Symmetric Hyperbolic System in the Self-dual Teleparallel Gravity

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In order to discuss the well-posed initial value formulation of the teleparallel gravity and apply it to numerical relativity a symmetric hyperbolic system in the self-dual teleparallel gravity which is equivalent to the Ashtekar formulation is posed. This system is different from the ones in other works by that the reality condition of the spatial metric is included in the symmetric hyperbolicity and then is no longer an independent condition. In addition the constraint equations of this system are rather simpler than the ones in other works.

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I. INTRODUCTION

As the closest alternative to general relativity (GR), teleparallel gravity can be traced back to Einstein [1] who regarded it as a unified field theory and attempted to use it to supersede GR. Teleparallel gravity can be regarded as a translational gauge theory [2-4], which makes it possible to unify gravity with other kinds of interactions in the gauge theory framework. Poincare gauge theory is a natural extension of the gauge principle to spacetime symmetry, and represents a alternative to GR (for more general attempts see [3]). In particular, teleparallel gravity was regarded as a promising alternative to GR until the work of Kopczynski [5], who found a hidden gauge symmetry which prevents the torsion from being completely determined by the field equations, and concluded that the theory is inconsistent. Nester [6] improved the arguments by showing that the unpredictable behavior of torsion occurs only for some very special solutions (see also [7]). Hecht et al. [8] investigated the initial value problem of teleparallel gravity and conclude that it is well defined if the undetermined velocity are dropped out from the set of dynamic velocities.

Teleparallel gravity possesses many salient features. Because of its simplicity and transparency teleparallel gravity seems to be much more appropriate than GR to deal with the problem of gravitational energy momentum [4, 9, 10]. Nester [9] succeeded in proving the positivity of total energy for Einstein’s theory in terms of teleparallel geometry. He found that special gauge features of teleparallel gravity, which are usually considered to be problematic, are quite beneficial for this purpose. Mielke [10] used the teleparallel approach to give a transparent description of Ashtekar’s new variables [11]. Andrade et al. [12] formulated a five-dimensional equivalent of Kaluza-Klein theory. Although quantum properties of the Poincare gauge theory are, in general, not so attractive, the related behavior in the specific case of the teleparallel theory might be better [13], and should be further explored.

The canonical Hamiltonian approach is the best way to clarify both the nature of somewhat mysterious extra gauge symmetries and the question of consistency of teleparallel gravity. It is found [7] that the presence of nondynamical torsion components is not a sign of an inconsistency, but a consequence of the constraint structure of the theory. In some works [7, 14, 15], the Hamiltonian formulation of teleparallel gravity has been developed. However, the well-posed initial value formulation of teleparallel gravity has not been discussed as yet. As is well known, hyperbolic formulation of the Einstein equation is one of the main research areas in GR [16]. This formulation is used in the proof of the existence, uniqueness, and stability of the solutions of the Einstein equation by analytical methods. Thus far, several first-order hyperbolic formulations are proposed [17-20]. The recent interest in hyperbolic formulation arises from its application to numerical relativity [21]. It is proved that the Einstein equation in Ashtekar’s variables constitutes a symmetric hyperbolic system [22, 23]. A question naturally arises whether there is a well-posed initial value formulation of teleparallel gravity which is equivalent to or different from the hyperbolic formulation of the Einstein equation. If it exists, can it give us some new perspectives and be applied to numerical relativity? An answer to this question will be given in this paper. A self-dual teleparallel formulation of general relativity which is equivalent to Ashtekar’s formulation has been developed, its canonical Hamiltonian analysis has been given and used to clarify the gauge structure of the theory [15]. In this paper a new symmetrical hyperbolic system of the Einstein equation will be posed in terms of the two-spinor formulation based on [15]. A new fact we find is that the reality condition of the spatial metric is included in the symmetric hyperbolicity and then is not independent. In addition, the constraint conditions take a rather simple form. All of these reduces the number of independent conditions imposed on the equations and then simplifies the relevant problems largely. In Sec. II, a canonical formulation of the self-dual teleparallel equivalent of GR is given, the Hamiltonian (evolution) equations and the constraint equations are written in terms of the two-spinor dyad and the canonical conjugate momenta. In Sec. III, by introducing a new variable the
evolution equations are rewritten as first-order forms. Then in Sec. IV, the conditions of the symmetric hyperbolicity of the evolution equations are discussed and the relations between the conditions of the symmetric hyperbolicity and the reality conditions of the spatial metric are obtained. Section V is devoted to some conclusions.

