Neutrinoless Double Beta Decay in Theories Beyond the Standard Model

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Neutrinoless double beta decay pops up almost in any extension of the standard model. It is perhaps the only process, which can unambiguously determine whether the massive neutrinos are Majorana or Dirac type particles. In addition from the lifetime of this decay, combined with sufficient knowledge of the relevant nuclear matrix elements, one can set a constraint involving the neutrino masses. Furthermore, if one incorporates the recent results of the neutrino oscillation experiments, one can determine or set a stringent limit on the neutrino mass scale. It is still of great theoretical and experimental interest since perhaps it is almost 70 years, which has been searched for, but not seen yet. It is still of great theoretical and experimental interest since perhaps it is the only process, which can unambiguously determine whether massive neutrinos can be Majorana or Dirac type particles. It can occur only if the mass eigenstates are Majorana particles (the particle coincides with its antiparticle).

In the weak basis \( \nu^0 \), \( \nu^0 \) the neutrino mass matrix takes the form:

\[
\begin{pmatrix}
\nu^0_L \\
\nu^0_R
\end{pmatrix} =
\begin{pmatrix}
m_L \\
M_D
\end{pmatrix}
\begin{pmatrix}
\nu^0_L \\
\nu^0_R
\end{pmatrix}
\]

where \( m_L \) is a 3 \times 3 left-handed (isotriplet) Majorana mass matrix, \( M_D \) is a Dirac 3 \times N mass matrix and \( m_N \) is the right-handed (isosinglet) \( N \times N \) Majorana mass matrix. In models motivated by GUT's \( N = 3, m_L = 0, m_D \) is of the order of the up quark mass matrix and \( m_N \) is very heavy (\( \geq 10^{10} \) GeV) so that light neutrinos with mass of order \( m_D(m_n)^{-1}m_D \) can occur. On the other hand in R-parity violating supersymmetry only \( m_L \) is non zero. Anyway after diagonalizing the above matrix the weak eigenstates are related to the mass eigenstates via the equation:

\[
\begin{pmatrix}
\nu^0_L \\
\nu^0_R
\end{pmatrix} =
\begin{pmatrix}
U_{11} & U_{12} \\
U_{21} & U_{22}
\end{pmatrix}
\begin{pmatrix}
\nu^c_R \\
\nu^c_R
\end{pmatrix}
\]

In the above notation \( U^{(11)} = U^{MNS} \) is the usual Maki, Nakagawa, Sakata charged leptonic matrix, which for convenience sometimes will merely be indicated by \( U \).

1.1. Left-handed currents only

In the presence of only left handed currents one has a contribution to neutrinoless double beta decay arising from the diagram shown in fig. \( \mathbb{H} \). One encounters two lepton violating parameters \( \eta_\nu \) and \( \eta_N^L \) given by \( \mathbb{H} \):

\[
\eta_\nu = \langle m_\nu \rangle / m_e
\]

(1)

\[
\langle m_\nu \rangle = \sum_j (U^{(11)}_{ej})^2 \xi_j m_j,
\]

(2)

\[
\eta_N^L = \sum_j (U^{(12)}_{ej})^2 \Xi_j \frac{m_p}{M_j}
\]

(3)

with \( m_j \) (\( M_j \)) the light (heavy) neutrino masses and \( \xi_j = e^{i\alpha_j} \) (\( \Xi_j = e^{i\Phi_j} \)) the Majorana phases of the corresponding mass eigenstates. The separation of these particle physics parameters from the nuclear physics holds, if the neutrino is very light \( m_j << m_e \) or much heavier than the proton \( M_j >> m_p \).
1.2. The leptonic right-left interference contribution

In the presence of right-handed currents one can have a contribution via light intermediate neutrinos, in which the corresponding amplitude does not explicitly depend on the neutrino mass \( \nu \) (see Fig. 2). We encounter two lepton violating parameters:

\[
\eta = \epsilon \eta_{RL}, \lambda = \kappa \eta_{RL}, \eta_{RL} = \sum_{j} (U_{\nu j}^{(21)} U_{\nu j}^{(11)}) \xi_j
\]

where \( \kappa = m_L^2/m_R^2 \), \( \epsilon = \tan \zeta \). \( m_L, m_R \) are the gauge boson masses and \( \zeta \) the gauge boson mixing angle.

