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**CAMERA BASED MONITORING OF THE RIGID BODY DISPLACEMENT
OF A MANDREL IN SUPERCONDUCTING CABLE PRODUCTION**

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We describe a machine vision measurement head that is used to monitor the mandrel in cable production. Two cameras are orthogonally aligned, viewing different sections of the cylindric mandrel surface. The use of telecentric lenses obviates the need for re-calibration after changing the mandrel. All parameters of rigid body motion are obtained in linear theory by using a multi-variate least squares fit procedure on dynamically corresponded sets of target points that vary due to obstruction by rotating wires. A rigorous analysis of measurement uncertainty is given.

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Introduction

The Large Hadron Collider (LHC) is under construction at CERN. This machine which is of very high technology will make use of circa 7 km of superconducting cables. The cables consist on 28 to 36 superconducting strands having diameters in the range 0.8-1mm. The strands are transposed and arranged in a cable of trapezoidal cross-section. The cable width and mid-thickness are respectively 15.1mm and 0.9mm.

The strands are stocked on spools installed in a rotating cage and converge towards a fixed mandrel preparing the cable shape before being compressed vertically and laterally between 4 shaped rollers located in a Turk's head. The cable is pulled via a caterpillar. The cable fabrication speed is of the order of 6m/minute. The tensile force of each strand as well as the absence of strand internal torsion are precisely controlled. The strands are roll up on the fixed mandrel which is submitted to rotation and axial force.

A very important feature for the high requested quality of the cables is the precise relative position of the mandrel compared to the shaped rollers. The mandrel has a thin shape at the extremity allowing to come close to the rollers within a strand diameter. It is of first importance to check the possible displacements of the mandrel in the 3 directions. The displacement of the cylindrical mandrel is parameterised as a rigid body motion with six degrees of freedom. The main challenges in monitoring the mandrel rigid body displacement during cable production are the space constraint - only a ring of 6mm width of the mandrel cylindrical surface is visible to the cameras, the rough environment, as well as the fact that wires cross the field of view shadowing part of the object and occluding periodically the target points.

To obviate the need of recalibrating the camera system after changing the mandrel or repositioning the system we use telecentric lenses. These provide so-called orthographic projection where the magnification remains constant throughout the depth of field. The accuracy of telecentric lens imaging compared to perspective projection is remarkably higher [schuster], but it is not possible to obtain all three coordinates of a target point to sufficient accuracy. Hence, the usual registration algorithms solutions relying on perspective information are not applicable when all six parameters are of interest [Fitzpatrick, Aloimonos, mostafavi]. In our case, the depth information must be obtained by using a second camera with an independent set of target points. Here we arrange cameras in perpendicular orientation.

Estimation of rigid-body motion parameters in computer vision is normally performed from image correspondences between two coordinate frames. A large number of methods and closed-form solutions have been proposed based on corresponded point sets [eggert]. By using triangulation with two (or more) cameras like in stereo vision or photogrammetry object point coordinates can be located in space. Commonly a set of at least three target points or fiducial markers (passive, coded or active) on the object is tracked [Maintz]. Then, the parameters of rigid body motion are extracted from the epipolar lines by a fit procedure. Epipolar geometry constrains corresponding points from different views of a rigidly moving object [steinbach,weber].

Some authors report correspondenceless procedures to determine parameters of rigid body motion, claiming that correspondent points do not have to be identified. But those techniques still imply that the two images

include the same set of points [Aloimonos, Lin, Liu, Steinbach]. For a single set of points, translation can be separated out by considering the centre of gravity. The rotation matrix then is usually obtained by the so-called singular value decomposition (or diagonalisation) of the sum of the outer product of the point coordinates [Arun]. Two (or three) parallelly aligned cameras and plane patches of an object are considered by [Aloimonos]. [LIU] discusses correspondenceless motion estimation with perspective projection.

However, stereovision or photogrammetry are not applicable in our case, because it is virtually impossible to observe the same target point simultaneously from two different directions when wires and their shadow cross the field of view during production. As the set of available target points is changing rapidly from image to image, we cannot use correspondence-less algorithms either. Therefore, the analysis is based on the relative displacements of the target points in the image planes of the cameras, the so-called optical flow. Correspondent points in a pair of images (reference and displaced) from the same camera are evaluated. The matching problem is separated into two steps. First, candidate points are identified in each frame. Then, correspondent points are matched. Because the mandrel is supposed to be stable during production, we may assume that displacements generally are small and zero on average. For small optical flow, the matching is easily done by scanning a small area around the target point.

To obtain a single set of parameters the two measurements are combined by using the fact that all target points lie on a cylinder and undergo the same rigid body motion. The analysis is simplified by the fact that rotations are small, and a linear approximation to rigid body rotation may be used instead of the general unit quaternion solution [Horn].

We combine the two distinct sets of target points by a multi-variate weighted least-squares procedure. All identified target points of both cameras are included. Because no prediction of the motion is possible, as movement is random around the equilibrium position, this global approach reduces sensitivity to noise and enhances the stability of the procedure. Weights are estimated from the analysis of measurement uncertainty. As they do not differ substantially for different target points, they are set equal in the implemented routine, and are not calculated recursively [Papadimitriou]. We first describe the measurement system and its components. Then the basic equations for the determination of the parameters of rigid body motion are given, and finally an uncertainty analysis is provided.

Measurement system

The monitoring system is designed for displacements of less than 0.4 mm with an accuracy of about 15 μm (2σ). The measurement head (Figure 1) is detachable from the cabling machine. In addition, an adjustment in all three Cartesian co-ordinate directions is provided in order to properly align the measurement head with the mandrel.

