An analysis on extrema and constrained bounds for the soft Pomeron intercept

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Abstract

We investigate some aspects and consequences of the extrema bounds for the soft Pomeron intercept, recently determined by means of global fits to $pp$ and $\bar{p}p$ total cross section data at both accelerator and cosmic-ray energy regions (scattering data). We also examine the effects of the secondary Reggeons by introducing fitted trajectories from Chew-Frautschi plots (spectroscopy data) and determining new constrained bounds for the Pomeron intercept. In both cases we extend the analysis to $baryon-p$, $meson-p$, $baryon-n$, $meson-n$, $gamma-p$ and $gamma-gamma$ scattering, presenting tests on factorization and quark counting rules. We show that in all the cases investigated, the bounds lead to good descriptions of the bulk of experimental data on the total cross sections, but with different extrapolations to higher energies. Our main conclusion is that the experimental information presently available on the above quantities is not sensitive to an uncertainty of 2% in the value of the soft Pomeron intercept. At 14 TeV (CERN LHC) the extrema and constrained bounds allow to infer $\sigma_{tot} = 114 \pm 25$ mb and $105 \pm 10$ mb, respectively.

Key words: elastic hadron scattering, total cross sections, Regge formalism

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1 Introduction

The total cross section is one of the most important physical quantities that characterizes the hadron-hadron scattering. From Unitarity (optical theorem) this quantity is expressed in terms of the forward elastic scattering amplitude and at high energies the relation reads

$$\sigma_{tot}(s) = \frac{\text{Im} F(s, t = 0)}{s}.$$  

(1)
where $s$ is the center-of-mass energy squared and $t$ is the four momentum transfer squared. From the experimental point of view, the general smooth increase of all hadronic total cross sections, above $\sqrt{s} \approx 20$ GeV, is a well established result [1]. However, since this rise is inherently a nonperturbative phenomena (soft diffractive process), the theoretical treatment is still essentially phenomenological.

Models rely on general principles of Quantum Field Theory and, among them, the Regge Pole formalism plays a fundamental role [2]. In this context, the elastic scattering amplitude in the $s$-channel is expressed as a descending asymptotic series of powers of $s$, each term representing a specific exchange in the $t$-channel [2,3]

$$F(s, t) = \sum_k \gamma_k(t) \zeta_k(t) s^{\alpha_k(t)},$$

(2)

where $\gamma_k(t)$ is the residue function, $\zeta_k(t)$ is the (complex) signature factor and $\alpha_k(t)$ the trajectory function.

The decreasing of the total cross sections below $\sqrt{s} \approx 20$ GeV is assumed to be a consequence of the exchange of meson resonances families, the Reggeons ($\mathbb{R}$), with the adequate quantum numbers in the $t$-channel process, and represented by trajectories interpolating the data on plots of spin $J$ versus the square of their masses (Chew-Frautschi plot). That scheme provides trajectories that are approximately linear in $t$, $\alpha_\mathbb{R}(t) \approx \alpha_\mathbb{R}(0) + \alpha_\mathbb{R}' t$, and for the known mesonic resonances we have intercepts $\alpha_\mathbb{R}(0) \approx 0.5$ and slopes $\alpha_\mathbb{R}' \approx 1.0$ GeV$^{-2}$. From Eqs. (1) and (2), and denoting $\gamma_k(t = 0) \zeta_k(t = 0) \equiv g_{\mathbb{R}_k}$ (the Reggeon strength), the total cross section reads

$$\sigma_{\text{tot}}(s) = \sum_k g_{\mathbb{R}_k} s^{\alpha_{\mathbb{R}_k}(0) - 1},$$

(3)

and therefore, decreases roughly as $1/\sqrt{s}$. On the other hand, the rise of the total cross sections above $\sqrt{s} \approx 20$ GeV is phenomenologically implemented by the introduction of an ad hoc trajectory, associated with a colourless state having the vacuum quantum numbers, the Pomeron ($\mathbb{P}$), with intercept slightly greater than one: $\alpha_\mathbb{P}(t) = \alpha_\mathbb{P}(0) + \alpha_\mathbb{P}' t$, $\alpha_\mathbb{P}(0) > 1$ (Supercritical Pomeron). Since there are no known particles related with the Pomeron trajectory (except for the glueball candidate $2^{++}$ [4]), it is necessary to perform fits to the available data in order to establish the Pomeron’s parameters, namely, its intercept $\alpha_\mathbb{P}(t = 0)$ and strength $g_\mathbb{P}$.

Donnachie and Landshoff have shown that the total cross sections on $pp$, $\bar{p}p$, $meson-p$, $\gamma-p$, $pn$, and $\bar{p}n$ scattering can be well described with the Pomeron and a degenerate Reggeon contribution (associated with the $a_2$, $f_2$, $\rho$, and $\omega$
families of particles) [5]. A remarkable fact is the result that all the data show agreement with the same value for the soft Pomeron intercept, namely \( \alpha_I(\tau = 0) = 1.0808 \). This means that the rise of the total cross sections is a direct consequence of the “object” exchanged, the Pomeron, and does not depend on the intrinsic structure of the hadrons involved in the scattering.

More recently, new data analyzes with extended models that allow the splitting of the Reggeon trajectories (non-degenerate \( C = +1 \) and \( C = -1 \) meson trajectories) or using different estimations for \( \sigma_{tot} \) from cosmic-ray experiments, have indicated larger values for the Pomeron intercept, namely 1.09 - 1.12 [6,7,8,9]. In general these analyzes are characterized by global fits of the Pomeron/Reggeon parameters to the forward scattering data, that is, without using as input the fitted trajectories from the Chew-Frautschi plots.

