Influence of the signal light on the transient optical properties of a four-level EIT medium

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General formulae for the transient evolution of the susceptibility (absorption) induced by the quantum interference effect in a four-level N-type EIT medium is presented. The influence of the signal light on the transient susceptibility for the probe beam is studied for two typical cases when the strength of the coupling beam is much greater or less than that of the signal field. An interesting level reciprocity relationship between these two cases is found.

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I. INTRODUCTION

Recently, many theoretical and experimental investigations have shown that the control of phase coherence in a multilevel atomic ensemble will give rise to many novel and striking quantum optical phenomena in the wave propagation of near-resonant light. These phenomena and effects include the atomic coherent population trapping (CPT) [1], laser without inversion [2,3] and electromagnetically induced transparency (EIT) [4]. The idea of the atomic CPT was first suggested by Orriols et al. in 1976 [5], and experimentally demonstrated by Gray et al. in 1978 [6] and by Alzetta et al. in 1979 [7]. In both CPT and EIT, two laser beams are involved in the quantum interference effect. In an EIT medium, if there is only one propagating resonant laser beam, it will be absorbed; but no laser beam will be absorbed when two appropriate laser beams propagate through the same medium (i.e., the opaque medium is turned into a transparent one). In CPT the two fields interacting with the atoms have nearly the same strength and the quantum interference effect arises from both fields. In EIT, however, one of the propagating laser beams is much weaker than the other [4,8]. Thus, the interference effect in EIT can be said to be driven by the stronger one of the two laser beams. This stronger beam is called the coupling beam, and the weaker beam termed the probe beam. Historically, the foundations of EIT were laid by Kocharovskaya and Khanin in 1988 [9] and independently by Harris in 1989 [2]. The first experimental observation of EIT was performed by Harris et al. in 1991 [10]. Besides the CPT explanation (in which the concept of dark state or non-coupling state is essential to the theoretical mechanism), EIT can also be interpreted by using the points of view of the interference between dressed states [11], the multiple routes to excitation (multi-pathway interference) model [12], and some other explanations such as the quantum-field formulation, where one uses Feynman diagram to represent the interfering process in EIT [13]. Due to its unusual quantum coherent characteristics, the discovery of EIT has so far led to many new peculiar effects and phenomena [2,3,14–18], some of which are believed to be useful for the development of new techniques in quantum optics [4].

More recently, the physical EIT effects observed experimentally include the ultraslow light pulse propagation [19,20], superluminal light propagation [21], light storage in atomic vapor [22,23], and atomic ground state cooling [24].

EIT arises from the atomic phase coherence and quantum interference in the atomic transition process. In 1995, Li et al. investigated the transient properties induced by the quantum interference effect [25]. These properties include the absorption for the probe field, transient gain without population inversion and enhancement of dispersion in a three-level EIT atomic medium when the coupling laser is switched on [25]. Recently, Greentree et al. studied the

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turn-on and turn-off dynamics (including the resonant and off-resonant transient behaviors) of EIT in a three-level medium [26]. Note that all the above investigations are associated with the three-level EIT system. Recent evidences have shown that the giant non-linearities (e.g., the enhancement of nonlinear absorption for the probe light) exist in a four-level coherent atomic medium [27,28]. The large nonlinear optical susceptibilities in a four-level EIT medium have some novel applications, e.g., the realization of an absorptive two-photon optical switch, in which a laser pulse controls the absorption of another laser field. In this process, the EIT system absorbs two photons, but not one photon [27]. Due to their novel effects and potential applications, four-level EIT media have attracted attention recently. For example, in 1998 Ling et al. considered the EIT effect in a four-level N-type Doppler broadened media [29]; Harris and Yamamoto described a four-level EIT atomic system that exhibits greatly enhanced third-order susceptibility, but has vanishing linear susceptibility [27]; Based on the suggestion of Harris and Yamamoto [27], Yan et al. reported in 2001 an experimental demonstration of absorptive two-photon switch by constructive quantum interference in a four-level atomic system (such as the cold $^{87}$Rb atoms) [28].

Little has been done in the literature on the investigation of the properties of the transient evolution in a four-level EIT medium. The consideration of the transient properties of EIT media is of importance due to their potential applications such as the absorptive optical switch [25], in which the transmission of a highly absorptive medium is considered. The enhancement of nonlinear absorption for the probe light) exist in a four-level N-type atomic ensemble interacting with three optical fields, namely, the coupling beam, the probe beam and the signal field, whose Rabi frequencies are denoted by $\Omega_c$, $\Omega_p$ and $\Omega_{24}$, respectively. The configuration of the four-level system is depicted in Fig. 1. In such a four-level atomic ensemble of “N” type, levels $|1\rangle$ and $|2\rangle$ are the ground states, $|3\rangle$ and $|4\rangle$ the excited states. The probe, coupling and signal fields couple the level pairs $|1\rangle$-$|3\rangle$, $|2\rangle$-$|3\rangle$, $|2\rangle$-$|4\rangle$, respectively. In the present paper, we assume that these three fields are all in resonance with the corresponding level transitions, i.e., there is no frequency detuning of $\Omega_c$, $\Omega_p$ or $\Omega_{24}$.

