$:

$$dP \equiv dP_a = -\rho_a \, d\Phi = -\rho_a \, dt \, v \cdot \nabla \Phi$$  \hspace{1cm} (12)

With the help of this equation we differentiate Equation (10) and plug the result into Equation (9): after some straightforward algebra we obtain

$$\frac{d \ln n}{dt} = \frac{n_a}{n} \, \frac{g \cdot v}{c_s^2} + \frac{2}{5} \, \frac{n_e}{n} \frac{n_H}{n} \frac{\Lambda(T)}{kT},$$  \hspace{1cm} (13)

where $c_s^2 = 5kT/3\mu m_p$ is the (squared) adiabatic sound speed inside the blob, $k$ is the Boltzmann constant, $g = -\nabla \Phi$ is the gravitational acceleration and $n_a = \rho_a/\mu m_p$ is the ambient total number particle density.

Using the definition 2, we may rewrite Equation (13) in terms of the over-density $\delta$:

$$\frac{d\delta}{dt} = \frac{g \cdot v}{c_s^2} + \frac{2}{5} \mu_e \mu_H n_a (1 + \delta)^2 \frac{\Lambda(T)}{kT} - (1 + \delta) \, v \cdot \nabla \ln n_a,$$  \hspace{1cm} (14)

where the coefficients $\mu_e$ and $\mu_H$ are defined by $\mu_e \equiv n_e/n$ and $\mu_H \equiv n_H/n$. The first term on the right-hand side of Equation (14) is the adiabatic compression owing to gravity; the second term describes the blob thickening at constant pressure as its temperature cools on account of the radiative losses; the last term is proportional to the slope on the background ambient gas, and is the reduction of the density contrast owing to the denser layers the blob sinks through. This last term is important in what it may hamper the growth of large over-densities in a strongly stratified ambient gas.

Equations (2, 3, 7, 8) and (14) provide a closed system of equations for the blob evolution, once we have specified the distribution of the ambient density $\rho_a$, the cluster gravitational potential $\Phi$ and a suitable set of initial conditions.

It is important to remark that in our derivation we have assumed that the ambient gas behaves as a static background without cooling. This approximation is justified as far as the blobs cool faster than the ambient gas. This assumption certainly holds if the blob remains over-dense, but may break down if $|\delta| \ll 1$ (Loewenstein 1989). Therefore, our results concerning blobs only slightly over-dense with respect to their surroundings require some care.
We apply the above equations to calculate the evolution of a blob in the cool core cluster M87. We choose this cluster because we know to a good approximation its gravitational mass, its temperature and density structure; besides, the central AGN here shows a complex interaction with the surrounding medium (e.g. Belsole et al. 2001; Molendi 2002). Where necessary, however, we shall extend our considerations beyond the particular features of M87. For the ambient density $\rho_a$, temperature $T_a$ and gravitational acceleration $g$ of M87 we assume the functional forms provided by Ghizzardi et al. (2004). We have employed the cooling function $\Lambda(T)$ given by Sutherland & Dopita (1993) for a solar-abundance thermal plasma. As far as the initial conditions, we assume that the blob has an initial over-density $\delta_0 > 0$, and is released from rest at the distance $r_0$ from the cluster center. We have solved numerically the system of Equations( 2, 3, 8, 14, 7) and these initial conditions with a step-adaptive fifth-order Runge-Kutta algorithm (Press et al. 1992).

Figure 3 plots the over-density evolution of some blobs, which only differ in their initial over-densities $\delta_0$; all of them have initial size $a_0 = 100$ pc, and have been released from rest at $r_0 = 20$ kpc from the cluster center. It is apparent that the qualitative behavior of the solution critically depends on $\delta_0$: if $\delta_0$ is below a critical threshold $\delta_C$ the over-density decreases, else if $\delta_0 > \delta_C$ it diverges. A blob with $\delta_0 < \delta_C$ soon evolves to $|\delta| \ll 1$, becoming hardly distinguishable from the ambient background gas, whose cooling and bulk velocity have been neglected in our approximation. Therefore, our result concerning these blobs may be looked at suspiciously, and for consistency we conservatively disregard their contribution to the feeding of the central black hole. These blobs are most likely to get pinned by the drag force to the average bulk motion of the ICM, and their successive evolution would require a more detailed analysis (see e.g. Nulsen 1986; Loewenstein 1989). To our concern, since the background medium has been assumed static, we may assume that such blobs have been thermally stabilized.