1.3. Right-handed neutrino mass term

In the presence of right-handed currents one can have additional mass dependent terms (see Fig. 3). The corresponding lepton violating parameter is given by:

\[
\eta_N^R = (\kappa^2 + \epsilon^2 + \epsilon \kappa) \sum_{j} (U_{\nu j}^{(22)})^2 \Xi_j m_p M_j
\]

1.4. Analysis of the data in terms of light neutrinos

In the presence of right handed currents the neutrinoless double beta decay lifetime is given by:

\[
[T_{1/2}^{00}]^{-1} = G_{01}^{00} |M_{GT}^{00}|^2 |X_L|^2 + \tilde{C}_{00} |\lambda| X_L \cos \psi_1 +
\]

Figure 1. neutrinoless double beta decay in left handed theories for light (a) and heavy (b) intermediate neutrinos.

Figure 2. Diagrams leading to neutrinoless double beta decay with amplitude not explicitly dependent on the neutrino mass.

Figure 3. The heavy neutrino contribution due to the right handed current.
and \( \lambda \) dominates and proceeds extracting a limit on that assumes that only one lepton violating parameter the lepton violating parameters. Normally one (see table 1) imposes only one constraint among tra dimensions, exotic Higgs scalars etc. by superstring models, unusual messengers in ex-
diate SUSY particles, unusual particles predicted
tions arising from other particles such as interme-
(neutrino), \( \psi \), where the subscript \( X \) indicates heavy particle (neutrino), \( \psi_1 \) is the relative phase between \( X_L \) and \( \lambda \) and \( \psi_2 \) is the relative phase between \( X_L \) and \( \eta \), while \( \tilde{C} \) depend on the energy and nuclear physics The ellipses \( \ldots \) indicate contributions arising from other particles such as intermediate SUSY particles, unusual particles predicted by superstring models, unusual messengers in extra dimensions, exotic Higgs scalars etc.

As we have already mentioned the experiment (see table 1) imposes only one constraint among the lepton violating parameters. Normally one assumes that only one lepton violating parameter dominates and proceeds extracting a limit on that one (see e.g. Table 3 below). In the most favored scenario one assumes that the light neutrino mass mechanism dominates. One can then employ the the neutrino oscillation data to get the constraints of Fig. 4. Clearly \( 0\nu\beta\beta \) decay can differentiate between the hierarchies. In fact the degenerate scenario supports the recent claims that \( 0\nu\beta\beta \) decay may even have been seen. In the inverted hierarchy, \( 0\nu\beta\beta \) can even set a limit on the lightest neutrino mass \( m_1 \).

A more reasonable procedure would be to consider multi-dimensional exclusion plots as, e.g., that involving \( < m_\nu > \) and \( \eta \), shown in Fig. 5 obtained with nuclear matrix elements found in the literature. From this plot one extracts the standard limits as the intersection of the exclusion

Table 1
Summary of the present experimental results.

