Neutrino Magnetic Moment Contribution to the Neutrino-Deuteron Reaction

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Abstract

We study the effect of the neutrino magnetic moment on the neutrino-deuteron breakup reaction, using a method called the standard nuclear physics approach, which has already been well tested for several electroweak processes involving the deuteron.

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The Sudbury Neutrino Observatory (SNO) has given precision data on the flux of the $^8$B solar neutrinos and on its flavor content, and these data have provided solid evidence for neutrino oscillations \cite{1, 2}. The SNO experiments measure the yield from the charged current (CC) reaction, $\nu_e + d \rightarrow e^- + p + p$, which occurs only for the electron neutrino, and the yield from the neutral current (NC) reaction, $\nu_x + d \rightarrow \nu_x + p + n$ \((x = e, \mu, \tau)\), which takes place with the same cross section for any neutrino flavor \(x\). In interpreting the SNO data the theoretical estimates of the $\nu-d$ cross sections play an important role. Detailed studies of the $\nu-d$ reaction in the solar neutrino energy region have been carried out using the standard nuclear physics approach (SNPA) \cite{3, 4}, pion-less effective field theory based on the power divergence subtraction scheme \cite{5, 6}, and EFT* (or MEEFT) \cite{7}, which is a “pragmatic” version of chiral perturbation theory in the Weinberg counting scheme \cite{8}. Apart from the radiative corrections, the uncertainties in the calculated values of the $\nu-d$ cross sections are estimated to be 1\%, see Ref.\cite{4}.

The above-mentioned calculations use the weak interaction Hamiltonian dictated by the standard model, in which the neutrinos interact with the deuteron only by exchanging the \(W\) and \(Z\) bosons. Meanwhile, if the neutrino has a magnetic moment (let it be denoted by $\mu_\nu$), it can interact with the deuteron via a photon exchange as well. This additional interaction, contributing to the $\nu_x d \rightarrow \nu_x p n$ reaction, can affect the interpretation of the NC data of SNO at some level of precision. In the following, the electromagnetic neutrino-deuteron interaction due to $\mu_\nu$ shall be simply referred to as the $\mu_\nu$-interaction. Since the contribution of the $\mu_\nu$-interaction to the $\nu_x d \rightarrow \nu_x p n$ reaction and the NC contribution do not interfere with each other (see below), we can decompose the cross section as

$$\sigma(\nu_d d \rightarrow \nu_d p n) = \sigma_{EM} + \sigma_{NC},$$

where $\sigma_{EM}$ stands for the contribution of the $\mu_\nu$-interaction. Akhmedov and Berezin (AB) \cite{9} estimated $\sigma_{EM}$, using the effective range approximation and the impulse approximation. Recently, Grifols, Masso and Mohanty (GMM) \cite{10} gave a new estimate of the $\mu_\nu$-interaction effect based on the equivalent photon approximation. GMM reported a significant difference between their value of $\sigma_{EM}$ and the value obtained by AB \cite{9}, but the origin of this discrepancy was left undiscussed.

We carry out here a calculation of $\sigma_{EM}$ which is free from the effective-range approximation, the impulse approximation and the equivalent photon approximation. To this end, we make an appropriate extension of SNPA used in Refs.\cite{3, 4} and calculate $\sigma_{EM}$, including both the impulse and exchange-current terms, and with the use of the two-nucleon wave functions generated from a high-precision realistic $NN$ potential. We shall also briefly discuss some consequences of our calculation in the context of placing upper limits to the neutrino magnetic moments.