The existence of a critical over-density $\delta_C$ is not difficult to understand (Balbus & Soker 1989; Loewenstein 1989; Tribble 1991). As a blob sinks, it crosses thicker and thicker layers of ambient gas, and its density contrast $\delta$ with respect to its surroundings would reduce. If this effect is not contrasted, the blob would stop its fall at an equilibrium distance from the center where its density is the same as the ambient gas, i.e. $\delta = 0$. The only way to overcome this effect is to cool fast. Since this process is isobaric, the blob density is enhanced, and the blob might reach the cluster center before its over-density has significantly reduced. Since the blob cools on a time-scale $t_{cool}$, a blob condenses and reaches the center if $t_{cool} \lesssim t_{fall}$. The equality between these time-scales defines the critical value $\delta_C$ for the blob over-density. This explains the qualitative behavior shown in Figure 3.

Figure 4 plots the evolution of the position and velocity of two blobs, one slightly over-
critical ($\delta \gtrsim \delta_C$) and one slightly under-critical ($\delta \lesssim \delta_C$). In both cases, the blob is initially in free-fall, but after a transition time of the order of few $\tau$—given by Equation (5)—the drag limits the velocity to the terminal value $v_t$ given by Equation (6). An under-critical blob attains with non-zero velocity its equilibrium position (where $\delta = 0$), which is overshot. The blob starts a series of oscillations about this position, which are quickly damped by the drag force (e.g., Balbus & Soker 1989; Loewenstein 1989; Tribble 1991). Eventually, the blob cools and becomes pinned to the bulk of the ambient gas. An over-critical blob is always denser than the surrounding gas, the drag plays a minor role, and the blob nearly free falls all the way down to the center.

The evolution of the blob temperature is presented in Figure 5. The under-critical blob has an initial radiative cooling time of $t_{cool} \approx 1.8 \times 10^8$ yr, and before this time the blob temperature does not change much. For such a blob the gravitational heating is more important than the radiative losses: the blob is heated up to the local ambient temperature, and becomes thermally stable. In a denser blob, on the contrary, the gravitational heating is insufficient to overcome the radiative cooling. After a time $t_{cool} \approx 1.2 \times 10^8$ yr, therefore, the blob cools very fast and almost free falls to the cluster center.

The initial release radius is important for the nonlinear development of a blob. Figure 6 plots the over-density evolution for some blobs with the same intrinsic properties, but released at different radii. It is apparent that the farthest blobs never develop large over-densities. This fact is related to the density profile of the ambient gas (shown in Figure 7). For a fixed over-density, a far blob has a relatively small density—in absolute terms—and as it sinks it soon gets embedded in the denser ambient gas: the over-density reduces and the blob is thermally stabilized. The situation is different if the blob has been released at a small distance to the center, where the ambient density profile is flat (Figure 7). As in our discussion following Equation (14), in this case the blob’s over-density growth is not hindered by the ambient gas gradient, and even a moderate initial over-density may evolve towards larger values. It is worth to stress the importance of this density stratification effect, as it relates the size of the central density plateau to the amount of allowed cold feedback in our model. Indeed, in the density plateau essentially all the over-densities may evolve to the non-linear regime, cool down and accrete on the AGN to provide the heating feedback. On the other hand, only the largest over-densities born outside the central plateau may evolve this way. In the case of M87 this plateau extends out only to $r \approx 5$ kpc (Figure 7): more typical cooling flow clusters, like A2052, have $r \approx 30$ kpc (Blanton et al. 2001); this allows the feedback between the AGN and a larger fraction of the cluster’s gas.

Following Tribble (1991) we estimate the critical over-density $\delta_C$ as a function of the other parameters. Equation (14) evaluated at $t = 0$ with our initial conditions yields $\delta > 0$. 

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The evolution of the blob temperature is presented in Figure 5. The under-critical blob has an initial radiative cooling time of $t_{cool} \approx 1.8 \times 10^8$ yr, and before this time the blob temperature does not change much. For such a blob the gravitational heating is more important than the radiative losses: the blob is heated up to the local ambient temperature, and becomes thermally stable. In a denser blob, on the contrary, the gravitational heating is insufficient to overcome the radiative cooling. After a time $t_{cool} \approx 1.2 \times 10^8$ yr, therefore, the blob cools very fast and almost free falls to the cluster center.

