Neutrino oscillations in a core-collapse supernova may be responsible for the observed rapid motions of pulsars. Given the present bounds on the neutrino masses, the pulsar kicks require a sterile neutrino with mass 2–20 keV and a small mixing with the active neutrinos. The same particle can be the cosmological dark matter. Its existence can be confirmed by the X-ray telescopes if they detect a 1–10 keV photon line from the decays of the relic sterile neutrinos. In addition, one may be able to detect gravity waves from a pulsar being accelerated by neutrinos in the event of a nearby supernova.

1. Introduction

There is an intriguing possible connection between two long-standing astrophysical puzzles: the origin of pulsar velocities and the nature of cosmological dark matter. The evidence for dark matter is extremely strong; its existence requires at least one new particle that is not a part of the Standard Model. If this new particle is a singlet fermion that has a small mixing with neutrinos, its emission from a supernova could be anisotropic.\(^1\)\(^2\) The anisotropy could explain the observed pulsar velocities. The purpose of this review is to explore this explanation of the pulsar kicks.

1.1. Pulsar velocities

The space velocities of pulsars are measured either by observation of their angular proper motions,\(^3\) or by measuring the velocity of an interstellar scintillation pattern as it sweeps across the Earth.\(^4\)\(^5\) Each of the two methods has certain advantages. Using the former method, one can get very precise measurements with the help of a high-resolution radio interferometer, but such observations take a long time. Measuring the velocity of a scintillation pattern can be done quickly, but the inference of the actual pulsar velocity must rely on some assumptions about the distribution of scattering material along the line of sight. (For instance, if the density of scatterers is higher near Earth, the pattern speed is less than the pulsars speed.) In addition to observational errors, one has to take into account various selection effects. For example, fast and faint pulsars are under-represented in the data as compared with the slow and bright ones. Therefore, one has to carefully model
the pulsar population to calculate the three-dimensional distribution of pulsar velocities corresponding to the observed two-dimensional projection of their proper motion.\textsuperscript{6,7}

Based on the data and population models, the average velocity estimates range from 250 km/s to 500 km/s.\textsuperscript{3-7} The distribution of velocities is non-gaussian, and there is a substantial population of pulsars with velocities in excess of 700 km/s. Some 15\% of pulsars\textsuperscript{7} appear to have velocities greater than 1000 km/s, while the fastest pulsars have speeds as high as 1600 km/s. Obviously, an acceptable mechanism for the pulsar kicks must be able to explain these very fast moving pulsars.

Pulsars are born in supernova explosions, so it would be natural to look for an explanation in the dynamics of the supernova\textsuperscript{13}. However, state-of-the-art 3-dimensional numerical calculations\textsuperscript{14} show that even the most extreme asymmetric explosions do not produce pulsar velocities greater than 200 km/s. Earlier 2-dimensional calculations\textsuperscript{15} claimed a somewhat higher maximal pulsar velocity, up to 500 km/s. Of course, even that was way too small to explain the population of pulsars with speeds (1000–1600) km/s. Recent three-dimensional calculations by Fryer\textsuperscript{14} show an even stronger discrepancy than the earlier numerical calculations of the supernova.

The hydrodynamic kick could be stronger if some large initial asymmetries developed in the cores of supernova progenitor stars prior to their collapse. Goldreich \textit{et al.}\textsuperscript{8} have suggested that unstable g-modes trapped in the iron core by the convective burning layers and excited by the $\epsilon$-mechanism may provide the requisite asymmetries. However, according to recent numerical calculations,\textsuperscript{9} the $\epsilon$-mechanism may not have enough time to significantly amplify the g-modes prior to the collapse. A different kind of the seed unisotropies may develop from the north-south asymmetry in the neutrino heating due to a strong magnetic field.\textsuperscript{10} If these asymmetries grow sufficiently during the later phases of the supernova, they may be relevant for the pulsar kicks.

Evolution of close binaries\textsuperscript{11} and asymmetric emission of radio waves\textsuperscript{12} have been considered as possible causes of the rapid pulsar motions. However, both of these explanations fail to produce a large enough effect.

Most of the supernova energy, as much as 99\% of the total $10^{53}$ erg are emitted in neutrinos. A few per cent anisotropy in the distribution of these neutrinos would be sufficient to explain the pulsar kicks. Alternatively, one needs (and, one is apparently lacking) a much larger asymmetry in what remains after the neutrinos are subtracted from the energy balance. The numerical calculations of the supernova assume that neutrino distribution is isotropic. What if this is not true?

Since the total energy released in supernova neutrinos is $E \sim 3 \times 10^{53}$ erg, the outgoing neutrinos carry the total momentum

\begin{equation}
    p_{\nu,\text{total}} \sim 1 \times 10^{43} \text{ g cm/s.}
\end{equation}
A neutron star with mass $1.4 M_\odot$ and $v = 1000 \text{ km/s}$ has momentum

$$p_\nu = (1.4 M_\odot) v \approx 3 \times 10^{41} \left( \frac{v}{1000 \text{ km/s}} \right) \text{ g cm/s} \approx 0.03 \left( \frac{v}{1000 \text{ km/s}} \right) p_{\nu, \text{total}}. \quad (2)$$

A few per cent asymmetry in the neutrino distribution is, therefore, sufficient to explain the observed pulsar velocities. What could cause such an asymmetry? The obvious suspect is the magnetic field, which can break the spherical symmetry and which is known to have an effect on weak interactions. We will examine this possibility in detail.

