Larmor radius effects on impurity transport in turbulent plasmas

M. Vlad and F. Spineanu
National Institute for Laser, Plasma and Radiation Physics,
Association Euratom-MEC, P.O.Box MG-36, Magurele, Bucharest, Romania

Abstract

Test particle transport determined by the Lorentz force in turbulent magnetized plasmas is studied. The time dependent diffusion coefficient, valid for the whole range of parameters, is obtained by developing the decorrelation trajectory method. The effects of Larmor gyration in the presence of trapping are analyzed and several transport regimes are evidenced.

Keywords: plasma turbulence, statistical approaches, test particle transport

1 Introduction

Impurity control in magnetically confined plasmas is a very important issue for the development of fusion reactors. Impurity behaviour is a complex problem related to confinement and transport of the bulk ions and electrons in plasma and to plasma-wall interaction. A very strong experimental effort (see e.g. [1]-[9]) lead to the conclusion that this process is far from being understood on the basis of the existing theoretical models. In particular, the experimental results for the diffusion coefficients are much larger than the neoclassical prediction, especially in the edge plasma, showing the presence of anomalous transport.

We analyze here the particular topic of impurity transport in turbulent plasmas using test particle approach. Particle motion in a stochastic potential was extensively studied in the guiding center approximation [10]-[15]. It is well known since many years [16] that, for slowly varying or large amplitude turbulence, the $E \times B$ drift determines a process of dynamical trapping of the trajectories. It consists of trajectory winding around the extrema of the stochastic potential and strongly influences the transport. Important progresses in the study of this nonlinear process were recently obtained. New statistical methods were developed [17], [18] that permitted to determine the asymptotic diffusion coefficient and also the correlation of the Lagrangian velocity and the time dependent (running) diffusion coefficient. It was shown that the trapping process determines the decrease of the diffusion coefficient and the change of its scaling in the parameters of the stochastic field. These methods were extended to more complex models, which consider, besides the $E \times B$
drift, particle collisions [19], an average velocity [20] or the parallel motion [21], and also to the study of the collisional particle diffusion in stochastic magnetic fields [22]. The conclusion of these studies is that the trapping combined with a decorrelation mechanism determines anomalous transport regimes. In these regimes the dependence of the diffusion coefficient $D$ on the parameters describing the decorrelation is inverted due to trapping in the sense that a decrease of $D$ appearing in the quasilinear regime is transformed into an "anomalous" increase of $D$ in the nonlinear regime [23].

All these studies are based on the guiding center approximation for particle motion that considers the Larmor radius negligible. This approximation is not adequate for the impurity ions which can have Larmor radii comparable or larger than the correlation length of the turbulence and cyclotron periods comparable with the turnover time of the $E \times B$ motion. In these conditions the trajectories have to be determined from the Lorentz force. The aim of this paper is to determine the effects of finite Larmor radius on particle transport in a turbulent magnetized plasmas. In particular, we analyze the influence on the trapping process and compare the characteristics of the transport induced by the Lorentz force (Lorentz transport) with those obtained in the guiding center approximation (drift transport). The time dependent diffusion coefficient is obtained as a function of the turbulence and ion's parameters by extending the decorrelation trajectory method developed for the drift transport [17], [24]. The transport regimes for a large range of parameters are determined.

The paper is organized as follows. Section 2 contains the basic equations, the statistical approach and the derivation of the general expression for the time dependent diffusion coefficient. The results are presented in Sections 3-6. First, in Section 3, a static potential is considered. We show that the trapping exists even at large values of the Larmor radius and that it determines a subdiffusive transport, as in the drift approximation, but with a time dependence of the diffusion coefficient strongly influenced by the gyration motion. Then we examine the dependence of the asymptotic diffusion coefficient on the three dimensionless parameters (see Section 2 for their definitions) that characterize this process and show that several transport regimes appear. The dependence on the first parameter (the Kubo number) that describes the effect of the time dependence of the stochastic potential is analyzed in Section 4 where two regimes are evidenced. The dependence of $D$ on the initial kinetic energy of the ions (the second parameter) is presented in Section 5 and in Section 6 the effect of the normalized cyclotron frequency that essentially describes the specific charge of the ions is examined. The conclusions are summarized in Section 7.