The initial release radius is important for the nonlinear development of a blob. Figure 6 plots the over-density evolution for some blobs with the same intrinsic properties, but released at different radii. It is apparent that the farthest blobs never develop large over-densities. This fact is related to the density profile of the ambient gas (shown in Figure 7). For a fixed over-density, a far blob has a relatively small density—in absolute terms—and as it sinks it soon gets embedded in the denser ambient gas: the over-density reduces and the blob is thermally stabilized. The situation is different if the blob has been released at a small distance to the center, where the ambient density profile is flat (Figure 7). As in our discussion following Equation (14), in this case the blob’s over-density growth is not hindered by the ambient gas gradient, and even a moderate initial over-density may evolve towards larger values. It is worth to stress the importance of this density stratification effect, as it relates the size of the central density plateau to the amount of allowed cold feedback in our model. Indeed, in the density plateau essentially all the over-densities may evolve to the non-linear regime, cool down and accrete on the AGN to provide the heating feedback. On the other hand, only the largest over-densities born outside the central plateau may evolve this way. In the case of M87 this plateau extends out only to $r \approx 5$ kpc (Figure 7): more typical cooling flow clusters, like A2052, have $r \approx 30$ kpc (Blanton et al. 2001); this allows the feedback between the AGN and a larger fraction of the cluster’s gas.

Following Tribble (1991) we estimate the critical over-density $\delta_C$ as a function of the other parameters. Equation (14) evaluated at $t = 0$ with our initial conditions yields $\delta > 0$. 

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If the blob is stable, this initial trend must be reversed, and there must be an instant \( t = \tilde{t} \) for which \( \dot{\delta}(t = \tilde{t}) = 0 \). If the blob is unstable, on the other hand, \( \dot{\delta} > 0 \) all the way. Equation (14) evaluated at \( t = \tilde{t} \) may be written

\[
(1 + \delta)^2 = \left| \frac{\mathcal{g} r}{c_a^2} + \frac{d \ln n_a}{d \ln r} \right| t_a \frac{v A(T_a)}{\Lambda(T)}.
\]

where \( c_a^2 = 5 k T_a/3 \mu m_p \) and \( t_a = 5 k T_a/2 \mu_e \mu_H n_a \Lambda(T_a) \) are respectively the sound speed and the isobaric cooling time of the ambient gas; in pressure equilibrium the blob temperature \( T \) and the ambient gas temperature \( T_a \) are related by \( T = T_a/(1 + \delta) \). We must evaluate the right-hand side of Equation (15) at \( t = \tilde{t} \). If \( \tau \ll \tilde{t} \) (where \( \tau \) is given by Equation 5), we may substitute \( v \) with its terminal value given by Equation (6). If we suppose that the other quantities on the right-hand side of Equation (15) are not too different from their initial values, the last equation may be rewritten

\[
\frac{(1 + \delta)^2}{\delta^{1/2}} = t_a \left( \frac{8}{3 C g r} \right)^{1/2} \left| \frac{\mathcal{g} r}{c_a^2} + \frac{d \ln n_a}{d \ln r} \right| \left( \frac{a}{r} \right)^{1/2} \frac{A(T_a)}{\Lambda(T)}.
\]

The value of \( \delta \) provided by this equation is the critical threshold \( \delta_C \) between a thermally stable and a thermally unstable blob. In the non-linear regime \( \delta \gg 1 \), by approximating the cooling function by a power law \( \Lambda \propto T^\alpha \), we obtain \( \delta_C \propto a^{1/(3-2\alpha)} \), which coincides with the expression given by Tribble (1991) if \( \alpha = 1/2 \), i.e. if thermal Bremsstrahlung is the chief coolant. Figure 8 plots \( \delta_C \) as a function of \( a \) for different release radii \( r \) (see also the Figure 1 of Tribble 1991). In the upper branch of the \( \delta_C - a \) plot we find \( \delta_C \propto a^{1/3} \) owing to the weak dependence of the cooling function on \( T \) in the temperature range typical of M87. It is interesting to note that the over-density required for a blob to be unstable is moderate: \( \delta_C \approx 0.8 - 3 \) if the blobs have sizes \( a \approx 10 - 100 \) pc. From Figure 8 we also note that the blobs have a minimum stable size. As it is seen from Equation (14), the heating term is proportional to the fall velocity, which is \( v_t \propto a^{1/2} \), where \( a \) is the blob radius. On the other hand, the radiative cooling term does not depend on the blob size. Therefore, we expect that if a blob is too small (i.e. below a critical threshold size) the gravitational term is insufficient to heat the blob: the radiative cooling prevails altogether, and the blob’s over-density increases monotonically with time.

