ON THE ORDER OF THE DECONFINING
TRANSITION IN $N_f = 2$ QCD

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A careful study is made on the lattice of the phase diagram of QCD with two staggered
flavors, to investigate the order of the chiral transition of $N_f = 2$ QCD. The specific
heat and the susceptibility of the chiral condensate are determined for different spatial
sizes of the system, and a finite size scaling analysis provides a determination of the
(pseudo)critical indices. The result is a strong indication that the chiral transition is
first order.

Keywords: Confinement; Lattice QCD; Chiral transition.

1. Introduction - Motivation

QCD with $N_f = 2$ is a specially interesting system to understand confinement. A
schematic view of the phase diagram is shown in Fig. 1, where for simplicity the
two quark masses have been put equal to $m$ and $\mu$ is the barion chemical potential.
At $\mu = 0$ the transition line between the region which is conventionally named
"confined" and the "deconfined" one is defined by the maximum of a number of
susceptibilities ($C_V, \chi_{\bar{\psi}\psi}, \ldots$) which happen to coincide within errors. As $m \to \infty$
the quenched case is recovered, and the transition is known to be first order, with
$\langle L \rangle$, the Polyakov loop, as order parameter, and $Z_3$ as symmetry. In principle $Z_3$
is explicitly broken by the coupling to the quarks and $\langle L \rangle$ is not an order parameter:
however the quenched description is valid empirically down to $m \simeq 2.5 - 3 \text{ GeV}$. At
$m \simeq 0$ a chiral phase transition takes place, from the low temperature phase where
chiral symmetry is spontaneously broken to a phase in which it is restored: there
$\langle \bar{\psi}\psi \rangle$ is a good order parameter. The mass term explicitly breaks chiral symmetry
but $\langle \bar{\psi}\psi \rangle$ is expected to work as an order parameter in a neighborhood of $m = 0$.

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At intermediate values of $m$ neither $\langle L \rangle$ nor $\langle \bar{\psi} \psi \rangle$ are expected to be good order parameters, even if their susceptibilities show a maximum along the transition line of Fig. 1. At $m \simeq 0$ another phase transition is expected to take place, with restoration of the $U_A(1)$ symmetry, which is broken by the anomaly at low temperatures. An independent definition of confinement is needed to ask the question whether deconfinement transition coincides with chiral and $U_A(1)$ transition. An effective critical free energy density $L_\phi$ can be written for the chiral transition assuming that the relevant critical excitation are the scalar and pseudoscalar particles.

$$\tilde{\phi} : \quad \phi_{ij} = \langle \bar{q}_i (1 + \gamma_5) q_j \rangle \quad (i, j = 1..N_f) \quad (1)$$

Under chiral and $U_A(1)$ transformations of the group $U_A(1) \otimes SU(N_f) \otimes SU(N_f)$, $\tilde{\phi}$ transforms as

$$\tilde{\phi} \rightarrow e^{i\alpha} U_+ \tilde{\phi} U_- \quad (2)$$

so that by the usual symmetry arguments, and neglecting irrelevant terms

$$L_\phi = \frac{1}{2} Tr \left[ \partial_\mu \phi^\dagger \partial^\mu \phi \right] - \frac{m_\phi^2}{2} Tr \{ \phi^\dagger \phi \} - \alpha^2 \left[ Tr \{ \phi^\dagger \phi \} \right]^2 - \frac{\alpha^2}{3} g_2 Tr \{ \phi^\dagger \phi \}^2. \quad (3)$$

Inclusion of anomaly brings in an additional term $L_\phi' = c [det \phi + det \phi^\dagger]$, which is $SU(N_f) \otimes SU(N_f)$ invariant, but not $U_A(1)$ invariant.