### 1.2. Neutrinos in the Standard Model and beyond

The number of light “active” left-handed neutrinos – three – is well established from the LEP measurements of the Z-boson decay width. In the Standard Model, the three active neutrinos fit into the three generations of fermions. In its original form the Standard Model described massless neutrinos. The relatively recent but long-anticipated discovery of the neutrino masses has made a strong case for considering right-handed neutrinos, which are SU(3)×SU(2)×U(1) singlets. The number of right-handed neutrinos may vary and need not equal to three.\(^{16,17}\) Depending on the structure of the neutrino mass matrix, one can end up with none, one, or several states that are light and (mostly) sterile, i.e., they interact only through their small mixing with the active neutrinos.

Unless some neutrino experiments are wrong, the present data on neutrino oscillations cannot be explained without sterile neutrinos. Neutrino oscillations experiments measure the differences between the squares of neutrino masses, and the results are:\(^{16}\) one mass squared difference is of the order of $10^{-5}\text{(eV}^2\text{)}$, the other one is $10^{-3}\text{(eV}^2\text{)}$, and the third is about $1\text{(eV}^2\text{)}$. Obviously, one needs more than three masses to get the three different mass splittings which do not add up to zero. Since we know that there are only three active neutrinos, the fourth neutrino must be sterile. However, if the light sterile neutrinos exist, there is no compelling reason why their number should be limited to one. We will see that a sterile neutrino required to explain the pulsar kicks and dark matter simultaneously must have a mass in the 2–20 keV range and a very small mixing.

In addition to explaining the neutrino oscillation data, pulsar kicks, and dark matter, theoretical models have invoked sterile neutrinos for various other reasons. For example, Farzan et al.\(^{18}\) used eV–keV sterile neutrinos to produce the mass matrices with certain properties, such as nearly bimaximal mixing\(^{19}\) of active neutrinos. Sterile neutrinos with mass around 200 MeV could reionize the universe even before the star formation,\(^{20}\) as early as the WMAP data suggest.\(^{21}\)

In general, the SU(3)×SU(2)×U(1) singlet (sterile) neutrino is not an eigenstate of the mass matrix. The mass eigenstates are linear combinations of the weak
eigenstates. Let us assume, for example, that the singlet neutrino has a non-zero mixing with the electron neutrino, but that the other mixing angles are zero or very small. Then one finds that the mass eigenstates have a simple expression in terms of the weak eigenstates:

\[ |\nu_1\rangle = \cos \theta_m |\nu_e\rangle - \sin \theta_m |\nu_s\rangle \]
\[ |\nu_2\rangle = \sin \theta_m |\nu_e\rangle + \cos \theta_m |\nu_s\rangle. \] (3)

If the mixing angle \(\theta_m\) is small, one of the mass eigenstates, \(\nu_1\), behaves very much like a pure \(\nu_e\), while the other, \(\nu_2\), is practically “sterile”, which means it has weak interactions suppressed by a factor \((\sin^2 \theta_m)\) in the cross section.

As discussed below, the sterile neutrinos that can kick the pulsars should have mass in the 2–20 keV range, and they should also have a small mixing \((\sin \theta \sim 10^{-4})\) with ordinary neutrinos, for example, the electron neutrino. Theoretical models of neutrino masses can readily produce a sterile neutrino with the required mass and mixing.\(^{18,22}\)

1.3. **Why a sterile neutrino can give the pulsar a kick**

Given the lack of a “standard” explanation for the pulsar kicks, one is compelled to consider alternatives, possibly involving new physics. One reason why the standard explanation fails is because most of the energy is carried away by neutrinos, which escape isotropically. The remaining momentum must be distributed with a substantial asymmetry to account for the large pulsar kick. In contrast, we saw that only a few per cent anisotropy in the distribution of neutrinos would give the pulsar a kick of required magnitude.

Neutrinos are always produced with an asymmetry, but they usually escape isotropically. The asymmetry in production comes from the asymmetry in the basic weak interactions in the presence of a strong magnetic field. Indeed, if the electrons and other fermions are polarized by the magnetic field, the cross section of the urca processes, such as \(n + e^+ \leftrightarrow p + \nu_e\) and \(p + e^- \leftrightarrow n + \nu_e\), depends on the orientation of the neutrino momentum:

\[ \sigma(\uparrow e^-, \uparrow \nu) \neq \sigma(\uparrow e^-, \downarrow \nu) \] (5)

Depending on the fraction of the electrons in the lowest Landau level, this asymmetry can be as large as 30\%, which is, seemingly, more than one needs to explain the pulsar kicks.\(^{23}\) However, this asymmetry is completely washed out by scattering of neutrinos on their way out of the star.\(^{24}\) This is intuitively clear because, as a result of scatterings, the neutrino momentum is transferred to and shared by the neutrons. In the approximate thermal equilibrium, no asymmetry in the production or scattering amplitudes can result in a macroscopic momentum anisotropy. This statement can be proved rigorously.\(^{24}\)

However, if the neutron star cooling produced a particle whose interactions with nuclear matter were even weaker than those of ordinary neutrinos, such a particle
Pulsar kicks from neutrino oscillations

Pulsar kicks from neutrino oscillations could escape the star with an anisotropy equal its production anisotropy. The state $\nu_2$ in equation (4), whose interactions are suppressed by $(\sin^2 \theta_m)$ can play such a role. It is intriguing that the same particle can be the dark matter.