We are concerned here with the reaction

$$\nu(k)_a + d(P) \rightarrow \nu(k')_b + p(p'_1) + n(p'_2)$$

at low energies, where \(a\) and \(b\) stand for neutrino flavors. The relevant transition amplitude $t_{fi}$ consists of two terms, one arising from the standard NC interaction and the other from the $\mu_\nu$-interaction. Thus

$$t_{fi} = -\frac{G_F}{\sqrt{2}} j_{\mu}^{NC} j_{\mu}^{NC,\mu} + \frac{1}{q^2} j_{\mu}^{EM} j_{\mu}^{EM,\mu},$$

where

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where
where \( q = k - k' \). The lepton neutral current matrix element, \( l^{NC,\mu} \), is diagonal in the weak-flavor eigen-states and is given as

\[
l^{\mu} = \bar{u}_b(k')\gamma^\mu(1 - \gamma_5)u_a(k)\delta_{ba},
\]

where the neutrino electromagnetic current is given in terms of the magnetic dipole moment \( \mu_{ba} \) and the electric dipole moment \( \epsilon_{ba} \),

\[
l^{EM,\mu} = \bar{u}_b(k')i\sigma^{\mu\nu}q_{\nu}(\mu_{ba} - \epsilon_{ba}\gamma_5)u_a(k),
\]

where \( b \) and \( a \) stand for the mass eigen-states of the neutrinos. In Eq.\( (3) \), \( j^{NC}_\mu \) and \( j^{EM}_\mu \) represent the nuclear matrix matrix elements:

\[
j^{X}_\mu = \langle p(p'_1)n(p'_2)|J^{X}_\mu|d(P)\rangle \quad (X = NC, EM),
\]

where \( J^{NC}_\mu \) is the weak NC current, and \( J^{EM}_\mu \) is the electromagnetic current. The explicit forms of \( J^{NC}_\mu \) and \( J^{EM}_\mu \) were described in detail in Refs.\( [3, 4] \). At the solar neutrino energies it is safe to apply non-relativistic approximation to the one-nucleon (impulse approximation) current. The two-nucleon exchange current is derived using the one-meson-exchange SNP A model\( [3, 4] \). We remark that this SNP A model describes very well the \( n + p \rightarrow d + \gamma \) reaction at low energies, and that, for \( E_n < 0.1 \text{ MeV} \), the magnetic dipole (M1) transition dominates, whereas for higher energies the electric dipole (E1) transition dominates; see Ref.\( [11] \) for details.

If the neutrino mass is neglected in Eqs.\( (4) \) and \( (5) \), then \( l^{NC,\mu} \) conserves the neutrino helicity while \( l^{EM,\mu} \) flips the helicity. This means that to a very good approximation there is no interference between the NC and EM contributions, and therefore the total cross section for the \( \nu_e d \rightarrow \nu_e pn \) reaction can be written as the incoherent sum of the NC and EM terms as in Eq.\( (1) \). An explicit expression for \( \sigma_{NC} \) and its elaborate numerical calculation can be found in Ref.\( [3] \); our goal here is to evaluate \( \sigma_{EM} \) to a comparable degree of accuracy. It is convenient to express \( \sigma_{EM} \) as an integral over the final two-nucleon relative kinetic energy, \( T_{NN} = p'^2/M \), where \( p' = (p'_1 - p'_2)/2 \),

\[
\sigma_{EM} = \int_0^{T_{max}} dT_{NN} \frac{d\sigma_{EM}}{dT_{NN}}.
\]

Here

\[
\frac{d\sigma_{EM}}{dT_{NN}} = (\mu_{\nu a})^2 \int_{-1}^{1} d\cos\theta \sum_{L, S, J, \ell'} \frac{4\alpha m_N \ell' k^3 k}{3(-q^2_a)} \left[ \frac{(k + k')^2(1 - \cos\theta)^2}{q^4} \sum_{J \geq 0} |<T^J>|^2\right.
\]

\[\left. + \frac{\sin^2\theta}{2q^2} \sum_{J \geq 1} \{|<T^J_M>|^2 + |<T^J_E>|^2\} \right],
\]

where \( q \) is the momentum transferred to the two-nucleon system; the neutrino scattering angle \( \theta \) is defined by \( \mathbf{k} \cdot \mathbf{k'} = kk'\cos\theta \) in the \( \nu-d \) center-of-mass system, \( \mu_{\nu a} \) is the effective neutrino magnetic moment for the incoming neutrino of flavor \( a \)\( [12, 13] \),