<table>
<thead>
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<th>Isotope</th>
<th>( T^{0\nu}_{1/2} ) (y)</th>
<th>Refs</th>
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</thead>
<tbody>
<tr>
<td>(^{48}\text{Ca})</td>
<td>&gt; 9.5 \times 10^{21} (76%)</td>
<td>[9]</td>
</tr>
<tr>
<td>(^{76}\text{Ge})</td>
<td>&gt; 1.9 \times 10^{25} (90%)</td>
<td>[10]</td>
</tr>
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<td>&gt; 1.6 \times 10^{25} (90%)</td>
<td>[11]</td>
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<td>&gt; 2.7 \times 10^{22} (68%)</td>
<td>[12]</td>
</tr>
<tr>
<td>(^{116}\text{Cd})</td>
<td>&gt; 5.5 \times 10^{22} (90%)</td>
<td>[13]</td>
</tr>
<tr>
<td>(^{128,130}\text{Te})</td>
<td>&gt; 7 \times 10^{22} (90%)</td>
<td>[14]</td>
</tr>
<tr>
<td>(^{128}\text{Te})</td>
<td>( R = (3.52 \pm 0.11) \times 10^{-4} ) (geochronal)</td>
<td>[15]</td>
</tr>
<tr>
<td>(^{130}\text{Te})</td>
<td>&gt; 7.7 \times 10^{24} (90%)</td>
<td>[16]</td>
</tr>
<tr>
<td>(^{130}\text{Xe})</td>
<td>&gt; 1.4 \times 10^{23} (90%)</td>
<td>[17]</td>
</tr>
<tr>
<td>(^{150}\text{Nd})</td>
<td>&gt; 1.2 \times 10^{21} (90%) &lt; 3 (90%)</td>
<td>[18]</td>
</tr>
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</table>

where

\[
\tilde{C}_{0\nu}|\eta|X_L \cos \psi_2 + \tilde{C}_{\lambda\lambda}|\lambda|^2 + \tilde{C}_{\eta\eta} + |X_R|^2 + |\eta|^2 + \tilde{C}_{\lambda\eta}|\lambda| |\eta| \cos (\psi_1 - \psi_2) \bigg] + |X_R|^2 + \text{Re}(\tilde{C}_{0\lambda} \lambda X_R + \tilde{C}_{0\eta} \eta X_R) = (6)
\]

and

\[
X_L = \frac{< m_\nu >}{m_e} (\chi_F - 1) + \eta^2 \chi_H + \ldots \quad X_R = \eta^2 X_H + \ldots
\]

\[
\chi_F = \left( \frac{g_Y}{g_A} \right)^2 \frac{M_F(0\nu)(0\nu)}{M_{GT}}
\]

\[
\chi_H = \left( \frac{g_Y}{g_A} \right)^2 \frac{M_{FH} - M_{GTH}}{M_{GT}}
\]
Figure 5. The excluded region in the space of the lepton violating parameters $<m_\nu>$ and $\eta$ entering neutrinoless double beta decay.

Figure 6. The constraint on the $\eta \lambda$ parameters extracted from $0\nu\beta\beta$ decay.

Curves from the axes. One may use neutrinoless double beta decay to extract information on the parameters of the right handed interaction $\kappa$ and $\epsilon$ from the parameters $\lambda$ and $\eta$ (see Fig. 5). From the definitions one finds:

$$\frac{\kappa}{\eta} = \frac{\lambda}{\eta}$$  \hspace{1cm} (7)

Furthermore Eq. 6 can be cast in the form 7, 8:

$$(\kappa^2 + \epsilon^2 + c_0 \epsilon \kappa) < \frac{m_p}{M_N} > = \eta_R^*, c_0 \approx 30$$  \hspace{1cm} (8)

Where $< \frac{m_p}{M_N} > = \sum_j (U_{ij}^H)^2 \Xi_j \frac{m_p}{M_N}$.

Using Eqs 7 and 8 one can extract limits on $m_{WR}$ and $\epsilon$. Thus, e.g., using:

$$\lambda = \frac{0.8}{\sqrt{2}} \times 10^{-6}, \eta = \frac{0.4}{\sqrt{2}} \times 10^{-8}, \eta_R^* = 0.8 \times 10^{-8}$$

we obtain the curves shown in Figs 7 and 8 as a function of the average heavy neutrino mass.

These should be compared with the values 6

$$m_{wR} \geq 715 GeV, \epsilon = 0.0016 \pm 0.0007$$

obtained from other experiments.

