The Revival of the Unified Dark Energy-Dark Matter Model  

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We consider the generalized Chaplygin gas (GCG) proposal for unification of dark energy and dark matter and show that it admits an unique decomposition into dark energy and dark matter components once phantom-like dark energy is excluded. Within this framework, we study structure formation and show that difficulties associated to unphysical oscillations or blow-up in the matter power spectrum can be circumvented. Furthermore, we show that the dominance of dark energy is related to the time when energy density fluctuations start deviating from the linear $\delta \sim a$ behaviour.

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I. INTRODUCTION

The GCG model \[1, 2\] is an interesting alternative to earlier proposals aiming to explain the observed accelerated expansion of the Universe such as an uncanceled cosmological constant \[3\] and quintessence \[4\], the latter being a variation of the idea that the cosmological term could evolve \[3\].

In the GCG approach one considers an exotic background fluid, described by the following equation of state

$$ p_{ch} = -\frac{A}{\rho_{ch}^\alpha}, $$

where $A$ and $\alpha$ are positive constants. The case $\alpha = 1$ corresponds to the Chaplygin gas. In most phenomenological analyses the range $0 < \alpha < 1$ has been considered. Within the framework of Friedmann-Robertson-Walker cosmology, this equation of state leads, after inserted into the relativistic energy conservation equation, to an energy density evolving as \[2\]

$$ \rho_{ch} = \left[ A + \frac{B}{a^{3(1+\alpha)}} \right]^{\frac{1}{\alpha}}, $$

where $a$ is the scale-factor of the Universe and $B$ an integration constant which should be positive for a well-defined $\rho_{ch}$ at all times. Hence, we see that at early times the energy density behaves as matter while at late times it behaves like a cosmological constant. This dual role is at the heart of the surprising properties of the GCG model. Moreover, this dependence with the scale-factor indicates that the GCG model can be interpreted as an entangled mixture of dark matter and dark energy.

The GCG model has been successfully confronted with various phenomenological tests: high precision Cosmic Microwave Background Radiation data \[6\], supernova data \[7\], and gravitational lensing \[8\]. In a recent work \[9\], it has been explicitly shown that regarding the latest supernova data \[10\], the GCG model is degenerate with a dark energy model involving a phantom-like equation of state (See also Ref \[11\] for a detail study with different Supernova data sets). This excludes the necessity of invoking an unphysical fluid violating the crucial dominant energy condition for theoretical model building of our Universe which leads to the big rip singularity in future. The GCG, on the other hand, can mimic such an equation of state, but without any such pathology as asymptotically it approaches to a well-behaved de-Sitter universe.

Despite all these pleasing features, the main concern with such an unified model is that it produces unphysical oscillations or even an exponential blow-up in the matter power spectrum at present \[12\]. This is expected from the behaviour of the sound velocity through the GCG fluid. Although, at early times, the GCG behaves like dark matter and its sound velocity is vanishing, as one approaches the present, the GCG starts behaving like a dark energy with a substantial negative pressure yielding a large sound velocity which, in turn, produces oscillations or blows-up in the power spectrum. In any unified approach this is unavoidable unless one can successfully identify the dark matter and the dark energy components of the fluid. Naturally, these components are interacting as both are entangled within a single fluid. This is the main motivation of our present investigation. We show that the GCG is a unique mixture of interacting dark matter and a cosmological constant-type dark energy, once one excludes the possibility of phantom-like dark energy. Due to the interaction between the components, there is a flow of energy from dark matter to dark energy. This energy transfer is vanishingly small until recent past, resulting in a negligible contribution from the cosmological constant at the time of gravitational

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collapse \((z_c \approx 10)\). This makes the model indistinguishable from a CDM dominated Universe in the past. Just before the present \((z \approx 2)\), the interaction starts to kick off producing a large energy transfer from dark matter to dark energy leading to its dominance at present. We have also shown that the epoch of this dark energy dominance is similar to that when dark matter perturbations start deviating from its linear behaviour. Moreover, in this approach, the Newtonian equations for small scale perturbations for dark matter do not involve any \(k\)-dependent term; hence, neither oscillations nor blow-up in the power spectrum are expected. We should mention that the decaying dark matter model has been previously considered as an interesting possibility to solve, within the CDM model, the problem of over-production of dwarf galaxies as well as the over-concentration of dark matter in halos \[14\]. Our results show that the GCG model is an interesting option for that scenario, such that the decay product is nothing but a cosmological constant.

