Non-BPS D-brane Near NS5-branes

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Abstract: We use tachyon field theory effective action to study the dynamics of a non-BPS Dp-brane propagating in the vicinity of $k$ NS5-branes. For the time dependent tachyon condensation we will concentrate on the case of the large tachyon and the case when a non-BPS D-brane is close to NS5-branes. For spatial dependent tachyon condensation we will argue that the problem reduces to the study of the motion of an array of D(p-1)-branes and D(p-1)-antibranes in the vicinity of $k$ NS5-branes.

Keywords: D-branes.
1. Introduction

In recent paper by Kutasov [1] the problem of the effective field theory description of the dynamics of BPS Dp-brane in the background of k NS5-branes has been analysed \(^1\). More precisely, Kutasov considered the stack of k parallel NS5-branes in type II string theory, stretched in the directions \((x^1, \ldots, x^5)\) and localised in \(x = (x^6, x^7, x^8, x^9)\) \(^2\). The D-branes that were studied are "parallel" to the fivebranes, i.e. they are extended in some of the fivebrane worldvolume directions \(x^\mu\) and pointlike in the directions transverse to the fivebranes \((x^6, x^7, x^8, x^9)\). Without loss of generality, we can take the worldvolume of the Dp-brane the directions \((x^0, x^1, \ldots, x^p)\). We will label the worldvolume of the Dp-brane by \(x^\mu\) as well, but here the index \(\mu\) only runs over the range \(\mu = 0, 1, 2, \ldots, p\) with \(p \leq 5\).

**1**For other works considering related problems, see [2, 3, 4, 5, 6, 7, 8].

**2**The directions along worldvolume of the fivebranes will be denoted by \(x^\mu, \mu = 0, 1, 2, \ldots, 5\); those transverse to the branes will be labelled by \(x^m, m = 6, 7, 8, 9\). We also use convention \(l_s = 1\).
to tachyon condensation on unstable D-brane \[1, 10, 11, 12\]. The very interesting outcome of the recent work on real time tachyon condensation is the observation that an effective action of Dirac-Born-Infeld (DBI) type for the tachyon \[14, 15, 16, 17\] captures surprisingly well many aspects of rolling tachyon solutions of the full open string theory \[18, 19\] and is thus very useful for studying these processes. The origin of this agreement was partially clarified in \[20, 21\], but see also \[22\]. Some important questions considering unstable D-branes in string theory were also recently discussed in \[3\].

As a next step in this research it seems to be natural to consider an unstable non-BPS Dp-brane in the background of $NS5$-branes and this paper is devoted to the study of this problem. We start in section \(2\) with the general description of parallel unstable non-BPS Dp-brane in the background of $k$ NS5-branes using the form of the tachyon effective DBI action proposed in \[14, 15, 16, 17\]. Then, following \[3\] we will concern on the radial motion of the non-BPS D-brane in the background of $k$ coincident $NS5$-branes. We will see that this motion depends on the form of the tachyon condensation. In the first case we will consider the time dependent tachyon condensation on the worldvolume of a non-BPS Dp-brane. We will analyse in section \(3\) this problem using the tachyon effective action proposed in \[19, 20, 21\]. We will generalise this action to the nontrivial spacetime metric and dilaton. We will show that in this form of the tachyon effective action the tachyon field appears as an additional embedding coordinate and a non-BPS Dp-brane action looks like the DBI action of Dp-brane embedded in eleven dimensional manifold with specific form of the metric and dilaton field. Using this form of the tachyon effective action we will study the time dependent tachyon condensation on the unstable D-brane in the case when the tachyon is large and non-BPS D-brane is close to NS5-brane. We will demonstrate that in this region of the tachyon and radial mode field theory space the effective action posses additional global symmetry. Then with the help of the corresponding conserved charge we will be able to find the relation between the tachyon and radial mode. Then from the expression for conserved energy we will get a differential equation that describes time evolution of the radial mode and that can be solved explicitly.