\[
(\mu_{\nu a})^2 = \sum_b |\mu_{ba} + \epsilon_{ba}|^2.
\]
In Eq. (8), \( < T_X^J > \) represents the nuclear reduced matrix element of a multipole operator \( T_X^J \) of rank \( J \):

\[
<T_X^J> = (L_f, S_f) J_f||T_X^J||d; J_f = 1 ,
\]

where \( X = C, M \) and \( E \) correspond to the Coulomb, magnetic and electric multipole operators, respectively. The explicit expressions for \( < T_X^J > \) can be found in Eqs. (57) and (38)-(40) in Ref. [3]. In calculating \( \sigma_{EM} \) at the solar neutrino energies, we need to pay particular attention to the sizeable final two-nucleon interactions. Here, as in Ref.[3], we use the final two-nucleon distorted S- and P-waves obtained with the use of the Argonne V18 potential \([14]\).

In order to illustrate the possible \( \mu_\nu \)-interaction effects on \( \sigma(\nu_\nu d \rightarrow \nu_\nu pn) \), we need to specify a “typical” value of \( \mu_\nu \). The radiative corrections within the standard model leads to an estimate \( \mu_\nu \sim 3 \times 10^{-10} m_\nu/1 \text{eV} [\mu_B] \), where \( \mu_B \) is the Bohr magneton \([12, 13]\); the value of \( \mu_\nu \) larger than this estimate would be an indication of new physics. The current upper limits obtained from reactor neutrino experiments are \( \mu_\nu < 1.0 \times 10^{-10} \mu_B \) \([15]\) and \( \mu_\nu < 1.3 \times 10^{-10} \mu_B \) \([16]\). The limit found using the \( \nu_\mu \) and \( \bar{\nu}_\mu \) from \( \pi^+ \) and \( \mu^+ \) decays is \( \mu_\mu < 6.8 \times 10^{-10} \mu_B \) \([17]\). Meanwhile, the limit for the tau neutrino is reported to be \( \mu_\tau < 3.9 \times 10^{-7} \mu_B \) \([18, 19]\). As for the astrophysical constraints, it is known that the avoidance of excessive stellar cooling requires \( \mu_\nu < (0.3 - 1) \times 10^{-11} \mu_B \) (see, e.g., Ref.[13]). For the sake of definiteness, we use here \( \mu_\nu = 10^{-10} \mu_B \) as a representative value. This completes the specification of all the quantities appearing in Eqs.[7], [8].

Our numerical results for \( \sigma_{EM} \) are shown in Table I and Fig. 1. In Fig. 1, the dotted curve represents the contribution of the Coulomb and E1 transitions leading to the \( ^3P_1 \) final two-nucleon state, while the dash-dotted curve gives the contribution of the M1 transition leading to the \( ^1S_0 \) state. The solid curve shows the value of \( \sigma_{EM} \) obtained by adding the contributions of all the multipoles in Eq. (8). We note that, except for the lowest energy region \( (E_\nu < 2.8 \text{ MeV}) \), \( \sigma_{EM} \) is dominated by the Coulomb and E1 transitions to the final P-wave states. This should be contrasted with the case of the NC reaction, where the Gamow-Teller transition to the final \( ^1S_0 \) state gives a dominant amplitude.

Fig. 2 shows the ratio \( R \equiv \sigma_{EM}/\sigma_{NC} \) calculated from \( \sigma_{EM} \) obtained in the present work and \( \sigma_{NC} \) given in Ref. [4]. \( R \) increases with the neutrino energy \( E_\nu \) and reaches \( \sim 5 \times 10^{-6} \) at \( E_\nu = 10 \text{ MeV} \), which is however still very small. The smallness of \( R \) reflects the fact that in the \( \nu-d \) disintegration reaction, the NC contribution comes from an allowed transition, while the EM contribution arises from a ‘first-forbidden’ transition. The latter is intrinsically suppressed by a small factor \( q < r > \), where \( < r > \) is a typical nuclear size. We remark that the contribution of the \( \mu_\nu \)-interaction is much larger for elastic \( \nu-d \) scattering than for the \( \nu-d \) breakup reaction, since the elastic scattering can occur via an allowed-type transition; it is unfortunate that \( \nu-d \) elastic scattering is at present practically impossible to detect.