Camera units (camera, lens, casing, illumination unit, and translation stage) are mounted to a vibration-damped sandwich plate (NEWPORT M-TD-11, machined to appropriate size). It has a total thickness of

28 mm with a composite core and skins of 3.4 mm stainless steel resulting in a density of $\rho = 60 \frac{\text{kg}}{\text{m}^3}$.

As the mandrel diameter may vary between 22 and 34 mm, the camera position must be adjustable to enable focusing. A total adjustment travel of 25 mm is provided by compact translation stages with adjustment screws with 1mm spindle pitch (Rose+Krieger).

In order to minimise the size of the measurement system, compact B/W CCIR-cameras with standard 768 x 512 pixel resolution have been chosen (jai CV-A60). They can be triggered and genlocked (synchronised).

A field of view (FOV) of $2a \times 2b = 6 \times 8 \text{ mm}^2$ covering the visible cylindrical part of the mandrel was aimed at. This FOV requires a magnification of 0.6X when using a camera chip size of $3.6 \times 4.8 \text{ mm}^2$ (corresponding to a 1/3"-CCD). A working distance of about 100 mm provides a safe distance to the running wires.

The objective was chosen to be a compact and lightweight telecentric C-mount lens (JENmetar 0.6x/8). Magnification is independent of object distance within the entire depth of focus, and is specified by a telecentricity of 0.625 mrad and a distortion < 0.05 %. Hence, there is no need to recalibrate the lateral resolution after repositioning the system or focusing the camera. The lens has no adjustable features, so mismanipulation or accidental mistuning are excluded. As no focus ring is provided on the lens, focussing is performed by using the translation stage. The depth of focus $2df$ is specified as $2df = 3.2 \text{ mm}$ (with $50\mu\text{m}$ allowable edge blur) compatible with the curvature of the cylindrical mandrel.

Because ambient light can strongly vary, a camera auto gain control (AGC) as well as a built in white-light LED illumination is provided. A shuttered operation is advisable to minimise blur due to the movement of the wires in front of the mandrel.

The camera and the lens described above result in a scaling of

$$\begin{aligned} (96.35 \pm 0.07) \text{ h-pixel/mm} \quad \text{or} \quad \kappa_{hor} &= (0.01038 \pm 0.00001) \text{ mm/h-pixel} \\ (96.0 \pm 0.1) \text{ v-pixel/mm} \quad \text{or} \quad \kappa_{vert} &= (0.01042 \pm 0.00001) \text{ mm/v-pixel} \end{aligned} \tag{1}$$

A National Instruments frame grabber (PCI-1409) was chosen, and the control software was developed with National Instruments LabView (Version 7). In order to obtain synchronised images from the two-camera system, the cameras are triggered to grab, but are read out sequentially by multiplexing two channels. A frame rate of ca. 2 Hz was realised for several hours of production.

The measurement system is connected to the PC by a single multiwire twisted-pair cable. The multi-wire cable transfers the two analogue video signals, HD/VD sync signals and the trigger signal, as well as the 12V DC power for the cameras and the LED-illumination.

Because the mandrel is polished and does not provide natural marks, we use a simple self-adhesive target. The target is based on a grating structure, the line crossings of which are used as target points. The targets (Figure 2) are fabricated to customer specification and printed onto a self-adhesive white foil of thickness 0.1 mm (Compucal foil). The target includes a scaling (1-50 mm) in order to allow for the determination of the so-called axis-offset described below and used in the evaluation procedure.

Determination of the parameters for rigid body motion

Definitions

If we define the origin of the co-ordinate system to be the centre of rotation, any point transforms in the linear approximation of small rotation angles according to

$$\mathbf{x}' = \mathbf{R} \mathbf{x} + \mathbf{t} = \begin{pmatrix} 1 & -\alpha_z & -\alpha_y \\ \alpha_z & 1 & -\alpha_x \\ \alpha_y & \alpha_x & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \\ t_z \end{pmatrix} \quad (2)$$

i.e. a linear transformation involving six parameters for the description of rigid body displacement

$$\mathbf{v}(\mathbf{x}) = \mathbf{x}' - \mathbf{x} = \begin{pmatrix} t_x - \alpha_z y - \alpha_y z \\ t_y + \alpha_z x - \alpha_x z \\ t_z + \alpha_y x + \alpha_x y \end{pmatrix} \quad (3)$$

We define the intersection of the mandrel axis with the plane spanned by the two camera optical axes as the origin of the co-ordinate system. The z-axis is defined as the mandrel axis pointing in cable direction. The y-axis is defined to point towards the half-angle of the camera axes. The x-axis (vertical direction) is perpendicular to y- and z-axes, forming a right hand system.

A point $\mathbf{P}$ on the surface of the cylinder is parametrised by (r, φ, z) . The z-co-ordinate remains unchanged in any parallel projection as with telecentric lenses onto a plane parallel to the z-axis. By definition, the cameras U and D point onto the y-axis at an angle of φ_U (ca. 45° from x-axis) and φ_D (ca. 135° from x-axis). They are, however, off from the z-axis by an amount s_U and s_D , respectively, called axis-offset, as sketched in Figure3. This axis-offset increases the depth span of the target points, which is vital to achieve a good separation of the various components of rigid body displacements.