In the interaction picture, the Hamiltonian for such a four-level N-type atomic ensemble has the following form (with $\hbar = 1$ for simplicity) [27]

$$H = -\frac{1}{2} (\Omega_p |1\rangle\langle 3| + \Omega_c |2\rangle\langle 3| + \Omega_{24} |2\rangle\langle 4|) + \text{h.c.},$$

where h.c. represents the Hermitian conjugation. The Hamiltonian associated with the level decay is assumed to take the form $H_\Gamma = i\Gamma$ with $\Gamma = \text{diag}[0, -\gamma_{21}/2, -\Gamma_3/2, -\Gamma_4/2]$, where $\Gamma_3$ and $\Gamma_4$ denote the spontaneous decay rates of levels $|3\rangle$ and $|4\rangle$, respectively, and $\gamma_{21}$ the dephasing rate (nonradiative decay rate) of level $|2\rangle$. Here $\gamma_{21}$, $\Gamma_3$ and $\Gamma_4$ are non-negative constants. Thus, in the interaction picture the Schrödinger equation governing the above four-level atomic ensemble is

$$i \frac{\partial}{\partial t} |\psi(t)\rangle = H_{\text{tot}} |\psi(t)\rangle$$

with the total Hamiltonian $H_{\text{tot}} = H + H_\Gamma$. Let $|\psi(t)\rangle = a_1(t)|1\rangle + a_2(t)|2\rangle + a_3(t)|3\rangle + a_4(t)|4\rangle$. Then according to the Schrödinger equation (2), the probability amplitudes $a_i(t)$ ($i = 1, 2, 3, 4$) satisfy the following set of equations

$$\begin{align*}
\dot{a}_1 &= \frac{i}{2} \Omega_p a_3, \\
\dot{a}_2 &= \frac{i}{2} (\Omega_c a_3 + \Omega_{24} a_4) - \frac{\gamma_{21}}{2} a_2, \\
\dot{a}_3 &= \frac{i}{2} (\Omega_p a_2 + \Omega_{24}^* a_1) - \frac{\Gamma_3}{2} a_3, \\
\dot{a}_4 &= \frac{i}{2} \Omega_{24}^* a_2 - \frac{\Gamma_4}{2} a_4,
\end{align*}$$

II. GENERAL TREATMENT FOR THE TRANSIENT EVOLUTION IN THE FOUR-LEVEL EIT MEDIA

In this section, we use a semiclassical theory to derive some general formulae for the transient behaviors of the probability amplitudes of a four-level EIT medium when a signal field is switched on. Consider a four-level atomic ensemble interacting with three optical fields, namely, the coupling beam, the probe beam and the signal field, whose Rabi frequencies are denoted by $\Omega_c$, $\Omega_p$ and $\Omega_{24}$, respectively. The configuration of the four-level system is depicted in Fig. 1. In such a four-level atomic ensemble of “N” type, levels $|1\rangle$ and $|2\rangle$ are the ground states, $|3\rangle$ and $|4\rangle$ the excited states. The probe, coupling and signal fields couple the level pairs $|1\rangle$-$|3\rangle$, $|2\rangle$-$|3\rangle$, $|2\rangle$-$|4\rangle$, respectively. In the present paper, we assume that these three fields are all in resonance with the corresponding level transitions, i.e., there is no frequency detuning of $\Omega_c$, $\Omega_p$ or $\Omega_{24}$.

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$$\begin{align*}
\dot{a}_1 &= \frac{i}{2} \Omega_p a_3, \\
\dot{a}_2 &= \frac{i}{2} (\Omega_c a_3 + \Omega_{24} a_4) - \frac{\gamma_{21}}{2} a_2, \\
\dot{a}_3 &= \frac{i}{2} (\Omega_p a_2 + \Omega_{24}^* a_1) - \frac{\Gamma_3}{2} a_3, \\
\dot{a}_4 &= \frac{i}{2} \Omega_{24}^* a_2 - \frac{\Gamma_4}{2} a_4,
\end{align*}$$

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where dot denotes the derivative of \( a_i(t) \) with respect to time.