II. DECOMPOSITION OF THE GCG FLUID

In Ref. \[2\], it has been shown that the GCG Lagrangian density has the form of a generalized Born-Infeld theory:

\[
\mathcal{L}_{GBI} = -A \left\{ 1 - (g^{\mu \nu} \theta_{,\mu} \theta_{,\nu})^{1/\alpha} \right\}^{1+\alpha},
\]

which clearly reproduces the Born-Infeld Lagrangian density for \(\alpha = 1\). The field \(\theta\) corresponds to the phase of a complex scalar field \([2]\).

Let us now discuss the decomposition of the GCG into components. Using Eqs. \[10, 12\] and introducing the redshift dependence, the pressure is given by

\[
p_{ch} = -\frac{A}{[A + B(1 + z)^{3(1+\alpha)}]^{\alpha} \left(1 + \alpha \right)}
\]

while the total energy density can be written as

\[
\rho_{ch} = \left[ A + B(1 + z)^{3(1+\alpha)} \right]^{1+\alpha}
\]

where one has set the present value of the scale-factor, \(a_0\), to 1.

Decomposing the energy density into a pressureless dark matter component, \(\rho_{dm}\), and a dark energy component, \(\rho_X\) with an equation of state \(w_X\), it follows that the equation of state parameter of the GCG can be written as

\[
w = \frac{p_{ch}}{\rho_{ch}} = \frac{\rho_X}{\rho_{dm} + \rho_X} = \frac{w_X \rho_X}{\rho_{dm} + \rho_X}.
\]

Thus, using \[14, 15\] and \[16\], one obtains for \(\rho_X\)

\[
\rho_X = -\frac{\rho_{dm}}{1 + w_X \left[1 + \frac{B}{A}(1 + z)^{3(1+\alpha)}\right]}.
\]

It is easy to see that, requiring that \(\rho_X \geq 0\) leads to the constraint \(w_X \leq 0\) for early times \((z \gg 1)\) and \(w_X \leq -1\) for future \((z = -1)\). Hence, one concludes that \(w_X \leq -1\) for the entire history of the universe. The case \(w_X < -1\) corresponds to the so-called phantom-like dark energy, which violates the dominant-energy condition and leads to an ill defined sound velocity (see however \[8\]). If one excludes this possibility, then the energy density can be split in an unique way:

\[
\rho = \rho_{dm} + \rho_{\Lambda}
\]

where

\[
\rho_{dm} = \frac{B(1 + z)^{3(1+\alpha)}}{[A + B(1 + z)^{3(1+\alpha)}]^{\alpha} \left(1 + \alpha \right)}
\]

and

\[
\rho_{\Lambda} = -\rho_{\Lambda} = \frac{A}{[A + B(1 + z)^{3(1+\alpha)}]^{\alpha} \left(1 + \alpha \right)}
\]

from which one obtains the scaling behaviour of the energy densities

\[
\frac{\rho_{dm}}{\rho_{\Lambda}} = \frac{B}{A}(1 + z)^{3(1+\alpha)}.
\]

One should note that this type decomposition based on the tachyon-like lagrangian has previously been considered by Padmanabhan and Choudhury \[15\]. Next we
express parameters $A$ and $B$ in terms of cosmological observables. From Eqs. (9) and (10), it follows that

$$\rho_{ch0} = \rho_{dm0} + \rho_{\Lambda0} = (A + B)^{\frac{1}{1+\alpha}} \ ,$$

where $\rho_{ch0}$, $\rho_{dm0}$ and $\rho_{\Lambda0}$ are the present values of $\rho_{ch}$, $\rho_m$ and $\rho_{\Lambda}$, respectively. Parameters $A$ and $B$ can then be written as a function of $\rho_{ch0}$

$$A = \rho_{\Lambda0} \rho_{ch0}^{\frac{1}{1+\alpha}} \ ; \ B = \rho_{dm0} \rho_{ch0}^{\frac{1}{1+\alpha}} \ .$$

It is also useful to express $A$ and $B$ in terms of $\Omega_{dm0}$, $\Omega_{\Lambda0}$, the present values of the fractional energy densities

$$\Omega_{dm(\Lambda)} = \frac{\rho_{m(\Lambda)}}{\rho_c} \ ,$$

where $\rho_c$ is the critical energy density, $\rho_c = 3H^2/8\pi G$. Using the Friedmann equation

$$3H^2 = 8\pi G \left[ A + B(1+z)^{3(1+\alpha)} \right]^{\frac{1}{1+\alpha}} + 8\pi G \rho_{0b} (1+z)^3 \ ,$$

one obtains

$$A \simeq \Omega_{\Lambda0} \rho_{c0}^{(1+\alpha)} \ , \ B \simeq \Omega_{dm0} \rho_{c0}^{(1+\alpha)} \ .$$