Let γ_{cam} be the off-axis angle which is given by

$$\gamma_{cam} = (\varphi_{P,cam} - \varphi_{cam}), \quad cam = U \text{ or } D \quad (4)$$

where $\varphi_{P,cam}$ is the angular co-ordinate of the point $\mathbf{P}_{cam}$ on the cylinder surface where the camera optical axis intersects (centre of the image). The axis-offset then is given by

$$s_{cam} = r \sin \gamma_{cam} \quad (5)$$

The camera co-ordinates of a point $\mathbf{P}=(x,y,z)$ on the surface of the cylinder are

$$\begin{aligned}\xi_{cam} &= z \\ \eta_{cam} &= r \sin(\varphi - \varphi_{cam}) - s_{cam} = y \cos \varphi_{cam} - x \sin \varphi_{cam} - s_{cam}\end{aligned}\quad (6)$$

In turn, the surface point co-ordinates can be recovered from the camera co-ordinates by

$$\begin{aligned}y &= \sqrt{r^2 - \tilde{\eta}_{cam}^2} \sin \varphi_{cam} + \tilde{\eta}_{cam} \cos \varphi_{cam} \\ x &= \sqrt{r^2 - \tilde{\eta}_{cam}^2} \cos \varphi_{cam} - \tilde{\eta}_{cam} \sin \varphi_{cam}\end{aligned}\quad (7)$$

where

$$\tilde{\eta}_{cam} = \eta_{cam} + s_{cam} \quad (8)$$

While the camera co-ordinates are read from the pixel positions, the axis-offset s_{cam} has to be determined from the actual field of view.

Determination of axis-offset s_{cam}

Considering Figure 3, the two cameras have axis-offsets s_U and s_D , respectively. These are induced by moving the measurement system away from the mandrel along the y-axis by an amount $(s_U + s_D)/\sqrt{2}$. The arc length between the two intersection points is, according to Eq. (4) given by

$$\|\mathbf{P}_D - \mathbf{P}_U\| = P_D - P_U = r(\varphi_D + \gamma_D - \varphi_U - \gamma_U) \quad (9)$$

where P_U and P_D denote the readings from the scaled target and correspond to the image centres. The target allows direct measurement of the arc length as it scales the circumference. As the camera axes are perpendicular, the arc length is approximatively

$$P_D - P_U = r \left(\frac{\pi}{2} + \gamma_D - \gamma_U \right) \approx \frac{\pi}{2} r + s_D - s_U \quad (10)$$

The axis offset should be maximized to achieve maximum depth sensitivity. However, as the exact position of the mandrel with respect to the measurement head is unknown, the values of s_U and s_D must be obtained from the field of view by using the position of the image centres as well as the image edges.

Consider Figure 4 to establish the relation between the angle offset γ and known image width b . Let Q and Q' be the lower and upper edge of the image. The arc length $\|\mathbf{Q} - \mathbf{P}\|$ is related to the subtended angle δ by:

$$\|\mathbf{Q} - \mathbf{P}\| = Q - P = \delta r \quad (11)$$

Further we have for the lower half of the image

$$\sin(\delta + \gamma_{cam}) = \frac{b + s_{cam}}{r} = \frac{b}{r} + \sin \gamma_{cam} \quad (12)$$

For the upper half of the image, the following similar formula applies.

$$\sin(\delta' - \gamma_{cam}) = \frac{b}{r} - \sin \gamma_{cam} \quad (13)$$

To evaluate the entire image content, we combine Equations (12) and (13), eliminating b , and using the addition formula of the angular functions.

$$\begin{aligned} \sin \delta' \cos \gamma_{cam} - \cos \delta' \sin \gamma_{cam} - \sin \delta \cos \gamma_{cam} - \cos \delta \sin \gamma_{cam} + 2 \sin \gamma_{cam} &= 0 \\ \tan \gamma_{cam} &= \frac{\sin \delta - \sin \delta'}{2 - \cos \delta' - \cos \delta} \end{aligned} \quad (14)$$

Replacing the angles δ and δ' by the readings, Eq. (11), we finally obtain

$$s_{cam} = r \sin \gamma_{cam} = r \frac{\sin \frac{Q-P}{r} - \sin \frac{P-Q'}{r}}{\sqrt{6 - 4 \cos \frac{P-Q'}{r} - 4 \cos \frac{Q-P}{r} + 2 \cos \left(\frac{Q-Q'}{r} \right)}} \quad (15)$$

Note that s_{cam} changes sign, when the arc length of the upper half image is larger than that of the lower half image.

Displacement in camera-co-ordinates and the parameters of rigid body motion

The camera co-ordinates (ξ_{cam}, η_{cam}) of the target points are determined by image correlation. A reference cross is scanned through the image, and all visible grid crossings are identified. Using a telecentric lens the camera co-ordinates are the parallel projection of the target points along the optical axis of the camera onto the CCD sensor plane. The co-ordinates of the measurement points are correlated to their reference positions in order to calculate the displacement values. We do not expect outliers, as we have chosen a target grating with pitch p much larger than the optical flow. Note that the sets of visible target points may vary from image to image due to the wires crossing the field of view during production.

The basic equations relating the displacement values of a target point to the parameters of rigid body motion are from Eqs. (3), (6) and (7):

$$\begin{aligned} \Delta \xi_{cam} &= t_z + \tilde{\eta}_{cam} (\alpha_x \cos \varphi_{cam} - \alpha_y \sin \varphi_{cam}) + \sqrt{r^2 - \tilde{\eta}_{cam}^2} (\alpha_x \sin \varphi_{cam} + \alpha_y \cos \varphi_{cam}) \\ \Delta \eta_{cam} &= (t_y \cos \varphi_{cam} - t_x \sin \varphi_{cam}) - \xi_{cam} (\alpha_x \cos \varphi_{cam} - \alpha_y \sin \varphi_{cam}) + \sqrt{r^2 - \tilde{\eta}_{cam}^2} \alpha_z \end{aligned} \quad (16)$$

Every measurement point provides two linearly independent equations for the determination of the six parameters of rigid body displacement. However, the last terms depend only weakly on the co-ordinates, if there is no axis-offset. In that case the corresponding parameters could be only separated in second order. On the other hand, the parameter combinations proportional to the camera co-ordinates ξ_{cam} and $\tilde{\eta}_{cam}$ can easily be extracted. However, the single parameters can only be isolated if two cameras are available.