In this paper, we study the transient behaviors and properties when the signal field (related to \( \Omega_{24} \)) is present, particularly shortly after it’s switched on. Thus, we should first give the initial conditions for the probability amplitudes when the signal light is absent. It is assumed that the present four-level N-type EIT system can be reduced to a standard three-level \( \Lambda \)-type EIT system (i.e., \( \Omega_{24} = 0 \)) before the signal light is switched on. Note that the coupling laser is strong, monochromatic and present for all time. Therefore, when \(|\Omega_p| \ll |\Omega_c| \) (always true in a standard three-level EIT system and the present four-level EIT system), due to the quantum interference effect, this three-level system will be nearly transparent (no absorption) to the probe beam even if the probe beam is resonant with the level transition \( |1 \rangle \rightarrow |3 \rangle \). In other words, if the intensity of the probe laser beam is sufficiently weak, virtually all the atoms remain in the ground state, i.e., the atomic population in level \( |1 \rangle \) is \( a_1 \approx 1 \) (and \( a_3 \) is nearly vanishing). Furthermore, \( a_1 = (i/2)\Omega_p a_3 \) is negligibly small since both \( \Omega_p \) and \( a_3 \) are small compared with \( \Omega_c \) and \( a_1 \), respectively. Thus, it is reasonable to assume that \( a_1 \approx 1 \) still holds when considering the transient behaviors in the four-level atomic medium induced by the quantum interference effect. In the rest of the paper we set \( a_1 = 1 \). The equations for the probability amplitudes \( a_2, a_3 \) and \( a_4 \) in (3) can be rewritten in the following matrix form

\[
\frac{\partial}{\partial t} \begin{pmatrix} a_2(t) \\ a_3(t) \\ a_4(t) \end{pmatrix} = \begin{pmatrix} -\frac{i}{2}\Omega_c & \frac{i}{2}\Omega_{24} & 0 \\ \frac{i}{2}\Omega_{24}^* & -\frac{\Gamma_2}{2} - \frac{\Gamma_3}{2} & 0 \\ 0 & 0 & -\frac{\Gamma_4}{2} \end{pmatrix} \begin{pmatrix} a_2(t) \\ a_3(t) \\ a_4(t) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad (4)
\]

In order to solve Eq. (4), one should first obtain the eigenvalues \( \lambda \) of the \( 3 \times 3 \) coefficient matrix in Eq. (4). These eigenvalues should satisfy

\[
\det \begin{pmatrix} -\frac{i}{2}\Omega_c - \lambda & \frac{i}{2}\Omega_{24} & 0 \\ \frac{i}{2}\Omega_{24}^* & -\frac{\Gamma_2}{2} - \lambda & 0 \\ 0 & 0 & -\frac{\Gamma_4}{2} - \lambda \end{pmatrix} = 0,
\]

where \( \det \) denotes the determinant of the matrix. Eq. (5) gives the following cubic equation

\[
\lambda^3 + 3b\lambda^2 + 3c\lambda + d = 0,
\]

where

\[
\begin{cases}
 b = \frac{\gamma_2}{2} + \frac{\Gamma_3 + \Gamma_4}{2}, \\
 c = \frac{\gamma_2}{2} + \frac{\Gamma_3 + \Gamma_4}{2} + \Omega_{12}^m + \Omega_{24}^m \Omega_{24}, \\
 d = \frac{\gamma_2}{2} + \frac{\Gamma_3 + \Gamma_4}{2} \Omega_{24}^m \Omega_{24}.
\end{cases}
\]

The three roots, \( \lambda_n \), of the cubic equation (6) are

\[
\begin{align*}
 \lambda_1 &= u + v - b, \\
 \lambda_2 &= uv + vw^2 - b, \\
 \lambda_3 &= vw^2 + uv - b,
\end{align*}
\]

where \( u = [(-q + \sqrt{\Delta})/2]^{-1/3}, \quad v = -p/u, \quad w = (-1 + i \sqrt{3})/2, \quad \Delta = 4p^3 + q^2, \quad p = c - b^2 \) and \( q = d - 3bc + 2b^3 \).

We need to solve first the homogeneous equation corresponding to Eq. (4). Insertion of \( a_2^{(n)}(t) = a_2^{(n)}(0) \exp(\lambda_n t), a_3^{(n)}(t) = a_3^{(n)}(0) \exp(\lambda_n t), a_4^{(n)}(t) = a_4^{(n)}(0) \exp(\lambda_n t) \) (\( n = 1, 2, 3 \)) into the homogeneous counterpart of Eq. (4) yields the following relations

\[
a_3^{(n)}(0) = \frac{i\Omega_{24}^m}{\Gamma_3 + 2\lambda_n} a_2^{(n)}(0), \quad a_4^{(n)}(0) = \frac{i\Omega_{24}^m}{\Gamma_4 + 2\lambda_n} a_2^{(n)}(0) \quad (n = 1, 2, 3)
\]

between the coefficients of the general solutions of the homogeneous equation. The above equations imply that both \( a_3^{(n)}(0) \) and \( a_4^{(n)}(0) \) (\( n = 1, 2, 3 \)) can be expressed in terms of \( a_2^{(n)}(0) \) (\( n = 1, 2, 3 \)). Therefore, the general solution of Eq. (4) takes the form

\[
\begin{pmatrix} a_2(t) \\ a_3(t) \\ a_4(t) \end{pmatrix} = \sum_{n=1,2,3} a_2^{(n)}(0) \left( \frac{1}{\Gamma_3 + 2\lambda_n} \right) \exp(\lambda_n t) + a_{(s)},
\]

