Einstein’s gravitational lensing and nonlinear electrodynamics

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(Dated: August 9, 2004)

PACS numbers: 42.65.-k, 03.50.-z, 04.20.Cv, 95.30.Sf, 95.30.-Gb, 95.30.Jd

In 1936 Einstein predicted the phenomenon presently known as gravitational lensing (GL). A prime feature of GL is the magnification, because of the gravitational field, of the star visible surface as seen from a distant observer. We show here that nonlinear electrodynamics (NLED) modifies in a fundamental basis Einstein’s general relativistic (GR) original derivation. The effect becomes apparent by studying the light propagation from a strongly magnetic (B) pulsar (SMP). Unlike its GR counterpart, the photon dynamics in NLED leads to a new effective GL, which depends also on the B-field permeating the pulsar. The apparent radius of a SMP appears then unexpectedly diminished, by a large factor, as compared to the classical Einstein’s prediction. This may prove very crucial in determining physical properties of high B-field stars from their X-ray emission.

Introduction.— Einstein’s general theory of relativity (GTR) has proved to be one of the most successful physical theories ever formulated. As a theory of the gravitational interaction, it has a number of predictions definitely corroborated by experiments or astronomical observations, which makes it the correct gravity theory here-to-fore. After readdressing an earlier study, Einstein [1] came up with the prediction of the gravitational lensing effect. The idea is that the gravitational field produced by a massive astrophysical object, the Sun for instance, can act as a convergent lens able to deviate light rays flying-by. Observations of far away quasars and galaxies and more familiar total solar eclipses confirm the reality of this phenomenon. More than half of the star’s surface may be seen by a distant observer. This means that a photon emitted at a given colatitude on the star’s surface, reaching the observer at infinity, must be emitted at a smaller angle with respect to the normal at that point. Because of this effect, a distant compact star must then appear to any observer larger then it actually is. The relation between the apparent radius $R_\infty$ of a spherical star, as seen by a distant observer, and its physical radius $R$ is

$$R_\infty = \frac{R}{(1 - \frac{R_S}{R})^{1/2}}.$$  \hspace{1cm} (1)

This relation is obtained from the photon trajectory in polar coordinates $(r, \theta)$ by using a Schwarzschild metric

$$ds^2 = \left(1 - \frac{R_S}{r}\right)dt^2 - \frac{dr^2}{\left(1 - \frac{R_S}{r}\right)} - r^2(d\theta^2 + \sin^2\theta d\phi^2).$$  \hspace{1cm} (2)

Here $R_S = 2GM/c^2$ is the Schwarzschild radius of the star of mass $M$. As is clear from this line-element no effects from physical fields other than the gravitational have been taken into account in prescribing the star apparent size. This Letter shows that the phenomenon changes in a fundamental fashion if NLED is called into play. The analysis below, based on well-known nonlinear Lagrangeans, as the Heisenberg-Euler (H-E, [2]) and the exact Born-Infeld (B-I, [3]), proves that when NLED is included to describe the photon dynamics, the trajectory depends on the background B-field pervading the star. We stress from the very beginning that the results below are obtained upon an idealization of the B-field structure. In a real star, the B-field is neither purely radial nor constant, so that the apparent radius hardly goes to zero at high B.

Apparent radius in NLED.— According to quantum electrodynamics (QED: see Delphenich 2003 [4] for a complete review on NLED and QED) a vacuum has nonlinear properties [2, 5], which affect the photon propagation. A noticeable advance in the realization of this theoretical prediction has been provided by Burke, Field, Horton-Smith, et al. [6], who demonstrated experimentally that the inelastic scattering of laser photons by gamma-rays is definitely a nonlinear phenomenon. The propagation of photons in NLED has been examined by several authors [7, 8, 9, 10, 11]. In the geometric optics approximation, it was shown by Novello et al. [12]; and Novello and Salim [13], that when the photon propagation is identified with the propagation of discontinuities of the EM field in a nonlinear regime, a remarkable feature floats: the discontinuities propagate along null geodesics of an effective geometry which depends on the EM field on the background. This means that the NLED interaction can be geometricized, in analogy to gravity in GTR. A key outcome of this formalism is introduced in this Letter.