### 2 Basic equations and statistical approach

We consider a constant confining magnetic field directed along $z$ axis, $B = B e_z$ (slab geometry) and an electrostatic turbulence represented by an electrostatic potential $\phi'(x, t)$, where $x \equiv (x_1, x_2)$ are the Cartesian coordinates in the plane perpendicular to $B$. The motion of an ion with charge $q$ and mass $m$ is determined by the Lorentz
force:
\[ m \frac{d^2x(t)}{dt^2} = q \left\{ -\nabla \phi(x, t) + u \times B \right\} \]  

(1)

where \( x(t) \) is the ion trajectory, \( u(t) = dx(t)/dt \) is its velocity and \( \nabla \) is the gradient in the \((x_1, x_2)\) plane. The initial conditions are

\[ x(0) = 0, \quad u(0) = u_0. \]  

(2)

This equation is transformed into a system of first order equations for the position and the velocity of the ion

\[ \frac{du_i}{dt} = -\frac{q}{m} \frac{\partial \phi(x, t)}{\partial x_i} + \Omega \varepsilon_{ij} u_j \]  

(3)

\[ \frac{dx_i}{dt} = u_i \]  

(4)

where \( \Omega = qB/m \) is the cyclotron frequency and \( \varepsilon_{in} \) is the antisymmetric tensor \( (\varepsilon_{12} = -\varepsilon_{21} = 1, \varepsilon_{11} = \varepsilon_{22} = 0) \). Introducing the instantaneous Larmor radius defined by

\[ \rho_i(t) \equiv -\frac{\varepsilon_{ij} u_j(t)}{\Omega}, \]  

(5)

the guiding center position \( \xi(t) \equiv x(t) - \rho(t) \) and \( \phi(x, t) \equiv \phi(x, t)/B \), the system becomes

\[ \frac{d\xi_i}{dt} = -\varepsilon_{ij} \frac{\partial \phi(\xi + \rho, t)}{\partial \xi_j} \]  

(6)

\[ \frac{d\rho_i}{dt} = \varepsilon_{ij} \left[ \frac{\partial \phi(\xi + \rho, t)}{\partial \xi_j} + \Omega \rho_j \right]. \]  

(7)

The electrostatic potential \( \phi(x, t) \) is a stochastic field and thus Eqs. (6-7) are Langevin equations. The solution consists, in principle, in determining the statistical properties of the ensembles of trajectories, each one obtained by integrating Eqs. (6-7) for a realization of the stochastic potential. We will determine here the mean square displacement and the time dependent diffusion coefficient for the guiding center trajectories \( \xi(t) \). These statistical quantities can also be determined for particle trajectories \( x(t) \) and for the Larmor radius \( \rho(t) \) but they are not physically relevant.

The potential is considered to be a stationary and homogeneous Gaussian stochastic field, with zero average. Such a stochastic field is completely determined by the two-point Eulerian correlation function, \( E(x, t) \), defined by

\[ E(x, t) \equiv \langle \phi(x', t') \phi(x + x', t + t) \rangle. \]  

(8)

The average \( \langle \ldots \rangle \) is the statistical average over the realizations of \( \phi(x, t) \). The statistical properties of the drift velocity components

\[ v_i^{dr}(x, t) \equiv -\varepsilon_{ij} \frac{\partial \phi(x, t)}{\partial x_j} \]  