However, independently of the parametrization (model) used, to select a correct or “secure” value for the intercept is a difficult task, mainly due to the well known disagreement between measurements of the \( \bar{p}p \) total cross sections at the highest energy reached in accelerators, \( \sqrt{s} = 1.8 \) TeV (CDF and E710/E811 results, [10,11]) . Experimental information on \( pp \) total cross sections from cosmic-ray experiments exist in the energy region \( \sqrt{s} : 6 - 40 \) TeV, but they are also characterized by discrepancies and large uncertainties [9].

Therefore, despite the fundamental role of the soft Pomeron intercept in the investigation of the rise of the hadron-hadron total cross sections, its “exact” or accepted value still remains an open problem and that is a consequence of the uncertainties in the experimental information at the highest energies.

Based on that fact, in a previous work we have developed a quantitative investigation of the effect of the discrepant data/information on the value of the Pomeron intercept [12]. By combining the highest or lowest results for the \( pp \) and \( \bar{p}p \) total cross sections from both accelerator and cosmic-ray experiments, and testing all the important variants of fits to the experimental data above 10 GeV through an extended Regge parametrization (non-degenerate trajectories), we have determined extrema upper and lower bounds for the intercept, 1.109 and 1.081, respectively. That permits to infer the fastest and slowest increase scenarios for the rise of the hadronic total cross sections, allowed by the experimental information presently available.

In this work, we investigate the effects of these bounds in a global study of the hadron-hadron elastic scattering. For each fixed extrema bound, previously determined from \( pp \) and \( \bar{p}p \) scattering, we extend the analysis to \( p^\pm n, \pi^\pm p, K^\pm p, K^\pm n, \Sigma^- p, \gamma p \) and \( \gamma \gamma \) elastic scattering (here \( p^\pm \) indicates \( p \) and \( \bar{p} \)), by means of simultaneous fits to the corresponding total cross section data (some partial results have already been presented in [13]). In addition, using as input the fitted secondary trajectories from the Chew-Frautschi plots, we determine
new constrained bounds for the intercept from $pp$ and $\bar{p}p$ scattering. These results are also extended to the above reactions. In all the cases investigated we present tests on factorization rule associated with $pp$, $\gamma p$, and $\gamma \gamma$ scattering and quark counting rule in $p^\pm p$, $p^\pm n$, $\pi^\pm p$, $K^\pm p$, $K^\pm n$, and $\Sigma^- p$ scattering. We show that both upper and lower extrema bounds for the intercept lead to good descriptions of the experimental data and information presently available. However, extrapolations to higher energies indicate different scenarios for the rise of the total cross sections.

The paper is organized as follows. In Section 2 we treat the bounds for the intercept from analyzes of $pp$ and $\bar{p}p$ data, by means of both global fits to the total cross section data (extrema bounds) and also using as input fitted secondary Reggeon trajectories (constrained bounds). In Section 3 we present the extensions to $p^\pm n$, $\pi^\pm p$, $K^\pm p$, $K^\pm n$, $\Sigma^- p$, $\gamma p$, and $\gamma \gamma$ scattering and tests concerning factorization and quark-counting rules. The conclusions and some final remarks are the contents of Section 4.

2 Bounds for the Pomeron intercept from $pp$ and $\bar{p}p$ scattering

In this Section we first introduce the extended parametrization (non-degenerate meson trajectories) to be used in all the fits. Afterwards we shortly review the results and notation used in the previous determination of the extrema bounds for the Pomeron intercept (scattering data), and then discuss the determination of constrained bounds obtained with fitted secondary Reggeon trajectories (spectroscopy data). We end the Section with a comparison between the results for the $pp$ and $\bar{p}p$ total cross sections obtained with both extrema and constrained bounds.

2.1 Extended Regge parametrization

In the extended Regge Pole model, the forward scattering amplitude is decomposed into three contributions [6,7]:

\[ F(s) = F_\Pi(s) + F_{a_2/f_2}(s) + \tau F_{\rho/\omega}(s), \]

where $F_\Pi$ represents the single Pomeron, $F_{a_2/f_2}$ the Reggeon with $C = +1$, $F_{\rho/\omega}$ the Reggeon with $C = -1$, and $\tau = +1 (-1)$ for antiparticle-particle (particle-particle) scattering. The intercepts of the Pomeron, the $C = +1$, and the $C = -1$ trajectories are expressed by $\alpha_\Pi(0) = 1 + \epsilon$, $\alpha_+(0) = 1 - \eta_+$, and $\alpha_-(0) = 1 - \eta_-$, respectively. With this amplitude, and through the optical theorem, Eq. (1), the total cross section for particle-particle and antiparticle-particle interactions reads

\[ \sigma_{tot}(s) = X s^\epsilon + Y_+ s^{-\eta_+} + \tau Y_- s^{-\eta_-}, \]
where the coefficients $X, Y_+, Y_-$, and the exponents $\epsilon, \eta_+, \eta_-$ are parameters to be determined by the fit to the data.

Two shortcomings must be recalled about this model. One refers to the intrinsic asymptotic character of Eq. (4), since it is intended for the region $s/s_0 \to \infty$, with $s_0 \approx 1$ GeV. Therefore one must be careful to infer physical meanings at finite energies. The other concerns the violation of the Froissart-Martini bound by power laws with exponents greater than one, as it is the case for the Supercritical Pomeron. However, it is understood that $\epsilon$ is an effective parameter, which may eventually depend on the energy. In spite of these shortcomings, the Regge formalism constitutes presently one of the most useful bridges to a well founded theoretical approach (non-perturbative QCD) for soft diffractive processes [2].