For the further discussion we introduce the dimensionless co-ordinates

$$\tau_{cam} = \frac{\tilde{\eta}_{cam}}{r} \quad \varsigma_{cam} = \frac{\xi_{cam}}{r} \quad (17)$$

Specifying the camera positions to $\varphi_U = 45^\circ$ and $\varphi_D = 135^\circ$ and setting $w = \sqrt{1/2}$, we may simplify notation for the evaluation procedure by introducing the parameter combinations

$$\begin{aligned} \delta_z &= \alpha_z r & \delta_+ &= \alpha_x r w + \alpha_y r w & t_+ &= w t_x + w t_y \\ \delta_- &= \alpha_x r w - \alpha_y r w & t_- &= w t_x - w t_y \end{aligned} \quad (18)$$

i.e. rotating the coordinate axes to an angle where they are parallel to the camera axes. The final set of four equations for the evaluation is

$$\begin{aligned} \Delta \xi_U &= t_z + \tau_U \delta_- + \sqrt{1 - \tau_U^2} \delta_+ \\ \Delta \eta_U &= -t_- - \varsigma_U \delta_- + \sqrt{1 - \tau_U^2} \delta_z \\ \Delta \xi_D &= t_z - \tau_D \delta_+ + \sqrt{1 - \tau_D^2} \delta_- \\ \Delta \eta_D &= -t_+ + \varsigma_D \delta_+ + \sqrt{1 - \tau_D^2} \delta_z \end{aligned} \quad (19)$$

Least-squares evaluation of the parameters of rigid body displacement

In order to take into account both sets of target points from the two cameras, a unified evaluation procedure based on a multivariate least-squares analysis [bevington] is applied minimising the following sum of residual squares [steinbach]:

$$\begin{aligned} & \sum_k g_k \left(\Delta \xi_{Uk} - t_z - \delta_- \tau_{Uk} - \delta_+ \sqrt{1 - \tau_{Uk}^2} \right)^2 + \sum_k h_k \left(\Delta \eta_{Uk} + t_- + \delta_- \varsigma_{Uk} - \delta_z \sqrt{1 - \tau_{Uk}^2} \right)^2 \\ & + \sum_m g_m \left(\Delta \xi_{Dm} - t_z + \delta_+ \tau_{Dm} - \delta_- \sqrt{1 - \tau_{Dm}^2} \right)^2 + \sum_m h_m \left(\Delta \eta_{Dm} + t_+ - \delta_+ \varsigma_{Dm} - \delta_z \sqrt{1 - \tau_{Dm}^2} \right)^2 = \min \end{aligned} \quad (20)$$

The sums on k and m extend through the entire set of identified target points N_U and N_D of the upper and lower camera, respectively. The weight factors are given by the variance of the single measurement values which are identical for both cameras

$$g_i = \frac{1}{u^2(\Delta\xi_i)} \quad h_i = \frac{1}{u^2(\Delta\eta_i)} \quad (21)$$

The variances are estimated below. The least squares procedure leads to the normal matrix equation

$$\underline{\mathbf{N}}\mathbf{A} = \mathbf{Y} \quad (22)$$

with the six parameters ($A_0, A_1, \dots, A_5$) of Eq. (24)

$$\mathbf{A} = (t_z \quad t_+ \quad t_- \quad \delta_z \quad \delta_+ \quad \delta_-)^T \quad (23)$$

and the measurement vector

$$\mathbf{Y} = \begin{pmatrix} [\Delta\xi_U] + [\Delta\xi_D] \\ -[\Delta\eta_D] \\ -[\Delta\eta_U] \\ [\Delta\eta_U\sqrt{1-\tau_U^2}] + [\Delta\eta_D\sqrt{1-\tau_D^2}] \\ [\Delta\xi_U\sqrt{1-\tau_U^2}] - [\Delta\xi_D\tau_D] + [\Delta\eta_D\varsigma_D] \\ [\Delta\xi_U\tau_U] - [\Delta\eta_U\varsigma_U] + [\Delta\xi_D\sqrt{1-\tau_D^2}] \end{pmatrix} \quad (24)$$

The square brackets are short for the respective weighted summations through the number of measurement points for cameras U and D. The normal matrix is given by

$$\underline{\mathbf{N}} = \begin{pmatrix} (N_U + N_D) & 0 & 0 & 0 & ([\sqrt{1-\tau_U^2}] - [\tau_D]) & ([\tau_U] + [\sqrt{1-\tau_D^2}]) \\ 0 & N_D & 0 & -[\sqrt{1-\tau_D^2}] & -[\varsigma_D] & 0 \\ 0 & 0 & N_U & -[\sqrt{1-\tau_U^2}] & 0 & [\varsigma_U] \\ 0 & -[\sqrt{1-\tau_D^2}] & -[\sqrt{1-\tau_U^2}] & ([1-\tau_U^2] + [1-\tau_D^2]) & [\varsigma_D\sqrt{1-\tau_D^2}] & -[\varsigma_U\sqrt{1-\tau_U^2}] \\ ([\sqrt{1-\tau_U^2}] - [\tau_D]) & -[\varsigma_D] & 0 & [\varsigma_D\sqrt{1-\tau_D^2}] & ([1-\tau_U^2] + [\tau_D^2] + [\varsigma_D^2]) & ([\tau_U\sqrt{1-\tau_U^2}] - [\tau_D\sqrt{1-\tau_D^2}]) \\ ([\tau_U] + [\sqrt{1-\tau_D^2}]) & 0 & [\varsigma_U] & -[\varsigma_U\sqrt{1-\tau_U^2}] & ([\tau_U\sqrt{1-\tau_U^2}] - [\tau_D\sqrt{1-\tau_D^2}]) & ([1-\tau_D^2] + [\tau_U^2] + [\varsigma_U^2]) \end{pmatrix} \quad (25)$$

Eq. (23) is solved for $\mathbf{A}$ by using the inverse of the normal matrix $\underline{\mathbf{N}}$.

