Effects of large field cutoffs in scalar and gauge models

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We discuss the notion of a large field cutoff for LGT with compact groups. We propose and compare gauge invariant and gauge dependent (in the Landau gauge) criteria to sort the configurations into “large-field” and “small-field” configurations. We show that the correlations between volume average of field size indicators and the behavior of the tail of the distribution are very different in the gauge and scalar cases. We show that the effect of discarding the large field configurations on the plaquette average is very different above, below and near $\beta = 5.6$ for a pure $SU(3)$ LGT.

A common challenge for quantum field theorists consists in finding accurate methods in regimes where existing expansions break down. In the RG language, this amounts to find acceptable interpolations for the RG flows in intermediate regions between fixed points. In a pure $SU(3)$ gauge theory near $\beta \approx 6$, the validity of weak and strong coupling expansions break down and the MC method seems to be the only reliable method. In the following, we discuss recent attempts to improve weak coupling expansions.

In the case of scalar field theory, the weak coupling expansion is unable to reproduce the exponential suppression of the large field configurations coming from the factor $\text{exp}(-\lambda \sum \phi^4)$ in the functional integral. This problem can be resolved [1,2] by introducing a large field cutoff $|\phi| < \phi_{\text{max}}$. One is then considering a slightly different problem, however a judicious choice of $\phi_{\text{max}}$ can be used to reduce or eliminate the discrepancy. This optimization procedure can be approximately performed using the strong coupling expansion and naturally bridges the gap between the weak and strong coupling expansions [3]. Can such a procedure be applied to gauge models?

The talk of K. Wilson about the early days of lattice gauge theory was a very inspirational moment of this conference. He stressed the importance of “butchering field theory” in the development of the RG ideas and recommended that we keep doing it. There exist many ways to cleave the large field configurations. For scalar fields, the configurations can be ranked, for instance, according to the largest absolute value of the field or according to the average over the sites of an even power of the field. The largest this power is, the more emphasis is put on the configurations with the largest field values. We expect correlations among these quantities. This is illustrated for a power 4 in the case of the harmonic oscillator in Fig. 1. The sample correlation is 0.82 (the

![Figure 1. Largest absolute value of the field versus average over all the sites of $\phi^4$, in $D = 1$ (harmonic oscillator), for 10,000 configurations (each point corresponds to a single configuration).](image-url)
maximal value being 1 for completely correlated quantities). In the right part of the figure, the set of configurations has been minced into 40 bins with different $\phi_{\text{max}}$. The central values can be fitted with a polynomial and the variance of the bins are relatively small (they would be smaller for higher correlations). Consequently, one would not expect too much change if one or the other method is used.

In order to understand how discarding the large field configurations changes the large order behavior of perturbative series, notice, for instance, that out of the 10,000 configurations of Fig. 1, only 56 have values of $|\phi|$ larger than 3. Neglecting these configurations affect the the order $\Lambda$ correction to the ground state energy ($\langle 0|\phi^4|0\rangle = 3/4$ without a field cut) by only 1 percent, however the same 56 account for about 90 percent of the sixth coefficient!

For gauge models, the closeness to the identity for a $SU(N)$ matrix $U$ can be measured in term of the quantity $(1/2N)Tr[(1-U)(1-U)^\dagger] = 1 - (1/N)ReTrU$. Due to the compactness of the group, this quantity is bounded. For instance, it is always smaller than 2 for $SU(2)$ and 3/2 for $SU(3)$. In these two cases, the “largest fields” correspond to the nontrivial elements of the center. For $U_L$ associated with a link $L$ in the $\mu$ direction, $1 - (1/N)ReTrU_L \propto A^\mu_\mu A^\mu_\mu$ near the identity and provides an indicator of the size of the field which is gauge dependent. However, in the Landau gauge, the average of this quantity is minimized making it a prime candidate as an indicator of field size. On the other hand, for the product of links along a plaquette $p$, $1 - (1/N)ReTrU_p$ provides a gauge invariant indicator which is proportional to the field strength near the identity. We would like to know how these indicators are correlated.

If no gauge fixing is imposed, the distribution of $1 - (1/N)ReTrU_L$ depends only on the Haar measure and not on $\beta$. This is illustrated in Fig. 2 for $SU(2)$. However, if the Landau gauge is used, the distribution will peak much closer to zero as shown in Fig. 3 for $SU(3)$. The average of this quantity for a large number of independent configurations is 0.139 . Note also that in the Landau gauge, there is a clear gap between the maximal value taken by $1 - (1/N)ReTrU_L$ (near $1.1 < 1.5$ in Fig. 3) and the largest possible value.

\[ P(A) = \int_0^{2\pi} d\omega \sin^2(\frac{\omega}{2})\delta(1 - \cos(\frac{\omega}{2}) - A) \]

\[ \text{Max} \{ 1 - (1/N)ReTrU_L \} \text{ could thus be considered as the analog of } \phi_{\text{max}}^2 \text{ in the scalar case. Is this quantity correlated with field size indicators based on volume average as in the scalar case? Apparently not. The tail of the distribution of } 1 - (1/N)ReTrU_L \text{ in the Landau gauge has low population and may not contain relevant information about the configuration. It may be dependent on the algorithm used to put the configuration in the Landau gauge.} \]

\[ SU(2) \]

\[ SU(3) \]

\[ 1 - 1/2ReTrU_L \]

\[ 1 - 1/3ReTrU_L \]

In Fig. 4, we show that the maximum value of the distribution has practically no correla-
Figure 4. (Absence of) correlations between $\text{Max}\{1 - \frac{1}{3} Re Tr U_L\}$ and $P$ for pure SU(3) configurations at $\beta = 6$ in the Landau gauge.

Figure 5. Correlation between $1 - \frac{1}{3} Re Tr U_L$ and $P$ for the same configurations as in Fig. 4

Figure 6. Relative change of the configuration average of $P$ when 80 percent of the large field configurations are discarded, for various values of $\beta$ in a pure SU(3) LGT on a $8^4$ lattice.

6. The effect of the cut is very small but of a different size below, near or above $\beta = 5.6$. The dependence on the volume of this quantity remains to be studied. It is conceivable that $\Delta P/P$ could be taken as an order parameter.

The results regarding the perturbative series [5] for $P$ presented at the Conference will be discussed in a separate preprint [6]. Most of our calculations used FERMIQCD and MDP [7]. The 221 configurations in the Landau gauge are from the public OSU configurations used in [8]. We thank A. Gonzalez-Arroyo, M. Creutz, F. Di Renzo, M. Ogilvie, D. Sinclair and P. van Baal for valuable conversations.

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