2. $0\nu\beta\beta$ DECAY IN R-PARITY NON-CONSERVING SUPERSYMMETRY

R-parity of supersymmetry is defined as

$$R_P = (-1)^{B+L+S}$$

($B$, $L$ and $S$ are the baryon, lepton numbers and the spin) and is assumed to be conserved. This allows the R-parity conserving superpotential

$$W_{Rp} = \lambda_i^c H_1 L_i E^c + \lambda_i^D H_1 Q_i D^c + \lambda_i^U H_2 Q_i U^c +$$
\[ \mu H_1 H_2, \] (9)

Once R-parity is not conserved one has additional lepton and baryon violating terms in the superpotential of the form

\[ W_{\text{noR}_p} = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \mu_j L_j H_2 + \lambda''_{ijk} U_i^c D_j^c D_k^c \] (10)

\[ Q_i = \left( \begin{array}{c} u_L \\ d_L \end{array} \right), \left( \begin{array}{c} \bar{u} \\ \bar{d} \end{array} \right), U_i^c = u_i^c(\bar{u}^*) \text{etc} \]

\[ L_i = \left( \begin{array}{c} \nu_L \\ e_L \end{array} \right), \left( \begin{array}{c} \bar{\nu} \\ \bar{e} \end{array} \right), E_i^c = e_i^c(\bar{e}^*) \text{etc} \]

One imposes a discrete symmetry, leading to \( \lambda''_{ijk} = 0 \) or \( \lambda''_{ijk} = 0 \), to avoid fast proton decay. In our case we will allow for lepton violation imposing \( \lambda''_{ijk} = 0 \). The bilinear and trilinear R-parity violating couplings give rise to massive Majorana neutrinos, which in turn contribute to \( 0\nu\beta\beta \) decay. Here we will consider additional contributions. The effective Lagrangian describing this process is \[ \mathcal{L}_{\text{eff}}^{\Delta L=2} = \frac{G_F^2}{2m_{\mu}} \bar{e}(1 + \gamma_5)e \epsilon[\eta_{PS} J_{PS} J_{PS} - \frac{1}{4} \eta_T J_{TT} J_{TT} + \eta_N J_{AT} J_{AT}], \]

The color singlet hadronic currents are

\[ J_{PS} = \bar{u}^c (1 + \gamma_5) d, J_{TT} = \bar{u}^c \sigma^{\mu\nu} (1 + \gamma_5) d \]

\[ J'_{AV} = \bar{u}^c \gamma^\mu (1 - \gamma_5) d \]

\( \alpha \) is a color index and \( \sigma^{\mu\nu} = (i/2) [\gamma^\mu, \gamma^\nu] \)

\[ \eta_{PS} = \eta_{\bar{e} e} + \eta_{\bar{\nu} \nu} + \eta_N + \eta_{\bar{\nu}} + 7 \eta_{\bar{\mu}}, \]

\[ \eta_T = \eta_{\bar{e} e} - \eta_{\bar{\nu} \nu} + \eta_N - \eta_{\bar{\mu}} \]

\[ \eta_{PS} = \eta_{\bar{e} e} + \eta_{\bar{\nu} \nu} + \eta_N + 7 \eta_{\bar{\mu}}, \]

\[ \eta_T = \eta_{\bar{e} e} - \eta_{\bar{\nu} \nu} + \eta_N - \eta_{\bar{\mu}} \]

with

\[ \eta_{\bar{\mu}} = \frac{\pi \alpha_s}{6} \lambda^\mu_{111} \frac{m_p}{G_F^2 m_{\tilde{\mu}}^2} \left[ 1 + \left( \frac{m_{\tilde{d}}}{m_{\tilde{\mu}}} \right)^4 \right] \]

\[ \eta_{\bar{e} e} = \frac{\pi \alpha_s}{2} \frac{\lambda^e_{111}}{G_F^2 m_{\tilde{\mu}}^2} \left[ \sum_{i=1}^{4} \frac{m_{\tilde{\mu}}}{m_{\chi_i}} \left( \frac{c_{\tilde{R}_i}(d) + c^2_{\tilde{L}_i}(u)}{m_{\tilde{d}} / m_{\tilde{\mu}}} \right)^4 \right] \]