(9)
are completely determined by those of the potential; they are stationary and homogeneous Gaussian stochastic fields like \( \phi (\mathbf{x}, t) \). The two-point Eulerian correlations of the drift velocity components, \( E_{ij}(\mathbf{x}, t) \equiv \langle v_{i}^{\text{dr}}(\mathbf{x}, t) \; v_{j}^{\text{dr}}(\mathbf{x} + \mathbf{x}, t + t) \rangle \), and the potential-velocity correlations, \( E_{\phi i}(\mathbf{x}, t) \equiv \langle \phi(\mathbf{x}, t) \; v_{i}^{\text{dr}}(\mathbf{x} + \mathbf{x}, t + t) \rangle \), are obtained using Eq. (9) as:

\[
E_{ij}(\mathbf{x}, t) = -\varepsilon_{in} \varepsilon_{jm} \frac{\partial^2 E(\mathbf{x}, t)}{\partial x_n \partial x_m}, \quad (10)
\]

\[
E_{\phi i}(\mathbf{x}, t) = -E_{i\phi}(\mathbf{x}, t) = -\varepsilon_{in} \frac{\partial E(\mathbf{x}, t)}{\partial x_n}. \quad (11)
\]

The Eulerian correlation of the drift velocity (10) evidences three parameters: the amplitude \( V = \sqrt{E_{11}(0, 0)} \), the correlation time \( \tau_c \), which is the decay time of the Eulerian correlation, and the correlation length \( \lambda_c \), which is the characteristic decay distance. These parameters combine in a dimensionless Kubo number

\[
K = \frac{V \tau_c}{\lambda_c} = \frac{\tau_c}{\tau_{fl}} \quad (11)
\]

which is the ratio of \( \tau_c \) to the average time of flight of the particles (\( \tau_{fl} = \lambda_c / V \)) over the correlation length. Using these parameters of the stochastic field, Eqs.(6-7) are written in dimensionless form as

\[
\frac{d\xi_i}{dt} = -\varepsilon_{ij} \frac{\partial \phi(\xi + \rho, t)}{\partial \xi_j}, \quad (12)
\]

\[
\frac{d\rho_i}{dt} = \varepsilon_{ij} \left[ \frac{\partial \phi(\xi + \rho, t)}{\partial \xi_j} + \overline{\Omega} \rho_j \right], \quad (13)
\]

where the normalization parameters are \( \tau_{fl} \) for time, \( \lambda_c \) for distances, \( V \) for the drift velocity and

\[
\overline{\Omega} = \Omega \tau_{fl}. \quad (14)
\]

The same notations are kept for the normalized quantities.

We note that the equation for the guiding center trajectory (12) is similar with that obtained in the guiding center approximation, with the difference that the argument of the potential is the particle trajectory \( \mathbf{x}(t) \equiv \xi(t) + \rho(t) \) instead of \( \xi(t) \). The equation for the Larmor radius (13) describes a cyclotron motion that has the radius and the frequency dependent of the stochastic potential. In the 2-dimensional case studied here, both equations are of Hamiltonian type (the two components of \( \xi(t) \) are conjugated variables as well as the two components of the Larmor radius \( \rho(t) \))

\[
\frac{d\xi_i}{dt} = -\varepsilon_{ij} \frac{\partial H(\xi, \rho)}{\partial \xi_j}, \quad \frac{d\rho_i}{dt} = \varepsilon_{ij} \frac{\partial H(\xi, \rho)}{\partial \rho_j}. \quad (15)
\]

They have the same Hamiltonian function

\[
H(\xi, \rho) = \phi(\xi + \rho) + \frac{\overline{\Omega}}{2} (\rho_1^2 + \rho_2^2), \quad (16)
\]

which is the energy of the particle. The Hamiltonian depends on \( \xi \) and \( \rho \) and thus the two Hamiltonian systems (12) and (13) are coupled. For each system the other
variable introduces a time dependence in $H(\xi, \rho)$ which perturbs the regular motion that is obtained in the absence of interaction. The perturbation can be very strong leading to a chaotic motion of the guiding centers.