One can inquire, by use of renormalization group plus $\epsilon$-expansion techniques, if infrared stable fixed points exist, which indicate the possible existence of a second or higher order transition. For $N_f \geq 3$ no such point exists, and the chiral transition is first order. At $N_f = 2$, in the absence of anomaly ($c = 0$) or if the $\eta'$ mass vanishes at $T_c$, no fixed point exists, the transition is first order, and also at $m, \mu \neq 0$ the transition is expected to be first order. In such a case no tricritical point exists in the plane $(\mu, T)$ of Fig. 1. If instead $m_{\eta'} \neq 0$ at $T_c$, and the $U_A(1)$ transition occurs at $T > T_c$, the symmetry group is $O(4)$ and the transition can be second order. In that case at $m, \mu \neq 0$ there is no transition but only a crossover, and a tricritical point is expected in the $(\mu, T)$ plane of Fig. 1.

The issue has fundamental implications for confinement. If the deconfining transition is order-disorder, so that an order parameter exists, a crossover is excluded. If instead a crossover exists, a state of free quarks can continously be deformed to the
confined phase, and can exist also there. The existing literature is not conclusive on this point, even if a preference is given to the second order plus crossover scenario. The problem deserves more attention. We are going to present preliminary results based on a $5 \times 10^9$ lattice simulations on APEmille computer.

2. Lattice Investigation

The order of the chiral transition, and, more generally, the transition line of Fig. 1 can be studied by lattice simulations and standard finite size scaling techniques. We have used staggered fermions on $L_t \times L_s^3$ lattices, with $L_t = 4$ and $L_s = 12, 16, 20, 24, 32$. The input parameters of the simulations are $\beta = 6/g^2$ and $am$, the quark mass in units of the inverse lattice spacing $a^{-1}$. The temperature is given by

$$T = \frac{1}{L_t a(\beta, m)}$$  

with $a(\beta, m)$ the lattice spacing in physical units. The reduced temperature $\tau \equiv (1 - T/T_c)$ is then given by

$$\tau = 1 - \frac{a(\beta_c, 0)}{a(\beta, m)}$$  

or, in a sufficiently small neighborhood of the critical point

$$\tau = \left. \frac{\partial \ln a}{\partial \beta} \right|_{(\beta_c, 0)} [\beta_c - \beta + km]$$  

with

$$k = \left. \frac{\partial \ln a}{\partial m} \right|_{(\beta_c, 0)}$$  

In the quenched case $k = 0$ and $\tau \propto (\beta_c - \beta)$.

The correlation length of the order parameter $\xi$ diverges at the critical point

$$\xi \to \tau^{-\nu} \quad \text{for} \quad \tau \to 0^+$$  

with a critical index known as $\nu$. For the specific heat $C_V$ and for the susceptibility $\chi$ of the order parameter, the following scaling laws are expected

$$C_V - C_0 = L_s^{\nu/\nu} \Phi_C(\tau L_s^{1/\nu}, am_q L_s^{y_h})$$  

$$\chi - \chi_0 = L_s^{\nu/\nu} \Phi_\chi(\tau L_s^{1/\nu}, am_q L_s^{y_h})$$  

Eqs. 9-10 are obtained by renormalization group arguments, holding when the lattice spacing is much smaller than $\xi$, so that $a/\xi \approx 0$. This is true for second order and weak first order transitions. The critical indices $\nu, \alpha, \gamma, y_h$ depend on the transition (see Table 1). The physics behind all that is that around $T_c$ the free energy can be written in terms of the order parameter, and the dependence is dictated by symmetry: irrelevant (higher dimensional) terms are neglected. Eq. 9 for the specific
heat is always valid. Eq (10) is only valid if the choice of the order parameter is correct. Therefore the specific heat is a reference to test the validity of any order parameter.