2.2 Extrema bounds

The extrema bounds for the Pomeron intercept have been determined in Ref. [12] from analyzes of the discrepancies in $pp$ and $\bar{p}p$ total cross section data at both accelerator and cosmic-ray energy regions. For $\bar{p}p$ scattering the discrepancies appear in the values of $\sigma_{tot}^{\bar{p}p}$ at $\sqrt{s} = 1.8$ TeV. The highest value concerns the measurement by the CDF Collaboration (CDF) [10] and the lowest values, the measurements by the E710 and E811 Collaborations (E710/E811) [11]. In the case of $pp$ total cross sections, extracted from $p$-air cross sections (cosmic rays) at $\sqrt{s} : 6 - 40$ TeV, the highest estimations concern the results by Nikolaev and also by Gaisser, Sukhatme and Yodh (NGSY) [14] and the lowest estimations, the results by Block, Halzen and Stanev (BHS) [15] (detailed discussion on the experimental uncertainties, together with numerical tables may be found in [9]).

The strategy and method used in [12] was the following. First, the accelerator data on $pp$ and $\bar{p}p$ scattering were split in two ensembles, where each ensemble displays one of the two possible scenarios for the total cross section behavior with energy:

Ensemble I - $\sigma_{tot}^{pp}$ and $\sigma_{tot}^{\bar{p}p}$ data ($10 \leq \sqrt{s} \leq 900 GeV$) + CDF datum ($\sqrt{s} = 1800 GeV$);

Ensemble II - $\sigma_{tot}^{pp}$ and $\sigma_{tot}^{\bar{p}p}$ data ($10 \leq \sqrt{s} \leq 900 GeV$) + E710/E811 data ($\sqrt{s} = 1800 GeV$).

The choice for the minimal energy at 10 GeV was based on an analysis showing that the parameters of Regge fits are stable for a cutoff at $\sqrt{s} \approx 9$ GeV [8]. We shall return to that point at the end of this Subsection.
In a second step, since with the extended Regge parametrization (4) one has 
\[ \sigma^\text{pp}_\text{tot}(s) - \sigma^\text{\bar{p}p}_\text{tot}(s) \to 0 \text{ as } s \to \infty, \]
the highest and lowest estimations for \( \sigma^\text{pp}_\text{tot} \) from
\textit{cosmic-ray} experiments have been adequately added to the above ensembles,
defining two other cases:

Ensemble I + NGSY (fastest increase scenario);

Ensemble II + BHS (slowest increase scenario).

Besides individual fits to the total cross sections, the available data on the
ratio of the real to the imaginary part of the amplitude, 
\( \rho(s) = \text{Re} F(s, t = 0)/\text{Im} F(s, t = 0) \), were also used in global fits involving \( \rho \) and \( \sigma_{\text{tot}} \). This has
been done by means of dispertion relations and either using the subtraction
constant \( K \) as a free fit parameter or assuming \( K = 0 \). Among the 16 vari-
ants of fits performed through the CERN-Minuit routine, the \textit{extrema values}
for the Pomeron intercept were obtained in the case of individual fits to \( \sigma_{\text{tot}} \),
the highest value with Ensemble I + NGSY and the lowest one with Ensemble II,
\( \alpha_{\text{IP}}^{\text{upper}}(0) = 1.104 \pm 0.005 \) and \( \alpha_{\text{IP}}^{\text{lower}}(0) = 1.085 \pm 0.004 \), respectively.
The \textit{extrema upper bound} was inferred by adding the corresponding error to
the central upper value and the \textit{extrema lower bound} by subtracting the cor-
responding error from the central lower value: 1.109 and 1.081, respectively.
The results of the fitting are displayed in Table 1, together with the extrema
bounds for the parameter \( \epsilon \).

Table 1
\textit{Extrema} values and bounds for the Pomeron intercept \( (\alpha_{\text{IP}}(0) = 1 + \epsilon) \): parameters
obtained through global fits to \( pp \) and \( \bar{p}p \) total cross section data [12].

<table>
<thead>
<tr>
<th>Ensemble:</th>
<th>I + NGSY</th>
<th>II</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon )</td>
<td>(upper)</td>
<td>(lower)</td>
</tr>
<tr>
<td>( \epsilon_{\text{extrema}} )</td>
<td>0.104±0.005</td>
<td>0.085±0.004</td>
</tr>
<tr>
<td>( X ) (mb)</td>
<td>16.4±1.2</td>
<td>20.47±0.88</td>
</tr>
<tr>
<td>( \eta_+ )</td>
<td>0.28±0.03</td>
<td>0.38±0.04</td>
</tr>
<tr>
<td>( Y_+ ) (mb)</td>
<td>51.2±4.2</td>
<td>62.2±7.5</td>
</tr>
<tr>
<td>( \eta_- )</td>
<td>0.42±0.04</td>
<td>0.42±0.04</td>
</tr>
<tr>
<td>( Y_- ) (mb)</td>
<td>17.4±3.8</td>
<td>17.2±3.9</td>
</tr>
<tr>
<td>No. ( F )</td>
<td>94</td>
<td>89</td>
</tr>
<tr>
<td>( \chi^2/F )</td>
<td>1.01</td>
<td>0.94</td>
</tr>
<tr>
<td>( \epsilon_{\text{extrema}} )</td>
<td>0.109</td>
<td>0.081</td>
</tr>
</tbody>
</table>

In that analysis, the secondary Reggeon intercepts were left as free parameters
in order to allow the necessary freedom for the Pomeron intercept to find its
extrema. The final result for the Reggeon intercepts, from Table 1, are:
\[ \alpha_{a_2/f_2}(0) = 1 - \eta_+ = 0.72 \pm 0.03, \]
\[ \alpha_{p/\omega}(0) = 1 - \eta_- = 0.58 \pm 0.04, \] (5)

for the upper bound, and

\[ \alpha_{a_2/f_2}(0) = 1 - \eta_+ = 0.62 \pm 0.04, \]
\[ \alpha_{p/\omega}(0) = 1 - \eta_- = 0.58 \pm 0.04, \] (6)

for the lower one. We shall return to these results in the next Subsection.

