The ultimate outcome of black hole – neutron star mergers

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ABSTRACT
We present a simple, semi-analytical description for the final stages of mergers of black hole (BH) – neutron star (NS) systems. Such systems are of much interest as gravitational wave sources and gamma–ray burst progenitors. Numerical studies show that in general the neutron star is not disrupted at the first phase of mass transfer. Instead, what remains of the neutron star is left on a wider, eccentric, orbit. We consider the evolution of such systems as they lose angular momentum via gravitational radiation and come into contact for further phases of mass transfer. During each mass transfer event the neutron star mass is reduced until a critical value where mass loss leads to a rapid increase in the stellar radius. At this point Roche lobe overflow shreds what remains of the neutron star, most of the mass forming a disc around the black hole. Such a disc may be massive enough to power a gamma–ray burst. The mass of the neutron star at the time of disruption (and therefore the disc mass) is largely independent of the initial masses of the black hole and neutron star, indicating that BH–NS star mergers may be standard candles.

Key words: binaries: close – gamma rays: bursts – stars: neutron.

1 INTRODUCTION

Binary systems consisting of a pair of compact objects (neutron stars (NS) or black holes (BH)) have long been of interest as tests of relativity and gravitational wave sources (Hulse and Taylor, 1975) or gamma-ray burst (GRB) progenitors (see e.g. Fryer, Woosley & Hartmann, 1999). Their formation requires that the system remains bound through the two supernovae creating the compact objects. While there exist many possible channels for the formation of NS–NS binaries, relatively few exist for the formation of BH–NS (or BH–BH) systems (Belczynski et al. 2002). The basic mechanism for the formation of BH–NS systems is non-conservative mass transfer between the two massive stars in a binary, common envelope evolution (either before or after the first SN event) to dissipate angular momentum and mass, and then finally a second SN. These systems subsequently lose further angular momentum via gravitational radiation and a substantial fraction merge within Hubble time (See Portegies Zwart & Yungelson 1998; Belczynski et al. 2002 for more information on the possible channels for BH–NS creation). While it is now thought that at least the majority of GRBs with durations longer than 2s result from core collapse of massive stars to supernovae (Hjorth et al. 2003; Stanek et al. 2003) binary mergers remain a likely candidate for the distinct class of GRBs with durations less than 2s (Kouveliotou et al. 1993). If this is the case then BH–NS mergers may contribute significantly to the short–GRB population.

High–resolution hydrodynamical simulations have proved a valuable tool in probing the last few orbits and final mergers of compact binary systems (Janka et al. 1999; Rosswog et al. 1999, Rosswog & Davies 2002; Rosswog & Liebendorfer 2003) and as their potential as short–GRB progenitors (e.g. Rosswog & Ramirez-Ruiz 2003). Much effort has gone into the understanding of double neutron star systems as progenitors, while BH–NS systems have also received attention (Kluzniak & Lee 1998; Portegies Zwart 1998; Lee 2000,2001; Janka et al. 1999; Rosswog, Speith & Wynn 2004). To be a viable GRB progenitor a compact object merger must make a massive torus (∼ 0.1 - 1 M⊙) surrounding a BH, or supramassive neutron star (e.g. Ruffert & Janka 1999; Veitrap & Stella 1998). The subsequent extraction of energy from this torus provides power for the GRB, possibly via neutrino–antineutrino annihilation. The formation of this torus is vital to the production of a GRB. In collapsar models the formation of a massive torus is straightforward since, in contrast to the mergers, a large mass (0.1 - 5 M⊙) reservoir is available from the progenitor star (MacFadyen, Woosley & Heger, 2001). In NS–NS systems of approximately equal mass it is also possible to form a massive disk (see e.g. Rosswog & Liebendorfer 2003). However in the case of BH–NS systems the large mass ratio can dramatically affect the outcome.
of the merger. For soft equations of state (e.g. polytropes with \( \Gamma = 2 \)) it is possible to have complete disruption on the first approach and thus is a reasonable mechanism for GRB production (Lee & Kluzniak 1999). However, for more realistic (harder) equations of state, such as that of Shen et al. (1998), the situation is less clear. When the neutron star spirals into contact with the black hole for the first time, a short burst of mass transfer on to the black hole follows, with mass transfer rates in excess of \( 100 \, M_\odot \, s^{-1} \). Crucially, much of the angular momentum of the transferred gas is fed back into what remains of the neutron star, which is then left on an eccentric, and wider, orbit. This orbit may be sufficiently wide that the neutron star no longer fills its Roche lobe (see eg Lee 2000). Angular momentum axis which is sufficiently large that the neutron star no longer