Hence, with

$$H^2 = H_0^2 \left[ \Omega_{\Lambda0} + \Omega_{dm0}(1+z)^{3(1+\alpha)} \right]^{\frac{1}{1+\alpha}} + \Omega_{b0}(1+z)^3 \ ,$$

one can express the fractional energy densities $\Omega_{dm}$, $\Omega_{\Lambda}$ and $\Omega_{b}$ as

$$\Omega_{dm} = \frac{\Omega_{dm0}(1+z)^{3(1+\alpha)}}{[\Omega_{\Lambda0} + \Omega_{dm0}(1+z)^{3(1+\alpha)}]^{\alpha/(1+\alpha)} X} \ ,$$

$$\Omega_{\Lambda} = \frac{\Omega_{\Lambda0}}{[\Omega_{\Lambda0} + \Omega_{dm0}(1+z)^{3(1+\alpha)}]^{\alpha/(1+\alpha)} X} \ ,$$

$$\Omega_{b} = \frac{\Omega_{b0}(1+z)^3}{X} \ ,$$

where

$$X = \left[ \Omega_{\Lambda0} + \Omega_{m0}(1+z)^{3(1+\alpha)} \right]^{1/(1+\alpha)} + \Omega_{b0}(1+z)^3 \ .$$

Finally, given that $\Omega_{dm0}$ and $\Omega_{\Lambda0}$ are order one quantities, one sees that at the time of nucleosynthesis, $\Omega_{\Lambda}$ is negligibly small, making the model consistent with the nucleosynthesis process.

Notice that there is an explicit interaction between dark matter and dark energy. This can be seen from the energy conservation equation, which in terms of the components can be written as

$$\dot{\rho}_{dm} + 3H\rho_{dm} = -\dot{\rho}_\Lambda \ .$$

Hence the evolution of dark energy and dark matter are linked so that energy is exchanged between these components (see Refs. [16, 17] for earlier work on the interaction between dark matter and dark energy). One can see from Figure 1, that until $z \simeq 2$, there is practically no exchange of energy and the $\Lambda$ term is approximatelly zero. However, around $z \simeq 2$, the interaction starts to kick off, resulting in an important growth of the $\Lambda$ term at the expense of the dark matter energy density. Around $z \simeq 0.2$, dark energy starts dominating the energy content of Universe. Obviously, these redshift values depend on the $\alpha$ parameter and, in Figure 1, we have assumed $\alpha = 0.2$. Nevertheless, the main conclusion is that in this unified model, the interaction between dark matter and dark energy is practically zero for almost the entire history of the Universe making it indistinguishable from the CDM model. The energy transfer starts just in the recent past resulting in a significant energy transfer from dark matter to the $\Lambda$-like dark energy. In the next section we shall see that this epoch of energy transfer is similar to the one when dark matter perturbations start departing from its linear behaviour.

### III. STRUCTURE FORMATION

In order to study structure formation, it is convenient to write the 0-0 component of Einstein’s equation as

$$3H^2 = 8\pi G (\rho_{dm} + \rho_b) + \Lambda \ ,$$

where $\Lambda$ is given by

$$\Lambda = 8\pi G \rho_\Lambda \ .$$

The energy conservation equation for the background fluid is given by Eq. (22). This is reminiscent of earlier work on varying $\Lambda$ cosmology [18, 19] where the cosmological term decays into matter particles. In our case, it is the opposite as $\alpha$ is always positive and hence the energy transfer is from dark matter to dark energy. This leads to the late time dominance of the latter and ultimately to the present accelerated expansion of the Universe.

Let us now consider the issue of energy density perturbations. We start by writing down the Newtonian equations for a pressureless fluid with background density $\rho_{dm}$, and density contrast $\delta_{dm}$, with a source term due to the energy transfer from dark matter to the cosmological constant type dark energy. Assuming that both, the density contrast $\delta_{dm}$ and peculiar velocity $v$ are small, i.e. $\delta_{dm} \ll 1$ and $v \ll u$, where $u$ is the velocity of a fluid element of volume, one can write the Euler, the continuity and the Poisson’s equations in the co-moving frame as follows [19]:
this occurs for perturbation in the ΛCDM case. One can also check that one recovers the standard equation for the dark matter baryation in the Newtonian limit when the scales are well to drive oscillations or blow up in the power spectrum.

