The Compact Muon Solenoid Experiment

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Sensitivity of Robust Vertex Fitting Algorithms

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Abstract

Robust vertex fitting algorithms are expected to improve the knowledge of the vertex position and of its uncertainty in the presence of mis-measured or mis-associated tracks. Such contaminations are likely to happen in real data as well as in realistic detector simulations. This note describes a simulation study of the sensitivity of two types of robust algorithms: a trimmed least-squares estimator and an adaptive estimator. The statistical properties of the algorithms are studied as a function of the source and the level of contamination, and compared to the results obtained with classical least-squares estimators. Two typical event topologies are studied, one resembling a high multiplicity primary vertex with a possible contamination from a nearby vertex, and one resembling a low multiplicity secondary vertex in a jet.

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1 Introduction

Vertex fitting usually relies on the least-squares (LS) minimization technique. This technique is statistically efficient and unbiased provided that:

1. the relation between the vertex measurements, i.e. the track parameters, and the fitted vertex parameters is linear;
2. all track parameters have Gaussian and perfectly known uncertainties;
3. all tracks do originate from the fitted vertex.

The first condition is not fulfilled in presence of a magnetic field, but a linear approximation in the neighbourhood of the vertex position is usually precise enough and has a negligible contribution to the uncertainty of the fitted parameters.

The two other conditions are more of a problem. In experiments or in detailed simulations, track parameter residuals very often depart from a single Gaussian distribution and exhibit tails. These tails are either due to non-Gaussian multiple Coulomb scattering and detector resolutions, or to imperfect modelling of the detector material and resolutions leading to an imperfect estimation of the track parameter covariance matrix. In addition, interesting physics, e.g. Higgs, SUSY and top physics, can manifest itself through short-lived particles like B mesons or τ leptons in the final state. The decay vertex of such particles is difficult to separate from the primary vertex, leading to mis-associations of tracks to vertices.

In estimation theory such contaminations are called outliers. Tracks with a wrong track parameter covariance matrix are further called Type-1 outliers, and tracks originating from a different vertex, Type-2 outliers. In contrast, tracks correctly associated and with a perfectly known uncertainty are called inliers. The least-squares estimator ceases to be unbiased as soon as the measurement sample is contaminated. Robust estimators, insensitive to outlying measurements, were thus proposed for vertex fitting in CMS [1]. First results with detailed simulation of di-jet events ($u\bar{u}$, $c\bar{c}$ and $b\bar{b}$) in the CMS detector are very encouraging. The estimation of the primary vertex position and uncertainty is significantly improved with respect to least-squares methods [1].

In this note, a detailed study is presented of the robustness of two types of robust vertex fitting algorithms, a trimmed least-squares estimator and an adaptive estimator. Their statistical properties are studied as a function of the source and the level of contamination, and compared to the results obtained with classical least-squares estimators. Outliers are introduced in the track sample in a controlled way, by using a simplified and configurable Monte Carlo track generator instead of detailed event reconstruction. Two typical event topologies are simulated. The first topology resembles a high multiplicity primary vertex, contaminated by badly measured tracks from that vertex, or by tracks from a nearby primary vertex. The second resembles a low multiplicity secondary vertex in a jet, distorted by mis-measured secondary tracks or by primary tracks.

The algorithms studied in the note are defined in Section 2. The simplified Monte Carlo track generator implemented to allow full control of the track sample (track parameter resolutions, amount of outliers of each type, etc.) and the analysis performed are described in Section 3. The results on the sensitivity of the algorithms in the high multiplicity topology are presented in Section 4. The results obtained in the low multiplicity case are discussed in Section 5. Conclusions are drawn in Section 6.

2 Vertex fitting algorithms

Fitting the vertex position consists in finding the vertex coordinates $\vec{x}$ which minimize a function F of the reduced distances between the fitted vertex and the N tracks associated to it. When all tracks have known Gaussian uncertainties and actually originate from the vertex, the function F which yields an efficient and unbiased estimate for $\vec{x}$ is the sum of the squared reduced distances. This is the well-known least sum of squares, or least-squares (LS) technique. In this case

$$F = \sum_i (\vec{x} - g(\vec{p}_i))^T \mathbf{G}^T \mathbf{C}_i^{-1} \mathbf{G} (\vec{x} - g(\vec{p}_i)); \quad i = 1 \rightarrow N \quad (1)$$

is a χ^2 variable with $2N - 3$ degrees of freedom, where $\vec{p}_i$ and $\mathbf{C}_i$ denote the parameters and covariance matrix of track i .