Measurement uncertainty

General

The estimation of the measurement uncertainty for the values of translation and rotation parameters is given by the variance and covariance due to the correlation algorithm. The uncertainty of the displacements in camera coordinates, $u^2(\Delta\xi_{cam})$ and $u^2(\Delta\eta_{cam})$ is needed for the calculation of the weight factors, Eq. (21). It is modularly composed [GUM] by 1) uncertainty of identification of the target points, 2) camera co-

ordinates, axis offset, radius, and 3) geometrical uncertainty due to the non-ideal real situation comprising uncertainty of camera angle φ_{cam} , non-coplanarity of the two cameras, and temperature effects.

To take into account the uncertainties of the dimensionless camera co-ordinates on the rhs of Eq. (19) we calculate an equivalent uncertainty, which adds to the identification uncertainty according to the error propagation rules. In summary, we have

$$u^2(\Delta\xi) = u^2(\Delta\xi, \text{ident}) + u^2(\Delta\xi, \text{equivalent}) + u^2(\Delta\xi, \text{geometry}) \quad (26)$$

and similar for the displacement $\Delta\eta$.

Covariance matrix

The variances and covariances of the fit parameters are given by the inverse of the normal matrix, Eq (25). For an idealised simulated example based on a regular grid and symmetric axis offsets of $s_U = s_D = r/3$, the covariance matrix is given in Table 1. It is block-diagonal showing correlation within the parameters $\{t_z, \delta_-, \delta_+\}$ and $\{\delta_z, t_-, t_+\}$. The uncertainty in t_- and t_+ as well as δ_- and δ_+ are identical, as expected. Back-transformation of the covariance matrix to the Cartesian co-ordinate system, Table 2, diagonalizes the x-y sub-space, while the correlation between t_x and δ_z as well as between t_z and δ_x are retained. The variance of the parameter t_x (i.e. the diagonal element of the covariance matrix) is higher by a factor of seven than that of t_z . The parameter t_y , i.e. translation towards the cameras has the highest accuracy.

A covariance matrix obtained during the calibration of the system is given in Table 3 for comparison. In this case, there were 35 and 32 target points identified in the images of the lower and upper camera, respectively. The matrix elements agree to a good extent with the idealised simulation. Since it is not reasonable to make an uncertainty analysis for each measurement, we assume that the idealised covariance matrix is valid for all measurements, but that its elements roughly scale with the inverse of the degree of freedom $N_U + N_D - 6$.

Uncertainty of displacement values and camera co-ordinates

The uncertainty of the camera co-ordinates of the measurement points is composed by the uncertainty due to telecentricity and distortion of the lens, the pixel-calibration, and the target point localisation. From the data sheet of the lens, the calibration uncertainty, Eq. (1) and an estimation of localisation uncertainty we have

$$\begin{aligned} u^2(\text{telecentricity}) &= (3.2\text{mm} \times 0.625\text{mrad})^2 = (0.002\text{mm})^2 \\ u^2(\text{distortion}) &< (4.0\text{mm} \times 0.05\%)^2 = (0.002\text{mm})^2 \\ r^2(\text{calibration}) &\approx (0.001)^2 \\ u^2(\text{localisation}) &\approx (0.2\text{pixel})^2 = (0.002\text{mm})^2 \end{aligned} \quad (27)$$

and therefore we set

$$\begin{aligned}
u^2(\xi) &= (0.0035 \text{ mm})^2 + (\xi \times 0.001)^2 < (0.0046 \text{ mm})^2 \\
u^2(\eta) &= (0.0035 \text{ mm})^2 + (\eta \times 0.001)^2 < (0.0053 \text{ mm})^2
\end{aligned} \tag{28}$$

For the measurement uncertainty of displacements being the difference of two localisations only the contribution due to identification uncertainty of the target point is essential, because distortion and telecentricity values may be assumed equal for the original and shifted position of a target point, and calibration uncertainty is negligible.

$$u^2(\Delta\xi, \text{ident}) \equiv u^2(\Delta\eta, \text{ident}) = (0.003 \text{ mm})^2 \tag{29}$$

Uncertainty of radius and axis offset

The uncertainty for the axis offset s is calculated by error propagation from Eq. (15). Each target reading is given to 0.1 mm. Assuming a rectangular distribution and including a target scale calibration error of 0.3%, the corresponding variance is,

$$u^2(\text{reading}) = \frac{(0.1 \text{ mm})^2}{12} + (0.003 \times 4 \text{ mm})^2 = (0.031 \text{ mm})^2 \tag{30}$$

Compared to the reading uncertainty, the contribution of the radius uncertainty

$$u^2(r) = (0.05 \text{ mm})^2. \tag{31}$$

is negligible. As the readings are independent from each other we have,

$$u^2(s) = \left[\left(\frac{\partial s}{\partial Q} \right)^2 + \left(\frac{\partial s}{\partial Q'} \right)^2 + \left(\frac{\partial s}{\partial P} \right)^2 \right] u^2(\text{reading}) \tag{32}$$