Before ending this Section, we must notice that for the very over-dense blobs the assumption of isobaric cooling may break down, since the sound waves inside the blob become too slow to pressurize it against the ambient gas. The evolution is now isochoric, and the blob density grows less than the amount predicted by our isobaric model (e.g. Burkert & Lin 2000). If this transition occurs before the blob has reached the inner density plateau, its density could be not high enough to avoid the thermal stabilization. In a more typical situation the transition to the isochoric regime occurs when the blob has already reached
a sizeable over-density, so this effect is generally not very important, and the qualitative conclusions drawn under the isobaric assumption are not altered; the quantitative results may be different by small amounts for the small blobs.

### 3.1. The Issue of Angular Momentum

In our qualitative sketch we have so far omitted any reference to the angular momentum of the infalling material. A too large angular momentum might prevent the flow from approaching the central black hole: the flux would merely stagnate, cool down and condense in filaments or stars. The AGN fueling is cut off altogether, which makes the feedback impossible (see Cowie et al. 1980, for a thorough analysis of the thermal instability of a high angular momentum flow). The existence of a circumnuclear disc around M87 shows that the flux possesses an amount of angular momentum, so the question is whether this angular momentum is high enough to jeopardize the feedback. We argue it is not.

First of all, as we will discuss below, the blobs are expected to form and accrete only in a region of the same extension as the inner gas density plateau ($5 - 30$ kpc). The distribution of angular momentum at larger radii is immaterial to the present discussion, and in the rest of this Section we only refer to the ICM within this central region.

The non-linear perturbation spectrum may stem directly from ICM disturbances driven by an early AGN activity, but also from galaxies mass-stripping (Soker et al. 1991). Since the galaxies do not have an ordered bulk motion, the blobs stripped from them are also unlikely to organize in an ordered flow with high net angular momentum. Besides, if a blob from a galaxy is injected with a high angular momentum, it is likely to loose most of it on account of its friction with the surrounding ICM. Therefore, even if a circular flow like a disc may form, it cannot be very large, as the example of M87 shows.

A flow with specific angular momentum $l$ circularizes at the radius $R_{\text{circ}} \approx \frac{l^2}{GM_{\text{BH}}}$, being $M_{\text{BH}}$ the central black hole mass. We estimate $R_{\text{circ}}$ as follows. Let us consider a cold blob of radius $a$ initially at the distance $R$ from the central black hole. The balance between gravitational attraction and the friction force quickly brings the blob to the terminal velocity $v_t$ given by Equation (6). Then, its orbital angular momentum is about $l \approx v_t R$ and Equation (6) gives $R_{\text{circ}} \approx a \delta$. The circularization radius is expected of the same size as the non-linear density blobs, since $\delta$ is expected to be of the order of a few. Rough as it is, this estimate is in fair agreement with the actual size of the circumnuclear disc of M87 $R_d \approx 10^2$ pc (Harms et al. 1994; Ford et al. 1994, see also below). We note that, statistically, many of the cold blobs will start with very low angular momentum $l \ll v_t R$. These will be
accreted directly to the black hole vicinity.

As an example we consider the CF cluster M87. First we note that M87 has a large Keplerian disc at its center. Harms et al. (1994) estimate the black hole mass to be $M_{\text{BH}} \approx 2.4 \times 10^9 \, M_\odot$ from their observation of a disc with a radius of $\approx 20 \, \text{pc}$. Their optical HST image shows the disc to be $\approx 3.5$ times larger, i.e., $R_d \approx 70 \, \text{pc}$ (Ford et al. 1994).

The cooler gas at the center of the Virgo cluster is at a temperature of $\approx 1 \, \text{keV}$, which implies for the above black hole mass a Bondi accretion radius of $\approx 80 \, \text{pc}$ (Churazov et al. 2002) \(^1\). A Bondi accretion radius as large as the disc around the black hole of M87 further suggests that the simple Bondi accretion flow (Churazov et al. 2002; Nulsen 2004) does not hold; the accreted material has a larger angular momentum, and may come from much larger radii.