(10)
where the column vector $a_{(s)}$ is a particular solution (steady solution) of Eq. (4), which can be easily obtained by setting $\dot{a}_2 = 0$, $\dot{a}_3 = 0$ and $\dot{a}_4 = 0$ in Eq. (3). The result is given as follows

$$a_{(s)} = \frac{1}{2\Omega_c^3}\frac{\gamma_{21}}{\gamma_3}\Gamma_4 + \frac{\gamma_{21}}{\gamma_3}\Gamma_4 + \frac{\gamma_{21}}{\gamma_3}\Gamma_4 + \frac{\gamma_{21}}{\gamma_3}\Gamma_4 \left( \begin{array}{c} \Omega_p^* \Omega_c \Gamma_4 \\ -\Omega_p^* \Omega_c \Gamma_4 \\ i\Omega_p^* \Omega_c \Gamma_4 \\ -i\Omega_p^* \Omega_c \Gamma_4 \\ \end{array} \right).$$

(11)

The three parameters $a_{(n)}^2(0)$ ($n = 1, 2, 3$) in the general solution (10) should be determined from the initial condition for the probability amplitudes $a_2(0)$, $a_3(0)$ and $a_4(0)$ of levels $|2\rangle$, $|3\rangle$ and $|4\rangle$ through the following matrix equation

$$\begin{pmatrix} a_2(0) - a_{2(1)}(0) \\ a_3(0) - a_{3(1)}(0) \\ a_4(0) - a_{4(1)}(0) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \Omega_p^* \Omega_c \Gamma_4 & 1 & 0 \\ -\Omega_p^* \Omega_c \Gamma_4 & -\Omega_p^* \Omega_c \Gamma_4 & 1 \end{pmatrix} \begin{pmatrix} a_2(1) \\ a_3(1) \\ a_4(1) \end{pmatrix}.$$  

(12)

First we consider the initial condition for the probability amplitudes before the signal field is switched on. In such a $\Lambda$-type EIT system, the equation of motion for probability amplitudes $a_2$ and $a_3$ may be rewritten as

$$\dot{a}_2 = \Omega_0 a_3 - \frac{\gamma_{21}}{2} a_2, \quad \dot{a}_3 = \frac{\Omega_0 a_2}{2} + \frac{\gamma_{21}}{2} a_3.$$  

(13)

The steady-state solution (for which $\dot{a}_2 = \dot{a}_3 = 0$) of the above set of equations is

$$a_{2(0)} = -\frac{\Omega_p^* \Omega_c}{\gamma_{21} \Omega_0} \simeq -\frac{\Omega_p^* \Omega_c}{\Omega_c}, \quad a_{3(0)} = \frac{\gamma_{21} \Omega_p^* \Omega_c}{\gamma_{21} \Omega_0}.$$  

(14)

Since the dephasing rate $\gamma_{21}$ is very small (less than $\Gamma_3, \Gamma_4$ and $|\Omega_c|$ by two orders of magnitude), the steady value for $a_3$ is negligibly small. Thus, we set zero initial value for $a_3$. From the dark-state condition for a three-level $\Lambda$-type EIT system, one obtains the initial condition for $a_2$: $a_2(0) = -\Omega_p^*/\Omega_c$. Obviously, the initial condition for $a_4$ is $a_4(0) = 0$. These initial conditions will be used in the following sections to determine the coefficients $a_{(n)}^2(0)$. Thus, based on Eqs. (10)-(12), one can obtain the time-dependent expression for the susceptibility at the probe frequency $[25]$

$$\chi_{31}(t) = \frac{2N|\mu_{31}|^2}{c_0 \hbar \Omega_p^*} \rho_{31}(t),$$  

(15)

where $N$ and $\mu_{31}$ denote the atomic number per volume and the atomic transition dipole between levels $|1\rangle$ and $|3\rangle$, respectively, and the density matrix element $\rho_{31}(t) = a_3(t)a_3^*(t) \simeq a_3(t)$.