The function g is the measurement function which transforms the vector of track parameters $\vec{p}_i$ into a two-dimensional constraint on the vertex coordinates. The matrix $\mathbf{G}$ is the Jacobian matrix $\frac{\partial \vec{f}}{\partial \vec{x}}$ and the matrix product $\mathbf{G}^T \mathbf{C}_i^{-1} \mathbf{G}$ represents the weight of the constraint $g(\vec{p}_i)$ in the vertex fit. Equation 1 is an application of the Kalman filter formalism to vertex fitting. A more complete formulation can be found in [2].

2.1 Least-squares, trimming and adaptive algorithms

To simplify the discussion the problem is further formulated in one dimension. The function to minimize reads in the least-squares case:

$$F = \sum_i \frac{(x - x_i)^2}{\sigma_i^2}; \quad i = 1 \rightarrow N.$$

The least-squares estimator ceases to be unbiased as soon as the sample contains one outlier of any type. The robustness of an estimator can be defined as the sensitivity of the fit properties (goodness-of-fit, resolution, uncertainty estimation and bias of the fitted parameters) to the presence of outliers. It is rated by the breakdown point b , i.e. the fraction of outliers above which the fitted parameters may suffer an arbitrary bias. To be more specific, if there are more than $b \times N$ outliers among the N vertex tracks, the bias in the vertex parameters will be proportional to the biases in the outlier parameters, which are *a-priori* unknown. The breakdown point of least-squares estimators is $1/N$, as a single outlier already introduces a bias.

In this note, two ways of robustifying least-squares estimators are studied, by trimming and by down-weighting distant tracks. The least trimmed sum of squares (LTS) algorithm discards the M out of N tracks which are the least compatible with the vertex. The ratio $k = M/N$ is called the trimming fraction and is an input parameter of the algorithm. The best choice of trimming fraction is the fraction of outliers in the vertex. When the outlier fraction cannot be estimated, 20% is an adequate starting value. In realistic physics analyses a higher outlier fraction is unlikely, and the maximum degradation of resolution in the case where all the tracks rejected are inliers is $1/\sqrt{0.8} = 1.12$.

The adaptive algorithm down-weights distant tracks by a factor which is a sigmoidal function of the reduced distance to the vertex:

$$w_i(r_i) = \frac{\exp(-\frac{r_i^2}{2T})}{\exp(-\frac{r_i^2}{2T}) + \exp(-\frac{r_{\text{cutoff}}^2}{2T})} \leq 1; \quad (2)$$

where $r_i = (x - x_i)/\sigma_i$ is the reduced distance of track i , r_{cutoff} is the distance where the weight function drops to 0.5 and T is a parameter controlling the sharpness of the drop. In realistic physics analyses, a value of r_{cutoff} around 4.0 should reject the most dangerous outliers while retaining almost 100% of the inliers. In this study the T parameter is chosen to obtain a sharp weight function. The effect of using a smoother weight function is not studied here.

Both robust algorithms are iterative. They need an initial estimate of the vertex position to compute the initial track distances, and iterate until the vertex position converges. Both can actually be formulated as iterative re-weighted least-squares algorithms, with the following function to be minimized at iteration k :

$$F^k = \sum_i (r_i^k)^2 \cdot w_i^k; \quad i = 1 \rightarrow N$$

with weights $w_i^k = w_i^k(r_i^{k-1})$ computed according to the vertex position at iteration $k - 1$. In the case of the LTS algorithm, the weights take discrete values, zero or one.

The adaptive algorithm has two advantages over the trimming algorithm: (1) the weights can be fractional (soft assignment), and (2) they are adapted in the course of iteration; there is thus no need to set the fraction of tracks rejected at algorithm initialization.

2.2 Initial guess of the vertex position

In robust estimation theory it is known that, in order to converge, robust algorithms must start with an initial estimate close enough to the global minimum. In particular, in the presence of many outliers, a first guess computed with a (non-robust) least-squares fit would not be reliable because it could be affected by an arbitrary bias.

Original methods were designed in order to find a coarse but unbiased initial estimate with little CPU overhead to the vertex fit. The average of the points of closest approach of a few track pairs (the five pairs of tracks with highest

p_T by default) is computed for each space coordinate. This average is made robust using e.g. a half-sample mode algorithm. These methods are described in [3].