In order to test the $O(4)$ option and second order, we have run at fixed value of the scaling variable $am_0L_s^n$ with $y_h = 2.49$, and different spatial sizes $L_s$. The expectation is then, from Eqs. (11-12) that the peak values scale as

\[
(C_V - C_0)_{\text{peak}} \propto L_s^{\alpha/\nu} \\
(\chi - \chi_0)_{\text{peak}} \propto L_s^{\gamma/\nu}.
\]

Since the value of $\alpha$ for $O(4)$ is negative the height of the peak for $C_V - C_0$ is expected to decrease at high $L_s$. Data show instead a rapid raise. The quantities $(C_V - C_0)_{\text{peak}} / L_s^{\alpha/\nu}$ and $(\chi - \chi_0)_{\text{peak}} / L_s^{\gamma/\nu}$, which should be constant if $O(4)$ were the symmetry, are displayed in Fig. 2. The $\chi^2/\text{dof}$ for a constant is very high: $O(4)$ symmetry is excluded. Since $y_h$ for $O(2)$ is equal within errors to that of $O(4)$, also $O(2)$ symmetry can be tested and it turns out to be equally bad. Our action is not “improved”; however we do not expect that an infrared property like Eqs. (11-12) are affected by ultraviolet improvement. We can safely state that $O(4)$ and $O(2)$ are excluded and with them the crossover scenario.

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<th>$\alpha$</th>
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Table 1. Critical exponents.
Continuity arguments and Eqs. 13, 14 require that, at small values of \( m \)

\[
C_V - C_0 \simeq (am_q)^{-\alpha/(\nu y_h)} \Phi_C(\tau L_s^{1/\nu}, am_q L_s^{y_h})
\]

(13)

\[
\chi - \chi_0 \simeq (am_q)^{-\gamma/(\nu y_h)} \Phi_\chi(\tau L_s^{1/\nu}, am_q L_s^{y_h})
\]

(14)

The positions of the peaks \( \beta_{max} \) scale then as

\[
\beta_c - \beta_{max} + km - k'L_s^{-1/\nu} = 0
\]

(15)

their heights as

\[
(C_V - C_0)^{\text{peak}}(am_q)^{\alpha/(\nu y_h)} = \text{const}
\]

(16)

\[
(\chi - \chi_0)^{\text{peak}}(am_q)^{\gamma/(\nu y_h)} = \text{const}
\]

(17)

Eqs. 13, 14, 15, 16 and 17 can be tested with the data. Eq. 15 is compatible both with first order (\( \nu = 1/3 \)) and \( O(4) \), in agreement with previous analyses.\cite{4,5} Eqs. 16 and 17 again exclude \( O(4) \) and \( O(2) \) and are consistent with a first order transition (see Fig. 3).

3. Dual Superconductivity of the Vacuum

A disorder parameter \( \langle \mu \rangle \) detecting dual superconductivity of the vacuum has been developed by our group during the last years and proved to be a good parameter for the quenched theory. The parameter can be defined equally well in the presence of dynamical quarks. \( \langle \mu \rangle \) is strictly zero in the deconfine phase of Fig. 1 and it drops to zero at the transition line, as shown by the fact that the susceptibility

\[
\rho = \frac{\partial}{\partial \beta} \ln \langle \mu \rangle
\]

(18)
has a sharp negative peak coincident within errors with the peak of $C_v$. Around $T_c$

$$\langle \mu \rangle = L_s^{k/\nu} \Phi(\mu) (\tau L_s^{1/\nu}, am_q L_s^\nu)$$

(19)

continuity arguments require

$$\langle \mu \rangle \simeq (am_q)^{-k/(\nu y_h)} \tilde{\Phi}(\mu) (\tau L_s^{1/\nu})$$

(20)

and therefore

$$\rho/L_s^{1/\nu} \simeq f(\mu) (\tau L_s^{1/\nu})$$

(21)

Independence on $m$ is expected. Eq. 21 allows a determination of $\nu$. If this agrees with the value obtained from $C_v$, a legitimation of $\langle \mu \rangle$ as order parameter follows. Eq. 21 is compatible with first order (Fig. 4).

4. Conclusions

Finite size scaling analysis of $N_f = 2$ QCD definitely excludes $O(4)$–and $O(2)$–symmetry at the chiral critical point, andfavours a first order transition. The investigation is continuing with larger lattices and improved actions, to better test the first order option. Dual superconductivity of the vacuum is confirmed as a good symmetry for the order parameter.

References