The time required to bring the system back into contact is

\[ T_{\text{contact}} = \frac{G(M_{\text{bh}} + M_{\text{ns}})}{c^2 a^3 (1 - e^2)^{7/2}} \times \left( 1 + \frac{121}{304} e^2 \right) \]  

(2)

Taking typical values of \( a = 150 \, \text{km} \) and \( e = 0.2 \), the timescale for the system to return to contact ranges from about 0.025 s (for \( M_{\text{bh}} = 14M_\odot \)) to 1 s (for \( M_{\text{bh}} = 2M_\odot \)). When the system comes into contact again, another pulse of mass transfer occurs. We note that the system is spiralling together rapidly when mass transfer begins, with typical values for \( \delta a/a \approx 0.05 - 0.10 \) in one orbital period; the subsequent mass transfer is therefore dissimilar from that encountered in the stable evolution of circular binaries. A large fraction of the material from the neutron star is stripped, and what remains gets a kick, leaving it (again) on a somewhat eccentric orbit with an increased semi-major axis.

3 EVOLUTION VIA MASS TRANSFER DOWN TO THE MINIMUM MASS.

In this section we model the evolution of a BH–NS binary using a two–body code, treating gravitational radiation in the quadrupole approximation, and allowing for mass transfer when the neutron star fills its Roche lobe, assuming the mass is transferred instantly to the black hole, but (crucially) returning some fraction of the angular momentum of the transferred material back to the neutron star on the timescale of the orbital period when mass transfer occurs.

The energy released in gravitational radiation is estimated by (see eg Zhuge et al 1994)

\[ L_{\text{gw}} = \frac{G}{5c^5} I_{ij}^{(3)} I_{ij}^{(3)} \]  

(3)

where \( I_{ij}^{(3)} \) is the third time derivative of the reduced quadrupole moment, \( I_{ij} \) and we sum over repeated indices. As the BH–NS binary can be defined to sit in the xy–plane, we need only consider three components of \( I_{ij} \), namely

\[ I_{xx} = \sum_{p=1}^{2} M_p \left( x_p^2 - r_p^2 / 3 \right) \]

(4)

\[ I_{yy} = \sum_{p=1}^{2} M_p \left( y_p^2 - r_p^2 / 3 \right) \]

(5)

\[ I_{xy} = \sum_{p=1}^{2} M_p x_p y_p \]

(6)

By computing the third time derivatives of the quadrupole moments, we can calculate the rate of energy loss via gravitational radiation. This energy loss rate can be applied to the two bodies via a drag force (see eg Zhuge et al 1994).

Mass transfer occurs when the neutron star fills its Roche lobe, ie when the separation is given by

\[ a_{\text{contact}} = \frac{R_{\text{ns}}}{0.49} \left( \frac{M_{\text{ns}} + M_{\text{bh}}}{M_{\text{ns}}} \right)^{1/3} \]

(7)

where we make the approximation that the neutron–star radius \( R_{\text{ns}} = 15 \, \text{km} \) for all neutron star masses. As we will see shortly, the rapid increase in the actual radius for a real neutron star once its mass falls below about 0.2 \( M_\odot \) plays a crucial role in the subsequent evolution of the system.
To calculate the mass transferred from the neutron star, $\delta M_{\text{ns}}$, we estimate the change in separation occurring over half an orbit, i.e.

$$\delta a = \frac{1}{2} (\dot{a} \times \tau_{\text{orb}})$$

and then assume that all material outside the Roche lobe at a separation $a_{\text{contact}} - \delta a$ is transferred. Given that the equation of state for the gas in a neutron star is very hard (i.e. the gas is very resistant to compression), we approximate a neutron star as a sphere of constant density. Hence the mass transferred is given by

$$\delta M_{\text{ns}} = M_{\text{ns}} \frac{a_{\text{contact}}^3 - (a_{\text{contact}} - \delta a)^3}{a_{\text{contact}}}$$

We make the approximation that the mass transfer is instantaneous, but return half of the material’s angular momentum to the neutron star over the timescale of one orbital period. This injection of angular momentum leaves the neutron star on a somewhat eccentric and, crucially, somewhat wider orbit. Typically, we find that the neutron star no longer fills its Roche lobe. We assume that no mass transfer occurs until the system has spiralled in to contact when we repeat the procedure described above. Thus the material within the neutron star is transferred to the black hole via discrete bursts of mass transfer separated by relatively long quiescent periods whilst the neutron star spirals in to contact via the emission of gravitational radiation. In reality, there may well be some mass loss from the neutron star during the quiescent periods. However, computer simulations show that this rate of mass loss is relatively small (e.g. Rosswog et al. 2004). Such low rates of mass loss may be due to oscillations within the neutron star induced by the more violent phases of mass transfer occurring when the neutron star fills its Roche lobe.