To facilitate comparison of our present results with those in the literature, we let \( \sigma_{EM}(\text{TNSKM}) \), \( \sigma_{EM}(\text{AB}) \), \( \sigma_{EM}(\text{GMM}) \) denote the values of \( \sigma_{EM} \) obtained by us, by AB \([9]\) and by GMM \([10]\), respectively. To compare \( \sigma_{EM}(\text{TNSKM}) \) and \( \sigma_{EM}(\text{AB}) \), we read off the numerical value of \( \sigma_{EM}(\text{AB}) \) from Fig. 1 in Ref.[3] and normalize it to the case of \( \mu_\nu = 10^{-10} \mu_B \), the value used in our present calculation. Comparison after this normalization indicates that \( \sigma_{EM}(\text{TNSKM}) \) is about 4 times larger than \( \sigma_{EM}(\text{AB}) \). As mentioned, our present calculation based on SNP A is free from the effective-range approximation and the impulse approximation, whereas the calculation in Ref.[3] does involve these two ap-
proximations. However, this difference in formalisms per se is not expected to lead to a discrepancy as large as a factor of \( \sim 4 \). For further comparison, we find it illuminating to simplify our full calculation by introducing the effective-range approximation and impulse approximation; the resulting simplified formula for the E1 contribution is found to be larger than the E1 part of Eq.(18) in Ref. [3] by a factor of 2. It is likely that the remaining discrepancy by a factor of \( \sim 2 \) is of the numerical origin. We have done a little numerical exercise of calculating “our version” of \( \sigma_{EM}(AB) \) directly from Eqs.(18) and (A1) of AB; let \( \sigma_{EM}(AB;\text{direct}) \) stand for the result of this exercise. If we use \( \sigma_{EM}(AB;\text{direct}) \) instead of \( \sigma_{EM}(AB) \) read off from the figure in AB, then the difference between our result and that of AB becomes a factor of \( \sim 2 \) (instead of \( \sim 4 \)), which is consistent with the above-mentioned difference in the analytic expressions.

As far as comparison with GMM [10] is concerned, we first discuss \( <\sigma_{EM}> \) defined as

\[
<\sigma_{\alpha}> = \int dE_{\nu} f(E_{\nu})\sigma_{\alpha}(E_{\nu}),
\]

(11)

where \( \alpha = EM, NC \) and \( f(E_{\nu}) \) is the normalized \( ^{8}\text{B} \) neutrino spectrum. If we use the value given in GMM, we find \( <\sigma_{EM}(\text{TNSKM})> \approx <\sigma_{EM}(\text{GMM})> \), but that the former is smaller than the latter by about 10%. To gain some insight into this difference, we have examined the analytic expression for the cross section in GMM and have confirmed that Eq.(16) in GMM is indeed valid in the long wavelength approximation and in a regime where the dominance of the electric and Coulomb dipole transitions holds. Furthermore, if we calculate \( \sigma_{EM} \) directly from Eqs.(16) and (17) of GMM, the result agrees with \( \sigma_{EM}(\text{TNSKM}) \) within 1%. Therefore, the above-mentioned \( \sim 10 \% \) discrepancy does not seem to be due to the equivalent-photon approximation used by GMM. We have not been able to identify the origin of the \( \sim 10 \% \) difference found in the comparison of \( <\sigma_{EM}> \) [22].

In these circumstances it is probably warranted to repeat that, if one applies the same method (SNPA) as used here to the calculation of the \( np \rightarrow \gamma \text{d} \) reaction cross section [11], the results agree very well with the experimental data [21] for both the M1 and E1 contributions. Furthermore, the calculation of the cross sections, \( \sigma(\nu_{e}\text{d} \rightarrow e^{-}\text{pp}) \) and \( \sigma(\nu_{\mu}\text{d} \rightarrow \nu_{\mu}\text{pn}) \), with the use of SNPA has been done by several groups (with different degrees of sophistication), and the results are known to be consistent among themselves at the level of precision relevant to the discussion of \( \sigma_{EM} \) here [23]. Thus it is our belief that the SNPA method used in the present work has been well tested in both formal and numerical aspects.