Particle motion is thus determined by three dimensionless parameters: $K$, $\bar{\rho}$ and $\bar{\Omega}$. The first one, the Kubo number $K$, does not appear in the equations, but only in the statistical description of the stochastic potential. It describes the effect of time variation of the stochastic potential. The second parameter $\bar{\rho}$ is the initial Larmor radius normalized with the correlation length and appears in the initial condition

\[ \rho(0) = \frac{|\rho(0)|}{\lambda c} \quad \rho(0) = \bar{\rho} \sin(\alpha) \quad \xi(0) = -\rho(0) \]  

where $\bar{\rho} = \frac{|\rho(0)|}{\lambda c} = |u_0| / V \bar{\Omega}$ and $\alpha$ determines the orientation of the initial velocity (the angle between $u_0$ and the $x_1$ axis is $\pi/2 - \alpha$). $\bar{\rho}$ is related to the initial kinetic energy of the particles. The third parameter $\bar{\Omega}$ defined in Eq.(14) is the cyclotron frequency normalized with $\tau_{fi}$ and describes the relative importance of the cyclotron and drift motion (second and respectively first term in 13) in the evolution of the Larmor radius.

Starting from the statistical description of the stochastic potential, we will determine the correlation of the Lagrangian drift velocity, defined by:

\[ L_{ij}(t) \equiv \langle v^{dr}_i [x(0), 0] v^{dr}_j [x(t), t] \rangle. \]  

The mean square displacement of the guiding center and its time dependent diffusion coefficient are integrals of this function:

\[ \left\langle \xi_i^2(t) \right\rangle = 2 \int_0^t d\tau \; L_{ii}(\tau) \; (t - \tau), \]  
\[ D_i(t) = \int_0^t d\tau \; L_{ii}(\tau), \]  

provided that the process is stationary [25].

The guiding center approximation obtained by taking $\rho = 0$ in Eq.(12) was recently studied by developing a semi-analytical approach, the decorrelation trajectory method [17], [24]. Using this approach an important progress was obtained in the understanding of the intrinsic trapping process specific to the $E \times B$ drift. We present here a generalization of the decorrelation trajectory method that applies to the Lorentz transport described by Eqs. (12-13).

The Langevin equations (12-13) for given values of the parameters $\bar{\Omega}$, $\bar{\rho}$ and $K$ is studied in subensembles (S) of realizations of the stochastic field, which are determined by given values of the potential and of the drift velocity in the starting point of the trajectories:

\[ \phi(0, 0) = \phi^0, \quad \mathbf{v}^{dr}(0, 0) = \mathbf{v}^0. \]  

All the trajectories contained in a subensemble have the same initial energy. The stochastic (Eulerian) potential and drift velocity in a subensemble (S) defined by condition (21) are Gaussian fields but non-stationary and non-homogeneous, with
space and time dependent averages. These averages depend on the parameters of the subensemble and are determined by the Eulerian correlation of the potential

\[ \Phi(x, t; S) \equiv \langle \phi(x, t) \rangle_S = \phi^0 \frac{E(x, t)}{E(0, 0)} + v_1^0 \frac{E_{1\phi}(x, t)}{E_{11}(0, 0)} + v_2^0 \frac{E_{2\phi}(x, t)}{E_{22}(0, 0)}, \quad (22) \]

\[ V_i(x, t; S) \equiv \langle v_i^{dr} [x, t] \rangle_S = -\varepsilon_{ij} \frac{\partial \Phi(x, t; S)}{\partial x_j}, \quad (23) \]

where \( \langle \ldots \rangle_S \) is the statistical average over the realizations that belong to \( S \). They are equal to the corresponding imposed condition \( (21) \) in \( x = 0 \) and \( t = 0 \) and decay to zero at large distance and/or time. The existence of an average Eulerian drift velocity in the subensemble determines an average motion, i.e. an average Lagrangian drift velocity \( \mathbf{V}^L(t; S) \equiv \langle \mathbf{v}^{dr} [x(t), t] \rangle_S \). The correlation of the Lagrangian drift velocity for the whole ensemble of realizations \( \langle \ldots \rangle \) can be written as

\[ L_{ij}(t) = \int \int d\phi^0 d\mathbf{v}^0 P_1(\phi^0, \mathbf{v}^0; 0, 0) v_i^0 v_j^0 \mathbf{V}^L(t; S) \quad (24) \]