II. THE CANONICAL FORMULATION OF THE SELF-DUAL TELEPARALLEL EQUIVALENT OF GR

We start with the Lagrangian of the self-dual (or chiral) teleparallel formulation of general relativity [15,10] which is equivalent to the Ashtekar Lagrangian [11] and is written in terms of the two-spinor formulation [24]

\[ V^+ = N\sigma [\omega_{(AB)}^{AC}\omega_{DC}^{DB} - \omega_{(AB)CD}\omega^{CBAD} - \sqrt{2}\omega_{(CE)}^{(CE)}E^D], \]

where \( \omega_{AB}^{CD} \) is the self-dual spin connection, \( \omega_{(AB)}^{CD} \) is the Ashtekar variable,

\[ \omega_{CD} = h^{AB}\omega_{ABC} = h^{AB}\omega_{[AB]CD}, \]

and

\[ h^{AB} = \frac{1}{N}(t^{AB} - N^{AB}), \]

with the determinant of the inverse soldering form \( \sigma = \text{det}(\mu^{AB} - \sqrt{-g}) \), the lapse \( N \) and the shift \( N^{AB} \).

In the two-spinor formalism [15] the basic variable is chosen to be the dyad \( \zeta_{aA} \):

\[ \zeta_0A = o_A, \zeta_1A = \iota_A, \]

and the self-dual spin connection \( \omega_{ABC} \) can be expressed by

\[ \omega_{ABC} = \zeta_b^{aA}\partial_{AB}\zeta_{bD}. \]

Using (2) (3) and (4) one gets

\[ \omega_{CD} = -\frac{1}{N}\zeta_b^{aA}\zeta_{bD}^{aA} - \frac{1}{N}N^{AB}\omega_{[AB]CD}, \]

where

\[ \zeta_{bD} = t^{AB}\partial_{AB}\zeta_{bD}. \]

Then the Lagrangian \( V_{||}^+ \) becomes

\[ V_{||}^+ = N\sigma [\omega_{(AB)}^{AC}\omega_{DC}^{DB} - \omega_{(AB)CD}\omega^{CBAD} - \sqrt{2}\omega_{(CE)}^{(CE)}E^D], \]

The canonical momentum conjugate to \( \zeta_{bD} \) is then

\[ \tilde{p}^{bD} = \frac{\partial V_{||}^+}{\partial \dot{\zeta}_{bD}} = -2\sqrt{2}\sigma\zeta_b^{aA}\omega^{(EC)}_E D, \]

which leads to

\[ \omega_{(AC)}^{CB} = \frac{\sqrt{2}}{4\sigma}\zeta_A \tilde{p}^{AB}. \]

The gravitational Hamiltonian can be computed

\[ H^+_G = \tilde{p}^{bD}\zeta_{bD} - V_{||}^+ = N\mathcal{H}_\perp + N^{AB}\mathcal{H}_{AB}, \]

where
\[ H_\perp = \frac{1}{8\sigma} \varepsilon_{abc} \bar{p}^a p^b C \gamma^C + \sigma \zeta^a C \zeta^b \partial_{(AB)} \zeta_a D \partial^{(CB)} \zeta_b D, \]  

and

\[ H_{AB} = \partial_{(AB)} \zeta_a D \bar{p}^a D. \]