To further elaborate on the proposed model, we plot in Figure 2 the free fall time in M87, as well as some relevant cooling times. In that Figure the crosses mark the cooling time of the ambient gas at several radii; the asterisks, the filled and the empty triangles are the cooling times of blobs with local over-densities with respect to the ambient of $\delta \rho/\rho_a = 1$, $\delta \rho/\rho_a = 3$ and $\delta \rho/\rho_a = 10$, respectively. Pressure equilibrium is assumed between the blobs and the ambient medium: since the blobs temperature is always above $T \approx 0.1 \, \text{keV}$, the sound waves are fast enough to ensure this equilibrium. The solid line represents the free fall time in the gravitational potential of M87; the dashed line is the fall time of a blob of radius $a = 300 \, \text{pc}$ and over-density $\delta \rho/\rho_a = 1$. In this case, the fall velocity is given by the balance between the gravitational acceleration and the drag force. The actual fall time lies in between these time-scales (see the next Section). The blobs for which $t_{\text{cool}} \approx t_{\text{fall}}$ have moderate over-densities $\delta \rho/\rho_a = 1 - 2$, and therefore temperatures of $1/2 - 1/3$ of the ambient gas. This value is not far from the values found by Molendi (2002) for the cool component in M87, and in good agreement with the cool component of NGC 5044 (Buote et al. 2003).

An important final remark is in order. As Nulsen (1986) pointed out, in the absence of a cohesive force a blob would be torn apart by the ram pressure in a characteristic time

$$t_d = \frac{a}{v_t} \left( \frac{\rho}{\rho_a} \right)^{1/2} \approx 10^7 \left( \frac{a}{100 \, \text{pc}} \right) \left( \frac{v_t}{10 \, \text{km s}^{-1}} \right)^{-1} \left( \frac{\rho}{\rho_a} \right)^{1/2} \, \text{yr}, \quad (17)$$

where $a$ is the blob’s radius and $v_t$ its terminal velocity, given by Equation (6). This may be considerably shorter than the time taken by the blob to fall to the center, and we must

\(^1\)Macchetto et al. (1997) estimate a somewhat larger mass for the central black hole in M87, namely $M_{\text{BH}} \approx (3.2 \pm 0.9) \times 10^9 \, M_\odot$. In this case, the Bondi accretion radius is slightly larger, about 100 pc.
therefore assume that some kind of cohesive force (like a magnetic tension) is at work to prevent the blob disruption. In any case, the proposed scenario works also for the smaller blobs forming from the fragmentation of a larger blob.

4. Summary

This paper deals with heating the intra-cluster medium (ICM) in galactic and cluster cooling flows (CF) by an active galactic nucleus (AGN) sitting at the cooling flow center. As was shown by many papers, the heating is most likely to take place via a feedback mechanism, where the ICM cooling enhances the AGN activity, which in turn heats the ICM and quenches the cooling flow.

Most previous papers (see § 1), assume that the central black hole accretes mass only from the ICM in its immediate neighborhood, basically via a Bondi-like accretion flow. In these models the feedback occurs as the ICM cools to a temperature of about 1 keV, and the ICM does not need to cool to low temperatures. We term these *hot-feedback models*. We examined three papers based on hot-feedback heating (Omma & Binney 2004; Ruszkowski & Begelman 2002; Hoeft & Brüggen 2004). We argued that the models worked out in these papers do not fit the general temperature profiles of CFs (Figure 1), and/or require fine tuning. We further argued that more generally, in the Bondi-type accretion flow of hot gas, the accretion rate is determined mainly by the conditions very close to the central black hole, and that this may result in unstable cooling of the regions further out.

We therefore proposed (§ 2) that the feedback occurs with the entire cool inner region, $r \lesssim 5 - 30$ kpc, in what we term a *cold-feedback model*. In the proposed scenario non-linear over-dense ($\delta \rho / \rho_a \gtrsim 2$, or $\rho / \rho_a \gtrsim 3$) blobs of gas cool fast and are removed from the ICM before the next major AGN heating event in their region. It is important to note that an AGN burst can take place and heat other regions, since the jets and/or bubbles may expand in other directions as well. The typical interval between such heating events at a specific region is $\approx 10^8$ yr. Some of these blobs cool and sink toward the central black hole, while others may form stars and cold molecular clouds.