In the present paper, in order to check the influence of the minimal energy (cutoff) in the determination of the extrema values of the intercept, we performed several fits to the total cross section with ensembles I + NGSY and II and varying \( \sqrt{s_{\text{min}}} \) around 10 GeV. The results for the upper and lower values of the parameter \( \epsilon \) are displayed in Fig. 1, together with the \( \chi^2/F \) obtained in each case. We see that although the lower values indicate more stability than the upper values, the cutoff at 10 GeV provide good statistical results (in accordance with the results obtained in [8]).

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Fig. 1. Behavior of the upper and lower extrema values of the parameter \( \epsilon = \alpha_p(0) - 1 \) (top) and the corresponding \( \chi^2/F \) (bottom) as function of the minimal energy (cutoff), when the secondary Reggeon intercepts are left as free fit parameters to the scattering data (Table 1).
2.3 Constrained bounds

As mentioned in our introduction, the secondary Reggeon intercepts can also be determined directly from the Chew-Frautschi plots of spin versus squared mass (spectroscopy data). In what follows we show that using these intercepts as inputs we restrict the upper and lower values and bounds of the Pomeron intercept. To that end, we gathered the available data of meson resonance families \( a_2 \), \( f_2 \), \( \rho \), and \( \omega \) in a Chew-Frautschi plot, as shown in Figure 2. A fit of a linear expression to the combined \( a_2/f_2 \) and \( \rho/\omega \) data provided the trajectories

\[
\alpha_{a_2/f_2}(t) = (0.548 \pm 0.016) + (0.847 \pm 0.009) t \quad (\chi^2/F = 258), \tag{7}
\]

\[
\alpha_{\rho/\omega}(t) = (0.442 \pm 0.003) + (0.912 \pm 0.005) t \quad (\chi^2/F = 63.7), \tag{8}
\]

which are also displayed in Fig. 2.

![Fig. 2. Chew-Frautschi plot of the \( a_2, f_2, \rho \), and \( \omega \) resonance families and the corresponding fitted trajectories for \( a_2/f_2 \) (solid line) and \( \rho/\omega \) (dashed line), Eqs. (7) and (8), respectively.](image)

For individual trajectories, the best fit provided

\[
\alpha_{a_2}(t) = (0.491 \pm 0.034) + (0.869 \pm 0.019) t \quad (\chi^2/F = 0.3), \tag{9}
\]

\[
\alpha_{f_2}(t) = (0.676 \pm 0.017) + (0.814 \pm 0.010) t \quad (\chi^2/F = 3.8), \tag{10}
\]

\[
\alpha_{\rho}(t) = (0.501 \pm 0.007) + (0.840 \pm 0.012) t \quad (\chi^2/F = 0.2), \tag{11}
\]

\[
\alpha_{\omega}(t) = (0.435) + (0.923) t \quad \text{(not fitted)}. \tag{12}
\]
Overall, these results show some agreement with previous analysis by Desgro-lard et al. [16].

From these fits, it is possible to establish a range for the $C = +1$ and $C = -1$ Reggeon intercept values, with Eqs. (7) and (8) providing the central values and Eqs. (9-12) providing the upper and lower bounds for each set. From that, we have:

\[
\alpha_{a_2/f_2}(0) = 1 - \eta_+ = 0.548^{+0.145}_{-0.091}, \tag{13}
\]

\[
\alpha_{\rho/\omega}(0) = 1 - \eta_- = 0.442^{+0.066}_{-0.007}. \tag{14}
\]

These results can be compared with those obtained from fits to scattering data in the determination of the upper and lower extrema bounds, Eqs. (5) and (6), respectively. For the $a_2/f_2$ trajectory, the Reggeon intercept in the case of the lower bound is compatible with the above spectroscopy result and for the upper bound the value is barely above the range of Eq. (13). For the $\rho/\omega$ trajectory the results with both upper and lower bounds are above the range of Eq. (14). These discrepancies may be associated with the particular variant that provided the extrema values for the Pomeron intercept in Ref. [12], that is, fits to the total cross section data only. As demonstrated in that paper, the inclusion of the $\rho(s)$ data constrains the asymptotic rise of the total cross section, restraining the evaluation of maxima bounds.

In order to investigate the effect that extracting the secondary Reggeon intercepts from spectroscopy data can have on the value of the Pomeron intercept, we performed new fits with both Reggeon intercepts fixed at their central values given by Eqs. (13) and (14) and letting free all the other parameters in Eq. (4). We used the same two sets of data that provided the extrema bounds of Table 1, i.e., Ensemble I + NGSY and Ensemble II, and $\sqrt{s_{\text{min}}} = 10$ GeV (see below). The results for these new fits are displayed in Table 2. As in the case of the extrema bounds, we can infer here the constrained bounds by adding and subtracting the uncertainties to the corresponding upper and lower central values, respectively (also displayed in Table 2).

As in the previous subsection, we have also checked the influence of the minimal energy on the constrained values of the intercept. The results are displayed in Fig. 3 showing, once more, that the stability in the intercepts and good statistical results are obtained for the cutoff at $\sqrt{s_{\text{min}}} = 10$ GeV.
Table 2

*Constrained* values and bounds for the Pomeron intercept \((\alpha_{IP}(0) = 1 + \epsilon)\): parameters obtained from fits to \(pp\) and \(\bar{p}p\) total cross section with fixed secondary Reggeon intercepts (central values in Eqs. (13) and (14)).