The simplest realization of this scenario is a model with only one singlet fermion in which the mass eigenstates are admixtures of active and sterile neutrinos, as in equation (4). For a sufficiently small mixing angle $\theta_m$ between $\nu_e$ and $\nu_s$, only one of the two mass eigenstates, $\nu_1$, is trapped in the core of a neutron star. The orthogonal state, $\nu_2$, escapes from the star freely. This state is produced in the same basic urca reactions ($n + e^+ \rightleftharpoons p + \bar{\nu}_e$ and $p + e^- \rightleftharpoons n + \nu_e$) with the effective Lagrangian coupling equal the weak coupling times $\sin \theta_m$. The production rate can be greatly enhanced if active neutrinos undergo a resonant conversion into the sterile neutrinos at some density. This effect will play an important role in some range of parameters, although this kind of enhancement is not necessary for the pulsar kick.

We will consider two ranges of parameters, for which the $\nu_e \rightarrow \nu_s$ oscillations occur on and off resonance. First, we will suppose that a resonant oscillation occurs somewhere in the core of a neutron star. Then the asymmetry in the neutrino emission comes from a shift in the resonance point depending on the magnetic field. The temperature gradient is then responsible for the asymmetry in the momentum carried by neutrinos. Second, we will consider a resonance outside the dense core, where the asymmetry has a somewhat different origin: it comes from the uncompensated momentum deposition by active neutrinos on one side of the star, while the corresponding neutrinos on the other side propagate the same layer of matter as sterile neutrinos. Finally, we will consider the off-resonance case, in which the asymmetry comes directly from the weak processes, as in eq. (5). However, before discussing the emission of sterile neutrinos from a supernova, let us briefly review their role in cosmology and the cosmological limits on their masses and mixing angles.

2. Relic sterile neutrinos as dark matter

Very few hints exist as to the nature of cosmological dark matter. We know that none of the Standard Model particles can be the dark matter, and we also know that the dark matter particles should either be weakly interacting or very heavy (or both). Theoretical models have provided plenty of candidates. For example, supersymmetric extensions of the Standard Model predict the existence of many new particles, including two dark matter candidates: the lightest supersymmetric particle (LSP) and the SUSY Q-balls. These are plausible, theoretically motivated dark-matter candidates, as are many others.

However, if one seeks a minimal solution to the dark matter problem, sterile neutrinos offer a unique possibility: one can add just one dark-matter particle to the Standard Model, as long as it is an $SU(3) \times SU(2) \times U(1)$ singlet. Gauge singlets can be produced in weak interactions through their mixing with ordinary neutrinos.
As discussed below, we are interested in masses in the 1–20 keV range and small mixing angles. Such sterile neutrinos, which interact with primordial plasma only via mixing, could never have been in thermal equilibrium in the early universe. They could be produced from active neutrinos through oscillations. However, at very high temperatures active neutrinos have frequent interactions in plasma. Matter and the quantum damping effects inhibit neutrino oscillations.27 The mixing of sterile neutrinos with one of the active species in plasma can be represented by an effective, density and temperature dependent mixing angle28,30,31:

$$\sin^2 2\theta_m = \frac{(\Delta m^2/2p)^2 \sin^2 2\theta}{(\Delta m^2/2p)^2 \sin^2 2\theta + (\Delta m^2/2p \cos 2\theta - V_m - V_T)^2},$$

(6)

Here $V_m$ and $V_T$ are the effective matter and temperature potentials. In the limit of small angles and small lepton asymmetry, the mixing angle can be approximated as

$$\sin 2\theta_m \approx \sin 2\theta_1 + 0.27 \zeta \left( \frac{T}{100 \text{MeV}} \right)^6 \left( \frac{\text{keV}^2}{\Delta m^2} \right),$$

(7)

where $\zeta = 1.0$ for mixing with the electron neutrino and $\zeta = 0.30$ for $\nu_\mu$ and $\nu_\tau$.

Obviously, thermal effects suppress the mixing significantly for temperatures $T > 150 \,(m/\text{keV})^{1/3}$ MeV. Since the singlet neutrinos interact only through mixing, all the interaction rates are suppressed by the square of the mixing angle, $\sin^2 \theta_m$. It is easy to see that these sterile neutrinos are never in thermal equilibrium in the early universe. Thus, in contrast with the case of the active neutrinos, the relic population of sterile neutrinos is not a result of a freeze-out. One immediate consequence of this observation is that the Gershtein–Zeldovich bound32 and the Lee–Weinberg bound33 do not apply to sterile neutrinos.

Sterile neutrinos are produced through oscillations of active neutrinos. The relation between their mass and the abundance is very different from what one usually obtains in freeze-out. One can trace the production of sterile neutrinos in plasma by solving the Boltzmann equation for the distribution function $f(p, t)$:

$$\left( \frac{\partial}{\partial t} - Hp \frac{\partial}{\partial p} \right) f_s(p, t) = xH \partial_p f_s - \Gamma_{(\nu_a \rightarrow \nu_s)} (f_a(p, t) - f_s(p, t)),$$

(8)

(9)

where $H$ is the Hubble constant, $x = 1 \,	ext{MeV}a(t)$, $a(t)$ is the scale factor, and $\Gamma$ is the probability of conversion. The solution28–31 of this equation in the relevant range of parameters gives the following expression for the cosmological density of relic sterile neutrinos:

$$\Omega_s \approx 0.3 \left( \frac{\sin^2 \theta}{10^{-9}} \right) \left( \frac{m_s}{10 \text{keV}} \right)^2$$

(10)

The band of the masses and mixing angles consistent with dark matter is shown in Fig.1.
Observations of the power spectrum of the Lyman-\(\alpha\) forest clouds at high redshift show a significant structure on small scales. This requires a small collisionless damping scale associated with a dark matter particle, in other words the dark matter must be sufficiently “cold”. This and other constraints force the sterile neutrino mass to be greater than 2.6 keV.\(^{30,34}\) In Fig.1, the region labeled “too warm” shows the boundary of the allowed range of masses.