Let us turn to the evolution for the baryon perturbation after decoupling for scales well inside the horizon. Since we are considering the evolutionary period after decoupling, the baryons are no longer coupled to photons, there is no significant pressure due to Thompson scattering, and one can effectively consider baryon as a pressureless fluid like the dark matter fluid. Of course, the interaction between baryons and dark energy is a relevant issue as it is related to the Equivalence Principle (see e.g. [20] and references therein). In what follows we shall assume that the interaction of dark energy with baryons vanishes. Thus, as baryons are uncoupled, the strong interaction between the dark matter and dark matter violates the Equivalence Principle. Given that it is the parameter α that controls this interaction (α = 0 means there is no interaction), it is a measure of the violation of the Equivalence Principle. One can also see from the behaviour of Ψ, that this violation also starts rather late in the history of the Universe. In the Newtonian limit, the evolution of the baryon perturbation after decoupling for scales well inside the horizon is similar to the one described earlier for dark matter, but without the source term as there is no energy transfer to or from baryons. It is given by

\[ \frac{\partial^2 \delta_{dm}}{\partial t^2} + 2 \frac{\dot{a}}{a} \frac{\partial \delta_{dm}}{\partial t} - 4\pi G \rho_{dm} \delta_{dm} = 0 \]  

where the third term in the l.h.s, we have dropped the contribution from baryons as it is negligible compared to the dark matter one. It is convenient to define for each component the linear growth function D(y) where y = log(a),

\[ \delta = D(y) \delta_0 \]  

Notice that, if \( \Psi = 0 \), i.e. without energy transfer, one recovers the standard equation for the dark matter perturbation in the ΛCDM case. One can also check that this occurs for \( \alpha = 0 \). Moreover, one can easily see that, in the above equation, there is no scale dependent term to drive oscillations or blow up in the power spectrum.

Let us turn to the evolution for the baryon perturbation in the Newtonian limit when the scales are well inside the horizon. Since we are considering the evolutionary period after decoupling, the baryons are no longer coupled to photons, there is no significant pressure due to Thompson scattering, and one can effectively consider baryon as a pressureless fluid like the dark matter fluid. Of course, the interaction between baryons and dark energy is a relevant issue as it is related to the Equivalence Principle (see e.g. [20] and references therein). In what follows we shall assume that the interaction of dark energy with baryons vanishes. Thus, as baryons are uncoupled, the strong interaction between the dark matter and dark matter violates the Equivalence Principle. Given that it is the parameter α that controls this interaction (α = 0 means there is no interaction), it is a measure of the violation of the Equivalence Principle. One can also see from the behaviour of Ψ, that this violation also starts rather late in the history of the Universe. In the Newtonian limit, the evolution of the baryon perturbation after decoupling for scales well inside the horizon is similar to the one described earlier for dark matter, but without the source term as there is no energy transfer to or from baryons. It is given by

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\[ \delta = D(y) \delta_0 \]  

where \( \delta_0 \) is the initial density contrast (assuming Gaussian distribution), as well as the growth exponent m(y) =

![Figure 2](image1.png)

**FIG. 2:** \( \delta_{dm} \) as function of scale factor. The solid, dotted, dashed and dash-dot lines correspond to \( \alpha = 0, 0.2, 0.4, 0.6 \), respectively. We have assumed \( \Omega_{dm} = 0.25, \Omega_b = 0.05 \) and \( \Omega_\Lambda = 0.7 \).

![Figure 3](image2.png)

**FIG. 3:** The growth factor \( m(y) \) as a function of scale factor a. The solid, dotted, dashed and dash-dot lines correspond to \( \alpha = 0, 0.2, 0.4, 0.6 \), respectively. We have assumed \( \Omega_{dm} = 0.25, \Omega_b = 0.05 \) and \( \Omega_\Lambda = 0.7 \).
D'(y)/D(y).

Asymptotically, as dark matter drives the evolution of the baryon perturbations, hence they grow with the same exponent m(y). However, their amplitudes may differ and their ratio corresponds to the so-called bias parameter, b ≡ δ_b/δ_dm.