In the next two sections, we will consider two special cases (for which simpler formulae and clearer physical interpretations can be obtained), namely, case when $|\Omega_{24}| \ll |\Omega_c|$ (i.e., with a weak signal field) and case when $|\Omega_{24}| \gg |\Omega_c|$ (i.e., with a strong signal field). Note that in these two cases the conditions ($|\Omega_p|, \gamma_{21}, \Gamma_3, \Gamma_4 \ll |\Omega_c|$) are always satisfied.

### III. INFLUENCE OF A WEAK SIGNAL FIELD ON THE SUSCEPTIBILITY FOR THE PROBE LIGHT

In this section we study the transient behaviors and properties when $|\Omega_{24}| \ll |\Omega_c|$. As discussed before, the nonradiative transition rate $\gamma_{21}$ between $|1\rangle$ and $|2\rangle$ is usually negligibly small. Thus, we can assume that $\gamma_{21} \ll \Gamma_3$ (in general, $\gamma_{21}$ is two orders of magnitude less than $\Gamma_3$). With this condition, the three eigenvalues $\lambda_n$ of the coefficient matrix of Eq. (4) become

$$\lambda_1 = -\frac{\Gamma_4}{2}, \quad \lambda_2 = -\frac{\Gamma_3 + i\Omega}{4}, \quad \lambda_3 = -\frac{\Gamma_3 - i\Omega}{4},$$  

(16)

where $\Omega = \sqrt{4\Omega_p^* \Omega_c - \Gamma_3^2}$.

Since $|\Omega_{24}| \ll |\Omega_c|$, the matrix elements $i\Omega_{24}/(\Gamma_4 + 2\Lambda_2)$ and $i\Omega_{24}/(\Gamma_4 + 2\Lambda_3)$ in Eq. (12) is very small (in the order of $|\Omega_{24}|/|\Omega_c|$). Thus, these two matrix elements can be set to zero. Eq. (12) can then be rewritten as

$$\begin{pmatrix} -\frac{\Omega_p^* \Omega_c}{\Gamma_0} - a_{2(0)}(0) \\ -a_{3(0)}(0) \\ -a_{4(0)}(0) \end{pmatrix} = \begin{pmatrix} \frac{\Omega_p^* \Omega_c}{\Gamma_0 + 2\Lambda_2} a_{2(0)}(0) \\ \frac{\Omega_p^* \Omega_c}{\Gamma_0 + 2\Lambda_2} a_{2(0)}(0) + \frac{\Omega_p^* \Omega_c}{\Gamma_0 + 2\Lambda_3} a_{2(0)}(0) \end{pmatrix}.$$  

(17)
which can be used to determine $a_2^{(n)}(0)$ ($n = 1, 2, 3$) in the solution (10). According to the steady-state solution (11), $a_2(s)$ is approximately $-\Omega_c^*/\Omega_c^*$ when $\gamma_{21} \ll \Gamma_3$ and $|\Omega_{24}| \ll |\Omega_c|$. Using this result and relations $\lambda_1 = -\Gamma_4/2$, $\Gamma_3 + 2\lambda_2 = -2\lambda_2$ and $\Gamma_3 + 2\lambda_2 = -2\lambda_3$, one obtains

\[
\begin{align*}
   a_2^{(1)}(0) &\approx \frac{i(\Gamma_4 + 2\lambda_1)a_2(0)}{\Omega_{24}} = 0, \\
   a_2^{(2)}(0) &\approx \frac{4a_{2(s)}\lambda_3 + 2i\Gamma_4\lambda_2}{\Omega_c^*\Omega} \left( \frac{\Omega_c^*}{\Omega} + a_2(s) \right) = \frac{4a_{2(s)}\lambda_3 + 2i\Gamma_4\lambda_2}{\Omega_c^*\Omega}, \\
   a_2^{(3)}(0) &\approx \frac{a_2^{(3)}(s) - a_2^{(3)}(0)}{\Omega_c^*} = -a_2^{(3)}(0).
\end{align*}
\]

(18)

It thus follows from Eq. (10) that the time-dependent expression for the probability amplitude of level |3⟩ is

\[
a_3(t) = -a_2^{(3)}(0)i\Omega_c^* \left[ \exp(\lambda_2t) - \exp(\lambda_3t) \right] + a_3(s) = a_3(s) \left\{ -\exp \left( -\frac{\Gamma_3}{4}t \right) \left[ \cos \left( \frac{\Omega}{4}t \right) - \frac{\Gamma_3}{\Omega} \sin \left( \frac{\Omega}{4}t \right) \right] + 1 \right\}. \tag{19}
\]

In a similar way, we obtain the following transient probability amplitude of ground state |2⟩

\[
a_2(t) = -2ia_{2(s)}\frac{\Omega}{\Omega} \exp \left( -\frac{\Gamma_4}{4}t \right) \sin \left( \frac{\Omega}{4}t \right) + a_2(s). \tag{20}
\]