If the (unknown \textit{a priori}) lepton asymmetry of the universe is sufficiently large, then the sterile neutrinos can be produced through resonant Mikheev-Smirnov-Wolfenstein\(^{35}\) (MSW) oscillations in the early universe.\(^{36}\) These neutrinos are non-thermal and cold because the adiabaticity condition selects the low-energy part of the neutrino spectrum.

Most of the cosmological constraints can be evaded if inflation ended with a low-temperature reheating.\(^{37}\) In this case, the cosmological upper bound on the mixing angle is weaker, and the allowed parameter space for the pulsar kicks extends to the lower masses and the larger mixing angles.\(^{37}\)

**Fig. 1.** The range of the sterile neutrino mass and mixing angle. Regions 1 and 2 correspond to parameters consistent with the pulsar kicks due to resonant MSW oscillations deep in the core (1) or outside the core (2). These two possibilities are discussed in sections 3.1 and 3.2, respectively. Region 3 corresponds to off-resonance active–sterile conversions in the core (see section 3.3). Cosmological bounds and the exclusion region due to X-ray observations are shown as well. The cosmological bounds shown here assume that the reheat temperature after inflation was higher than 1 GeV and that the lepton asymmetry of the universe is small \(L \ll 10^{-3}\).
3. Pulsar kicks from active–sterile neutrino oscillations

We will now examine the range of parameters in which the emission of sterile neutrinos from the core is sufficiently strong and anisotropic to give the pulsar a kick. There are three possible regimes for the sterile neutrino emission. Depending on the mass and the mixing angle, there may or may not be a resonant conversion of the active to sterile neutrinos at some density in a hot neutron star. If there is an MSW resonance, the position of the resonance point depends on the density and the magnetic field. The latter introduces the required anisotropy. In the absence of the MSW resonance, an off-resonance emission from the entire volume of the neutron star core is possible. We will see that this emission is efficient only after the matter potential has evolved from its initial value to nearly zero. This important evolution requires some time, which, in turn imposes constraints on the masses and mixing angles. We will consider the following three possibilities for the pulsar kick:

- MSW resonance in the core ($\rho > 10^{14} \text{g/cm}^3$)
- MSW resonance outside the core ($\rho < 10^{14} \text{g/cm}^3$)
- an off-resonance emission from the core

We will see that the three regimes are probably mutually exclusive. For example, for all the masses that are consistent with the resonance, the matter potential evolves very slowly, and there is no significant emission from the core off-resonance.

3.1. MSW resonance in the core

Let us consider neutrino cooling during the first 10–15 seconds after the formation of a hot proto-neutron star. For simplicity we will assume that it has a uniform (dipole) magnetic field $\vec{B}$. Neutrino oscillations in a magnetized medium are described by an effective potential that depends on the magnetic field in the following way:

$$V(\nu_e) = 0$$
$$V(\bar{\nu}_e) = V_0 (3 Y_e - 1 + 4 Y_{\nu_e})$$
$$V(\nu_{\mu,\tau}) = -V(\bar{\nu}_{\mu,\tau}) = V_0 (Y_e - 1 + 2 Y_{\nu_e}) + \frac{eG_F}{\sqrt{2}} \left( \frac{3N_e}{\pi^2} \right)^{1/3} \frac{\vec{k} \cdot \vec{B}}{|\vec{k}|}$$

where $Y_e$ ($Y_{\nu_e}$) is the ratio of the number density of electrons (neutrinos) to that of neutrons, $\vec{B}$ is the magnetic field, $\vec{k}$ is the neutrino momentum, $V_0 = 10 eV \frac{10^{14} \text{g/cm}^3}{\rho}$. The magnetic field dependent term in equation (13) arises from polarization of electrons and not from a neutrino magnetic moment, which in the Standard Model is small and which we will neglect. (A large neutrino magnetic moment can result in a pulsar kick through a somewhat different mechanism, discussed below.)

The condition for a resonant MSW conversion $\nu_i \leftrightarrow \nu_j$ is

$$\frac{m_i^2}{2k} \cos 2\theta_{ij} + V(\nu_i) = \frac{m_j^2}{2k} \cos 2\theta_{ij} + V(\nu_j)$$

(14)
where $\nu_{i,j}$ can be either a neutrino or an anti-neutrino.

In the presence of the magnetic field, the resonance condition (14) for $\nu_a \rightarrow \nu_s$ ($a = \mu, \tau$) conversions is satisfied at different distances $r$ from the center, depending on the value of the $(\vec{k} \cdot \vec{B})$ term in (13). The average momentum carried away by the neutrinos depends on the temperature of the region from which they escape. The deeper inside the star, the higher is the temperature during the neutrino cooling phase. Therefore, neutrinos coming out in different directions carry momenta which depend on the relative orientation of $\vec{k}$ and $\vec{B}$. This causes the asymmetry in the momentum distribution.