Four conditions should be met in the inner region participating in the feedback heating.

1. In order for the blob not to reach a point where its density equals the ambient density as it sinks, the ICM density profile should not be too steep. This implies that the relevant dense blobs form in the cluster core, where the density profile is shallow. In the quantitative example used here for M87 this is the region $r \lesssim 5$ kpc, while in more typical clusters it is larger, e.g., $r \lesssim 30$ kpc in A2052. We note that the lower
segment of magnetic flux loops can be prevented from reaching the stabilizing point by the upward force of the magnetic tension inside the loop (Soker 2004). Therefore, some perturbations can be formed at large distances, where density profile is steep, and still cool to low temperature and feed the central black hole.

2. Non-linear perturbations are required. These presumably formed mainly by previous AGN activity, e.g. jets and radio lobes.

3. The cooling rate of these non-linear perturbations is short relative to few times the typical interval between successive AGN outbursts.

4. The blobs must not be evaporated by thermal conduction before they are delivered to the AGN. This requires a strong suppression of thermal conduction, which may be done by the magnetic fields observed in the radio lobes in several cooling flow clusters.

The first and the third condition, which are not completely independent of each other, require that the initial ICM cools by a factor of a few before the feedback starts operating, and the second condition requires that the inner region must be disturbed.

Finally, in § 3 we have calculated the falling time and cooling time of dense blobs. The results have then been applied to the cooling-flow cluster M87.

The cold-feedback model has the following implications and predictions (for more details and references see § 1).

1. In the cold-feedback models the optical filaments observed in many CF-clusters and the cooler molecular gas detected via CO observations come from cooling ICM (with some amount possibly from stripping from galaxies).

2. In cold feedback models, some X-ray emission from gas at temperatures \( \lesssim 10^7 \) K is predicted to exist, much more than in many other AGN heating models. but at a level more than an order of magnitude below that in old versions of the CF model, but compatible with the moderate CF model. We stress that in the cold-feedback heating, cooling flows do exist. Such gas cooling to below X-ray emitting temperatures was found recently in two CF clusters (Abell 2597: Morris & Fabian 2005; Abell 2029: Clarke et al. 2004).

3. The feeding of the central black hole with cold gas in the cold feedback models makes the process similar in some aspects to that of AGN in spiral galaxies. Therefore, the outflow can be similar (Soker & Pizzolato 2005).
4. It is possible that in the cold feedback model a substantial fraction of gas that cooled to low temperatures and was accreted to the accretion disc around the central black hole, is injected back to the ICM at non-relativistic velocities (Soker & Pizzolato 2005).