where \( P_1(\phi^0, \mathbf{v}^0; 0, 0) \) is the probability that a realization belongs to the subensemble \( (S) \). The average Lagrangian drift velocity \( \mathbf{V}^L(t; S) \) is determined using an approximation that essentially consists in neglecting the fluctuations of the trajectories in \( (S) \). This approximation is validated in \( [18] \) where the fluctuations of the trajectories in \( (S) \) are taken into account in a more complicated and precise method. It is shown that they lead to a weak modification of the diffusion coefficients \( D(t) \), although they strongly change \( \mathbf{V}^L(t; S) \). Introducing the average guiding center trajectory in \( (S) \), \( \Xi(t; S) \equiv \langle \xi(t) \rangle_S \), and the average Larmor radius in \( (S) \), \( \Pi(t; S) \equiv \langle \rho(t) \rangle_S \), the equations of motion can be averaged over the realizations in \( (S) \) in this approximation and yield

\[ \frac{d \Xi_i}{dt} = -\varepsilon_{ij} \frac{\partial \Phi(\Xi + \Pi, t; S)}{\partial \xi_j}, \quad (25) \]

\[ \frac{d \Pi_i}{dt} = \varepsilon_{ij} \left[ \frac{\partial \Phi(\Xi + \Pi, t; S)}{\partial \xi_j} + \Omega \Pi_j \right]. \quad (26) \]

The initial conditions for the two components of the subensemble average trajectory are obtained from Eq. \( (17) \)

\[ \Pi_1(0) = \mathbf{\Pi} \cos(\alpha), \quad \Pi_2(0) = \mathbf{\Pi} \sin(\alpha), \quad \Xi(0) = -\Xi(0). \quad (27) \]

Since the orientation of the initial velocity \( \mathbf{u}_0 \) is arbitrary, we will consider in each realization in \( (S) \) initial conditions with uniform distribution of \( \alpha \) over the interval \( [0, 2\pi] \).

This approximation ensures the conservation of the subensemble average energy of the particles.

Considering for simplicity an isotropic stochastic potential with the Eulerian correlation depending \( |x| \), a diagonal correlation tensor is obtained for the Lagrangian drift velocity, \( L_{ij}(t) = \delta_{ij} L(t) \), and the following expressions for the time dependent (running) diffusive coefficient:

\[ D(t) \equiv D_B F(t), \quad (28) \]
\[
L(t) = V^2 \frac{dF(t)}{dt}
\]

where
\[
F(t) = \frac{1}{2} \frac{1}{(2\pi)^{3/2}} \int_{-\infty}^{\infty} d\phi' \exp \left( -\frac{(\phi')^2}{2} \right) \int_0^\infty dv v^2 \exp \left( -\frac{v^2}{2} \right) \int_0^{2\pi} d\alpha \Xi_1(t; S)
\]

and \( \Xi_1(t; S) \) is the component of the solution of Eqs. (25-26) along the initial average drift velocity \( v^0 \) and \( v = |v^0| \).

We have thus determined the correlation of the Lagrangian drift velocity (for ions with mass \( m \), charge \( q \) and given initial kinetic energy) corresponding to given Eulerian correlation \( E(x, t) \) of the stochastic potential. Explicit results for \( L(t) \) and \( D(t) \) are obtained by effectively calculating the average trajectories in (S), solutions of Eqs. (25-27), and the weighted average (30). This procedure appears to be very similar with a direct numerical study of the simulated trajectories. There are however essential differences. The average trajectories are obtained for a rather smooth and simple potential and the number of trajectories is much smaller than in the numerical study due to the weighting factor determined analytically. This reduced very much the calculation time, such that it can be performed on PC. A computer code is developed for explicit calculation of \( D(t) \) for given values of the parameters \( K, \varpi, \varpi' \) and prescribed Eulerian correlation of the potential.

The results presented in next sections are for \( E(x, t) = 1/(1 + x^2/2) \exp(-t/K) \). As shown in \cite{23} the shape of the Eulerian correlation of the potential determines the strength of the trapping represented by the exponent of the time decay of the diffusion coefficient in the static case. However the general behavior of the decorrelation trajectories and of \( L(t) \) and \( D(t) \) are the same for all correlations.