<table>
<thead>
<tr>
<th>Ensemble: I + NGSY</th>
<th>II (upper)</th>
<th>II (lower)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\epsilon_{\text{values}}) constrained</td>
<td>0.087(\pm)0.002</td>
<td>0.082(\pm)0.002</td>
</tr>
<tr>
<td>(X) (mb)</td>
<td>20.49(\pm)0.30</td>
<td>21.30(\pm)0.29</td>
</tr>
<tr>
<td>(\eta_+) (fixed)</td>
<td>0.452 (fixed)</td>
<td>0.452 (fixed)</td>
</tr>
<tr>
<td>(Y_+) (mb)</td>
<td>86.3(\pm)2.2</td>
<td>81.2(\pm)2.1</td>
</tr>
<tr>
<td>(\eta_-) (fixed)</td>
<td>0.558 (fixed)</td>
<td>0.558 (fixed)</td>
</tr>
<tr>
<td>(Y_-) (mb)</td>
<td>36.22(\pm)0.76</td>
<td>36.17(\pm)0.76</td>
</tr>
<tr>
<td>No. (F)</td>
<td>96</td>
<td>91</td>
</tr>
<tr>
<td>(\chi^2/F)</td>
<td>1.26</td>
<td>1.03</td>
</tr>
<tr>
<td>(\epsilon_{\text{bounds}}) constrained</td>
<td>0.089</td>
<td>0.080</td>
</tr>
</tbody>
</table>

2.4 Bounds for the rise of the \(pp\) and \(\bar{p}p\) total cross sections

The results for the \(pp\) and \(\bar{p}p\) total cross sections obtained by means of the extended Regge parametrization (4) with both the *extrema* and *constrained* bounds (Tables 1 and 2) are displayed in Fig. 4, together with the experimental data and estimations used in the ensembles (see [9] and [12] for references and discussions on these and others numerical values).

From Tables 1 and 2, we see that the best statistical results \((\chi^2/F)\) were obtained with the extrema bounds and from the curves in Fig. 4 we notice the following aspects: (1) fixing the secondary Reggeon intercepts to the spectroscopy data (non-degenerate \(a_2/f_2\) and \(\rho/\omega\) with linear parametrization) curbs the rising of the total cross sections by limiting the freedom of the Pomeron intercept; (2) the region delimited by the constrained bounds favors the E710/E811 and BHS results, indicating a “conservative” scenario; (3) the curve obtained with the extreme upper bound is compatible with the NGSY result but is above the CDF result; (4) from the regions delimited by the upper and lower extrema and constrained bounds we can infer averaged values for the total cross sections at the BNL RHIC and CERN LHC energies, 200 GeV and 14 TeV, respectively:

\[
\sigma_{\text{tot}}^{\text{constrained}}(200 \text{ GeV}) = 51.8 \pm 2.1 \text{ mb}, \quad \sigma_{\text{tot}}^{\text{extrema}}(200 \text{ GeV}) = 51.9 \pm 3.8 \text{ mb},
\]
Fig. 3. Behavior of the upper and lower constrained values of the parameter \( \epsilon = \alpha P(0) - 1 \) (top) and the corresponding \( \chi^2/F \) (bottom) as function of the minimal energy, when the secondary Reggeon intercepts are fitted to the spectroscopy data (Table 2).

\[
\sigma_{\text{tot}}^{\text{constrained}}(14 \text{ TeV}) = 105 \pm 10 \text{ mb}, \quad \sigma_{\text{tot}}^{\text{extrema}}(14 \text{ TeV}) = 114 \pm 25 \text{ mb}.
\]

We notice that the above range provided by the constrained bound at 14 TeV is in agreement with the QCD-inspired model prediction by Block, Halzen, and Stanev, namely \( \sigma_{\text{tot}} = 108.0 \pm 3.4 \text{ mb} \) [15].

3 Extensions to \( p^\pm n, \pi^\pm p, K^\pm p, K^\pm n, \Sigma^- p, \gamma p \) and \( \gamma\gamma \) scattering

Besides \( pp \) and \( \bar{p}p \), several other hadronic reactions have been measured through the last decades [1]. Although none of them has the energy range of \( pp \) and \( \bar{p}p \), some reactions have been measured up to considerably high energy values. For meson-proton, the total cross section data for \( K^\pm p \) have been recorded up to \( \sqrt{s} = 24.1 \text{ GeV} \), whereas for \( \pi^\pm p \) the top energy is \( \sqrt{s} = 34.7 \text{ GeV} \). For baryon-proton, the \( \Sigma^- p \) has been measured up to this same energy, whereas for \( \gamma p \) and \( \gamma\gamma \) there are measurements reaching 200 GeV and data on \( p^\pm n \) and \( K^\pm n \) are available up to \( \sqrt{s} \approx 25 \text{ GeV} \). Therefore, these reactions provide a good ground for investigating the effects of the bounds obtained for the soft Pomeron intercept. In this Section we present the results of global fits to the
3.1 Global fits to total cross sections

Making use of parametrization (4) we perform fits to $\sigma_{tot}$ data from $p^\pm n$, $\pi^\pm p$, $K^\pm p$, $K^\pm n$, $\Sigma^- p$, $\gamma p$ and $\gamma\gamma$ scattering, exploring the effects of the extrema and constrained upper and lower bounds.

First, from Table 1, we fix the intercepts, $\eta_+$ and $\eta_-$ at their central values and use for $\epsilon$ the extrema bounds, namely $\epsilon = 0.081$ and $\epsilon = 0.109$. With this procedure we have 18 free parameters: the strengths $X$, $Y_+$ and $Y_-$ for $p^\pm n$ $\pi^\pm p$, $K^\pm p$, $K^\pm n$ and $X$, $Y_+$ for $\Sigma^- p$, $\gamma p$ and $\gamma\gamma$ scattering. The total number of data points are 196 and therefore we have $F = 178$. The fit results for the extrema lower bound were:

$$
\sigma_{tot}^{p^\pm n} = (21.02 \pm 0.19) s^{0.081} + (63.2 \pm 2.6) s^{-0.38} \mp (15.19 \pm 0.82) s^{-0.42},
$$
$$
\sigma_{tot}^{\pi^\pm p} = (13.33 \pm 0.05) s^{0.081} + (25.18 \pm 0.60) s^{-0.38} \mp (3.76 \pm 0.20) s^{-0.42},
$$
with $\chi^2/F = 0.83$. With the extrema upper bound we obtained