Computing the variation, we obtain the Hamiltonian equations

\[ \dot{\zeta}_{aA} = \frac{\delta H_G}{\delta p^a A} = -\frac{N}{4\sigma} \bar{p}_{aA} + N^C B \partial_{(CB)} \zeta_{aA}, \]  

\[ \dot{\bar{p}}^a A = -\frac{\delta H_G}{\delta \zeta_{aA}} \]  

\[ = N^C B \partial_{(BC)} \bar{p}^a + 2 N \sigma \zeta^a D \zeta^b B \partial_{(BC)} \partial^{(DC)} \zeta_b A - \frac{N}{2\sigma} \zeta^a A \bar{p}^B \bar{p}_{bB} \]  

\[ -4 N \sigma \zeta^a A \zeta^b B \partial_{(EB)} \zeta_{cD} \partial^{(CB)} \zeta_b D + 8 N \sigma \zeta^a D \zeta^b B \zeta^c E \partial_{(BC)} \zeta_{cE} \partial^{(DC)} \zeta_b A \]  

\[ -2 N \sigma \zeta^a C \partial_{(CB)} \zeta^b D \partial^{(AB)} \zeta_{bD} + 2 N \sigma \zeta^b B \partial_{(BC)} \zeta^a D \partial^{(DC)} \zeta_b A \]  

\[ +2 N \sigma \zeta^a D \partial_{(BC)} \zeta^b B \partial^{(DC)} \zeta^A_b + \partial_{(BC)} N^C B \bar{p}^a A - 2 \partial_{(BC)} N \sigma \zeta^a D \zeta^b B \partial^{(DC)} \zeta_b A, \]

and the constraint equations

\[ H_\perp = 0, H_{AB} = 0. \]

The detail of the constraint structure of the theory can be found in [15].

III. THE FIRST-ORDER EVOLUTION EQUATIONS

Since the Hamiltonian equations of \( \bar{p}^a A \) include the second-order terms \( 2 N \sigma \zeta^a D \zeta^b B \partial_{(BC)} \partial^{(DC)} \zeta_b A \), in order to get the first-order evolution equations we decompose \( \omega_{(AC)}^{DB} \) into its trace-free and trace part

\[ \omega_{(AC)}^{DB} = \omega_{\text{trf}}_{(AC)}^{DB} + \frac{1}{4\sqrt{2} \sigma} \varepsilon_{C}^{D} \zeta_{aA} \bar{p}^B \]  

and introduce a new variable \( \bar{q}_{AB} \) and a real constant spatial 1-form \( \psi_{CD} \) by

\[ \omega_{\text{trf}}_{(AC)}^{DB} = \frac{\sqrt{2}}{8\sigma} \psi_{D(C\bar{q})A}, \]  

with the properties

\[ \psi_{CD} = \psi_{(CD)}, \bar{q}_{AB} = \bar{q}_{[AB]}. \]

Substituting (14) into (13) and using (4) we obtain

\[ \partial_{(AC)} \zeta_{aB} = \frac{\sqrt{2}}{8\sigma} [\bar{q}_{a(C\bar{q})A} - \zeta_a (C\bar{q})A]. \]

Then we can compute

\[ \dot{\zeta}_{aA} = -\frac{N}{4\sigma} \bar{p}_{aA} - \frac{\sqrt{2} N^C B}{8\sigma} \zeta_{aB} \bar{p}_{CA} + \frac{\sqrt{2} N^C B}{8\sigma} \psi_{aB} \bar{q}_{CA}, \]  

and

\[ \dot{\sigma} = -[N \zeta^a A \bar{p}_{aA} + \frac{\sqrt{2} N^C B}{2} \psi^A B \bar{q}_{CA}], \]

Using these results we can rewrite (11) as
\[ \tilde{p}^A = \frac{\sqrt{2}}{8} \zeta^a_8 \zeta^b_8 C \partial_{(BC)} \tilde{\rho}^A + N^{CB} \partial_{(BC)} \tilde{p}^A \]
\[ - \frac{\sqrt{2}}{8} \zeta^a_8 \psi^{BC} \partial_{(BC)} \tilde{\rho}^A + \frac{\sqrt{2}N}{8} \psi^a_8 \partial_{(BC)} \tilde{q}^C \]
\[ + Q_p(p, q), \]  
(19)  

where source term \( Q_p(p, q) \) is a quadratic polynomial of \( \tilde{p}^A \) and \( \tilde{q}_{AB} \).