3 Simplified Monte Carlo study

3.1 The Vertex Gun

The interpretation of the envisaged results is eased by using a simplified and configurable Monte Carlo track generator rather than detailed event reconstruction. This simplified Monte Carlo generator is further called the Vertex Gun. It is available in the `VertexFastSim` package of the CMS reconstruction and analysis software ORCA [4]. The number of vertices, their position and the total number of tracks originating from each vertex are configurable. Track reconstruction is simulated by smearing the track parameters with a five-dimensional Gaussian distribution. In this study only the two impact parameters (R_{IP} , z_{IP}) are smeared. The smearing for inliers is done with a fixed spread of $50 \mu\text{m}$.

The fraction of Type-1 outliers is configurable for each vertex. Type-1 outliers are simulated by smearing the impact parameters with a spread of $k_{\text{Type-1}} \cdot 50 \mu\text{m}$. The same covariance matrix is used for both inliers and outliers in the computation of the vertex position. This method generates tails in the distribution of the impact parameter residuals and pulls, by simulating tracks which are $k_{\text{Type-1}}$ times less precise than they are thought to be.

Type-2 outliers are simulated by generating another vertex near the vertex of interest. Two topologies are evaluated. The first topology resembles two primary vertices near each other (Fig. 1). The tracks are uniformly generated within a cone of aperture 2θ equal to 1.6 rad around the x axis. The second vertex is separated from the vertex of interest by a distance Δy along the y axis.

In the second topology, the vertex of interest is a secondary vertex in a jet, and the source of outliers is the primary vertex. The tracks are generated in a cone of aperture $2\theta = 0.5$ rad around the x axis. The separation Δx of the two vertices is along the cone axis and is fixed to 3 mm.

Figure 1: Illustration of some relevant parameters in the Vertex Gun setup.

3.2 Analysis

In this note the impact of Type-1 and Type-2 outliers on the fit results is studied for the least-squares (LS), the least-trimmed-squares (LTS) and the adaptive vertex fitters. The fit properties considered are the distributions of the fit χ^2 probability, of the differences between reconstructed and simulated vertex coordinates (residuals), and of the residuals divided by the corresponding fitted uncertainty (pulls).

For a perfect fit the observed χ^2_{obs} should follow a uniform probability distribution $P(\chi^2_{\text{ndf}} \geq \chi^2_{\text{obs}})$ ranging between 0 and 1. The average value of this distribution over an event sample should be 0.5. The pull distribution should follow a Gaussian distribution centred at zero and with a Root Mean Square (RMS) equal to unity. The bias is estimated as the average value of the residual distribution. The resolution is estimated as the width of a Gaussian fit to this distribution. This resolution is compared with the Minimum Variance Bound (MVB) resolution which reflects the use of all the unbiased information available in an event. This Minimum Variance Bound is estimated by fitting a vertex with the LS fitter using the inlying tracks only.

For each set of Vertex Gun parameters 10000 events are simulated. Unless otherwise stated, the r_{cutoff} parameter of the adaptive fitter is equal to 4.0, and the trimming fraction of the LTS algorithm, to 20%.

4 High multiplicity primary vertex topology

4.1 Sensitivity to mis-measured tracks

In this section the average χ^2 probability, the width of the pull distribution and the resolution of the vertex position obtained from the three vertex fitting algorithms are compared. This study is done as a function of the fraction of Type-1 outliers in the track sample. The total number of tracks, including inliers and outliers, is fixed and equal to 20.

The average χ^2 probability as a function of the fraction of Type-1 outliers is shown in Fig. 2 (left). The results obtained when the smearing of outliers is ten times broader than that used for inliers ($k_{\text{Type-1}} = 10$) are indicated by closed markers. The results when the smearing is only three times broader ($k_{\text{Type-1}} = 3$) are indicated by open markers. As expected, the LS fitter is very sensitive to Type-1 outliers. When the covariance matrix of outlying tracks is underestimated by a factor ten, the presence of one outlier brings the average χ^2 probability below 5%. When the covariance matrix is underestimated by a factor three, the average χ^2 probability still drops sharply when the fraction of outliers increases, reaching values below 10% for more than two outliers.

Figure 2: Average χ^2 probability of the vertex fitters studied as a function of the fraction of Type-1 (left) and Type-2 (right) outliers. On the left plot, closed (open) marks show results for outliers smeared 10 (3) times broader than inliers.