3 Transport by Lorentz force

We consider here a static potential \( (K, \tau_c \rightarrow \infty) \) and compare the results with those obtained in the guiding center approximation (drift transport) \cite{17}, \cite{23}. The aim is to identify the effects of the Larmor radius.

In the frame of the decorrelation trajectory method, the difference between the Lorentz and the drift transport consists in the equations for the decorrelation trajectories. The trajectories obtained from Eqs. (25-26) are much complicated than in the guiding center approximation. The important simplification introduced in the latter case that actually reduces the number of parameters at one and eliminates the time dependence of the stochastic potential (see \cite{17}, \cite{23}) cannot be applied in the present case. The trajectories effectively depend on the six parameters \( \alpha, u, \phi^0, \varpi, \varpi' \) and \( K \). In the static case there are closed periodic trajectories of the guiding center \( \xi(t) \), even at large Larmor radii which shows that trapping exists. The initial drift velocity does not influence only the period but also the size and the shape of the paths. The orientation of the initial velocity of the particle, \( \alpha \), strongly influences the trajectories obtained with the initial condition (27) because the average produced by the gyration is different for different values of \( \alpha \).
Figure 1: The contour plot of the function $f(\phi^0, t)$ for the drift (a) and Lorentz (b) transport.

The function $F(t)$ in Eq. (30) is obtained by summing the contribution, $f(\phi^0, t)$, of all the decorrelation trajectories that start from a point where the potential is $\phi^0$. This function $f(\phi^0, t)$ gives details of the diffusion process showing the trapping. The contour plot of this function is represented in Fig. 1.a. for the drift transport and in Fig. 1.b. for the Lorentz transport. One can see that in the first case the contributions of the large values of $|\phi^0|$ are eliminated progressively due to trapping and $f(\phi^0, t)$ shrinks continuously as time increase such that thinner and thinner intervals of $\phi^0$ centered around zero contribute to $D(t)$. The Lorentz transport is characterized by a completely different pattern for this function (Fig. 1.b). At small time it has a Gaussian shape that increase up to a maximum. This increase is not smooth but performed in steps. At later times, $f$ becomes unsymmetrical and has a minimum at $\phi^0 = 0$. It decays continuously in time maintaining a large range of $\phi^0$. Thus, the trapping process is completely different for the Lorentz
transport. At each value of $\phi^0$ in a large interval, the cyclotron motion determines large, open trajectories as well as small, closed ones, depending on the values of $\alpha$ and $v$. The contributions of small trajectories are progressively eliminated by mixing determining the decay of the function $f(\phi^0, t)$.

The time dependent diffusion coefficient is presented in Fig. 2 for the Lorentz transport (continuous line) and for the drift transport (dashed line). The time dependence of the Lorentz diffusion coefficient is rather complex and a strong influence of the Larmor radius can be observed. At small time the diffusion coefficient increases nonuniformly, in steps. Averaging these steps a linear time dependence can be observed, similar with that obtained in the drift transport. This behavior extends to times much longer than the flight time. The maximum of $D(t)$ is at about $7\tau_{fl}$. At later times a decay of $D(t)$ appears with a time dependence that is approximately the same as in the drift case, but with $D(t)$ larger with a factor of about 2. Thus the transport in the static case is subdiffusive.

![Figure 2: The time dependent diffusion coefficient for the Lorentz transport (blue line) and for the drift transport (dashed line) for $\Omega = 10$, $\overline{\rho} = 1$, $K = \infty$.](image)

The correlation of the Lagrangian drift velocity is presented in Fig. 3 for the Lorentz transport (continuous line) compared with drift approximation (dashed line). It decays very fast (in a time much smaller than $\tau_{fl}$) and then it presents a series of peaks with decreasing amplitude and eventually has a negative tail. The peaks appear around multiples of the cyclotron gyration period $T = 2\pi/\Omega$. 
Figure 3: The correlation of the Lagrangian drift velocity for the Lorentz transport (blue line) and for the drift transport (dashed line) for $\Omega = 10$, $\bar{\rho} = 1$, $K = \infty$.