$$
\sigma_{\text{tot}}^{p^\pm n} = (15.52 \pm 0.19) s^{0.109} + (55.7 \pm 1.7) s^{-0.28} \mp (15.43 \pm 0.84) s^{-0.42}, \\
\sigma_{\text{tot}}^{\pi^\pm p} = (10.13 \pm 0.05) s^{0.109} + (24.73 \pm 0.40) s^{-0.28} \mp (3.82 \pm 0.21) s^{-0.42}, \\
\sigma_{\text{tot}}^{K^\pm p} = (9.27 \pm 0.07) s^{0.109} + (13.85 \pm 0.57) s^{-0.28} \mp (6.96 \pm 0.23) s^{-0.42}, \\
\sigma_{\text{tot}}^{K^\pm n} = (9.24 \pm 0.11) s^{0.109} + (13.25 \pm 0.99) s^{-0.28} \mp (3.67 \pm 0.39) s^{-0.42}, \\
\sigma_{\text{tot}}^{\Sigma^\pm p} = (15.51 \pm 0.52) s^{0.109} + (25.1 \pm 4.0) s^{-0.28}, \\
\sigma_{\text{tot}}^{\gamma p} = (0.051 \pm 0.001) s^{0.109} + (0.105 \pm 0.009) s^{-0.28}, \\
\sigma_{\text{tot}}^{\gamma\gamma} = (0.00015 \pm 0.00001) s^{0.109} + (0.00023 \pm 0.00011) s^{-0.28},
$$

with $\chi^2/F = 0.77$.

These parametrizations for the $p^\pm n$, $\pi^\pm p$, $K^\pm p$, $K^\pm n$, $\Sigma^\pm p$, $\gamma p$ and $\gamma\gamma$ are displayed in Fig. 5 together with the experimental data. We see that, with the exception of the highest energy data points in $\pi^- p$, $\gamma p$ and $\gamma\gamma$ scattering, the experimental data are well described in both cases (upper and lower bounds). We note that the highest points have the largest errors bars and, therefore, little influence in global fits. We shall return to this point in Sec. 4.

Second, in order to check the effect of fixing the $C = +1$ and $C = -1$ trajectories to the spectroscopy data, new fits to the $p^\pm n$, $\pi^\pm p$, $K^\pm p$, $K^\pm n$, $\Sigma^\pm p$, $\gamma p$ and $\gamma\gamma$ were performed using the values from Table 2. For this case, we also fixed the intercepts $\eta_+$ and $\eta_-$ at their central values and used for $\epsilon$ the constrained bounds.

The fit results with the lower constrained bound were:

$$
\sigma_{\text{tot}}^{p^\pm n} = (21.64 \pm 0.16) s^{0.080} + (82.4 \pm 3.3) s^{-0.452} \mp (32.2 \pm 1.7) s^{-0.558}, \\
\sigma_{\text{tot}}^{\pi^\pm p} = (13.71 \pm 0.04) s^{0.080} + (30.83 \pm 0.77) s^{-0.452} \mp (7.72 \pm 0.42) s^{-0.558}, \\
\sigma_{\text{tot}}^{K^\pm p} = (12.13 \pm 0.06) s^{0.080} + (13.2 \pm 1.1) s^{-0.452} \mp (14.40 \pm 0.48) s^{-0.558}, \\
\sigma_{\text{tot}}^{K^\pm n} = (12.06 \pm 0.10) s^{0.080} + (12.12 \pm 1.9) s^{-0.452} \mp (7.73 \pm 0.82) s^{-0.558}, \\
\sigma_{\text{tot}}^{\Sigma^\pm p} = (20.45 \pm 0.48) s^{0.080} + (24.4 \pm 8.0) s^{-0.452}, \\
\sigma_{\text{tot}}^{\gamma p} = (0.069 \pm 0.001) s^{0.080} + (0.119 \pm 0.016) s^{-0.452}, \\
\sigma_{\text{tot}}^{\gamma\gamma} = (0.00020 \pm 0.00001) s^{0.080} + (0.00021 \pm 0.00024) s^{-0.452},
$$
with $\chi^2/F = 0.70$. With the upper constrained bound we obtained

$$
\sigma_{p^\pm n}^{tot} = (20.21 \pm 0.15) s^{0.089} + (89.6 \pm 3.2) s^{-0.452} \mp (32.2 \pm 1.7) s^{-0.558},
$$

$$
\sigma_{\pi^\pm p}^{tot} = (12.83 \pm 0.04) s^{0.089} + (34.98 \pm 0.75) s^{-0.452} \mp (7.63 \pm 0.42) s^{-0.558},
$$

$$
\sigma_{K^\pm p}^{tot} = (11.35 \pm 0.05) s^{0.089} + (16.9 \pm 1.1) s^{-0.452} \mp (14.39 \pm 0.48) s^{-0.558},
$$

Fig. 5. Results of the global fits to proton-neutron, sigma-proton, pion-proton, kaon-proton, kaon-neutron, gamma-proton and gamma-gamma total cross section data, with fixed *extrema bounds*: lower (dashed) and upper (solid), Eqs. (13) and (14), respectively. For visualization, the $K^\pm n$ total cross sections have been multiplied by a factor 0.5.
\[ \sigma_{K^\pm n}^{K^\pm n} = (11.28 \pm 0.09) s^{0.089} + (15.9 \pm 1.9) s^{-0.452} \mp (7.71 \pm 0.82) s^{-0.558}, \]
\[ \sigma_{\Sigma^*}^{p} = (19.07 \pm 0.45) s^{0.089} + (31.7 \pm 7.8) s^{-0.452}, \]
\[ \sigma_{p}^{p} = (0.064 \pm 0.001) s^{0.089} + (0.139 \pm 0.016) s^{-0.452}, \]
\[ \sigma_{\gamma}^{\gamma} = (0.00019 \pm 0.00001) s^{0.089} + (0.00031 \pm 0.00023) s^{-0.452}, \]

with \( \chi^2/F = 0.64 \). The corresponding curves are similar to those in Fig. 5 with narrower limit regions and will not be displayed here.