According to Eq. (3), the transient probability amplitude of excited level |4⟩ in the presence of the signal field $\Omega_{24}$ satisfies $\dot{a}_4 = (i/2)\Omega_{24}^*a_2 - (\Gamma_4/2)a_4$. Substituting expression (20) into this equation, we obtain

\[
\begin{align*}
   a_4(t) &= \exp \left( -\frac{\Gamma_4}{2}t \right) \left\{ a_{3(s)} \frac{\Omega_{24}^*\Omega_c}{\Omega} \int_0^t \exp \left( -\frac{\Gamma_4}{4}t' - \frac{\Gamma_4}{4}t' \right) \sin \left( \frac{\Omega}{4}t' \right) dt' + a_{2(s)}i\frac{\Omega_{24}^*}{\Omega} \left[ \exp \left( -\frac{\Gamma_4}{2}t \right) - 1 \right] + \mathcal{C} \right\} \\
   &= a_{3(s)} \frac{\Omega_{24}^*\Omega_c}{\Omega} \left\{ \exp \left( -\frac{\Gamma_4}{4}t \right) \left[ \left( \frac{\Gamma_4}{4} - \frac{\Gamma_4}{4} \right) \sin \left( \frac{\Omega}{4}t \right) - \frac{\Gamma_4}{\Omega} \cos \left( \frac{\Omega}{4}t \right) \right] + \frac{\Omega}{4} \exp \left( -\frac{\Gamma_4}{2}t \right) \right\} \\
   &\quad + a_{2(s)} \frac{i\Omega_{24}^*}{\Omega} \left[ 1 - \exp \left( -\frac{\Gamma_4}{2}t \right) \right], \tag{21}
\end{align*}
\]

where the integral constant $\mathcal{C}$ should be zero due to the initial condition $a_4(0) = 0$. It is readily verified that the steady value (when $t \to \infty$) of $a_4$ is $(i\Omega_{24}^*/\Omega_4)a_{2(s)}$, which agrees with the steady value (11). Thus we obtain the explicit expressions for the probability amplitudes of levels |2⟩, |3⟩ and |4⟩ in the transient evolution process of the four-level N-type EIT media. Such a transient evolution process commences as the signal laser is switched on.

Next we discuss the influence of the signal field on the induced polarizability of the EIT medium due to the |1⟩-|3⟩ transition and the nonlinear absorption for the probe light. Insertion of Eq. (19) into Eq. (15) yields

\[
\chi_{31}(t) = \frac{2N|\mu_{31}|^2}{\epsilon_0\hbar\Omega_{24}^*} a_{3(s)} \left\{ -\exp \left( -\frac{\Gamma_3}{4}t \right) \left[ \cos \left( \frac{\Omega}{4}t \right) - \frac{\Gamma_3}{\Omega} \sin \left( \frac{\Omega}{4}t \right) \right] + 1 \right\}, \tag{22}
\]

where the steady probability amplitude $a_{3(s)}$ is given by (11) (the second element).

As an illustrative example, we choose the following values (typical for transitions in hyperfine-split Na D lines [30]): $\Gamma_3 = 1.2 \times 10^8$ s$^{-1}$, $\Gamma_4 = 2.5 \times 10^8$ s$^{-1}$ and $\gamma_{21} \approx 3 \times 10^6$ s$^{-1}$. The susceptibility $\chi_{31}(t)$ at the probe frequency is purely imaginary (see the above equation; note that $a_{3(s)}$ is purely imaginary) and the transient behavior of $\text{Re} \{\chi_{31}\}$ is shown in Fig. 2 for various values of $\Omega_c$ ($\Omega_{24} = 0.1\Omega_c$). At $t = 0$, the susceptibility is zero since before the signal field $\Omega_{24}$ is switched on the four-level N-type atomic ensemble can be reduced to a three-level A-type EIT system. However, once the signal field is switched on, the susceptibility $\chi_{31}(t)$ (and hence the nonlinear absorption) for the probe beam in such a four-level N-type system is oscillating (with damped oscillating amplitude) but will finally reach the steady-state value. If the Rabi frequency $\Omega_c$ of the coupling beam becomes greater, the susceptibility curve for the probe laser will oscillate more significantly (cf. the solid curve of Fig. 2). When $\Omega_c$ decreases to $2\Gamma_3$, one has $\Omega = 0$ and thus the oscillation in the susceptibility curve vanishes (cf. the dotted curve of Fig. 2).

