A finite temperature investigation of the Georgi-Glashow model in 3D

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We study the SU(2) gauge theory with scalar matter in the adjoint representation in 3D at finite temperature. We find evidence for a finite temperature phase transition both in the symmetric and in the broken phase; such transitions are consistent with the universality class of Ising 2D, in agreement with recent analytical arguments.

1. INTRODUCTION

The Georgi-Glashow model in 3D is a useful toy model to investigate mechanisms of confinement related to topological degrees of freedom. The model is an SU(2) Yang-Mills theory with a scalar field in the adjoint representation of the gauge group. Its Euclidean action in the continuum is written as

\[ S = \int d^3 x \left( \frac{1}{2g^2} \text{Tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{1}{2}g^2 (D_\mu \phi D_\mu \phi) + m_0^2 \text{Tr}(\phi^2) + \frac{\mu^2}{2} (\text{Tr}(\phi^2))^2 + m_0^2 \right), \]

where \( D_\mu \phi = \partial_\mu \phi + i[A_\mu, \phi] \) with \( \phi = \phi^a \sigma^a/2 \).

The model has 3 dimensionful bare parameters, \( g^2 = \text{[mass]}, m_0 = \text{[mass]}, \mu = \text{[mass]} \), which can be rewritten in terms of 1 dimensionful parameter, \( y = m_0^2/g^4 \), and 2 dimensionless, \( x = \mu^2/g^2 \). When the adjoint scalar field acquires a non-zero vacuum expectation value (vev), the Higgs mechanism takes place and SU(2) is dynamically broken to U(1); as a consequence two gauge bosons, \( W^{\pm} \), acquire a mass. At tree level, one finds \( M_{W^\pm}^2/g^4 = -x/y \) and \( M_H^2 = 0 \), while the Higgs boson has a mass \( M_H^2/g^2 = 2y \) and a vev \( \phi^a \phi^a = -y g^2/x \).

Magnetic monopoles solutions \cite{1,2} are instantons in the 3D case. Monopoles influence both the charged and neutral sector of the theory: they provide at the same time a mechanism of confinement of \( W^{\pm} \) bosons and a mechanism of mass generation for the photon field. In the semiclassical approximation one finds \cite{3}:

\[ M_{\gamma}^2 \sim \left( \frac{-x}{y} \right)^7 \exp \left( -4\pi \sqrt{-x/y} \right), \quad \sigma = \frac{g^2 M_\gamma}{2\pi^2}, \]

respectively for the photon mass \( M_\gamma \) and the string tension \( \sigma \). In our investigation we will use lattice simulations in order to understand whether the system undergoes a finite temperature deconfinement phase transition and if the magnetic monopoles are the only relevant degrees of freedom necessary to describe correctly the critical behaviour of the system near the critical temperature \( T_c \).

2. THE LATTICE MODEL

The discretized action of the Georgi-Glashow model can be written as

\[ S = \beta \sum_{x,\mu,\nu,\sigma} \left( 1 - \frac{1}{2} \text{Tr} U_{\mu\nu}(x) \right) + \sum_{x} \text{Tr}(\Phi(x)\Phi(x)) \]

\[ - 2\kappa \sum_{x,\mu} \text{Tr}(\Phi(x)U_{\mu}(x)\Phi(x + \hat{\mu}a)U_{\mu}^\dagger(x)) \]

\[ + \lambda \sum_{x} (2\text{Tr}(\Phi(x)\Phi(x)) - 1)^2, \]

where \( U_{\mu\nu}(x) \) is the plaquette and the scalar field \( \Phi(x) \) is in the adjoint representation of SU(2). In our simulations we used a standard Kennedy-Pendleton heatbath algorithm for the pure gauge
part of the action, modified with a Metropolis step to take into account the hopping term, which is quadratic in the link. For the scalar update instead we adopted the Bunk algorithm [4]. We used lattices with $N_\tau = 4, 6$ and $N_s = 24, 32, 40, 44, 48, 54, 64$.

The model in 3D is superrenormalizable and only a finite number of counterterms are needed to renormalize the action. The matching of the lattice theory to the continuum one has been computed in Ref. [5]. Such equations specify how to approach the continuum limit along lines of costant physics.