Introducing a constant spatial vector \( \varphi^{AB} \) satisfying
\[ \psi_{AC} \varphi^{AB} = \epsilon^B \]
(20)  
and using (16) one can obtain
\[ \tilde{q}_{AB} = \frac{8\sqrt{2}}{3} \sigma \zeta^b_8 \varphi_D^C \partial_{(AC)} \tilde{\zeta}_{bB} - \frac{1}{3} \varphi_{AC} \zeta_C^C \tilde{p}^A. \]  
(21)  

Taking the time derivative leads to
\[ \dot{\tilde{q}}_{AB} = \frac{8\sqrt{2}}{3} \zeta^b_8 \varphi_D^C \partial_{(AC)} \dot{\zeta}_{bB} - \frac{1}{3} \varphi_{AC} \zeta_C^C \dot{\tilde{p}}^A \]
\[ + \frac{8\sqrt{2}}{3} \zeta^b_8 \varphi_D^C \partial_{(AC)} \tilde{\zeta}_{bB} + \frac{8\sqrt{2}}{3} \sigma \zeta^b_8 \varphi_D^C \partial_{(AC)} \tilde{\zeta}_{bB} \]
\[ - \frac{1}{3} \varphi_{AC} \zeta_C^C \tilde{p}^A. \]
(22)  

Using (17), (18), and (21) we get the evolution equation of \( \tilde{q}_{AB} \):
\[ \dot{\tilde{q}}_{AB} = - \frac{2\sqrt{2}}{3} \zeta^a_8 \varphi_D^C \partial_{(AC)} \tilde{p}_B - \frac{\sqrt{2}N}{24} \zeta^a_8 \varphi_A^C \partial_{(CD)} \tilde{p}_B + \frac{2}{3} N^{DE} \varphi_D^E \zeta^a_8 \partial_{(DE)} \tilde{p}_B + \frac{1}{3} N^{CD} \varphi_D^E \zeta^a_8 \partial_{(DE)} \tilde{p}_B \]
\[ - \frac{\sqrt{2}N}{24} \varphi_{AC} \zeta^D_8 \partial_{(DE)} \tilde{q}_C - \frac{\sqrt{2}N}{24} \varphi_{CD} \partial_{(DE)} \tilde{q}_B + \frac{2}{3} N^{CD} \partial_{(AC)} \tilde{q}_B \]
\[ + Q_q(p, q), \]
(23)  

where source term \( Q_q(p, q) \) is another quadratic polynomial of \( \tilde{p}^A \) and \( \tilde{q}_{AB} \). The source terms \( Q_p(p, q) \) and \( Q_q(p, q) \) do not contain any derivatives of the fundamental variables other than the lapse \( N \) and the shift \( N^{AB} \).

Substituting (16) into (8) and (9) one can obtain:
\[ \mathcal{H}_{AB} = \frac{\sqrt{2}}{16\sigma} [\psi_{CA} \tilde{q}_{BD} + \psi_{CB} \tilde{q}_{AD}] \tilde{p}^{CD}, \]
and
\[ \mathcal{H}_\perp = - \frac{13}{128\sigma} \tilde{p}_{AB} \tilde{p}^{AB} - \frac{1}{64\sigma} \psi^A B \tilde{p}_{AC} \tilde{q}^{BC} - \frac{25}{384\sigma} \tilde{q}_{AB} \tilde{q}^{AB}. \]
And then the constraint equations (12) leads to
\[ \tilde{q}_{AB} = - \tilde{q}_{BA}, \]
(24)  

and
\[ 39 \tilde{p}_{AB} \tilde{p}^{AB} + 6 \psi^A B \tilde{p}_{AC} \tilde{q}^{BC} + 25 \tilde{q}_{AB} \tilde{q}^{AB} = 0 \]
(25)  
The equations (16), (24) and (25) constitute the constraint equations. However, the equation (24) just confirms the equation (15) and then is not a independent constraint. The independent constraints are only (16) and (25).
IV. THE SYMMETRIC HYPERBOLICITY OF THE EVOLUTION EQUATIONS