It is worth to note that, differently from the \( pp \) and \( \bar{p}p \) case, fixing the secondary Reggeon intercepts and reducing the Pomeron intercept produced a decrease on the \( \chi^2/F \). Also, differently from the \( pp \) and \( \bar{p}p \) case, in all scenarios for the hadrons the \( \chi^2/F \) remains lower than one, which seems to indicate that these data, at least for their current energy range, are not quite sensitive to the behavior of the Pomeron and Reggeon intercepts.

### 3.2 Factorization and quark counting

The Pomeron contribution to the total cross section is characterized by two parameters, the intercept \( 1 + \epsilon \) and the coefficient \( X \) (also referred to as strength or coupling). In the previous sections we have focused the discussion only on the investigation of bounds for the intercepts (extrema and constrained). The analysis was based on the selection of the fastest and slowest increase scenarios for the total cross sections, allowed by the experimental data presently available, and on fit procedures. Since in the fitting process both parameters, \( \epsilon \) and \( X \), are statistically correlated (and also correlated with the secondary Reggeon parameters), we expect that different bounds on \( \epsilon \) may, in some way, imply in different limit values for the coefficients (see, for example, Tables 1 and 2).

In order to investigate the possible consequences and range of this effect we consider here tests on two important properties related to the strength of the soft Pomeron: factorization and the additive-quark rule. To that end, we first quickly review the conditions under which the above properties are expected to hold [2,3] and then present the numerical tests followed by a discussion on the obtained results.

According to the quark-counting rule the strength of the soft Pomeron to hadrons \( A \) and \( B \) is proportional to the number of valence quarks inside each hadron, \( N_A, N_B \): \( X \propto N_A N_B \). Therefore, from Eq. (4), if only a single Pomeron exchange dominates, we expect that

\[ \frac{\sigma_{\text{baryon}} - \text{baryon}}{\sigma_{\text{baryon}} - \text{baryon}} \approx 1, \quad \frac{\sigma_{\text{meson}} - \text{baryon}}{\sigma_{\text{baryon}} - \text{baryon}} \approx \frac{2}{3}. \]  

(19)
meaning that the soft Pomeron couples to single quarks in a hadron, instead of to the whole hadron.

In an elastic process mediated by the exchange of a single Pomeron, the factorization of the coupling, as the product of the couplings at each vertex, corresponds to the factorization of the associated residues functions, \( \gamma_k(t) \) in Eq. (2). This means that if we consider only the Pomeron trajectory (or one Reggeon trajectory) the total cross sections for different processes can be correlated since the same Pomeron coupling can occur in different reactions. From Eqs. (1-3) and once one exchanged trajectory is assumed, the total cross sections for elastic processes like \( p + p \rightarrow p + p \), \( \gamma + \gamma \rightarrow \gamma + \gamma \), and \( \gamma + p \rightarrow \gamma + p \) should be related by

\[
\sigma_{pp} \sigma_{\gamma\gamma} \approx \left[ \sigma_{\gamma p} \right]^2. \tag{20}
\]

Based on the above considerations, in order to test quark counting and factorization we must consider only one contribution and therefore the leading one (Pomeron). Physically that means to investigate the region of large enough energy where the contributions from the secondary reggeons can be neglected as compared with the Pomeron contribution. In other words, we must assume that our parametrizations may be valid even beyond the energy region with available data, so that the total cross section in Eq. (4) can be expressed by

\[
\sigma_{\text{tot}}(s) \approx X s^\epsilon.
\]

With this formula and the values of the Pomeron coefficients from Tables 1 and 2 and Eqs. (15-18), we have calculated several ratios involving total cross sections with the corresponding propagated errors. The results are displayed in Table 3 in a form so that both factorization and quark counting are fully verified for the ratios equal to 1. We have the following comments on these results.

Roughly, the best agreements with (19) and (20) have been obtained with the extrema bounds. As expected, the corresponding results with the constrained bounds lie inside the error bars with the extrema bounds.

Concerning the quark counting rule (19), for the meson-baryon cross section we have that the ratio of the pion-proton to proton-proton cross section, obtained from the extrema bounds, agrees better with the expected value of \( 2/3 \) for the lower \( \epsilon \) value, although both results matched the expected ratio when the errors from the fit parameters are taken into account. For the kaon-proton reaction, the ratio to the proton-proton cross section does not agree with the \( 2/3 \) value for both the extrema and constrained cases, falling some 15% lower.
than that, on average. The same happens with the kaon-neutron to neutron-proton ratio, which is about 14% lower, on average. However, it is advocated that the Pomeron coupling with strange quarks have a weaker strength as to light quarks [2], so that such deviations should be expected. We notice that for all the bounds the central values of the ratio $X_{K\pm p}/X_{\pi\pm p}$ are equal or greater than 0.88, which is a bit above the value 0.87 obtained by Donnachie and Landshoff with degenerate Reggeon trajectories [5]. We also notice that the kaon-neutron to kaon-proton ratio show a very stable pattern, been equal to unity, or very close to that, for all bounds considered. As for the baryon-baryon cross section, we have that the agreement between the ratio of neutron-proton to proton-proton and the expected unit value from the quark counting rule is quite noticeable, for all $\epsilon$. The other baryon-baryon reaction available, sigma-proton, provides a ratio to proton-proton that is systematically lower than one, in agreement with the hypothesis of a weaker coupling of the Pomeron with strange quarks mentioned above.

Concerning the factorization rule (20), both constrained and extrema upper bounds provided ratios closer to 1 than the corresponding lower bounds. Despite the fact that, taking into account the error bars, the only result consistent with the value 1 had been obtained with the extreme upper bound, roughly, we can say that the factorization rule is barely verified in all the cases.