\[ \eta_{\bar{\nu} \nu} = \frac{2 \pi \alpha_s}{12} \frac{\lambda^\nu_{111}}{G_F^2 m_{\tilde{\mu}}^2} \left( \frac{m_{\tilde{d}}}{m_{\tilde{\nu}}} \right)^2 \]

\[ \eta_{\bar{\nu}} = \frac{\pi \alpha_s}{2} \frac{\lambda^\nu_{111}}{G_F^2 m_{\tilde{\mu}}^2} \left( \frac{m_{\tilde{d}}}{m_{\tilde{\nu}}} \right)^2 \left( \sum_{i=1}^{4} \frac{m_{\tilde{\mu}}}{m_{\chi_i}} \left[ c_{\tilde{R}_i}(d) + c_{\tilde{L}_i}(u) \right] \right) \]

where

\[ \alpha_2 = g_2^2 / (4\pi), \quad \alpha_s = g_3^2 / (4\pi) \]

are SU(2)_L and SU(3)_c gauge coupling constants.

\[ m_{\tilde{\mu}}, m_{\tilde{d}}, m_{\tilde{\nu}} \] are masses of the u-squark, d-squark, gluino \( \tilde{g} \) and of the lightest neutralino \( \chi \), respectively.

\[ \chi = Z_\mu \hat{B} + Z_\mu \hat{W}^3 + Z_{H_1^0} \hat{H}_1^0 + Z_{H_2^0} \hat{H}_2^0 \]

Here \( \hat{W}^3 \) and \( \hat{B} \) are neutral SU(2)_L and U(1) gauginos while \( \hat{H}_1^0, \hat{H}_2^0 \) are higgsinos which are the superpartners of the two neutral Higgs boson fields \( H_1^0 \) and \( H_2^0 \). The neutralino couplings are defined as

\[ \epsilon_{\bar{e} e} = -T_3(\psi) Z_{\tilde{W}} + \tan \theta_W (T_3(\psi) - Q(\psi)) Z_{\tilde{B}}, \epsilon_{\bar{\nu} \nu} = Q(\psi) \tan \theta_W Z_{\tilde{B}}. \]

Here \( Q \) and \( T_3 \) are the electric charge and the weak isospin of the fields \( \psi = u, d, e \).

The lifetime for neutrinoless double beta decay now becomes:

\[ [T_{1/2}^{0\nu}]^{-1} = \frac{G_F |\eta_T M^{2N}_{\tilde{g}} + (\eta_{PS} - \eta_T) M^{2N}_{\tilde{f}} + \frac{3}{8} (\eta_T + \frac{5}{8} \eta_{PS}) M \pi N + \eta_N M \pi N|^2}{4 (\eta_T + \frac{5}{8} \eta_{PS}) M \pi N + \eta_N M \pi N} \]
where
\[
\mathcal{M}^2_{\tilde{q}} = c^2 \left[ a^0_{\tilde{p} \tilde{q}} M_{F_2} + a^0_{\tilde{A} \tilde{q}} M_{G_{T2}} + a^1_{\tilde{A} \tilde{q}} M_{G_{T1}} + a^1_{\tilde{A} \tilde{q}} M_{G_{T3}} \right],
\]
\[
\mathcal{M}^2_{\tilde{f}} = c^2 \left[ a^0_{\tilde{p} \tilde{f}} M_{F_2} + a^0_{\tilde{A} \tilde{f}} M_{G_{T2}} + a^1_{\tilde{A} \tilde{f}} M_{G_{T1}} + a^1_{\tilde{A} \tilde{f}} M_{G_{T3}} \right],
\]
\[
\mathcal{M}^\pi \! N = c^2 \left[ \frac{4}{3} a^1_{\tilde{p} \pi} (M_{G_{T-1\pi}} - M_{T-1\pi}) + a^2_{\tilde{A} \pi} (M_{G_{T-2\pi}} - M_{T-2\pi}) \right],
\]
\[
\mathcal{M}_N = c^2 \left[ \frac{g_2^2}{g_A^2} M_{F_N} - M_{G_{TN}} \right],
\]
\[
c^2 = \frac{m_e}{m_\nu} \left( \frac{m_A}{m_\nu} \right)^2.
\]