After a lengthy derivation the variance can be estimated to be

$$u^2(s) < \left(\frac{r^4}{b^4} \right) u^2(\text{reading}) \tag{33}$$

Uncertainty of the dimensionless camera co-ordinates

Because the uncertainty in s , Eq. (33), scales with the square of the radius, it dominates the uncertainty of the shifted camera-co-ordinates $\tilde{\eta}_U = \eta_U + s_U$ and the dimensionless co-ordinate τ , defined in Eq. (17)

$$u^2(\tau) \approx \frac{u^2(s)}{r^2} \tag{34}$$

From Eq. (17) the uncertainty for ς is with Eq. (28) and (33),

$$u^2(\varsigma) = \frac{1}{r^2} \left[(0.0035 \text{ mm})^2 + \varsigma^2 (u^2(r) + (0.001)^2 r^2) \right] \quad (35)$$

which depends on the co-ordinate ς itself. In order to give a reasonable estimate of the uncertainty, we replace ς^2 by its expectation value $\langle \varsigma^2 \rangle = \frac{3 \text{ mm}^2}{r^2}$, and we find as an upper limit for the minimum radius of $r = 11 \text{ mm}$

$$u^2(\varsigma) = \frac{1}{r^2} \left[(0.0035 \text{ mm})^2 + 3 \text{ mm}^2 \left(\frac{(0.05 \text{ mm})^2}{(11 \text{ mm})^2} + (0.001)^2 \right) \right] = \frac{1}{r^2} (0.009 \text{ mm})^2 \quad (36)$$

Equivalent uncertainty

Now having all necessary uncertainty contributions at hand, we tackle the equivalent uncertainty of the displacement values. From Eq. (19) we deduce the equivalent uncertainties to be

$$u^2(\Delta \xi_{cam}, \text{equivalent}) = \left(\delta_{\mp} \mp \sqrt{\frac{\tau_{cam}^2}{1 - \tau_{cam}^2}} \delta_{\pm} \right)^2 u^2(\tau_{cam}) \quad (37)$$

$$u^2(\Delta \eta_{cam}, \text{equivalent}) = \delta_{\mp}^2 u^2(\varsigma_{cam}) + \delta_z^2 \frac{\tau_{cam}^2}{1 - \tau_{cam}^2} u^2(\tau_{cam})$$

where the upper sign is for $cam=U$ and the lower for $cam=D$, respectively. As the fit-parameters δ_i themselves appear in these expressions, the procedure calls for iteration, i.e. to perform a least squares fit while neglecting the equivalent uncertainties, then using the fit parameters to calculate the equivalent uncertainties according to Eq.(37), and iterating the procedure. For our purpose it is sufficient to estimate the equivalent uncertainty by using the expectation values. The expectation values of the squared rotations, assumed to be uncorrelated, are from Eq. (18)

$$\begin{aligned} \langle \delta_+^2 \rangle &= \frac{r^2}{2} (\langle \alpha_x^2 \rangle + \langle \alpha_y^2 \rangle) \equiv \langle \delta_-^2 \rangle & \langle \delta_+ \delta_- \rangle &= 0 = \langle \alpha_x \alpha_y \rangle \\ \langle \delta_z^2 \rangle &= r^2 \langle \alpha_z^2 \rangle \end{aligned} \quad (38)$$

From symmetry considerations we further assume that the two expectation values of rotation around x and y are equal, and we define the rotation in radial direction

$$\langle \alpha_x^2 \rangle \equiv \langle \alpha_y^2 \rangle = \langle \alpha_r^2 \rangle \quad (39)$$

The equivalent uncertainties are then equal for both cameras and expressed as:

$$\begin{aligned} u^2(\Delta\xi, \text{equivalent}) &= r^2 \langle \alpha_r^2 \rangle \left(\frac{1}{1-\tau^2} \right) u^2(\tau) \\ u^2(\Delta\eta, \text{equivalent}) &= r^2 \langle \alpha_r^2 \rangle u^2(\varsigma) + r^2 \langle \alpha_z^2 \rangle \frac{\tau^2}{1-\tau^2} u^2(\tau) \end{aligned} \quad (40)$$

Inserting again average values for the expressions of co-ordinates into the expressions for the equivalent uncertainties, one may show that

$$\left\langle \frac{1}{1-\tau^2} \right\rangle = \frac{r}{4b} \ln \left(\left| \frac{(b+r)^2 - s^2}{(r-b)^2 - s^2} \right| \right) \approx 1 + \frac{s^2}{r^2} \quad (41)$$

and

$$\left\langle \frac{\tau^2}{1-\tau^2} \right\rangle = \left\langle \frac{1}{1-\tau^2} \right\rangle - 1 \approx \frac{s^2}{r^2} \quad (42)$$

Substituting these expressions into the equivalent uncertainties Eq. (40) and substituting the co-ordinate uncertainties from Eq.(34) and (36) finally yields

$$\begin{aligned} u^2(\Delta\xi, \text{equivalent}) &= \langle \alpha_r^2 \rangle \left(1 + \frac{s^2}{r^2} \right) u^2(s) \\ u^2(\Delta\eta, \text{equivalent}) &= \langle \alpha_r^2 \rangle u^2(\varsigma) + \langle \alpha_z^2 \rangle \frac{s^2}{r^2} u^2(s) \end{aligned} \quad (43)$$