We now briefly discuss consequences of our calculation in the context of obtaining information about the neutrino magnetic moments. In considering the SNO data, we find it useful to follow the argument of GMM [10]. The total “NC” deuteron dissociation events observed at SNO is the sum of the standard NC events and those due to the \( \mu_{\nu} \)-interaction:

\[
N^\text{exp} = N_{d} T \Phi_{\text{SUN}}[<\sigma_{NC}> + <\sigma_{EM}>],
\]

(12)

where \( N_{d} \) is the number of target deuterons and \( T \) is the exposure time; \( \Phi_{\text{SUN}} \) is the total \( ^{8}\text{B} \) solar neutrino flux. The empirical \( ^{8}\text{B} \) neutrino flux, \( \Phi_{\text{SNO}} \), obtained at SNO assumes that the total dissociation events arise solely from the standard NC interaction. Thus \( \Phi_{\text{SNO}} = N^\text{exp} / [N_{d} T <\sigma_{NC}>] \), which implies \( \Phi_{\text{SNO}} = \Phi_{\text{SUN}}(1 + \delta) \) with \( \delta \equiv <\sigma_{EM}> / <\sigma_{NC}> \). To extract information on \( \mu_{\nu} \), GMM employed \( \sigma_{EM} = \sigma_{EM}(\text{GMM}) \) and the value of \( \sigma_{NC} \) obtained by Nakamura et al. [4], and furthermore they assumed \( \Phi_{\text{SUN}} = \Phi_{\text{SSM}} \), where \( \Phi_{\text{SSM}} \) is the \( ^{8}\text{B} \) solar neutrino flux predicted by the standard solar model [24]. GMM then arrived
at \( \Phi_{SNO} = \Phi_{SSM}(1 + 6.06 \times 10^{14} \mu^2) \), where \( \tilde{\mu} \) is the operational neutrino magnetic moment defined by \( \tilde{\mu}^2 = \mu_{21}^2 + \mu_{22}^2 + \mu_{23}^2 \). The use of \( \Phi_{SNO} = (4.90 \pm 0.37) \times 10^6 \text{cm}^2\text{s}^{-1} \), and \( \Phi_{SSM} = (5.87 \pm 0.44) \times 10^6 \text{cm}^2\text{s}^{-1} \) leads to \( \tilde{\mu}^2 = (-2.76 \pm 1.46) \times 10^{-16} \), where the errors in \( \Phi_{SNO} \) and \( \Phi_{SSM} \) have been added quadratically. Inflating the error up to 1.96\( \sigma \), GMM deduced the upper limit \( |\tilde{\mu}| < 3.71 \times 10^{-9} \mu_B \) (95\% CL). If we repeat a similar analysis using \( \sigma_{EM}(\text{TNSKM}) \), we arrive at \( \Phi_{SNO} = \Phi_{SSM}(1 + 5.46 \times 10^{14} \tilde{\mu}^2) \). We then should obtain practically the same conclusion on \( |\tilde{\mu}| \) as in GMM. This implies that, if the upper limit \( \sim 10^{-10} \mu_B \) to \( \mu_\nu \) is adopted, the interpretation of the existing SNO data is not likely to be influenced by the possible existence of \( \mu_\nu \).

Recently, an extensive global analysis of the totality of the data from the Super-Kamiokande, SNO, and KamLAND has been carried out, with the possible existence of \( \mu_\nu \) taken into account [25]. This analysis reports an upper limit \( \mu_{\text{eff}} < 1.1 \times 10^{-10} \mu_B \) (90\% CL). The value of \( \sigma_{EM} \) described in our present communication has been used as input in the global analysis in Ref. [25].