We show the evolution of one BH–NS binary in Figs 1 and 2. Here we have taken the black hole mass, $M_{\text{bh}} = 14M_\odot$, and $M_{\text{ns}} = 1.4M_\odot$. We begin the integration of the orbit with a separation of 300 km. In Fig. 1 we show the separation as a function of time around the onset of the first contact. Over half of the material of the neutron star is transferred to the black hole at a time of $\sim 0.56$ s. What remains of the neutron star is left on an eccentric and slightly wider orbit with the neutron star no longer filling its Roche lobe. The behaviour of the separation as a function of time shown in Fig. 1 is very similar to that seen in hydrodynamical numerical simulations (e.g. Lee 2000; Rosswog et al. 2004). One limitation of essentially all hydrodynamical numerical simulations of BH–NS binaries is that they are only able to integrate for up to 0.1 s or so after the first phase of mass transfer. With our much simpler two–body code, we are able to follow the evolution of the BH–NS system through subsequent phases of mass transfer. This is shown in Fig. 2, where we plot the separation as a function of time in the top half of the figure, and the neutron–star mass as a function of time in the lower half of the figure. From this figure we see that the neutron star quickly loses most of its mass. The mass transfer occurring at about 0.5 s and 2 s gives the neutron star a relatively large kick, placing it into very eccentric orbits and increasing the pericentric separation to over 150 km. Once the mass falls to about 0.2 $M_\odot$, the amount of mass transferred declines (because of a lower inspiral rate, $\dot{a}$, when the system comes into contact). The separation also shows very little increase when mass transfer occurs at this point. Also at such low masses, the neutron star radius increases rapidly as the mass approaches the minimum mass (e.g. Figure 1, Davies et al. 2002). We find that, after allowing for an increase in the neutron star radius once the mass drops below 0.2 $M_\odot$, a brief phase of mass transfer no longer detaches the system, i.e. mass transfer continues as the NS is still filling...
its Roche lobe. Rather than having a discrete burst of mass flow onto the black hole, the neutron star will experience continuous mass loss once its mass drops to below about 0.2 $M_\odot$. We find that this result is true for all black–hole masses we considered. (3.5$M_\odot \lesssim M_{bh} \lesssim 14.0M_\odot$).

In the results shown earlier, we assumed that half of the angular momentum was returned to the donor. This is a reasonable figure; we would expect some angular momentum to be returned to the donor although some might well be carried into the black hole with the gas. Also, our results shown earlier are in good overall agreement with hydrodynamical simulations (Lee 2000; Ruffert & Janka 2003, Rosswog et al 2004). We see similar amounts of mass transfer from the neutron star to the black hole during the first phase of mass transfer and what remains of the donor is left on a similar orbit in our calculations and hydrodynamical simulations. Changing the fraction of angular momentum returned to the donor does affect the subsequent trajectory of the donor; if more angular momentum is returned to the donor, it will be left on a wider orbit. However the system will still spiral in to contact again, hence the overall picture of the evolution of the system remains unchanged. The neutron star mass is reduced to 0.2 $M_\odot$ in discrete phases of mass transfer on a timescale of $\sim 1 - 10$ s.

4 EVOLUTION AT THE MINIMUM MASS

To investigate what is likely to happen next, we must consider the response of the neutron star to mass loss and compare this to the response of the Roche lobe. As given in equation (7), the Roche lobe radius can be written as

$$R_L \propto a \left( \frac{M_{ns}}{M} \right)^{1/3}$$  \hspace{1cm} (10)

Differentiating this with respect to time results in

$$\frac{\dot{R}_L}{R_L} = \frac{\dot{a}}{a} + \frac{1}{3} \frac{\dot{M}_{ns}}{M_{ns}}$$  \hspace{1cm} (11)

For what follows we assume conservative mass transfer (i.e. $M_{bh} + M_{ns} = \text{constant}$: all of the mass lost by the NS is accreted by the BH). While in practice this may not be the case any deviations away from conservative mass transfer act to destabilise the mass transfer further and encourage the rapid formation of a disc.