For comparison we write again the amplitude \(\eta_N|\mathcal{M}_N|^2\) associated with the heavy neutrino. The other terms come from SUSY, the first two correspond to the two nucleon mode, while the third term \(\mathcal{M}^\pi \! N\) corresponds to the pion mode. The pion mode is dominant [1], [8], recently confirmed with a different method [6]. One gets:
\[
a^1_{\tilde{p} \pi} = -0.044, \quad a^2_{\tilde{A} \pi} = 0.2 \quad \text{(Elementary particle treatment [8])}
\]
\[
a^1_{\tilde{p} \pi} = -0.012, \quad a^2_{\tilde{A} \pi} = 0.15 \quad \text{(Quark model [1])}
\]

Using these ME one obtains from the data the limits:
\[
\eta_N \leq \frac{1}{|\mathcal{M}_N|} \frac{1}{\sqrt{G_{01} T_{1/2}^\exp}}
\]
\[
\eta_{\text{SUSY}} \equiv \frac{3}{8} \left( \eta_T^2 + \frac{5}{8} \eta_{PS}^2 \right) \leq \frac{1}{|\mathcal{M}^\pi \! N|} \frac{1}{\sqrt{G_{01} T_{1/2}^\exp}}
\]

By employing standard techniques and working in the allowed SUSY parameter space one can obtain the masses and couplings involving the intermediate SUSY particles (gluinos, neutralinos, squarks and s-leptons) and their couplings, which enter the expression for \(\eta_{\text{SUSY}}\). Thus from the limits on this lepton violating parameter, one can constrain and obtain limits on the unknown R-parity violating parameter \(\lambda'_{111}\).

3. BRANE WORLD

It must have become clear from the above discussion that \(0\nu\beta\beta\) decay has something to say in any model, which predicts massive Majorana
neutrinos. We cannot possibly discuss all such models. In models involving extra dimensions, however, one has the special feature that they predict Dirac neutrinos. Furthermore such models predict sterile neutrinos with a mass in the keV-MeV region. In such cases the particle and nuclear physics get entangled. So the effective neutrino mass extracted may depend on the nucleus.

### 4. NUCLEAR MATRIX ELEMENTS

The discussion of the nuclear matrix elements is outside the scope of this talk. So we will limit ourselves to some brief comments. The first step towards their evaluation consists in deriving the effective transition operator. Particle physics dictates the structure of the operator at the quark level. The next step is going from the quark to the nucleon level. This is quite straightforward in the case of light neutrinos. Since, however, the momenta of the intermediate neutrinos are, by nuclear standards, quite high, \( \approx 70 \text{MeV/c} \), one must consider corrections to the structure of the nucleon current \( \Pi \) (induced pseudoscalar etc), which tend to decrease the nuclear \( |ME|^2 \) by \( 20 - 30\% \). This causes a corresponding increase in the extracted neutrino mass, regardless of the nuclear model. For heavy intermediate particles the situation is worse. The effective operator is short ranged. So, if one considers only nucleons, the nucleon must not be point like, but as a bag of quarks or with a suitable form factor. Other mechanisms must also be considered, e.g. \( 0\nu\beta\beta \) decay induced by pions in flight between nucleons (pion mode). This mechanism tends to dominate in the case of SUSY contributions. The next step is to obtain the needed nuclear wave functions. This is a formidable task, since the nuclear systems undergoing \( 0\nu\beta\beta \) decay have complicated structure. Furthermore the obtained nuclear matrix elements are quite small compared to a canonical value (sum rule) like \( \sum_f |ME(i \to f)|^2 \) (the summation is over all final states, not just those energetically allowed). So normally small effects maybe important here. The basic methods employed in the evaluation of these matrix elements \( \Pi, \Omega \) are:

1. The Quasiparticle Random Phase Approximation (QRPA). One builds the needed intermediate states as RPA excitations on a quasiparticle vacuum (one for each ground state). One can include a large number of single particle states, since the number of the resulting configurations is manageable. This method has been applied in almost all systems. Various refinements have been incorporated (proton neutron-pairing, Pauli principle corrections, \( g_{pp} \) renormalization factors etc). The thus obtained matrix elements (ME) by various authors still have a spread larger than a factor of 2.

2. The Large Basis Shell Model. With modern computers it has been possible to use the nuclear shell model in obtaining the nuclear wave function. One uses a single particle model space more restricted than that employed in the QRPA, since soon the number of resulting configurations becomes prohibitively large. With increasing interest in \( 0\nu\beta\beta \) decay we expect the nuclear theory groups will improve on this in the remainder of the decade, forming consortia, if necessary.

<table>
<thead>
<tr>
<th>1/(a)</th>
<th>(\langle m\rangle) [eV]</th>
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<tr>
<td>[GeV]</td>
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<tr>
<td>(10^{10})</td>
<td>(\lesssim 10^{-6})</td>
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</table>

Table 2

**Numerical estimates of \(\langle m\rangle\) for different nuclei in a 5-dimensional brane-shifted model.** (For choice of parameters and further details see [21].)
We thus hope that by the time the ongoing and planned experiments give the anticipated results the nuclear matrix elements will be known with an accuracy better than a factor of 2. The reliability of these matrix elements can be tested, once the experimental results become available, by taking ratios, which, assuming that one mechanism dominates, are independent of the lepton violating parameters (ratios of lifetimes of different nuclei or ratios of transitions to different final states of the same nucleus).

5. RESULTS

$\nu\beta\beta$ decay is our best hope to set the scale for neutrino mass. Based on the available experimental results and using the range of uncertainty in the nuclear matrix elements the following limits emerge:

$$<m_\nu> <0.45-1.4 \text{eV} \left[ ^{76}\text{Ge}, \frac{2}{5}1-1.5 \text{eV} \right]$$

$$\lambda < (0.8-2.1) \times 10^{-6} \left[ ^{76}\text{Ge}, (2.5-7.3) \times 10^{-6} \right] \left[ ^{136}\text{Xe} \right]$$

$$\eta < (0.4-1.8) \times 10^{-8} \left[ ^{76}\text{Ge}, (0.9-3.7) \times 10^{-8} \right] \left[ ^{128}\text{Te} \right]$$

Other mechanisms may dominate $\nu\beta\beta$ decay. One popular scenario is to be mediated not by neutrinos, but by SUSY particles. From $\eta_{\text{SUSY}}$ for each target in the order of table 3, we extract the value of $\lambda_{\eta_{111}}$:

$$(1.0,1.8,1.8,0.8,1.8,0.9,2.3) \times 10^{-3}$$

We should not forget, however, that these values depend on the SUSY parameter space, in particular the SUSY scale, see Figs 4 and 10.

Even if the neutrinos do not dominate in $\nu\beta\beta$ decay, its observation will demonstrate that they are massive Majorana particles.

REFERENCES

3. A. Faessler and F. Šimkovic,
4. P. Vogel, Double Beta Decay: nucl-th/0005020 nucl-th/9904065

<table>
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<tr>
<th>(A,Z)</th>
<th>$\langle m_\nu \rangle$ [eV]</th>
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<th>$\langle \eta \rangle \times 10^{-8}$</th>
<th>$\langle \eta_{\text{SUSY}} \rangle \times 10^{-8}$</th>
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