Geometrical uncertainty

The uncertainty due to uncertainty in **camera position angle** φ_{cam} affects the displacement values from Eq (16) according to

$$\begin{aligned} \frac{\partial \Delta\xi_{cam}}{\partial \varphi_{cam}} &= \tau(-\alpha_x r \sin \varphi_{cam} - \alpha_y r \cos \varphi_{cam}) + \sqrt{1-\tau^2}(\alpha_x r \cos \varphi_{cam} - \alpha_y r \sin \varphi_{cam}) \\ \frac{\partial \Delta\eta_{cam}}{\partial \varphi_{cam}} &= (-t_y \sin \varphi_{cam} - t_x \cos \varphi_{cam}) - \varsigma(-\alpha_x r \sin \varphi_{cam} - \alpha_y r \cos \varphi_{cam}) \end{aligned} \quad (44)$$

Squaring the expressions to obtain the contribution to the uncertainty and taking into account again the average uncorrelated values for the rotation angles, Eq. (39)

$$u^2(\Delta\xi, \varphi_{cam}) = \left(\frac{\partial \Delta\xi}{\partial \varphi_{cam}} \right)^2 u^2(\varphi_{cam}) = r^2 \langle \alpha_r^2 \rangle u^2(\varphi_{cam}) \quad (45)$$

For the displacement in η we find with similar symmetry arguments for the translation:

$$u^2(\Delta\eta, \varphi_{cam}) = \left(\frac{\partial \Delta\eta}{\partial \varphi_{cam}} \right)^2 u^2(\varphi_{cam}) = \left[\langle t_r^2 \rangle + \langle \varsigma^2 \rangle r^2 \langle \alpha_r^2 \rangle \right] u^2(\varphi_{cam}) \quad (46)$$

The uncertainty due to **non-coplanar camera orientation** is taken into account by considering an extra z-component to one of the cameras, which affects the ξ -co-ordinate. Only the displacement in η is affected according to equation (16):

$$u^2(\Delta\eta, z) = \left(\frac{\partial \Delta\eta}{\partial \xi} \right)^2 u^2(z) = \langle (\alpha_x \cos \varphi - \alpha_y \sin \varphi)^2 \rangle u^2(z) = \langle \alpha_r^2 \rangle u^2(z) \quad (47)$$

To take into account **temperature effects** we assume a homogeneous temperature variation of the entire installation by $\pm 5^\circ\text{C}$. The mount of the measurement system and the cabling machine are made of steel and therefore undergo the same thermal expansion. Hence, the relative z-position between camera and mandrel tip adds to the measurement uncertainty. It may be assumed to remain within

$$u^2(\Delta\xi, \text{temperature}) = (\alpha \times L_{Mandrel} \times \Delta T)^2 = (10^{-5} \times 100\text{mm} \times 5)^2 = (0.005\text{mm})^2 \quad (48)$$

The temperature expansion of the breadboard influences the camera position in the x-y-plane. As a shift along the camera axis does not influence the measurement with telecentric lenses, only a net shift perpendicular to the camera axis, i.e. the η -co-ordinate, must be taken into account. However, since the entire set-up is made of steel, the expansion in y-direction is compensated by the expansion in x-direction.

It is appropriate to compile and quantify the different contributions for the combined uncertainty of $\Delta\xi$, see Table 4, and $\Delta\eta$, see Table 5. Considering the value of the combined uncertainty of the camera displacement values, we may use $u(\Delta\xi) = u(\Delta\eta) = 0.008\text{mm}$ for $r = 17\text{mm}$ and $u(\Delta\xi) = u(\Delta\eta) = 0.006\text{mm}$ for $r = 11\text{mm}$ as the 1σ -value. The covariance matrix elements given in Table 2 must be multiplied by this value to quantify the results. Usually the diagonal matrix elements are taken as an estimate of the measurement uncertainty of the parameters. They are given in Table 6. The expanded uncertainty is calculated for an expansion factor $k = 2$ and covers a range of 95% when a normal distribution is assumed. Note that for the uncertainty of the angles of rotation α the uncertainty of the radius, Eq. (31) can be neglected.

After having calculated the parameters of rigid body motion, the displacement of the centre of the mandrel tip $\mathbf{T} = (0, 0, L_{mandrel})^T$ is given by

$$\mathbf{v}(\mathbf{T}) = \begin{pmatrix} t_x - \alpha_y L_{mandrel} \\ t_y - \alpha_x L_{mandrel} \\ t_z \end{pmatrix} \quad (49)$$

Its components have an expanded uncertainty of

$$\begin{aligned} U^2(v_x(\mathbf{T})) &= U^2(t_x) + L_{mandrel}^2 U^2(\alpha_y) \\ U^2(v_y(\mathbf{T})) &= U^2(t_y) + L_{mandrel}^2 U^2(\alpha_x) \\ U^2(v_z(\mathbf{T})) &= U^2(t_z) \end{aligned} \quad (50)$$

and the covariances are of no importance.

Conclusion

We have described the application of machine vision in cable production tool monitoring. Rigid body displacement of the cylindrical mandrel is obtained from two cameras and based on a multi-variate least squares algorithm applied to displacement of target point camera coordinates. The main advantages of the technique are that it can handle varying sets of target points in two distinct views, and that no re-calibration is needed.

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Table 1: Covariance matrix for an idealised set of target points.

Parameter	Covariance Matrix					
t_z	1.838	0.000	0.000	0.000	-1.437	-1.437
t_+	0.000	6.717	6.688	7.163	0.000	0.000
t_-	0.000	6.688	6.717	7.163	0.000	0.000
δ_z	0.000	7.163	7.163	7.671	0.000	0.000
δ_+	-1.437	0.000	0.000	0.000	1.169	1.096
δ_-	-1.437	0.000	0.000	0.000	1.096	1.169

Table 2: Covariance matrix for an idealised set of target points, transformed back to Cartesian co-ordinates.