The average χ^2 probability of the LTS fitter is close to 0.5 in a narrow range of contamination around 20%. This

value corresponds to the trimming fraction chosen. When the fraction of outliers increases above the trimming fraction, the average χ^2 probability of the LTS fitter behaves in the same way as that of the LS fitter. This behaviour indicates that the breakdown point of the LTS fitter is equal to the trimming fraction, as expected from the theory. For contaminations below the trimming fraction, some inliers are also rejected, and only the subset of inliers which are the most compatible among themselves is retained. The distribution of the χ^2 probability is thus truncated, low probabilities being cut away. The fraction of outliers in an event could be estimated by tuning the trimming fraction in order to reach a χ^2 probability of 0.5.

The average χ^2 probability of the adaptive fitter is the most stable with respect to the contamination level. When the smearing factor $k_{\text{Type-1}}$ is large compared to the cutoff parameter r_{cutoff} of the algorithm, the χ^2 probability is almost not perturbed by the presence of Type-1 outliers.

In Fig. 3 the width of the pull distribution is shown as a function of the fraction of Type-1 outliers, for $k_{\text{Type-1}} = 3$ and 10. A distinction is made between the direction of the cone in which the tracks are generated and the direction perpendicular to the cone axis. If the outliers are not too different from the inliers ($k_{\text{Type-1}} = 3$), the pull of all three vertex fitters is non-unity whenever the fraction of outliers is non-zero. The pull of the robust algorithms, however, is less affected than that of the LS fitter.

Figure 3: RMS of the pull distributions of the vertex fitters studied as a function of the fraction of Type-1 outliers. The directions parallel and transverse to the cone direction are shown. The track multiplicity is 20 including outliers. The triangles indicate the results obtained at trimming fractions of 10%, 30% and 40%, different from the default value of 20%. On the left (right) the coordinates of the outliers are smeared 3 times (10 times) broader than the inliers.

The RMS of the pulls of the adaptive fitter is the closest to unity. The LTS fitter performs worse for all fractions of outliers, except when the fraction of outliers is equal to the trimming fraction. This property is verified when setting the trimming fraction equal to the fraction of outliers (triangular markers). For outlier fractions above the trimming fraction, the pull of the LTS fitter behaves similarly to that of the LS fitter. This behaviour is more prominent when the smearing of the outliers is broader. Below the trimming fraction, the RMS of the pulls of the LTS fitter is above unity because the fitter also rejects some inliers. The position of the fitted vertex is pulled towards the set of inliers which are the most compatible among themselves. This internal compatibility introduces a bias in the track sample, leading to a resolution which is worse than the fitted vertex uncertainty.

For the resolution on the vertex position similar observations are made (Fig. 4). The performance of the adaptive fitter is closer to the MVB when the $k_{\text{Type-1}}$ factor increases: outliers with a covariance matrix which is mis-estimated by a larger amount are easier to reject. When $k_{\text{Type-1}}$ is closer to 1, the resolution of the LS fitter is less degraded, while the resolution of the adaptive fitter departs from the MVB. The adaptive fitter, however, still performs best.

The robustness of the LTS fitter is comparable to that of the adaptive fitter only when the trimming fraction equals

Figure 4: RMS of the residuals distributions of the vertex fitters studied as a function of the fraction of Type-1 outliers. The directions parallel and transverse to the cone direction are shown. The track multiplicity is 20 including outliers. The triangles indicate the results obtained at trimming fractions of 10%, 30% and 40%, different from the default value of 20%. On the left (right) the coordinates of the outliers are smeared 3 times (10 times) broader than the inliers. The results are compared to the Minimum Variance Bound.

the fraction of outliers. For contamination levels below the trimming fraction, the resolution of the LTS fitter is degraded by two effects: less tracks are used in the vertex fit, and the subset of tracks which are the most compatible among themselves is biased.

In Fig. 5 the effect of the r_{cutoff} parameter of the adaptive fitter is illustrated when changing the value from 4 to 2, hence rejecting more outliers but also inliers. As expected, the resolution becomes worse. The width of the pull distribution becomes closer to unity for large contamination levels. For small contamination levels, the pull RMS is larger than 1 for the same reason as for the LTS fitter: some inliers are rejected and the subset of most compatible inliers is biased.

4.2 Sensitivity to mis-associated tracks

In this section the average observed χ^2 probability, the bias and resolution on the vertex position are studied for the three vertex fitting algorithms, as a function of the fraction of Type-2 outliers in the track sample. All Type-2 outliers originate from the same vertex, separated from the vertex of interest by a distance Δy along the y-axis. The total number of tracks, including inliers and outliers, is fixed and equal to 20.