We conclude that, in the context of the procedure used, the effects of the extrema bounds are not in disagreement with what is generally expected from both quark counting and factorization rules.

4 Conclusions and final remarks

Disagreements between different experimental estimations of the total cross sections at the highest energies ($pp$ and $\bar{p}p$ scattering), lead to uncertainties in the determination of the soft Pomeron intercept. By means of an extended Regge parametrization (non-degenerated meson trajectories) the effect of these discrepancies has been translated into the estimation of extrema bounds for the intercept [12]. In this work, we have investigated the consequences of these bounds in the study of the total cross sections from $p^\pm n$, $\pi^\pm p$, $K^\pm p$, $K^\pm n$, $\Sigma^- p$, $\gamma p$ and $\gamma\gamma$ scattering and have presented tests on factorization and quark counting rules. The effects of secondary Reggeon constraints from Chew-Frautschi plots (spectroscopy data) have also been treated.

We have shown that both extrema bounds, 1.109 and 1.081 lead to good descriptions of the bulk of the experimental data presently available on total cross sections for all the reactions investigated. Tests on factorization and quark counting indicate that the results with all the bounds are not in dis-
Table 3
Results for the quark counting rule and factorization using the Pomeron strengths obtained with both extrema and constrained upper and lower bounds.

<table>
<thead>
<tr>
<th>Bounds:</th>
<th>extrema lower</th>
<th>extrema upper</th>
<th>constrained lower</th>
<th>constrained upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{\mathcal{P}}(0)$</td>
<td>1.081</td>
<td>1.109</td>
<td>1.080</td>
<td>1.089</td>
</tr>
<tr>
<td>$\frac{\sigma_{np}}{\sigma_{pp}}$</td>
<td>1.03±0.04</td>
<td>0.94±0.07</td>
<td>1.02±0.02</td>
<td>0.99±0.02</td>
</tr>
<tr>
<td>$\frac{\sigma_{\Sigma^-p}}{\sigma_{pp}}$</td>
<td>0.98±0.05</td>
<td>0.94±0.08</td>
<td>0.96±0.03</td>
<td>0.93±0.03</td>
</tr>
<tr>
<td>$\frac{3}{2} \frac{\sigma_{++}}{\sigma_{pp}}$</td>
<td>0.98±0.04</td>
<td>0.93±0.07</td>
<td>0.96±0.01</td>
<td>0.94±0.01</td>
</tr>
<tr>
<td>$\frac{3}{2} \frac{\sigma_{K^+p}}{\sigma_{pp}}$</td>
<td>0.87±0.04</td>
<td>0.85±0.06</td>
<td>0.85±0.01</td>
<td>0.83±0.01</td>
</tr>
<tr>
<td>$\frac{3}{2} \frac{\sigma_{K^+n}}{\sigma_{np}}$</td>
<td>0.85±0.04</td>
<td>0.90±0.07</td>
<td>0.84±0.01</td>
<td>0.84±0.01</td>
</tr>
<tr>
<td>$\frac{\sigma_{K^+}}{\sigma_{\gamma\gamma}}$</td>
<td>1.00±0.04</td>
<td>1.00±0.08</td>
<td>0.99±0.02</td>
<td>0.99±0.02</td>
</tr>
<tr>
<td>$\frac{\sigma_{K^+}}{\sigma_{\pi^+}}$</td>
<td>0.89±0.006</td>
<td>0.92±0.008</td>
<td>0.88±0.005</td>
<td>0.88±0.005</td>
</tr>
<tr>
<td>$\frac{\sigma_{pp}}{\sigma_{\gamma\gamma}} \left(\frac{\sigma_{np}}{\sigma_{pp}}\right)^2$</td>
<td>0.91±0.06</td>
<td>0.96±0.09</td>
<td>0.91±0.04</td>
<td>0.93±0.04</td>
</tr>
</tbody>
</table>

agreement with what is generally expected from standard fits to scattering data. This means that, if we consider the average of the extrema bounds, $\alpha_{\mathcal{P}}(0) = 1.095\pm 0.020$, the experimental data investigated are not sensitive to typical uncertainties of 2% in the value of the intercept. We understand that this conclusion brings novel information on the numerical interval that could be associated with a truthful value for the soft Pomeron intercept. However, extrapolation beyond the regions with available data indicate different scenarios for the rise of the total cross sections. Certainly, future data might select the best bound.

The constrained bounds imposed by the fitted secondary trajectories (Chew-Frautschi plots) reduce the previous intercept bounds, allowing the estimation $\alpha_{\mathcal{P}}(0) = 1.085\pm 0.006$ (average). From Fig. 4 the constrained bounds show qualitative agreement with a slower increase scenario for the total cross section, as represented by Ensemble II + BHS (the E710 and E811 results for $\bar{p}p$ and the results by Block, Halzen and Stanev for $pp$ at cosmic-ray energies). However, it should be noted that the best statistical results were obtained in the case of the extrema bounds, as indicated in Tables 1 and 2.

We note that, as in other approaches, the highest data points from $\pi^- p$, and $\gamma\gamma$ scattering are not described (Fig. 5). Although the extrapolations predict different behaviors for $\sigma_{tot}(s)$, neither bound are able to describe the above points in a reasonable way, specially in the $\gamma\gamma$ scattering. This last case may suggest the necessity of an additional component, as the hard Pomeron [4].
In this paper we have considered a specific extended parametrization characterized by degenerate secondary trajectories $a_2/f_2$ and $\rho/\omega$, Eq. (4). Another aspects that may affect the value of the bounds and even the minimal energy for stable fits are the possibility of nondegenerate $\rho$ and $\omega$ trajectories and also a non-linear $f_2$ trajectory. We are presently investigating these aspects with both scattering and spectroscopy data.

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References