Parameter	Back transformed Covariance Matrix					
t_z	1.838	0.000	0.000	0.000	-2.032	0.000
t_x	0.000	13.405	0.000	10.130	0.000	0.000
t_y	0.000	0.000	0.029	0.000	0.000	0.000
δ_z	0.000	10.130	0.000	7.671	0.000	0.000
δ_x	-2.032	0.000	0.000	0.000	2.265	0.000
δ_y	0.000	0.000	0.000	0.000	0.000	0.073

Table 3: Covariance matrix obtained during calibration, transformed back to Cartesian co-ordinates.

Parameter	Back transformed Covariance Matrix					
t_z	1.882	0.044	-0.052	0.014	-2.107	0.390
t_x	0.044	14.680	0.810	11.066	-0.049	0.011
t_y	-0.052	0.810	0.076	0.614	0.059	-0.011
δ_z	0.014	11.066	0.614	8.359	-0.016	0.003
δ_x	-2.107	-0.049	0.059	-0.016	2.381	-0.427
δ_y	0.390	0.011	-0.011	0.003	-0.427	0.141

Table 4: Combined uncertainty of $\Delta\xi$ for the range of radii considered.

Influence parameter	Eq.	Assumed values	Contribution to $u^2(\Delta\xi)$ r=11 mm	Contribution to $u^2(\Delta\xi)$ r=17 mm
Identification of target point	(29)	$u^2(\Delta\xi, ident) = (0.003mm)^2$	$9 \times 10^{-6} \text{ mm}^2$	$9 \times 10^{-6} \text{ mm}^2$
Equivalent uncertainty	(30) (33)	$u^2(s) < (0.24mm)^2$ r = 11 mm $u^2(s) < (0.56 \text{ mm})^2$ r = 17 mm		
	(43)	Assume: $\langle \alpha_r^2 \rangle = \langle \alpha_z^2 \rangle = (8 \text{ mrad})^2$ Assume: $\frac{s^2}{r^2} = \frac{1}{9}$	$4 \times 10^{-6} \text{ mm}^2$	$22 \times 10^{-6} \text{ mm}^2$
Camera position angle	(45)	Assume: $u^2(\varphi_{cam}) = (17 \text{ mrad})^2$	$2 \times 10^{-6} \text{ mm}^2$	$5 \times 10^{-6} \text{ mm}^2$
Temperature change	(48)	$\Delta T = 5^\circ\text{C}$	$25 \times 10^{-6} \text{ mm}^2$	$25 \times 10^{-6} \text{ mm}^2$
Total		$u^2(\Delta\xi)$	$40 \times 10^{-6} \text{ mm}^2$	$61 \times 10^{-6} \text{ mm}^2$
		$u(\Delta\xi)$	0.0063 mm	0.0078 mm

Table 5: Combined uncertainty of $\Delta\eta$ for the range of radii considered.

Influence parameter	Eq.	Assumed values	Contribution to $u^2(\Delta\eta)$ r=11 mm	Contribution to $u^2(\Delta\eta)$ r=17 mm
Identification of target point	(29)	$u^2(\Delta\eta, ident) = (0.003 mm)^2$	$9 \times 10^{-6} mm^2$	$9 \times 10^{-6} mm^2$
Equivalent uncertainty	(36) (43)	$u^2(\varsigma) = \frac{1}{r^2} (0.009 mm)^2$ $\langle \alpha_i^2 \rangle = (8 mrad)^2 \quad \frac{s^2}{r^2} = \frac{1}{9}$	$1 \times 10^{-6} mm^2$	$3 \times 10^{-6} mm^2$
Camera position angle	(46)	Assume: $\langle t_r^2 \rangle = (0.13 mm)^2$ $u^2(\varphi_{cam}) = (17 mrad)^2$	$5 \times 10^{-6} mm^2$	$8 \times 10^{-6} mm^2$
Non-coplanarity	(47)	$u^2(z) = (0.5 mm)^2$	$16 \times 10^{-6} mm^2$	$16 \times 10^{-6} mm^2$
Total		$u^2(\Delta\eta)$	$31 \times 10^{-6} mm^2$	$36 \times 10^{-6} mm^2$
		$u(\Delta\xi)$	0.0056 mm	0.0060 mm

Table 6: Combined and expanded uncertainty of the parameters of rigid body displacement.

	Combined uncertainty				Expanded Uncertainty			
	r = 11 mm	r = 17 mm				r = 11 mm	r = 17 mm	
$u(t_z) =$	8.1	11	[μm]		$U(t_z) =$	16	22	[μm]
$u(t_x) =$	22	29	[μm]		$U(t_x) =$	44	58	[μm]
$u(t_y) =$	1.0	1.4	[μm]		$U(t_y) =$	2	3	[μm]
$u(\delta_z) =$	17	22	[μm]					
$u(\delta_x) =$	9.0	12	[μm]					
$u(\delta_y) =$	1.6	2.2	[μm]					
$u(\alpha_z) =$	1.5	1.3	[mrad]		$U(\alpha_z) =$	3	2.6	[mrad]
$u(\alpha_x) =$	0.8	0.7	[mrad]		$U(\alpha_x) =$	1.6	1.4	[mrad]
$u(\alpha_y) =$	0.15	0.13	[mrad]		$U(\alpha_y) =$	0.3	0.3	[mrad]

Figure 1: Measurement head with breadboard, two cameras in their casing, electrical connector box, and attachment tube.

Figure 2: (a): Back end of the mandrel with attached target. Note the mm-grid with the scale.

(b) View of the target by one camera. Note the wires obstructing part of the target.

Figure 3: Definition of axes and camera-co-ordinates.

Figure 4: Relation between axis offset s , the offset angle γ and image width b .