Euler-Heisenberg approach.— The H-E [2] Lagrangean
\[ L(F, G) = -\frac{1}{4} F + \mu \frac{F}{4} \left( F^2 + \frac{7}{4} G^2 \right), \]  

(3)

where \( \mu = \frac{8 \pi^2}{3 \hbar c} \) is a quantum parameter, is a gauge invariance description of the photon propagation in NLED that uses two invariants

\[ F = F_{\mu\nu} F^{\mu\nu}, \quad G = F_{\mu\nu}^* F^{\mu\nu} = \frac{1}{2} \eta_{\mu\nu}^\alpha \beta F_{\alpha\beta} F^{\mu\nu}, \]  

(4)

constructed upon the Maxwell EM tensor and its dual 20. The method of characteristic surfaces or shock waves followed here (introduced by Hadamard 14) can be applied to any field theory having hyperbolic field equations, including electrodynamics. Hence, the result in Eq. (2) (see appendix), implies two possible paths of propagation or polarization modes, according to the double solution \( \Omega_{\pm} \). Using Eq. (3) to compute the Lagrangean derivatives \( L_F, L_{FF}, L_{GG} \), one arrives to the couple of effective metrics (first order in \( \mu \))

\[ g_{\mu\nu}^{\text{eff}} = g^{\mu\nu} + 8 \mu F^{\mu\alpha} F_{\alpha\nu}, \quad g_{\mu\nu}^{\text{eff}} = g^{\mu\nu} + 14 \mu F^{\mu\alpha} F_{\alpha\nu}. \]  

(5)

In terms of electric (\( E \)) and magnetic (\( B \)) fields the tensor \( F_{\mu\nu} \) can be written as

\[ F_{\mu\nu} = E_{\mu} V_{\nu} - E_{\nu} V_{\mu} + \eta_{\mu\nu} \alpha \beta V_{\alpha} B_{\beta}, \]  

(6)

where \( V^\mu \) is the normalized (\( V^\mu V_\mu = 1 \), Eq. (4)) velocity of the reference frame where the fields are measured. Our main concern here is the behavior of photons, in the NLED context, emitted from a highly magnetized neutron star. Corotating charges in the pulsar magnetosphere or a rotating magnetic dipole lead to induced \( E \)-fields in the star surface. We consider here slowly rotating neutron stars in order that the \( E \)-field contribution could be neglected. Since \( E_\alpha = 0 \), the above expression simplifies to: \( F_{\mu\nu} = \eta_{\mu\nu} \alpha \beta V_{\alpha} B_{\beta} \). Its self-product reads

\[ F^{\mu\alpha} F_{\alpha\nu} = -B^\alpha B^\nu - B^2 (g^{\mu\nu} - V^\mu V^\nu), \]  

(7)

where \( B^2 = B^\alpha B_\alpha \). After discarding a nonphysical conformal factor the effective metrics become

\[ g_{\mu\nu}^{\text{eff}} = g^{\mu\nu} - \frac{\beta_\pm^2}{1 - \beta_\pm^2 B^2} B^\mu B^\nu + \frac{\beta_\pm^2 B^2}{1 - \beta_\pm^2 B^2} V^\mu V^\nu. \]  

(8)

The inverse (covariant) metric of Eq. (8) is obtained from the relation \( g_{\mu\nu} g^{\nu\alpha} = \delta_{\mu}^\alpha \), which then reads

\[ g_{\mu\nu}^{\text{eff}} = g_{\mu\nu} + \beta_\pm^2 B^\mu B^\nu - \beta_\pm^2 B^2 V^\mu V^\nu. \]  

(9)

\[ B_\mu = |B| l_\mu, \quad l_\mu = \sqrt{-g_{rr} \delta_\mu^r}, \quad V_\mu = \sqrt{g_{00}} \delta_\mu^0, \]  

(10)

and consequently one arrives to the effective metric components

\[ g_{00}^{\text{eff}} = (1 - \beta_\pm^2 B^2) g_{00}, \quad g_{rr}^{\text{eff}} = (1 - \beta_\pm^2 B^2) g_{rr}. \]  

(11)

Notice the structural similarity of both metric components. From Eq. (11) is straightforward to rewrite the expression for the apparent radius of the spherical star in Eq. (14) in the potutative background metric given by Eq. (2). After doing so, one can compute the ratio: \( \mathcal{N} \equiv \frac{\beta_\pm^2}{(\frac{g_{rr}^{\text{eff}}}{g_{rr}})} \), of the star area with H-E NLED effects included to the area without (Einstein’s derivation), as seen from the distant observer. The results are presented in Fig.1. This anomalous behavior invalidates the Denisov et al. 11 results; which advocate for crucial changes in using the H-E approximation (see discussion below).