$$\Delta k_s \approx \frac{2e}{3\pi^2} \left( \frac{\mu_e}{T} \frac{dT}{dN_n} \right) B.$$
To calculate the derivative in (19), we assume approximate thermal equilibrium. Then one can use the relation between the density and the temperature of a non-relativistic Fermi gas:

\[ N_n = \frac{2(m_n T)^{3/2}}{\sqrt{2\pi^2}} \int \frac{\sqrt{z}dz}{e^{z} - \mu_n / T + 1} \]

where \( m_n \) and \( \mu_n \) are the neutron mass and chemical potential. The derivative \((dT/dN_n)\) can be computed from (20). Finally,

\[ \frac{\Delta k_s}{k_s} = \frac{8e\sqrt{2}}{\pi^2} \frac{\mu_e \mu_n^{1/2}}{m_n^{3/2} T^2} B \]

We have assumed that only one of the neutrino species undergoes a resonance transition into a sterile neutrino. The energy, however, is shared between 6 species of active neutrinos and antineutrinos. Therefore, the final asymmetry due to anisotropic emission of sterile neutrinos is 6 times smaller:

\[ \frac{\Delta k_s}{k} = \frac{1}{6} \frac{\Delta k_s}{k_s} = \frac{4e\sqrt{2}}{3\pi^2} \frac{\mu_e \mu_n^{1/2}}{m_n^{3/2} T^2} B = \]

\[ = 0.01 \left( \frac{\mu_e}{100 \text{ MeV}} \right) \left( \frac{\mu_n}{80 \text{ MeV}} \right)^{1/2} \left( \frac{20 \text{ MeV}}{T} \right)^2 \left( \frac{B}{3 \times 10^{16} \text{ G}} \right) \]

This estimate\(^1\) can be improved by considering a more detailed model for the neutrino transport and by taking into account time evolution of chemical potentials discussed below. However, it is clear that the magnetic field inside the neutron star should be of the order of \(10^{16} \text{ G}\). The approximation used in equation (17) holds as long as the resonant transition occurs deep in the core, at density of order \(10^{14} \text{ g cm}^{-3}\). This, in turn, means that the sterile neutrino mass must be in the keV range. We note that theoretical models of neutrino masses can readily produce a sterile neutrino with a required mass.\(^{18,22}\) The corresponding region of parameters is shown as region “1” in Fig. 1.

### 3.2. Resonance at densities below \(10^{14} \text{ g/cm}^3\)

For smaller masses, the resonance occurs at smaller densities. Outside the core, fewer neutrinos are produced, while there is a flux of neutrinos diffusing out of the core. Therefore, the approximation (17) is not valid.

Outside the core, the active neutrinos can interact with matter and deposit momentum to the neutron star medium. After an active neutrino is converted into a sterile neutrino, it no longer interacts with matter and comes out of the star. The cross section for active neutrino interactions in matter is \(\sigma \sim G_F^2 E_\nu^2\), where \(E_\nu\) is the neutrino energy. If the resonant conversion \(\nu_a \rightarrow \nu_s\) occurs at different depths for different directions, the neutrinos may spend more time as active on one side of the star than on the other side of the star, as shown in Fig. 3. Hence, they deposit more momentum through their interactions with matter on one side than
Pulsar kicks from neutrino oscillations

on the other side. Let us estimate this difference. (This argument was used before in application to active neutrino transport. Here we adopt it to sterile neutrino.)

Depending on the magnetic field, the resonance lies at different depths, eq. (15). Hence, the neutrinos on one side of the star pass an extra layer of thickness \(2\delta \cos \phi\) as active, while the neutrinos on the other side pass this layer as sterile. The active neutrinos going through a layer of nuclear matter with thickness \(2\delta\) have an extra probability

\[ P_\delta = (2\delta) \sigma N_n \]  

(23)
to interact and deposit momentum \(k \sim E_\nu\) to the neutron star. This momentum is not balanced by the neutrinos on the other side of the star because they go through the this layer as sterile neutrinos.

![Diagram of neutron star with MSW resonance](image)

Fig. 3. For MSW resonance outside the core, the neutrino passes between \(r_- = r_0 - \delta \cos \phi\) and \(r_+ = r_0 + \delta \cos \phi\) as a sterile \(\nu_s\) on one side of the star, while it still propagates as an active \(\nu_a\) on the other side. The active neutrinos interact and deposit some extra momentum on the right-hand side, between \(r_-\) and \(r_+\). Since the neutron star is a gravitationally bound object, the momentum deposited asymmetrically in its outer layers gives the whole star a kick.

Obviously, the neutron star as a whole is a gravitationally bound object, so any momentum deposited on one side of the star gives the whole neutron star a kick.

The difference in the momentum deposition per active neutrino between the directions \(\phi\) and \(-\phi\) is

\[ \frac{\Delta k_s}{k} \sim (2\delta \cos \phi) N_n \sigma \sim G_F^2 E_\nu^2 \frac{\mu_e}{Y_e} \frac{eB}{h_{N_e}} h_{N_e} \cos \phi, \]  

(24)

where we have used eq. (16) and introduced the scale height of the electron density \(h_{N_e} = [d(\ln N_e)/dr]^{-1}\).

We take \(Y_e \approx 0.1\), \(E_\nu \approx 3T \approx 10\) MeV, \(\mu_e \approx 50\) MeV, and \(h_{N_e} \approx 6\) km. We assume \(T \approx 3\) MeV because it is a realistic average temperature around the neutrinosphere, in agreement with theoretical models as well as observations of the
supernova SN1987A.\textsuperscript{41} (This temperature is lower than the core temperature used earlier.)