The total angular momentum in the system is given by

$$J = M_{bh} M_{ns} \left( \frac{G a}{M} \right)^{1/2},$$  \hspace{1cm} (12)

which can be differentiated to give,

$$\frac{\dot{J}}{J} = \frac{M_{bh}}{M_{bh}} + \frac{\dot{M}_{ns}}{M_{ns}} + \frac{\dot{a}}{2a}$$  \hspace{1cm} (13)

Thus combining equations (11) and (13) gives us,

$$\frac{\dot{R}_L}{R_L} = 2 \frac{\dot{J}}{J} - 2 \frac{\dot{M}_{ns}}{M_{ns}} \left[ \frac{5}{6} - \frac{M_{ns}}{M_{bh}} \right]$$  \hspace{1cm} (14)

Including the response of the neutron star as it loses mass, and assuming the neutron star continues to just fill its Roche lobe as mass is transferred, we arrive at the following equation

$$\frac{M_{ns}}{M_{ns}} = \frac{J / J}{\left( \frac{5}{6} - \frac{M_{ns}}{M_{bh}} \right)}$$  \hspace{1cm} (15)

where $\zeta$ relates the change in the neutron–star radius to the change in its mass and is given by

$$\frac{\dot{R}_{ns}}{R_{ns}} = \zeta \times \frac{\dot{M}_{ns}}{M_{ns}}$$  \hspace{1cm} (16)

For a given amount of angular momentum loss $\dot{J}$, equation (15) gives the required value of $\dot{M}_{ns}$ for the system to transfer material in a stable fashion (i.e. where the neutron star just fills its Roche lobe). A key feature of equation (15) is that as $\zeta$ decreases towards $-5/3 - 2M_{ns}/M_{bh}$, the required mass transfer rate becomes increasingly large, to the point where the neutron star transfers all its mass to the black hole in less than one orbital period. In other words, the neutron star gets shredded. The exact neutron-star mass at which this occurs depends on the mass–radius relation but must be at a value slightly lower than 0.2 $M_\odot$ where the radius begins to expand rapidly (eg see Figure 1 in Davies et al 2002).

Rosswog, Speith and Wynn (2004) calculate the conditions under which the flow from a neutron star form an accretion disc (rather than directly accreting on to the BH). They find that for co–rotating systems and low mass ratios ($M_{bh}/M_{ns} < 5$) disc formation is unlikely. However in our scenario the final mass of the neutron star is almost always $< 0.2M_\odot$, so the mass ratio is almost always $> 5$. Under these circumstances it is inevitable that the circularisation radius is greater than both the Schwarzschild radius and the innermost stable orbit.

We conclude that in BH–NS systems where mass transfer is initially episodic, the neutron star is shredded once its mass is reduced to about 0.15 – 0.20 $M_\odot$, producing a disc containing a large fraction of the material (some mass loss may occur) with a radius in the range 150 – 250 km (depending on the black–hole mass). The mass within this disc is sufficient for GRB formation. We note that the energy released by the formation of nuclei (of order 10 MeV/nucleon) is insufficient to eject most of the material from the system provided $M_{bh} \geq 3M_\odot$.

An interesting consequence of the evolution described above is that the mass of the final, GRB–producing disc is largely independent of the initial mass of the NS (or BH). Since formation occurs at some minimum NS mass the majority of this mass is used in the formation of the disc and be approximately the same for each system. Figure 2 shows that the fractional loss of mass per encounter is sufficiently small (for the later mass transfer events which occur closer to the minimum mass) that the range of NS mass on the contact at which disc formation occurs is low. The disc in BH–NS mergers could thus offer a standard energy reservoir, making the events standard candles.

5 CONCLUSIONS

We have considered the final merger of BH–NS systems. The entire process, beginning with the first phase of mass transfer and ending with the disruption of the neutron star, has a relatively long duration (a few seconds, or equivalently
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many thousands of orbital periods). Complete hydrodynamical modelling of the entire merger process is therefore impossible. Instead we have used a simpler, semi-analytical treatment for the episodes of mass transfer from the neutron star to the black hole. In our model we evolve the BH and NS as point masses whose orbits circularise and shrink via gravitational radiation losses. This drag force brings the NS and BH into contact as the NS fills its Roche lobe. We assume instantaneous mass transfer from the NS to the BH, with some fraction of angular momentum being transferred back to the NS on an orbital timescale. This places what remains of the neutron star on an eccentric and somewhat wider orbit with the neutron star no longer filling its Roche lobe. The system then spirals together via the emission of gravitational radiation until the neutron star fills its Roche lobe for a second time. More material is transferred and the neutron star receives a second kick. This process repeats until the neutron star reaches a mass slightly below 0.2 M⊙ when its radius increases rapidly on mass loss. At this point, the neutron star remains in contact with the black hole and a (brief) continuous phase of mass transfer probably ensues. The mass–radius relation for the neutron star is so steep at this point that the mass transfer is dynamically unstable and the neutron star is shredded and forms a disc (with a radius in the range 150 – 250 km) around the black hole. This disc is sufficiently massive (ie around 0.1M⊙) for GRB production and therefore the prospect that BH–NS systems can produce observable GRBs is good. Furthermore the independence of disc mass on the initial NS mass indicates that GRBs produced via this method may have an approximately standard energy.

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