Born-Infeld NLED.- The propagation of light from hypermagnetized neutron stars can also be viewed within the framework of the Born-Infeld Lagrangean

\[ L = -b^2 \left( \sqrt{1 + \frac{F}{b^2} - G^2} - 1 \right) \]  

(12)

where \( b^2 = -\frac{c}{m_0^2} \rightarrow c = 9.8 \times 10^{15} \text{ e.s.u.} \). As is well known, this is an exceptional Lagrangean. One of its remarkable properties is that it does not exhibit birefringence 12, 12. In this case the deduction we present in the appendix fails since the quantities \( \Omega_\mu \), with
\[ g_{\mu \nu}^{\text{eff}} = g_{\mu \nu} + \frac{2}{b^2} F_{\mu \nu} F^{\mu \nu}. \]  

Using the self-product of the tensor \( F_{\mu \nu} \) given by (14), and noting that in our case \( F = F_{\mu \nu} F^{\mu \nu} = 2B^2 \), the effective metric then reads (see reference [12] for details)

\[ g_{\mu \nu}^{\text{eff}} = g_{\mu \nu} + \frac{2B^2}{b^2} (V^\mu V^\nu - l^\mu l^\nu). \]  

By computing the inverse metric via \( g_{\mu \nu}^{\text{eff}} g^{\text{eff}}_{\alpha \beta} = \delta_{\alpha \beta} \), the covariant form of this effective metric is obtained as

\[ g_{\mu \nu}^{\text{eff}} = g_{\mu \nu} - \frac{2B^2/b^2}{(2B^2/b^2 + 1)} V_\mu V_\nu + \frac{2B^2/b^2}{(2B^2/b^2 + 1)} l_\mu l_\nu. \]  

After recalling the relations for \( l_\mu \) and \( B^\mu \) given in Eq.(10), one can verify that the covariant \( rr \) effective metric component is then written as

\[ g_{rr}^{\text{eff}} = g_{rr} - \frac{2B^2/b^2}{(2B^2/b^2 + 1)} g_{rr} = \frac{1}{1 + 2B^2/b^2} g_{rr}. \]  

By following steps similar to the H-E case, the area ratio is presented in Fig. 2. Although our hypothesis about the field geometry maximizes the NLED effect on the observed properties of the star, they should properly be taken into account when analyzing the X-ray emission. (This point will be considered in a forthcoming paper). It should also be relevant in pondering the effects on supernova dynamics of photons radiated either by highly magnetized proto-neutron stars (Miralles et al. [18]) or stellar-mass black holes (van Putten et al. [19]).

**Photon trajectories as seen from a distance.** – In NLED the Lagrangean defined by Eq.(2) becomes

\[ L(q^k, q^k) \equiv 1 = -\mu(B) \left( 1 - \frac{R_S}{r} \right) \left( \frac{dt}{ds} \right)^2 + \frac{\mu(B)}{1 - \frac{R_S}{r}} \left( \frac{dr}{ds} \right)^2 + r^2 \left( \frac{d\theta}{ds} \right)^2 + r^2 \sin^2 \theta \left( \frac{d\phi}{ds} \right)^2, \]  

where \( \mu(B) = \frac{1}{1 + 2B^2/b^2} \). The resulting Euler-Lagrange equations read:

\[ \frac{d}{ds} \left( -2g_{tt}^{\text{eff}} u^t \right) = 0 \rightarrow g_{tt}^{\text{eff}} u^t = E_0, \quad \frac{d}{ds} \left( 2g_{rr}^{\text{eff}} u^r \right) = 0 \rightarrow g_{rr}^{\text{eff}} u^r = C_2, \text{ and} \]

\[ \frac{d}{ds} \left( 2g_{\theta \theta}^{\text{eff}} u^\theta \right) = 0 \rightarrow g_{\theta \theta}^{\text{eff}} u^\theta = L_0, \quad \frac{d}{ds} \left( 2g_{\phi \phi}^{\text{eff}} u^\phi \right) = 0 \rightarrow g_{\phi \phi}^{\text{eff}} u^\phi = \tilde{h}, \]  