The principal parts of the evolution equations (17), (19) and (23) are, respectively

\[
\begin{align*}
\dot{\zeta}_{aA} & \approx 0, \\
\dot{p}^{\alpha A} & \approx \sqrt{2 \over 8} N \zeta^{\alpha B} \zeta_{bC} \theta_{(BC)} \bar{p}^{\alpha A} + N^{CB} \theta_{(BC)} \bar{p}^{\alpha A} \\
& \quad - \sqrt{2 \over 8} N \zeta^{\alpha D} \psi^{BC} \theta_{(BC)} \bar{q}^{DA} + \sqrt{2 \over 8} N \psi^{Ba} \theta_{(BC)} \bar{q}^{CA} \\
& = \phi^{aADC} \epsilon_{nN} \theta_{(DC)} \bar{p}^{\alpha A} + \psi^{aADC} \epsilon_{MN} \theta_{(DC)} \bar{q}^{\alpha MN},
\end{align*}
\]

and

\[
\begin{align*}
\dot{\tilde{q}}_{AB} & \approx - \frac{2\sqrt{7} N}{3} \varphi^{\alpha C} \theta_{(AC)} \bar{p}_{aB} - \frac{\sqrt{2}}{24} N \varphi^{A} \epsilon_{C} \epsilon^{\alpha C} \theta_{(DC)} \bar{p}_{aB} + \frac{2N^{DE}}{3} \varphi_{F} \epsilon^{\alpha C} \epsilon_{E} \theta_{(AC)} \bar{p}_{aB} + \frac{1}{3} N^{CD} \varphi^{A} \epsilon_{nN} \theta_{(DC)} \bar{p}_{aB} \\
& \quad - \frac{\sqrt{2}}{24} N \theta_{(AC)} \bar{q}_{B} - \sqrt{2} \frac{2}{24} N \varphi_{C} \psi^{ED} \theta_{(ED)} \bar{q}_{B} + 2N^{DC} \frac{2}{3} \theta_{(AD)} \bar{q}_{CB} \\
& = \Theta_{AB}^{CDnN} \theta_{(CD)} \bar{p}_{nN} + \Pi_{AB}^{CDMN} \theta_{(CD)} \bar{q}_{MN},
\end{align*}
\]

where

\[
\begin{align*}
\phi^{aADC} \epsilon_{nN} &= \frac{\sqrt{7}}{8} N \zeta^{\alpha D} \zeta_{nA} + N^{CD} \epsilon_{nA} \epsilon_{nA}, \\
\psi^{aADC} \epsilon_{MN} &= -\frac{\sqrt{7}}{8} N \zeta_{M} \psi^{DE} \epsilon_{nA} + \frac{\sqrt{2}}{8} N \psi^{Da} \epsilon_{M} \epsilon_{nA}, \\
\Theta_{AB}^{CDnN} &= -\frac{2\sqrt{7}}{3} N \varphi^{nD} \epsilon_{C} \epsilon^{D} \epsilon_{B} - \frac{\sqrt{2}}{24} N \varphi^{A} \epsilon_{C} \epsilon^{nC} \epsilon^{B} + \frac{2N^{FE}}{3} \varphi_{F} \epsilon_{A} \epsilon^{nB} + \frac{1}{3} N^{CD} \varphi^{A} \epsilon_{nB}, \\
\Pi_{AB}^{CDMN} &= \frac{\sqrt{2}}{24} N \epsilon_{A} \epsilon^{DM} \epsilon_{B} + \sqrt{2} \frac{2}{24} N \varphi^{A} \epsilon^{CD} \epsilon_{B} + \frac{2N^{DM}}{3} \epsilon_{A} \epsilon_{B}.
\end{align*}
\]