In Fig. 2 (right) the average χ^2 probability is plotted as a function of the fraction of Type-2 outliers. The separation between the two vertices is 0.3 mm. Similar features as for the Type-1 outliers are observed. Again the adaptive fitter is the most robust for all fractions of outliers.

In Fig. 6 the average bias of the vertex position estimator is shown as a function of the fraction of Type-2 outliers, for values of Δy equal to 0.1 mm, 0.3 mm and 0.5 mm. When the two vertices are closest to each other, there is an almost linear dependence of the average bias on the fraction of outliers, indicating that outliers contribute to the fit with a weight about equal to the weight of inliers. This is due to the fact that the vertex separation is only twice the width of the Gaussian used for the smearing of the track parameters, and primary and secondary tracks cannot be separated.

The discriminating power of the robust fitters becomes more visible when the distance Δy is increased to a value of 0.3 mm. The LTS fitter has a negligible bias up to the breakpoint where the fraction of outliers equals the trimming fraction of 20%. For higher outlier fractions the bias of LTS fitter has the same dependency as that of the LS fitter. The bias of the adaptive fitter remains below ten micrometers up to about 35-40% of outliers. Beyond this point, the fit sometimes converges to the position of the second vertex.

Figure 5: RMS of the residuals distribution (left) and the pull distribution (right) of the adaptive vertex fitter as a function of the fraction of Type-1 outliers. The directions parallel and transverse to the cone direction are shown. The track multiplicity is 20 including outliers. Two different values of the $r_{\text{cut-off}}$ parameter are used.

Resolution plots in Fig. 7 reveal an intriguing but irrelevant feature. For some fractions of outliers and short separations between the vertices, one or the other of the algorithms shows resolutions better than the MVB. This is because the distance between the vertices is fixed, and thus the fitters measure a virtual vertex between the two generated vertices. This vertex is fitted with both inliers and outliers, thus with more tracks than the primary vertex alone.

5 Low multiplicity secondary vertex topology

As the track multiplicity decreases, a larger sensitivity to outliers is expected, since a smaller subsample of tracks can be used by the algorithm in order to check the compatibility of the other tracks. In this section the relative improvement of using the adaptive fitter compared to the LS fitter is studied, for vertices with a total track multiplicity ranging from 3 to 10. The sensitivity is checked for mis-measured and mis-associated tracks. In the latter case, the vertex of interest is a secondary vertex, separated by 3 mm from a primary vertex which is the source of Type-2 outliers.

5.1 Sensitivity to mis-measured tracks

The dependency of the width of the pull distribution on the fraction of Type-1 outliers is shown in Fig. 8 for both the LS and the adaptive vertex fitters. The outlier covariance matrix is underestimated by a factor $k_{\text{Type-1}} = 3$. The performance of the adaptive fitter becomes worse when decreasing the number of tracks in the events: it departs from the MVB and gets closer to the performance of the LS fitter. The same observation can be made from Fig. 9, where the resolution is shown as a function of the fraction of outliers.

Similar observations are seen when the covariance underestimation factor is $k_{\text{Type-1}} = 10$ (Fig. 10 and Fig. 11). As expected, however, it becomes easier for the adaptive fitting algorithm to reject the Type-1 outliers when $k_{\text{Type-1}}$ increases. The resulting width of the pull distribution is closer to unity and the estimated resolution approaches better the MVB.

Still, down to the smallest possible multiplicities, the adaptive fitter always performs better than the LS fitter.

5.2 Sensitivity to mis-associated tracks

The average bias along the cone axis is shown in Fig. 12 as a function of the fraction of Type-2 outliers, for the LS and adaptive vertex fitters. The jet cone aperture is $2\theta = 0.5$, and Type-2 outliers originate from a primary vertex

Figure 6: Average bias on the vertex position in the y direction as a function of the fraction of Type-2 outliers, for several separations Δy between the vertex of interest and the source of Type-2 outliers. The track multiplicity is 20 including outliers.

3 mm behind the secondary vertex of interest. The separation between the vertices is along the cone axis. The robustness of the adaptive fitter is reduced when the vertex multiplicity is lower. Still, in several cases which are of interest for physics analyses (three- or four-prong secondary vertices plus one outlier), the bias is largely reduced as compared to the LS fitter.