where \( \tilde{h} \) is defined as the impact parameter of the photon propagating from the NS surface to an observer at \( r \rightarrow \infty \). In analogy with the planetary system one can chose a particular “orbital” plane at \( \theta = \pi/2 \), so that \( u^\theta = 0 \) and \( \sin^2 \theta = 1 \), and thus \( g_{tt}^{\text{eff}} = r^2 \). As photons travel along null geodesics one can also write

\[ g_{tt}^{\text{eff}} (u^t)^2 + g_{rr}^{\text{eff}} (u^r)^2 + g_{\phi \phi}^{\text{eff}} (u^\phi)^2 \equiv 0, \]  

from which the photon propagation equation reads (after the usual change \( u = 1/r, \ t = d/dr, \text{ and } A = 2B^2/b^2 \) 

\[ E_0^2 - \left( \frac{1}{A + 1} \right)^2 \tilde{h}^2 (u^t)^2 - (1 - R_S u) \left( \frac{1}{A + 1} \right) \tilde{h}^4 u^2 = 0, \]  

which one can in turn write as \( (E_0, \tilde{h} \text{ constants}) \)

\[ (u^t)^2 + u^2 = E_0 \tilde{h}^2 + R_S u^3 + \left( 2 \frac{E_0}{\tilde{h}^2} - u^2 - R_S u^3 \right) A + \frac{E_0}{2} A^2. \]  

Eq.(20) shows very clearly that NLED modifies the standard propagation of photons as compared to that one in a pure Schwarzschild gravitational field \( \text{sG} \), as described in most textbooks of GTR. (A full detailed solution will be given elsewhere). A first look at Eq.(20) suggests that very large \( B \)-fields do reduce the effective star’s visible area.
Discussion and conclusion. – By comparing Eqs. 11 and 12, one can verify that: a) Eq. 11 clearly exhibits a divergence. Such a behavior, in turn, invalidates the results by Denisov et al. 12 obtained within the H-E approach, since they extended to the case of magnetars their formula to estimate the light-ray bending angle beyond the QED B-field limit. That is inconsistent. b) Both approaches, the H-E and the B-I NLED, induce critical changes in the $\nu\nu$ metric component. c) More relevant, the area of a putative star may appear to a distant observer nonphysically diminished in H-E NLED, whereas it is physically largely reduced, and may even “disappear”, for fields $B \geq 10^{17}$ G in the B-I approach. The impact of this on the star flux must be dramatic.

Thence, the effect introduced in Ref. [17] and this new here crucially alters the dynamics of photons from very high $B$-field stars, while entangles (makes it difficult) the inference of their physical properties.

Appendix: effective geometry formalism. – Following Hadamard [14], the surface of discontinuity 21 of the EM field is denoted by $\Sigma$. The field is continuous when crossing $\Sigma$, while its first derivative presents a finite discontinuity. These properties are specified as follows: $[F_{\mu\nu}]_{\Sigma} = 0$, $[F_{\mu\nu}\chi]_{\Sigma} = f_{\mu\nu}\kappa_{\lambda}$, where the symbol $[F_{\mu\nu}]_{\Sigma} = \lim_{\delta \to 0+} (J_{\Sigma})_{\Sigma + \delta} - (J_{\Sigma})_{\Sigma - \delta}$ represents the discontinuity of the arbitrary function $J$ through the surface $\Sigma$. The tensor $f_{\mu\nu}$ is called the discontinuity of the field, $\kappa_{\lambda}\partial_{\lambda}\Sigma$ is the propagation vector, and the symbol “$|$” stands for partial derivative.

Hereafter we investigate the effects of nonlinearities of very strong $B$-fields in the evolution of EM waves; described onwards as the surface of discontinuity of the EM field (represented here-to-fore by $F_{\mu\nu}$). Extremizing the Lagrangian with respect to the potentials $A_{\mu}$ yields the following field equation: $\nabla_{\nu}(L_{F}F^{\mu\nu} + L_{G}F_{*}^{\mu\nu}) = 0$. Besides this, we have the cyclic identity: $\nabla_{\nu}F^{\mu\nu} = 0 \Leftrightarrow F_{\mu|\nu} + F_{\nu|\mu} + F_{\alpha|\beta\nu\mu} = 0$. The field equation can be written explicitly as: $L_{F}\nabla_{\nu}F^{\mu\nu} + 2N^{\nu\alpha\beta}\nabla_{\nu}F_{\alpha\beta} = 0$, where the tensor $N$ is defined as

$$N^{\nu\alpha\beta} = L_{FF}F_{\mu\nu}F_{\alpha\beta} + L_{FG}(F_{\mu\nu}F_{*}^{\alpha\beta} + F_{*}^{\mu\nu}F_{*\alpha\beta}) + L_{GG}F_{*\alpha\beta}F_{*\mu\nu}.$$ \hspace{1cm} (21)