A clear story of the physical process can be deduced from the time evolution of $D(t)$ and $L(t)$. Starting from $t = 0$, at very small time ($t \ll T$) $D(t)$ is equal with the drift diffusion coefficient. Then, the cyclotron motion with a large Larmor radius ($\bar{\rho} = 1$ in Figs. 2, 3) averages the stochastic field along the trajectory. Consequently the guiding center has a very small displacement and $D(t)$ is much reduced compared to the drift case. After a period the trajectories come back near the initial position (all in phase because $T$ is a constant), a coherent motion of the guiding centers appears during the passage of the particles and a step in $D(t)$ is produced. Thus the evolution of the guiding centers is determined mainly by short coherent kicks appearing with period $T$. Their displacement is thus slower. Consequently, the trapping appears at later time and the linear increase of $D(t)$ extends to longer times leading to values of $D(t)$ that are higher than for the drift diffusion. When the displacements of the guiding centers increase the coherence of the periodic kicks is progressively lost and the peaks of $L(t)$ becomes smaller and thicker.

The diffusion coefficient obtained for the Lorentz transport is a function of the dimensionless parameters $K$, $\bar{\rho}$ and $\bar{\Omega}$. They contain the physical parameters of the stochastic field (amplitude, correlation length, correlation time) and of the impurity ions (mass, charge, kinetic energy). The dependence of the diffusion coefficient on these parameters is analyzed in the next Sections.

4 $K$ dependence

We consider here time dependent stochastic potentials and determine the dependence of the diffusion coefficient on the parameter $K$ defined in Eq. (11).

In the case of drift transport, a change of variable can be done in the equation for the decorrelation trajectories in order to introduce the time factor of the Eulerian correlation of the potential in the time variable. The diffusion coefficient for the time dependent potential is so determined from $D(t)$ obtained for the static potential with
the same space dependence in the Eulerian correlation [24]. The equations for the decorrelation trajectories (25-26) obtained for the Lorentz transport do not have this property: due to the cyclotron motion (second term in Eq. (26)), the time factor in the average potential \( \Phi \) cannot be introduced in the time variable. Thus the \( K \) dependence of the diffusion coefficient must be determined by performing the calculations of \( D(t) \) for each value of \( K \).

![Figure 4: The evolution of the diffusion coefficient for the Lorentz transport in a time dependent stochastic potential (red line) compared to the results obtained in a static potential for Lorentz transport (blue line) and for the drift transport (dashed line).](image)

However, we have shown in [18], using general considerations based on the shape of the correlation of the Lagrangian velocity, that at large \( K \) the asymptotic diffusion coefficient can be approximated by the diffusion coefficient determined for the static potential at \( t = \tau_c \), (or in dimensionless units at \( t = K \)). These considerations are not dependent on the specific type of motion or on the statistical method used to obtain
the Lagrangian correlation. We examine here the accuracy of this approximation for the Lorentz transport.

Typical examples for the evolution of the diffusion coefficient in time dependent stochastic fields are presented in Fig. 4 (continuous line) compared to $D(t)$ obtained in the static potential for Lorentz (dotted line) and drift (dashed line) transport. One can see that the diffusion coefficient saturates showing that the transport is diffusive in time dependent stochastic potentials. A large $K$ case is considered in Fig. 4. a which shows that the above approximation is rather accurate. For smaller $K$ values that are not situated on the tail of the Lagrangian correlation, the demonstration presented in [18] does not apply. However, as seen in Fig. 4. b for $K = 2$ the above approximation is valid and so was in many other cases we have considered. Thus the asymptotic diffusion coefficient in a time dependent stochastic potential with Kubo number $K$ is

$$D(\infty|K,\overline{\rho},\overline{\Omega}) \approx D(K|\infty,\overline{\rho},\overline{\Omega})$$

where $D(t|K,\overline{\rho},\overline{\Omega})$ is the time dependent diffusion coefficient obtained for the parameters $K,\overline{\rho},\overline{\Omega}$, $D(\infty|K,\overline{\rho},\overline{\Omega})$ is its asymptotic value and $D(t|\infty,\overline{\rho},\overline{\Omega})$ is the diffusion coefficient obtained in the static potential.

