Mass Spectrum and Decay Constants in the Continuum Limit

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We present first results for light hadron masses, quark masses and decay constants in the continuum limit using $O(a)$ improved fermions at three different values of the gauge coupling $\beta$.

1. INTRODUCTION

It has become standard to reduce, or indeed eliminate completely, the $O(a)$ cut-off effects of Wilson fermions by adding the local Sheikholeslami-Wohlert counterterm. The QCDSF collaboration has simulated quenched QCD with improved fermions at three different values of the gauge coupling $\beta = 6.0$, 6.2, and 6.4, which covers the range of lattice spacings $a^{-1} \approx 2 - 3.5$ GeV. The simulations have been done on lattices of size $16^3 \times 32$ ($\beta = 6.0$), $24^3 \times 48$ ($\beta = 6.0$, 6.2), $32^3 \times 64$ ($\beta = 6.2$), and $32^3 \times 48$ ($\beta = 6.4$). The spatial size of the lattices varies between 1.7 - 2.5 fm. We have generated $O(200-1000)$, $O(300)$, $O(100)$ gauge field configurations at $\beta = 6.0$, 6.2 and 6.4, respectively. They have been evaluated for 3-8 different values of the hopping parameter $\kappa$ with $m_\pi / m_p$ approximately in the range of 0.4 - 0.9.\footnote{Talk given by D. Pleiter at Lattice 98, Boulder, U.S.A.}\footnote{For more simulation details, see [1].}

2. CHIRAL EXTRAPOLATION

In order to extrapolate the masses and decay constants to the chiral limit, we use the phenomenological ansatz

$$m_X^2 = b_0 + b_1 m_\pi^2 + b_3 m_N^3.$$ (1)

Fits with only three parameters using this ansatz give smaller $\chi^2/dof$ than the ansatz based on predictions of chiral perturbation theory, e.g. for $m_p$ and $m_N$ \cite{3}. Fig. 1 shows $m_\pi^2$ and $m_N^2$ as a function of $m_\pi^2$ at $\beta = 6.2$. We have used a two parameter fit (keeping $b_3 = 0$ fixed) at $\beta = 6.4$, since we currently have results for three $\kappa$ values only.

Searching for quenched chiral logarithms predicted by the chiral perturbation theory we
looked at the logarithm of the ratio $m^2_\pi / m_q$ as function of the quark mass $m_q$. Since the standard method for determination of $\kappa_c$ depends on the presence of these singularities, we use the quark masses determined by the Ward identity method. Using the ansatz \[ \ln \left( \frac{am_\pi}{am_q} \right)^2 = c_0 - \frac{\delta}{1 + \delta} \ln(am_q) + c_1 am_q + c_2 (am_q)^2 \] we find $\delta \approx 0.1 - 0.2$ for $\beta = 6.0$ and 6.2. Our data and the fits are plotted in fig. 2.

3. HADRON MASSES

In fig. 3 we plot our results for the dimensionless mass ratio $m_N/m_p$ versus $(m_{\pi}/m_p)^2$. Within errors there are no visible cut-off effects. For $\beta = 6.0$ and 6.2 we find the ratio $m_N/m_p$ in the chiral limit to be in agreement with the experimental value.

We use a linear $a^2$ in order to extrapolate our results for the hadron masses to the continuum limit. We use the string tension $K$, which has cut-off errors of $O(a^2)$, to fix the scale and $\sqrt{K} = 427$MeV to express the experimental values in terms of $\sqrt{K}$. As shown in fig. 4 our preliminary data agrees with the experimental values. For the $a_0$ and $b_1$ mesons we find large $O(a^2)$ effects.

4. QUARK MASSES AND DECAY CONSTANTS

We define the bare quark mass using the Ward identity method. While the renormalization constant $Z_A$ and the coefficient $c_A$ are known non-perturbatively [2], we use tadpole improved values for $Z_A^{\overline{MS}}(am)$, $b_A$ and $b_P$ [1.6]. To determine $m_s$ we proceed as described in [1] and use the physical pion and kaon masses as input.
To calculate the decay constants $f_\pi$ and $1/f_\rho$ we use the improved renormalized operators $A_\mu = (1 + b_A am) Z_A (A_\mu + c_A \bar{a}_b P)$ and $V_\mu = (1 + b_V am) Z_V (V_\mu + ic_V \bar{a}_b T_{\mu \lambda})$. $Z_A$, $b_A$ and $c_A$ are known non-perturbatively [2]. While $f_\pi$ scales very well, this is less obvious for $1/f_\rho$. For other matrix elements using improved axial and vector currents see [7].

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REFERENCES

7. R. Horsley, this conference.