The steady value of the susceptibility for the probe beam is

\[
\chi_{31}(\infty) = \frac{i2N|\mu_{31}|^2 (\Omega_{24}^*\Omega_{24} + \gamma_{21}\Gamma_4)}{\epsilon_0\hbar(\Omega_c^*\Omega_c\Gamma_4 + \gamma_{21}\Gamma_3\Gamma_4 + \Omega_{24}^*\Omega_{24}\Gamma_3)}. \tag{23}
\]
The four-level EIT medium is absorptive for the probe beam since the susceptibility at the probe frequency is purely imaginary (cf. the above two equations). From expression (23) one sees that if the dephasing rate $\gamma_{21} \rightarrow 0$ the linear absorption for the probe beam vanishes. However, due to the presence of the signal field, there appears a nonlinear absorption effect may dominate the optical behaviors and properties of such a multilevel EIT medium. Thus, such a system may function as a two-photon absorptive optical switch by turning on and off the signal field [27].

IV. INFLUENCE OF A STRONG SIGNAL FIELD ON THE SUSCEPTIBILITY FOR THE PROBE LIGHT

In this section, we will consider the transient evolutional behavior of the four-level EIT medium in another case when $|\Omega_{24}| \gg |\Omega_c|$ (i.e., with a strong signal field). For this case, the eigenvalues of the coefficient matrix in Eq. (4) are

$$\lambda_1 = -\frac{\Gamma_3}{2}, \quad \lambda_2 = -\frac{\Gamma_4 + i\Omega'}{4}, \quad \lambda_3 = -\frac{\Gamma_4 - i\Omega'}{4},$$

where $\Omega' = \sqrt{4\Omega^*_{24}\Omega_{24} - \Gamma_3^2}$. Substituting the initial conditions (i.e., $a_1(0) = 1$, $a_2(0) = -\Omega^*_p/\Omega_c$, $a_3(0) = 0$ and $a_4(0) = 0$) for the probability amplitudes into Eq. (12), we obtain the following equations

$$
\begin{align*}
-\frac{\Omega^*_p}{\Omega_c} - a_{2(s)} &= a_{2(1)}(0) + a_{2(2)}(0) + a_{2(3)}(0), \\
-\bar{a}_{3(s)} &= \frac{\Omega^*_p}{\Omega_c}a_{2(1)(0)}, \\
-\bar{a}_{4(s)} &= a_{2(0)}(0) + a_{2(2)}(0) + a_{2(3)}(0) - \Omega^*_p \Omega_c
\end{align*}
$$

for the coefficients $a_{2(n)}(0)$ ($n = 1, 2, 3$). Insertion of the eigenvalues (27) into Eqs. (28) gives the explicit expressions for the three coefficients $a_{2(n)}(n = 1, 2, 3)$, i.e.,

$$
\begin{align*}
a_{2(1)}(0) &= i\frac{\Omega^*_p}{\Omega_c}a_{3(s)} = 0, \\
a_{2(2)}(0) &= -2a_{4(s)}\Omega_{24} - 2a_{2(s)}\Omega' + i(\Gamma_4 - i\Omega')\left(\frac{\Omega^*_p}{\Omega_c} + a_{2(s)}\right) - \Omega^*_p \Omega_c, \\
a_{2(3)}(0) &= \frac{2a_{4(s)}\Omega_{24} - i(\Gamma_4 - i\Omega')\left(\Omega^*_p \Omega_c + a_{2(s)}\right)}{2i\Omega'}.
\end{align*}
$$
Substitution of expressions (29) into solutions (10) will yield the explicit expressions for the probability amplitudes of the atomic levels of the four-level EIT media under the condition $|\Omega_c| \ll |\Omega_{24}|$. For example, the probability amplitude of level $|3\rangle$ is

$$a_3(t) = -i \left[ \frac{\Omega^*_c a_2(2)(0)}{2\lambda_2 + \Gamma_3} + \frac{\Omega^*_c a_2(3)(0)}{2\lambda_3 + \Gamma_3} + \frac{\Omega^*_c a_{2(s)} + \Omega^*_p}{\Gamma_3} - \frac{\gamma_21\Omega^*_p}{\Omega^*_c + \gamma_21}\right] \exp\left(-\frac{\Gamma_3}{2}t\right)$$

$$+ i\Omega^*_c \left[ \frac{a_2(0)}{2\lambda_2 + \Gamma_3} \exp(\lambda_2 t) + \frac{a_2(0)}{2\lambda_3 + \Gamma_3} \exp(\lambda_3 t) \right] + i \left( \frac{\Omega^*_c a_{2(s)} + \Omega^*_p}{\Gamma_3} \right).$$