It is known that a non-zero value of \( \mu_\nu \) can affect the shape of the recoil electron spectrum in electron-neutrino scattering. A similar effect is expected in the \( \nu -d \) elastic scattering (which is however at present practically impossible to measure). For the \( \nu -d \) disintegration reaction, the effect of the \( \mu_\nu \) interaction is much smaller than for the elastic scattering, but it might be of some (academic) interest to discuss to what extent one can distinguish the NC and \( \mu_\nu \)-interaction contributions to the neutron energy spectrum in the \( \nu -d \) breakup reaction. As an example, we consider the \( T_{NN} \) dependences of \( d\sigma_{EM}/dT_{NN} \) and \( d\sigma_{NC}/dT_{NN} \), where \( T_{NN} \) is the final two-nucleon relative kinetic energy. Fig.3 gives these differential cross sections calculated for \( E_\nu = 10 \text{ MeV} \), and for \( \mu_{\text{eff}} = 10^{-7}\mu_B, 3 \times 10^{-8}\mu_B, \) and \( 10^{-8}\mu_B \). These values of \( \mu_{\text{eff}} \) are much larger than the upper limits known for \( \nu_e \) and \( \nu_\mu \), but within the upper limit pertaining to \( \nu_\tau \). Fig.3 shows the NC contribution is shown by the solid line, while the \( \mu_\nu \) contribution is given by the dotted, dashed and dot-dashed curves corresponding to \( \mu_{\text{eff}} = 10^{-7}\mu_B, 3 \times 10^{-8}\mu_B \), and \( 10^{-8}\mu_B \), respectively. The NC contribution has a sharp peak in the low \( T_{NN} \) energy region, owing to the final \( ^1S_0 \) state dominance for the Gamow-Teller transition. By contrast, the \( \mu_\nu \) contribution exhibits a maximum away from threshold, reflecting the fact that the \( \mu_\nu \)-interaction leads to the final P-wave states. As illustrated, \( \mu_{\text{eff}} \) of the order of \( 10^{-7}\mu_B \) can affect the shape of the energy spectrum of the recoil nucleon in the \( \nu -d \) breakup reaction.

To summarize, we have studied the effect of the neutrino magnetic moment on the neutrino-deuteron breakup reaction, using a method called the standard nuclear physics approach (SNPA), which has been well tested for a number of electroweak processes involving the deuteron. The present calculation is free from the various approximations (mentioned earlier in the text) that were used in the previous estimations.

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[19] When neutrino oscillations are taken into account, a limit deduced from scattering experiments should be understood as a limit on $\mu_f = \sum_j |\sum_k U_{f k} \mu_k e^{-iE_k L}|^2$, where $f$ is the flavor of the beam neutrino, $k$ and $j$ denote mass eigenstates, and $U$ is the mixing matrix [20].
[22] In GMM’s article [10], there is the statement that $\sigma_{EM}(GMM)$ is smaller than $\sigma_{EM}(AB)$ by a factor of $\sim 2$. If this is indeed the case, our value of $\sigma_{EM}$, which is five times larger than that of Ref.[1] should be larger than $\sigma_{EM}(GMM)$ by a factor of 8. However, as mentioned in the text, we find $\sigma_{EM}(TNSKM) \sim \sigma_{EM}(GMM)$. It seems that the above-mentioned statement of the factor of 2 difference between $\sigma_{EM}(GMM)$ and $\sigma_{EM}(AB)$ needs to be re-examined.
FIG. 1: The cross section, $\sigma_{EM}$, for the $\nu_x d \rightarrow \nu_x pn$ reaction due to the $\mu_\nu$-interaction. $\sigma_{EM}$ has been calculated for $\mu_{eff} = 10^{-10} \mu_B$, and is given in units of cm$^2$.

FIG. 2: The ratio $R \equiv \sigma_{EM}/\sigma_{NC}$ calculated for $\mu_{eff} = 10^{-10} \mu_B$ as a function of the incoming neutrino energy.
FIG. 3: The neutrino deuteron disintegration cross sections, $\sigma_{NC}$ and $\sigma_{EM}$, as functions of $T_{NN}$. See the text for details.

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