5 \( \overline{\rho} \) dependence

The parameter $\overline{\rho} = |\rho(0)|/\lambda_c$ essentially describes the effect of the initial kinetic energy of the ions on their Lorentz diffusion. It does not appear in the drift transport, which is determined only by the Kubo number $K$.

The dependence of the asymptotic diffusion coefficient on the Kubo number for several values of $\overline{\rho}$ is presented in Fig. 5. One can see that the Larmor radius produces observable effects even for rather small values (at $\overline{\rho} = 0.1$). As expected, the effect strongly increase with the increase of $\overline{\rho}$. This modification of the diffusion coefficient due to Larmor radius is complex and it may consist of a strong decrease as well as of a strong increase, depending on the conditions. Thus, the general idea that the effective diffusion is reduced due to the cyclotron motion which averages the stochastic potential, is not always true.
At small Kubo numbers the diffusion coefficient is much smaller than in the drift approximation. It increases in steps that appear, for all values of $\overline{p}$, at values of $K$ which are multiples of the cyclotron period $T = 2\pi/\Omega$. Apart from these steps that are attenuated at larger $K$, there is a global increase with $K$ as $D(\infty|K,p,\Omega) \sim D_B K = (\lambda_c^2/\tau_c)K^2$ in this regime. This is similar with the quasilinear regime of the drift transport and corresponds to initial ballistic motion of the guiding centers. This regime extends to $K > 1$, up values that increase with the increase of $\overline{p}$. The diffusion coefficient at this value of $K$ has a value much larger than for the drift transport.

At larger values of $K$, the trapping becomes effective and the diffusion coefficient has approximately the same $K$ dependence as in the drift transport. The effect of the Larmor radius consists in an amplification factor in the diffusion coefficient that is independent of $K$ in this regime. It increases with the increase of $\overline{p}$.

6 $\overline{\Omega}$ dependence

The parameter $\overline{\Omega} = \Omega \tau_{fi}$ describes the effect of the specific charge of the ions $q/m$ on their Lorentz diffusion. It determines the moments of the steps appearing in the $K$ dependence of the diffusion coefficient. Apart this, there is no strong influence of this parameter when $\overline{\Omega} \gg 1$ (see Fig. 6). For small $\overline{\Omega}$ the trajectories become chaotic and the diffusion coefficient has an irregular dependence on the parameters. Note that the chaotic variations seen in this Figure at large $K$ for $\overline{\Omega} = 1$ are not
calculation errors (they are not changed by increasing the number of calculated trajectories, thus the accuracy).

Figure 6: The asymptotic diffusion coefficient for the Lorentz transport as a function of $K$ for the values of $\Omega$ that label the curves and for $\rho = 1$; the result obtained in the guiding center approximation is also represented (dashed line).

7 Conclusions

We have studied the impurity ion transport produced by the Lorentz force in a turbulent magnetized plasma. Expressions for the time dependent diffusion coefficient and for the correlation of the Lagrangian drift velocities are obtained in terms of a class of smooth, deterministic trajectories by developing a generalization of the decorrelation trajectory method. This statistical approach is compatible with the invariance of particle energy.

We have shown that the Larmor radius has a strong effect on impurity ion transport in turbulent plasmas. The generally accepted idea that the effective diffusion is reduced due to the cyclotron motion which averages the stochastic potential, is not always true. The cyclotron motion can also determine the build up of correlation of the Lagrangian drift velocity by bringing the particles back in the correlated zone of the stochastic potential. The correlation $L(t)$ shows a series of periodic peaks, which lead to increased diffusion coefficients in slowly varying potentials. Consequently, at given Larmor radius, the transport can be reduced or increased, depending essentially on the value of the Kubo number.

References