The distribution of residuals is shown in Fig. 13 for a few low-multiplicity topologies. Tails or even a second peak appear in the distributions of residuals for the adaptive fitter. The second peak is due to events where the adaptive fitter has converged to the primary vertex. These imperfections indicate where the adaptive algorithm could be further improved. In particular, the smoothness of the weight function as well as the evolution of the T parameter during the course of iteration (i.e. the annealing schedule of the algorithm) could be tuned so as to increase the fraction of events converging to the vertex of interest.

Eventually, in physics analyses, events in the second peak can be flagged by comparing the position of the fitted vertex to the beam spot or to the position of the primary vertex estimated at vertex finding stage. Such a test is less decisive when using the LS algorithm.

6 Conclusion and outlook

A study of the statistical properties of the least-trimmed-squares (LTS) and the adaptive vertex fitting algorithms has been performed as a function of the type and the level of contamination present in the track sample. In order to perform this study, a simplified Monte Carlo setup has been developed, the so-called Vertex Gun, which allows the introduction of a known amount of outlying tracks, either mis-measured (Type-1 outliers), or mis-associated (Type-2 outliers) to the vertex. In the case of Type-2 outliers, two event topologies have been tested, one typical of a high multiplicity primary vertex, and the other typical of a low multiplicity secondary vertex. The results on the

Figure 7: RMS of the residuals distributions as a function of the fraction of Type-2 outliers, for several separations Δy between the vertex of interest and the source of Type-2 outliers. The directions parallel and transverse to the cone direction are shown. The track multiplicity is 20 including outliers. The results are compared to the MVB.

width of the pull distribution, the average χ^2 probability, the estimated resolution and the possible bias, obtained with samples of 10000 Vertex Gun events, are compared with those obtained with a least-squares vertex fitter.

The adaptive fitter is found to be the algorithm for which the fit results are the least affected by the presence of outliers. The adaptive fitter is thus the most robust of the algorithms tested. This good performance has been observed for both event topologies, in particular for multiplicities typical of secondary vertices of physics interest (B-meson and τ -lepton decays). The LTS fitter performs equally well only when the trimming fraction is set equal to the *a-priori* unknown number of outliers associated to the vertex.

The improvement of using robust fitters is more pronounced when the track parameters of the outliers are more different from the inliers as it becomes easier to reject them from the inlier sample. This has been observed for track multiplicities associated to the vertex ranging from three to twenty. When the outliers are not very distinct, the degradation of the performance of the LS fitter due to the presence of outliers is of course less severe. The results of the adaptive fitter remain however always better as long as the cutoff parameter is chosen so as to retain almost all inliers (four times the impact parameter resolution in this study).

Figure 8: RMS of the pull distributions as a function of the fraction of Type-1 outliers, for different track multiplicities (10, 5 and 3 tracks including outliers). The directions parallel and transverse to the cone direction are shown. The results are compared to the MVB. The Type-1 outliers are smeared with a Gaussian which is three times broader than that used for the inliers.

This study has been performed with simplified event topologies in order to understand the statistical properties and features of the different vertex fitting algorithms. The application of robust vertex fitting algorithms in a complete analysis of fully simulated LHC events remains to be studied.

Figure 9: RMS of the residuals distributions as a function of the fraction of Type-1 outliers, for different track multiplicities (10, 5 and 3 tracks including outliers). The directions parallel and transverse to the cone direction are shown. The results are compared to the MVB. The Type-1 outliers are smeared with a Gaussian which is three times broader than that used for the inliers.

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Figure 10: RMS of the pull distributions as a function of the fraction of Type-1 outliers, for different track multiplicities (10, 5 and 3 tracks including outliers). The directions parallel and transverse to the cone direction are shown. The results are compared to the MVB. The Type-1 outliers are smeared with a Gaussian which is 10 times broader than that used for the inliers.

Figure 11: RMS of the residuals distributions as a function of the fraction of Type-1 outliers, for different track multiplicities (10, 5 and 3 tracks including outliers). The directions parallel and transverse to the cone direction are shown. The results are compared to the MVB. The Type-1 outliers are smeared with a Gaussian which is 10 times broader than that used for the inliers.

Figure 12: Average bias along the cone axis (absolute value) for the LS and adaptive vertex fitters, as a function of the fraction of Type-2 outliers, for different track multiplicities (10, 5, 4 and 3 tracks including outliers). The cone aperture 2θ amounts to 0.5 and Type-2 outliers are originating from a primary vertex located 3 mm behind the vertex of interest along the cone axis.

Figure 13: Distributions of residuals along the cone axis for the LS and adaptive vertex fitters. The numbers of inliers and Type-2 outliers are specified on the histograms.