After integrating over angles and taking into account that only one neutrino species undergoes the conversion, we obtain the final result for the asymmetry in the momentum deposited by the neutrinos.

\[
\frac{\Delta k_s}{k} = 0.03 \left( \frac{T}{3 \text{ MeV}} \right)^2 \left( \frac{\mu}{50 \text{ MeV}} \right) \left( \frac{h}{6 \text{ km}} \right) \left( \frac{B}{10^{15} \text{ G}} \right),
\]

This is, clearly, a sufficient asymmetry for the pulsar kick. The corresponding region of parameters is shown as region “2” in Fig. 1.

The above estimates are valid as long as \( \delta \) is much smaller than the mean free path. One can also describe the propagation of neutrinos in this region using the so called diffusion approximation.\textsuperscript{42} It was used for the neutrino transport by Schinder and Shapiro\textsuperscript{43} in planar approximation and was applied to the pulsar kicks by Barkovich \textit{et al.}\textsuperscript{44,45}

### 3.3. Off-resonance transitions

Let us now consider the case of the off-resonance emission from the core. This possibility was discussed qualitatively in section 1.3. Now we want to determine the neutrino parameters consistent with the kick mechanism.

For masses of a few keV, the resonant condition is not satisfied anywhere in the core. In this case, however, the off-resonant production of sterile neutrinos in the core can occur through ordinary urca processes. A weak-eigenstate neutrino has a \( \sin^2 \theta \) admixture of a heavy mass eigenstate \( \nu_2 \). Hence, these heavy neutrinos can be produced in weak processes with a cross section suppressed by \( \sin^2 \theta \).

Of course, the mixing angle in matter \( \theta_m \) is not the same as it is in vacuum, and initially \( \sin^2 \theta_m \ll \sin^2 \theta \). However, as Abazajian, Fuller, and Patel\textsuperscript{30} have pointed out, in the presence of sterile neutrinos the mixing angle in matter quickly evolves toward its vacuum value. When \( \sin^2 \theta_m \approx \sin^2 \theta \), the production of sterile neutrinos is no longer suppressed, and they can take a fraction of energy out of a neutron star.

We note in passing that time evolution of the matter potential may be important for a number of other reasons.\textsuperscript{46}

Following Abazajian, Fuller, and Patel, one can estimate the time it takes for the matter potential to evolve to zero from its initial value \( V^{(0)}(\nu_e) \simeq (-0.2... + 0.5)V_0 \). The time scale for this change to occur through neutrino oscillations off-resonance is

\[
\tau_{\nu}^{\text{off-res}} \approx \frac{4\sqrt{2} \pi^2 m_n}{G_F^3 \rho} \frac{(V^{(0)}(\nu_e))^3}{(\Delta m^2)^2 \sin^2 2\theta \mu^3} \frac{1}{(\Delta m^2)^2},
\]

As long as this time is much smaller than 10 seconds, the mixing angle in
mater approaches its vacuum value in time for the sterile neutrinos to take out some fraction of energy from a cooling neutron star.

The urca processes produce ordinary neutrinos with some asymmetry depending on the magnetic field. The same asymmetry is present in the production cross sections of sterile neutrinos. However, unlike the active neutrinos, sterile neutrinos escape from the star without rescattering. Therefore, the asymmetry in their emission is not washed out as it is in the case of the active neutrinos. Instead, the asymmetry in emission is equal the asymmetry in production.

The number of neutrinos $dN$ emitted into a solid angle $d\Omega$ can be written as

$$
\frac{dN}{d\Omega} = N_0(1 + \epsilon \cos \Theta_\nu),
$$

where $\Theta_\nu$ is the angle between the direction of the magnetic field and the neutrino momentum, and $N_0$ is some normalization factor. The asymmetry parameter $\epsilon$ is equal

$$
\epsilon = \frac{g_V^2 - g_A^2}{g_V^2 + 3g_A^2} \frac{k_0}{E_{\text{tot}}} (E_s/E_{\text{tot}}),
$$

where $g_V$ and $g_A$ are the axial and vector couplings, $E_{\text{tot}}$ and $E_s$ are the total neutrino energy and the energy emitted in sterile neutrinos, respectively. The number of electrons in the lowest Landau level, $k_0$, depends on the magnetic field and the chemical potential $\mu$ as shown in Fig. 4.

![Fig. 4. The fraction of electrons in the lowest Landau level as a function chemical potential. The value of the magnetic field is shown next to each curve.](image-url)
The momentum asymmetry in the neutrino emission is

$$\epsilon \sim 0.02 \left( \frac{k_0}{0.3} \right) \left( \frac{r_E}{0.3} \right), \quad (29)$$

where $r_E$ is the fraction of energy carried by the sterile neutrinos. To satisfy the constraint based on the observation of neutrinos from supernova SN1987A, we require that $r_E < 0.7$. As can be seen from Fig. 4, the asymmetry in equation (29) can be of the order of a few per cent, as required, for magnetic fields $10^{15} - 10^{16}$ G.