Then from the explicit expression (30) for $a_3$ and Eq. (15) we can obtain the transient behavior of the susceptibility $\chi_{31}(t)$ at the probe frequency. It is shown that when $t \to \infty$ the steady value of the susceptibility for the probe beam is

$$\text{Im}\{\chi_{31}\}(\infty) \approx \text{Im}\{\chi_{31}^{(1)}\} + \text{Im}\{\chi_{31}^{(3)}\} + \text{Im}\{\chi_{31}^{(5)}\} + \ldots = \frac{2N|\mu_{31}|^2}{\epsilon_0\hbar\Omega_p} \left[ 1 - \frac{\Omega^*_c\Omega_c}{\Omega^*_c\Omega_{24}\Omega_{24}} + \left( \frac{\Omega^*_c\Omega_c}{\Omega_{24}\Omega_{24}} \right)^2 + \ldots \right].$$

(31)

It follows from the above formula that both the linear absorption and nonlinear (various orders) absorption for the probe light exist for the case of $|\Omega_{24}| \gg |\Omega_c|$. This is different from the absorptive behavior in the case of $|\Omega_{24}| \ll |\Omega_c|$ where the linear absorption vanishes.

Fig. 3 shows the transient behavior of $\text{Im}\{\chi_{31}\}$ for various values of $\Omega_{24}$ (with $\Omega_p = 0.1\Omega_c$). The other parameters are the same as those used in Fig. 2. As $\Omega_{24}$ becomes very large, $\text{Im}\{\chi_{31}\}$ approaches the same steady value $2N|\mu_{31}|^2/(\epsilon_0\hbar\Gamma_3)$ (cf. Eq. (31)), which means the linear absorption for the probe light. In other words, if the signal field is much stronger than the coupling light, the nonlinear absorption for the probe light may be greatly inhibited.

Comparing Eq. (28) with (17), one sees that the roles of $a_3(t)$ and $a_4(t)$ in the case of $|\Omega_{24}| \gg |\Omega_c|$ are equivalent to those of $a_4(t)$ and $a_3(t)$ in the case of $|\Omega_{24}| \ll |\Omega_c|$, respectively, namely, the transient behaviors and properties of level $|3\rangle$ (or level $|4\rangle$) in the case of $|\Omega_{24}| \gg |\Omega_c|$ is similar to those of level $|4\rangle$ (or level $|3\rangle$) in the case of $|\Omega_{24}| \ll |\Omega_c|$. Such a property may be called level reciprocity, which is an interesting phenomenon between the two excited states in the two cases in the four-level N-type EIT system.

V. CONCLUDING REMARKS

In the present paper, we have considered the transient evolutionary process of a four-level EIT medium when the signal field is switched on. Once the signal light is switched on, the atomic population and the susceptibility (absorption) of the initial state (i.e., the three-level A-type EIT system) oscillatorily approaches the steady values of a four-level EIT system at a time scale of lifetime (several nanoseconds) of the excited atomic levels. Both the linear and the nonlinear optical properties of a multilevel atomic system can be modified by the phase coherence and the quantum interference that utilizes EIT. We have considered the transient process of establishing large enhancement of the nonlinear polarizability in the four-level EIT system: specifically, in the case of $|\Omega_{24}| \ll |\Omega_c|$, the linear absorption for the probe light vanishes and the only retained absorption is of the third-order nonlinearity; in the case of $|\Omega_{24}| \gg |\Omega_c|$, however, both the linear and the various-order nonlinear absorptions exist. But once the signal field becomes stronger, the various-order nonlinear absorptions will be inhibited and the retained absorption is of the linearity only. In addition, we have also found an interesting level-reciprocity relationship between the two excited states in the two cases of $|\Omega_{24}| \gg |\Omega_c|$ and $|\Omega_{24}| \ll |\Omega_c|$.

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Fig. 1. Schematic diagram for a four-level N-type atomic system. Levels $|2\rangle$ and $|3\rangle$ are coherently coupled by the coupling beam with Rabi frequency $\Omega_c$. The signal field is switched on at $t = 0$.

Fig. 2. The susceptibility $\text{Im}\{\chi_{31}\}$ (in the unit of $\frac{2N|\mu_{31}|^2}{\epsilon_0 \hbar}$) as time increases for the case $\Omega_{24} \ll \Omega_c$. Here $\Omega_{24} = 0.1\Omega_c$.

Fig. 3. The susceptibility $\text{Im}\{\chi_{31}\}$ (in the unit of $\frac{2N|\mu_{31}|^2}{\epsilon_0 \hbar}$) as time increases for the case $\Omega_{24} \gg \Omega_c$. Here $\Omega_p = 0.1\Omega_c$.

\[ \text{Im}(\chi_{31}) \]

- \( \Omega_c = 2\Gamma_3 \)
- \( \Omega_c = 5\Gamma_3 \)
- \( \Omega_c = 100\Gamma_3 \)

Time (s) \( \times 10^{-8} \)

\( x \times 10^{-10} \)