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Fig. 1.— The temperature profiles for a sample of Clusters observed with Chandra (C) or XMM-Newton (X): A478 (Sun et al. 2003b, C), A496 (Tamura et al. 2001a, X), A1068 (Wise et al. 2004, C), A1795 (Ettori et al. 2002, C), A1835 (Schmidt et al. 2001, C), A1991 (Sharma et al. 2004, C), A2029 (Lewis et al. 2003, C), A2052 (Blanton et al. 2001, C), A2199 (Johnstone et al. 2002, C), A4059 (Choi et al. 2004, C), M87 (Ghizzardi et al. 2004, C+X), Perseus (Schmidt et al. 2002, C), and finally the groups NGC 1550 (Sun et al. 2003a, C) and NGC 5044 (Buote et al. 2003, C). For the sake of readability, the clusters with an approximatively power-law temperature profiles and those with a central temperature floor have been plotted in two different panels (upper and lower, respectively). Where necessary, the radii from the original papers have been corrected for a cosmology with $H_0 = 70$ km/s/Mpc.
Fig. 2.— A comparison between the free fall time, the terminal-velocity fall time and the isobaric cooling times for M87. The free fall time scale $t_{\text{ff}} \approx (2r/g)^{1/2}$ is shown by the solid line; the terminal-velocity fall times $t_t \approx r/v_t$ refer to a blob of radius $a = 300$ pc and an over-density $\delta \rho/\rho_a = 1$ (dashed line) and $\delta \rho/\rho_a = 3$ (dotted line). The gravitational acceleration has been calculated from the deprojected density and temperature profiles, with the additional hypothesis of hydrostatic equilibrium. For the data and the procedure, see Ghizzardi et al. (2004), and references therein. The cooling times have been calculated with the deprojected data of Ghizzardi et al. (2004), assuming an average metal abundance of $Z/Z_\odot = 1.0$ (expressed in Anders & Grevesse 1989, solar units), with the cooling function provided by Sutherland & Dopita (1993). The crosses refer to the ambient gas, asterisks to gas blobs with local over-densities $\delta \rho/\rho_a = 1$, the filled and empty triangles respectively to gas blobs with local over-densities $\delta \rho/\rho_a = 3$ and $\delta \rho/\rho_a = 10$ with respect to the ambient gas. Pressure equilibrium between the blobs and the ambient is assumed. The plot does not extend beyond 0.4 kpc because the density and temperature profiles we have taken from Ghizzardi et al. (2004) and that we used to calculate the characteristic time scales do not push to smaller radii.
Fig. 3.— The evolution of blobs with $a_0 = 100$ pc, $r_0 = 20$ kpc, but different initial over-densities $\delta_0 = (\rho - \rho_a)/\rho_a$: $\delta_0 = 1.5$ (solid line), $\delta_0 = 2.0$ (dashed line), $\delta_0 = 2.5$ (dot-dashed line), $\delta_0 = 3.0$ (dotted line). Below a critical threshold $\delta_C$ of the initial over-density $\delta_0$ (in this case $2 < \delta_C < 2.5$), the blobs are stabilized, above this limit they condense in a short (cooling) time. Here and in the following Figures the ICM properties reproduce those of M87, whose data have been derived from Ghizzardi et al. (2004).
Fig. 4.— The distance from the cluster center (upper panel) and the fall velocity (lower panel) of two blobs with initial over-densities $\delta_0 = 2.0$ (solid line) and $\delta_0 = 2.5$ (dashed line). Both the blobs have the initial radius $a_0 = 100$ pc, and have been released from rest at the distance $r_0 = 20$ kpc from the cluster center.
Fig. 5.— The temperature evolution of two blobs with the same characteristics as in Figure 4. Also the line styles are the same as in Figure 4.

Fig. 6.— The evolution of the over-density as a function of the release radius: \( r = 2 \) kpc (solid line), \( r = 5 \) kpc (dashed line), \( r = 10 \) kpc (dash-dotted line), and \( r = 20 \) kpc (dotted line). In all cases the blob has an initial over-density \( \delta_0 = 2 \), and an initial radius \( a_0 = 100 \) pc.
Fig. 7.— The total number particle density (upper panel) and the temperature profile (lower panel) of the ambient gas in M87. These profiles have been borrowed from Ghizzardi et al. (2004).
Fig. 8.— The critical density $\delta_C$ as a function of the blob size $a$ for different initial positions of the blob: $r = 2$ kpc (solid line), $r = 5$ kpc (dashed line), $r = 10$ kpc (dash-dotted line), and $r = 40$ kpc (dotted line). The blobs on the left part of the plot are unstable, and evolve to larger values of $\delta \rho/\rho_a$. The blobs on the right part are stable, and evolve to smaller over-densities. The blobs with sizes in the range $a \approx 10 - 100$ pc are unstable provided that their over-densities are in the moderately non-linear regime $\delta \rho/\rho_a \approx 0.8 - 3$, or that they have been released from very small radii. For consistency with our neglect of the cooling of the ambient gas, we should consider only the part of the curves above the horizontal dashed line, corresponding to a blob cooling time of about 30% of the ambient gas. Blobs with over-densities below this (rather arbitrary) limit have cooling times closer to the ambient gas. The 30% confidence limit has been calculated by comparing the cooling time $t_a \propto T_a/n_a\Lambda(T_a)$ for the ambient gas with the corresponding expression for the blob cooling time $t_b$. We assume pressure equilibrium between the ambient and the blob, so $T_a = (1 + \delta) T_b$, where $T_b$ is the blob temperature. By approximating the cooling function with a power law $\Lambda \propto T^\alpha$, we obtain $t_a/t_b = (1 + \delta)^{2-\alpha}$. Our approximation $t_b \ll t_a$ requires $\delta \gg 1$. If we demand $t_b < 30\% t_a$ and take $\alpha \approx 0$ in the temperature range considered here, we obtain the plotted confidence limit $\delta > 0.8$. 

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