In what follows, we study the behaviour of δ_dm, m(y) and b as function of the scale factor a. While solving the differential equations for the linear perturbation, the initial conditions are chosen such that at a = 10^{-3}, the standard linear solution D ∼ a, is reached. In Figure 2, we have plotted the linear density perturbation for the dark matter, δ_dm, as a function of α. One can see that while for α = 0 (the ΛCDM case) the perturbation stops growing at late times, for models with α > 0, the perturbation starts departing from the linear behaviour around a ∼ 0.8 (we have assumed for scale factor at present a_0 = 1) i.e. z ∼ 0.25 which is similar to the epoch when the Λ term starts dominating (cf. Figure 1). In view of this behaviour, it is tempting to conjecture that, in our unified model, the interaction between dark matter and Λ-like dark energy is related with structure formation, so that sufficiently high density contrast (δ_dm >> 1) can result in a large energy transfer from the dark matter to the dark energy due to which acceleration of the Universe sets in. We should mention that this kind of scenario has been earlier discussed in Ref. [21]. Thus our study shows that there is a link between the structure formation scenario and the dominance of dark energy which ultimately results in the acceleration of the Universe expansion. This may give possible hint to the solution of so-called Cosmic Coincidence problem.

The behaviour of the growth factor m(y) is also quite interesting. One can see from Figure 3, that between the present and z ∼ 5, the growth factor is quite sensitive to the value of α. With α = 0.2, it increases up to 40% at present in relation to the value of the ΛCDM case. Notice that m(y) governs the growth of the velocity fluctuations in linear perturbation theory as the velocity divergence evolves as θ = -H am δ_dm; therefore, large deviations of the growth factor with changing α are detectable via precision measurements of large scale structure, through joint measurements of the redshift-space power spectrum anisotropy and bi-spectrum from z = 0 to z ∼ 2.

We have also studied the behaviour of the bias parameter in our model. In Figure 4, its evolution is shown. The plot suggests that the bias parameter also changes sharply in recent past with increasing α. This bias extends to all small scales allowing for the Newtonian limit, being hence distinguishable from the hydrodynamical or nonlinear bias which takes place only for collapsed objects. Thus, from the observation of large scale clustering one can distinguish the non-zero α case from the α = 0 (ΛCDM) case.

One can also see from Figure 2 that there is no suppression of δ_dm at late times for any positive value of α, and thus one should not expect the corresponding suppression in the power spectrum normalization, σ_8, for the total matter distribution. This was one major problem in the previous GCG model approach which, as pointed out by Sandvik et al. [13], cannot be solved even after the inclusion of baryons. In the new approach we propose here, one can successfully overcome this difficulty.

Another interesting cosmological probe for our model comes from galaxy cluster M/L ratios. The most recent average value Ω_m = 0.17 ± 0.05, has been determined by Bahcall and Comerford [22] by observing 21 galaxy clusters around z ∼ 1. The fact that nearby cluster data seem to prefer smaller values for Ω_m than the one obtained from WMAP data at z ∼ 1100, can be regarded as a signal in favour of a decaying dark matter model like the GCG.

Finally, one should expect a smaller ISW effect at early times, as the gravitational potential is practically constant up to z ∼ 2 but, at late times, there will be a larger ISW effect due to the energy transfer from dark matter to dark energy resulting in a large variation of the gravitational potential.

IV. DISCUSSION AND CONCLUSIONS

In this work, we have considered a decomposition of the GCG in two interacting components. The first one can be regarded as dark matter since it is pressureless. The second one has an equation of state, p_X = ω_X ρ_X. It has been shown that ω_X ≤ -1. Thus, once phantom-like behaviour is excluded our decomposition is unique. This shows that in a unified model like GCG one does not need anything more than a simple cosmological constant as dark energy which is consistent with the recent study by Jassal et. al [23] where it has been shown that a combination of WMAP data and observations of high
redshift supernovae, is fairly consistent with a cosmological constant like dark energy.

It has been demonstrated that in this model the so-called dark energy dominance is related with the time when matter fluctuations become large (\(\delta_{dm} > 1\)). Furthermore, we have shown that in what concerns structure formation, the linear regime (\(\delta_{dm} \sim a\)) is valid fairly close to the present, meaning that at the time of structure formation begins, \(z_c \simeq 10\), the dark energy component was irrelevant and clustering occurred very much like in the CDM model. We have shown that the growth factor as well as the bias parameter have a noticeable dependence on the \(\alpha\) parameter. We have implemented a model where there is a violation of the Equivalence Principle, as dark energy and baryons are not directly coupled. This may turn out to be a distinct observational signature of the present approach.

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