The principal parts of the evolution equations can be expressed as a "matrix form"

\[
\begin{align*}
\partial_t \begin{pmatrix}
\zeta_{aA} \\
p^{\alpha A} \\
\tilde{q}_{AB}
\end{pmatrix}
&= \begin{pmatrix}
0 & 0 & 0 \\
0 & \phi^{aADC} \epsilon_{nN} & \psi^{aADC} \epsilon_{MN} \\
0 & \Theta_{AB}^{CDnN} & \Pi_{AB}^{CDMN}
\end{pmatrix}
\partial_{(CD)} \begin{pmatrix}
\zeta_{bB} \\
p_{nN} \\
\tilde{q}_{MN}
\end{pmatrix},
\end{align*}
\]

and then they are symmetric hyperbolic if the conditions

\[
\begin{align*}
\phi^{aADC} \epsilon_{nN} &= \Phi^{aNCDa}, \\
\Theta_{AB}^{CDnN} &= \Theta^{ABCDnN}, \\
\Pi_{AB}^{CDMN} &= \Pi^{MNCAB},
\end{align*}
\]

are satisfied. Using the formulas

\[
\epsilon_{nA} = \epsilon_{mb} \zeta_{m} A, \quad \epsilon^{nA} = \epsilon^{mb} \zeta_{m} A
\]

and

\[
\Phi^{aNCDa} = \frac{\sqrt{2}}{8} N \epsilon_{nD} \epsilon^{AC} \epsilon^{AN} + N^{CD} \epsilon_{nA} \epsilon^{AN}
\]

are satisfied.
we can find easily that the condition
\[ \Phi^{aACDnN} = \Phi^{nCDaA} \]
leads to
\[ \frac{\sqrt{2}}{8} N \zeta^{aD} \epsilon^{nC} \epsilon^{NA} + N^{CD} \epsilon^{na} \epsilon^{NA} = \frac{\sqrt{2}}{8} N \zeta^{aD} \epsilon^{AN} + N^{CD} \epsilon^{m} \epsilon^{AN}. \]  
(34)

Supposing
\[ N^{CD} = N^{CD}, \]  
(35)
and using
\[ \epsilon^{mb} = \epsilon^{mb}, \]
we find from (34) that
\[ \epsilon^{NA} = \epsilon^{NA}, N = \overline{N}. \]  
(36)
Since
\[ \Psi^{ABCDMN} = -\frac{\sqrt{2}}{8} N \epsilon^{MA} \psi^{DC} \epsilon^{BN} + \frac{\sqrt{2}}{8} N \psi^{DM} \epsilon^{AC} \epsilon^{BN}, \]
and
\[ \Theta^{ABCDMN} = -\frac{2\sqrt{2}N}{3} \varphi^{MD} \epsilon^{AC} \epsilon^{BN} - \frac{\sqrt{2}}{24} N \varphi^{AD} \epsilon^{MC} \epsilon^{BN} - \frac{2N^{EM}}{3} \varphi^{E} \epsilon^{AC} \epsilon^{BN} + \frac{1}{3} N^{CD} \varphi^{AM} \epsilon^{BN}, \]
the condition
\[ \Theta^{ABCDMN} = \Psi^{ABCDMN} \]
leads to
\[ -\frac{2\sqrt{2}N}{3} \varphi^{MD} \epsilon^{AC} \epsilon^{BN} - \frac{\sqrt{2}}{24} N \varphi^{AD} \epsilon^{MC} \epsilon^{BN} - \frac{2N^{EM}}{3} \varphi^{E} \epsilon^{AC} \epsilon^{BN} + \frac{1}{3} N^{CD} \varphi^{AM} \epsilon^{BN} \]
\[ = -\frac{\sqrt{2}}{8} N \epsilon^{MA} \psi^{DC} \epsilon^{BN} + \frac{\sqrt{2}}{8} N \psi^{DM} \epsilon^{AC} \epsilon^{BN}, \]
which means
\[ \varphi^{AC} = 3 \psi^{AC}; \]  
(37)
and
\[ 2N^{A} B + N^{CD} \varphi^{A} C \psi^{BD} = 0. \]  
(38)
Using (37) and (20) we obtain
\[ \psi^{CA} \psi^{AB} = \frac{1}{3} \epsilon^{CB}. \]  
(39)
And, (38) leads to trivial result 0 = 0.
Similarly,
\[ \Pi^{MNCDA} = -\frac{\sqrt{2}}{24} N \epsilon^{MC} \epsilon^{DA} \epsilon^{NB} + \frac{\sqrt{2}}{24} N \varphi^{MA} \psi^{CD} \epsilon^{NB} + \frac{2N^{DA}}{3} \epsilon^{MC} \epsilon^{NB}, \]
and
\[ \Pi^{ABCDMN} = \Pi^{MNCDA} \]
lead to $\epsilon^{AM} = -\epsilon^{MA}$, and $N^{DC} = N^{DC}$, which are trivial.