Surface magnetic fields of ordinary radio pulsars are estimated to be of the order of $10^{12} - 10^{13}$ G. However, the magnetic field inside a neutron star may be much higher,\(^\text{47,48,49}\) probably up to $10^{16}$ G. The existence of such a strong magnetic field is suggested by the dynamics of formation of the neutron stars, as well as by the stability of the poloidal magnetic field outside the pulsar.\(^\text{48}\) Moreover, the discovery of soft gamma repeaters and their identification as magnetars,\(^\text{49}\) i.e., neutron stars with surface magnetic fields as large as $10^{15}$ G, gives one a strong reason to believe that the interiors of many neutron stars may have magnetic fields as large as $10^{15} - 10^{16}$ G. There are also plausible physical mechanisms that can generate such a large magnetic field inside a cooling neutron star.\(^\text{47,48,50}\)

### 3.4. Pulsar kicks from the active neutrinos alone?

One can ask whether the sterile neutrino is necessary and whether the oscillations of active neutrinos alone could explain the pulsar kicks. The interactions of muon and tau neutrinos in nuclear matter are characterized by a smaller cross section than those of the electron neutrinos. This is because the electron neutrinos $\nu_e$ interact through both charged and neutral currents with electrons, while $\nu_\mu$ and $\nu_\tau$ interact with electrons through neutral currents alone. Therefore, nuclear matter is more transparent to $\nu_e$ than to $\nu_{\mu,\tau}, \bar{\nu}_{\mu,\tau}$. As a result, the surface of last scattering for $\nu_{\mu,\tau}$ and $\bar{\nu}_{\mu,\tau}$ lies (about a kilometer) deeper than that of $\nu_e$. The electron antineutrino can interact through charged currents with positrons while they are present in nuclear matter. The $\bar{\nu}_e$ mean free path starts out closer to that of $\nu_e$, but, as the number of positrons diminishes during the cooling period, this mean free path increases and becomes comparable to that of $\nu_{\mu,\tau}$ and $\bar{\nu}_{\mu,\tau}$.

Since the $\mu$- and $\tau$-neutrinospheres lie inside the electron neutrinosphere, it is possible that neutrino oscillations could convert a $\nu_e$ into $\nu_\mu$ or $\nu_\tau$ at some point between the two neutrinospheres, where the $\nu_e$ is trapped, but the $\nu_{\mu,\tau}$ is free-streaming. Then the shift in position of the MSW resonance would result in an anisotropy of the outgoing momentum. This mechanism could explain the pulsar kicks, but it would require one of the active neutrino masses to be of order $100$ eV.\(^\text{51}\) This is not consistent with the present data.

### 3.5. What if neutrinos have a large magnetic moment?

The neutrino magnetic moment in the Standard Model is very small, $\mu_\nu \approx 3 \times 10^{-10} (m_\nu/\text{eV}) \mu_B$, where $\mu_B = e/2m_e$ and $m_\nu$ is the (Dirac) neutrino mass. This
is why we have so far neglected any effects of direct neutrino interactions with the magnetic field.

However, the present experimental bounds allow the neutrino magnetic moments to be as large as $10^{-12} \mu_B$. If, due to some new physics, the neutrino magnetic moment is large, it may open new possibilities for the pulsar kick. Voloshin\(^{39}\) has proposed an explanation of the pulsar kick based on the resonant spin-flip and conversion of the left-handed neutrinos into the right-handed neutrinos, which then come out of the neutron star. Voloshin argued that the magnetic field inside a neutron star may be irregular and may have some asymmetrically distributed “windows”, through which the right-handed neutrinos could escape. The resulting asymmetry may, indeed, explain the pulsar velocities.

Other kinds of new physics may cause the pulsar kicks as well\(^ {52}\).

### 3.6. Spin-kick from neutrinos

Spruit and Phinney\(^ {53}\) have argued that the pulsar rotational velocities may also be explained by the kick received by the neutron stars at birth. The core of the progenitor star is likely to co-rotate with the whole star until about 10 year before the collapse. This is because the core should be tied to the rest of the star by the magnetic field. However, then the angular momentum of the core at the time of collapse is $10^3$ times smaller than the angular momentum of a typical pulsar. Spruit and Phinney have pointed out that the kick that accelerates the pulsar can also spin it up, unless the kick force is exerted exactly head-on.

The neutrino kick can be strongly off-centered, depending on the configuration of the magnetic field. If the magnetic field of a pulsar is offset from the center, so will be the force exerted on the pulsar by the anisotropic emission of neutrinos. This mechanism may explain simultaneously the high spatial velocities and the unusually high rotation speeds of nascent neutron stars.\(^ {53}\) E.S. Phinney has suggested\(^ {54}\) that a highly off-centered magnetic field could be generated by a *thin-shell* dynamo in a hot neutron star. Since the neutron star is cooled from the outside, a convective zone forms near the surface and, at later times, extends to the interior. While convection takes place in the spherical shell, the dynamo effect can cause a growth in the magnetic field. Thin-shell dynamos are believed to be responsible for generating magnetic fields of Uranus and Neptune,\(^ {55,56}\) which, according to the Voyager 2 measurements,\(^ {57}\) are both off-centered and tilted with respect to the axis of rotation. Unlike other planets, which have convection in the deep interior and end up with a well-centered dipole field, Uranus and Neptune have thin spherical convective zones near the surface, which explains the peculiarity of their dynamos. During the first few seconds after the supernova collapse, convection in a neutron star also takes place in a spherical layer near the surface. The thin-shell dynamo can, in principle, generate an off-center magnetic field in a neutron star, just like it does in Uranus and Neptune.\(^ {54}\)
4. Observational consequences and experimental searches

The mixing angle $\theta$, consistent with the neutrino kick and also with dark matter, is very small: $\sin \theta \sim 10^{-5} - 10^{-4}$. Hence, it is unlikely that the existence of the sterile neutrino with the required mixing can be tested in a laboratory experiment. However, some astrophysical observations can be used to verify or rule out our scenario.

4.1. Search for the relic sterile neutrinos with mass 2–20 keV.

The parameter space allowed for the pulsar kicks overlaps nicely with that of dark-matter sterile neutrinos. Relic sterile neutrinos in this range should make up the galactic halos. For smaller mixing angles, some part of dark matter is sterile neutrinos. In any case, there should exist some population of sterile neutrinos left over after the Big Bang.