3. THE PHASE DIAGRAM

Lattice studies at $T = 0$ indicate that the symmetric and Higgs phase are analytically connected [6,7], i.e. they are separated by a first order phase transition or crossover depending on the value of $\lambda$. What should one expect at finite $T$? The scalar dynamical field is in the adjoint representation, so it cannot screen static charges in the fundamental representation. Therefore one should expect that these charges are really confined at zero and low temperatures and that they undergo a phase transition at a certain critical temperature. Moreover the action is $Z_2$-invariant and the Polyakov loop can be used to look for the signature of a finite temperature phase transition. In the symmetric phase we studied the Polyakov loop in the fundamental representation $L_F$ and its susceptibility

$$\chi_{L_F} = N_s^2 \left( \langle L_F^2 \rangle - \langle L_F \rangle^2 \right)$$

across the phase transition, at fixed $N_\tau$ and different spatial lattice sizes $N_s$. The results are reported in Fig. 1 (up). The ratio $\chi_{L_F}/N_s^{\gamma/\nu}$ are expected to be a universal function of the scaling variable, i.e.

$$\chi_{L_F}/N_s^{\gamma/\nu} = f((\beta - \beta_c) N_s^{1/\nu})$$

indeed the susceptibilities for different spatial volumes collapse on the same curve (Fig. 1 (down)) when rescaled with the critical indices of the 2D Ising model ($\nu = 1, \gamma = 1.75$), suggesting that the Georgi-Glashow model in 3D in the symmetric phase is in the same universality class of Ising 2D.

Even if the symmetric and the Higgs phase are analytically connected, the Higgs phase needs particular care both in the analytical and in the numerical investigation. Following Ref. [8], one can assume that, at low temperatures, monopoles are the only relevant degrees of freedom. At finite $T$ one dimension is compactified, with length $\beta = 1/T$, and monopoles at distances larger than $\beta$ feel a 2-dimensional interaction; hence the model is described by a 2-dimensional Coulomb potential. A 2-dimensional Coulomb gas undergoes a BKT phase transition; the conclusion in [8] is thus that the model is in the universality class of U(1) 2d.

The analysis presented in [9], reaches a different conclusion by taking into account the effect of the charged $W^\pm$ bosons. Dimensional reduction
is considered to be a valid approximation but the starting point is an effective Lagrangian written in terms of a field $V(x)$, which is the creation operator of a magnetic vortex with flux $2\pi/g$; this picture allows for a description of the charged sector of the model. The monopoles play an essential role in this scenario; indeed the effective theory without the monopole-induced term would be equivalent to an $XY$ model and thus it would be again in the $U(1)$ universality class. Including the monopole effects, the original $U(1)$ symmetry becomes anomalous and only the $\mathbb{Z}_2$ subgroup is conserved. The conclusion in Ref. [9] is that the model is in the Ising 2D universality class.

In order to check these two scenarios, we used a modified Polyakov loop [10] and its susceptibility

$$L^m = \frac{1}{N_s^2} \sum_{\vec{x}} \left( \text{Tr}_F \left( \Pi_{\tau=1}^{N_s} U_\tau (\vec{x}, t) \right) \right)^2, \quad (7)$$

$$\chi_{L^m} = N_s^2 \left( \langle L^m \rangle - \langle L^m \rangle \right)^2. \quad (8)$$

In the Higgs phase the gauge links are aligned along the direction of the scalar field in color space and the Polyakov loops with $\phi$-insertions give a better signal. We made simulations deep in the Higgs phase at couplings which correspond to the perturbative continuum ratios $M_H/g^2 \sim 1.5$ and $M_W/g^2 \sim 0.5$. As one can see from Fig. (2)(up), the susceptibility increases by increasing the spatial volume; by rescaling the results at different volumes with the Ising critical indices, one can see again a nice collapse on the same curve (Fig. (2)(down)), thus providing evidence that in the Higgs phase the model is also in the Ising 2D universality class.

4. CONCLUSIONS

In our lattice investigation we have found that the Georgi-Glashow model in 3D undergoes a finite temperature confinement/deconfinement phase transition characterized by the critical indices of the Ising 2D universality class, both in the symmetric and in the Higgs phase, suggesting that at $T \neq 0$ both magnetic monopoles and vortices must be taken into account in order to get a correct description of the theory.

REFERENCES