In summary, the conditions for the evolution equations of $\zeta_{aA}$, $\bar{p}^a A$ and $\bar{q}_{AB}$ together with the assumption (35) reduce to the reality conditions (36) on the metric and a condition (39) on the constant spatial vector $\psi^{AB}$.

In this case the polynomial $Q_p(p, q)$ in the equation (19) has the form

$$Q_p(p, q) = -2\sqrt{2}N\zeta^D\psi^C\partial (AC)\bar{p}_{aB} - \frac{\sqrt{2}N}{8}\zeta^D\psi_A^C\partial (CD)\bar{p}_{aB} + 2N^{DE}\psi^C\zeta^a D\partial (AC)\bar{p}_{aB} + N^{CE}\psi AD\zeta^D \partial (CE)\bar{p}_{aB}$$

$$+ \sqrt{2N} \partial (AC)\bar{q}_{BC} - \frac{\sqrt{2N}}{8} \psi AC^D\partial (DE)\bar{q}_{BC} + \frac{2}{3} N^{CD}\partial (AC)\bar{q}_{BD}$$

and the evolution equation (23) becomes

$$\partial AB = -2\sqrt{2}N\zeta^D\psi^C\partial (AC)\bar{p}_{aB} - \frac{\sqrt{2}N}{8}\zeta^D\psi_A^C\partial (CD)\bar{p}_{aB} + 2N^{DE}\psi^C\zeta^a D\partial (AC)\bar{p}_{aB} + N^{CE}\psi AD\zeta^D \partial (CE)\bar{p}_{aB}$$

$$+ Q_q(p, q)$$

where

$$Q_q(p, q)$$

$$= \frac{-25N}{64\sigma}\bar{p}_{aB}^2\bar{p}_{BC}\bar{p}_{BD} - \frac{17N}{64\sigma}\bar{p}_{aB}^{CD}\bar{p}_{BD} + \frac{65N}{64\sigma}\bar{p}_{AC}\bar{p}_{BD} + \frac{85N}{64\sigma}\bar{p}_{AB}\bar{D}$$

$$+ \frac{\sqrt{7N}}{8\sigma}\psi A^D\bar{p}_{BC}\bar{p}_{ED} + \frac{\sqrt{7N}}{4\sigma}\psi A^C\bar{p}_{CD}\bar{p}_{EB} + \frac{\sqrt{7N}}{8\sigma}\psi A^D\bar{p}_{ABC} + \frac{\sqrt{7N}}{8\sigma}\psi A^D\bar{p}_{ABC}.$$
V. CONCLUSIONS

Now we have proved that the evolution equation (17), (19) and (41) constitute a symmetric hyperbolic system under the conditions (35), (36) and (39), and the constraint equations are (16) and (25). Here we suppose only the reality of the shift vector $N^{CD}$ and then obtain naturally the reality of the spatial metric as one of the conditions of the symmetric hyperbolic system rather than an independent one as in [23].

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[16] See the recent review article by O. A. Reula, Living Rev. Relat. 1998-3 at http://www.livingreviews.org/Articles/ for an extensive survey and a more complete list of references.
[21] See the review article by T. W. Baumgate and S. L. Sharpio gr-qc/0211028.