The relic sterile neutrinos with mass in the 2–20 keV range can decay into three lighter neutrinos, or into a lighter neutrino and a photon. The Feynman diagrams for the latter channel of decay are shown in Fig. 5. The rate of the radiative decay is

$$\Gamma_\gamma \approx 6.8 \times 10^{-33} \text{ s}^{-1} \left( \frac{\sin^2 2\theta}{10^{-10}} \right) \left( \frac{m_s}{1 \text{ keV}} \right)^5.$$ (30)

Although $\tau = \Gamma^{-1} \sim 10^{25} - 10^{33}$ s is much longer than the age of the universe, there are, nevertheless, enough decays in the clusters of galaxies for the photons to be observed. Since $\nu_2 \to \nu_1 \gamma$ is a two-body decay, the photon energy is equal $(m_s/2)$, which is in the 1–10 keV range for the masses of interest to us. These photons should be detectable by the X-ray telescopes. Chandra and XMM-Newton can exclude part of the parameter space shows in Fig. 1. The future Constellation-X can probably explore the entire allowed range of parameters.

To detect the relic sterile neutrinos, one should look for an isolated line that cannot be identified with another source, such as interstellar gas. One can try to distinguish the lines in the gas emission from the one due to dark matter by comparing the strengths of signals from regions with different temperatures. The detection strategy is discussed in detail by Abazajian, Fuller, and Tucker.
4.2. Gravity waves from a pulsar kick due to neutrinos

In the event of a nearby supernova, the neutrino kick can produce gravity waves that could be detected by LIGO and LISA.\textsuperscript{59,60} These gravity waves can be produced in several ways.

Obviously, the departure from spherical symmetry is a necessary condition for generating the gravity waves. A neutron star being accelerated by neutrinos is not moving fast enough to generate gravitational waves from its own motion. However, the anisotropy in the outgoing neutrinos turns out to be sufficient to produce an observable signal in the event of a nearby supernova.

Most of the neutrinos come out isotropically and can be neglected. However, a few per cent of asymmetrically emitted neutrinos move along the direction of the magnetic field. In general, the magnetic field is not aligned with the axis of rotation, and, therefore, the outgoing neutrinos create a non-isotropic source for the waves of gravity. (Water jet produced by a revolving lawn sprinkler is probably a good analogy for the geometry of this source.) The signal was calculated by L. Loveridge.\textsuperscript{59} It can be observed by advanced LIGO or LISA if a supernova occurs nearby, as shown in Fig. 6. Alternatively, the neutrino conversion itself may cause gravity waves coming out of the core.\textsuperscript{60}

![Fig. 6. Gravity waves signal at LIGO and LISA calculated by L.C. Loveridge.](image)

### 4.3. $B - v$ correlation?

Unfortunately, the neutrino kick mechanism does not predict a correlation between the direction of the surface magnetic fields and the pulsar velocity. The kick velocity is determined by the magnetic field inside the hot neutron star during the first seconds after the supernova collapse. Astronomical observations can be used to infer the surface magnetic fields of pulsars some millions of years later. The relation
between the two is highly non-trivial because of the complex evolution the magnetic field undergoes in a cooling neutron star. Let us outline some stages of this evolution.

Immediately after the formation of the hot neutron star the magnetic field is expected to grow due to differential rotation, thermal effects, and convection. The dynamo effect can probably account for the growth of the magnetic field to about $10^{15} - 10^{16}$ G.

The growth of the magnetic field takes place during the first ten seconds after the supernova collapse, in part because the neutrino cooling causes convection. At the same time, during the neutrino cooling phase, the neutron star receives a kick. The magnetic field relevant for the kick is the average interior magnetic field during the first 10 seconds. There is no reason to believe that it has the same direction or magnitude as the surface field at the end of the neutrino cooling phase. This, however, is only one of several stages in the evolution of the magnetic field. Next, at some temperature below 0.5 MeV, the nuclear matter becomes a type-II superconductor. The magnetic field lines form flux tubes, reconnect, and migrate. Next, over millions of years, the pulsar rotation converts the magnetic field energy into radio waves and causes the field to evolve even further. The end result of this evolution is, of course, a configuration of magnetic fields that is very different from what it was five seconds after the onset of the supernova.

Clearly, the magnetic field inside a hot young neutron star is not expected to have much correlation with the surface field of a present-day pulsar. Some naive analyses of the $B - v$ correlation have ignored the magnetic field evolution and have reached incorrect conclusions.

5. Conclusion

An asymmetric neutrino emission from a cooling neutron star can explain the observed pulsar velocities. The asymmetry may be caused by neutrino oscillations in the magnetized nuclear matter if there is a sterile neutrino with mass in the 2–20 keV range and a small mixing with ordinary neutrinos. It is intriguing that the same particle is a viable dark matter candidate. We know that at least one particle beyond the Standard Model must exist to account for dark matter. This particle may come as part of a “package”, for example, if supersymmetry is realized in nature. However, it may be that the dark matter particle is simply an SU(3)×SU(2)×U(1)-singlet fermion, which has a small mixing with neutrinos.

Future observations of X-ray telescopes may be able to discover the relic sterile neutrinos by detecting keV photons from the sterile neutrino decay in clusters of galaxies. Finally, if gravitational waves are detected from a nearby supernova, the signal may show the signs of a neutron star being accelerated by an asymmetric neutrino emission.
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