Taking the discontinuities of the field equation we get: $f_{\beta\lambda}\kappa_{\beta} = -\frac{1}{2}N^{\mu\nu\rho\lambda}f_{\rho\mu}k_{\nu}k_{\nu}$. The discontinuity of the Bianchi identity yields: $f_{\alpha\beta}\kappa_{\gamma} + f_{\alpha\gamma}\kappa_{\beta} + f_{\beta\gamma}\kappa_{\alpha} = 0$. From these equations we obtain (see 10) for details

$$\chi k^{2} = \frac{4}{L_{F}}F^{\mu\nu}F_{\mu\nu}k_{\nu}k_{\nu}(L_{FF} + L_{FG}) - \frac{G}{L_{F}}k^{2}(L_{FG} + L_{GG})$$ \hspace{1cm} (22)

where we introduce the notation: $\chi = F^{\alpha\beta}f_{\alpha\beta}, \chi^{*} = F_{*}^{\alpha\beta}f_{\alpha\beta}, k^{2} = g^{\mu\nu}k_{\mu}k_{\nu}$. As the H-E QED Lagrangean is not a functional of the product $F \ast G$, then one obtains

$$\chi k^{2} = \frac{4}{L_{F}}F^{\mu\nu}F_{\mu\nu}k_{\nu}k_{\nu}(L_{FF} + L_{FG}) - \frac{G}{L_{F}}k^{2}(L_{FG} + L_{GG}) + \frac{2F}{L_{F}}k^{2}(L_{GG})^{*}.$$ \hspace{1cm} (24)

We seek thus for a master relation representing the propagation of field discontinuities, which should be independent of the quantities $f_{\alpha\beta}$, that is, independent of $\chi$ and $\chi^{*}$. There is a simple way to achieve such a goal. We firstly isolate the common term $F^{\mu\nu}F_{\mu\nu}k_{\nu}k_{\nu}$, which appears in both Eqs. 24 and 25. Then, by assuming that $k^{2} \neq 0$, the difference of these equations can be put in the form of an algebraic linear relation between $\chi$ and $\chi^{*}$: $\Omega^{2} + \Omega \chi^{2} + \chi^{*} \Omega \chi^{2} = 0$, where we define: $\Omega = \frac{n}{2}L_{FF}^{2}$, and $\Omega = L_{FG} + \frac{2F}{L_{F}}L_{FG}G_{G}$, and $\Omega = -\frac{G}{L_{F}}L_{FG}$. Solving the quadratic equation for $\chi^{*}$ we obtain: $\chi^{*} = \Omega \pm \chi$, with: $\Omega = -\frac{2F}{L_{F}} + \frac{4\Omega^{2}}{2\Omega^{2}} - \frac{4G}{L_{F}}G_{G}^{2}$. Using this solution in Eqs. 24 and 25, and assuming $\chi \neq 0$, after some algebra one gets the dispersion relation

$$g_{\mu\nu}^{\text{eff}} = \frac{L_{FF}}{L_{F} + L_{GG}}F^{\lambda\mu}F^{\nu\lambda}k_{\mu}k_{\nu} = 0.$$ \hspace{1cm} (26)

Thence, one concludes that the discontinuities will follow geodesics in this effective metric $g_{\mu\nu}^{\text{eff}}$.

Acknowledgements. – JMS thanks CNPq (Brazil). HJMC thanks FAPERJ (Brazil).

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[20] The attentive reader must notice that this first order approximation is valid only for \( B \)-fields smaller than \( B_q = \frac{m_e^3 c^3}{\bar{e}} = 4.41 \times 10^{13} \) G (Schwinger’s critical \( B \)-field).
[21] Of course, the entire discussion onwards could alternatively be phrased using concepts more familiar to the astronomy community as that of light rays used for describing propagation of EM waves in geometric optics.