In section \(5\) we will study the situation when the tachyon depends on the spatial coordinate on the worldvolume of non-BPS D-brane while the radial mode is function of time. In this case the radial mode and the tachyon decouple and can be studied separately. In particular, the spatial dependent tachyon condensation leads to the emergence of an array of codimension one $D(p-1)$-branes and $D(p-1)$-antibranes \[18, 23, 24, 25\]. On the other hand the dynamics of the radial mode describes the collective motion of this system and its time dependence is the same.

\footnote{For recent review of the relation between tachyon condensation, matrix models and Liouville theory, see \[13\] where more extensive list of relevant papers can be found.}
as in the case of BPS D-brane \[1\].

Finally, in conclusion \((3)\) we outline our results and suggest possible directions for further research.

2. The effective action for non-BPS D-brane in the \(NS5\)-brane background

We restrict to the case of weak coupling, when \(NS5\) branes are much heavier than D-branes—their tension goes as \(1/g_s^2\) while that of D-branes as \(1/g_s\). As in the paper \([1]\) we study the dynamics of a D-brane in vicinity of fivebranes when we regard the fivebranes as static and study the motion of non-BPS D-branes in their gravitation potential.

The background field around \(k\) \(NS5\)-branes are given be the solution \([26]\), where the metric, dilaton and NS \(B\)-field are

\[
  ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + H(x^m)dx^m dx^m, \\
  e^{2(\Phi - \Phi_0)} = H(x^n), \\
  H_{mnp} = -\epsilon_{mnop}\partial_q\Phi. 
\]

(2.1)

Here \(H(x^n)\) is the harmonic function describing \(k\) fivebranes and \(H_{mnp}\) is field strength of the \(NS B\)-field. For fivebranes at generic positions \(x_1, \ldots, x_k\) we have

\[
  H = 1 + \sum_{j=1}^{k} \frac{1}{|x - x_j|^2}. 
\]

(2.2)

For coincident fivebranes, where an \(SO(4)\) symmetry group of rotation is preserved, the harmonic function (2.2) reduces to

\[
  H = 1 + \frac{k}{r^2}, r = |x| .
\]

(2.3)

Our goal is to study non-BPS \(D_p\)-brane stretched in directions \((x^1, \ldots, x^p)\). We will label the worldvolume of the D-brane by \(\xi^\mu, \mu = 0, \ldots, p\) and use reparametrization invariance of the worldvolume of the D-brane to set \(\xi^\mu = x^\mu\). As in case of the stable D-brane the position of the D-brane in the transverse directions \((x^6, \ldots, x^9)\) gives to rise to scalar fields on the worldvolume of D-brane, \((X^6(\xi^\mu), \ldots X^9(\xi^\mu))\). Following \([14, 15, 16, 17]\) we can then presume that the dynamics of the non-BPS \(D_p\)-brane is governed by the action

\[
  S = -\tau_p \int d^{p+1}\xi V(T)e^{-\Phi - \Phi_0} \sqrt{-\det(G_{\mu\nu} + B_{\mu\nu} + \partial_\mu T \partial_\nu T)},
\]

(2.4)
where $\tau_p$ is tension of non-BPS Dp-brane and where $V(T)$ is tachyon potential. It was conjectured in [19, 20] that this potential has the form

$$V(T) = \frac{1}{\cosh \frac{T}{\sqrt{2}}}.$$  \hspace{1cm} (2.5)

We must also mention that the determinant in (2.4) runs over the worldvolume directions $\mu = 0, \ldots, p$, $G_{\mu\nu}$ and $B_{\mu\nu}$ are induced metric and $B$ field on the non-BPS Dp-brane

$$G_{\mu\nu} = \frac{\partial X^A}{\partial \xi^\mu} \frac{\partial X^B}{\partial \xi^\nu} G_{AB}(X),$$

$$B_{\mu\nu} = \frac{\partial X^A}{\partial \xi^\mu} \frac{\partial X^B}{\partial \xi^\nu} B_{AB}(X),$$  \hspace{1cm} (2.6)

where the indices $A, B = 0, \ldots, 9$ run over the whole ten dimensional spacetime so that $G_{AB}$ and $B_{AB}$ are metric and $B$-field in ten dimensions.

As in the paper [1] we will consider the situation when we place NS5-branes at $x = 0$ and restrict to the pure radial fluctuations of non-BPS Dp-brane in the transverse $R^4$ labelled by $x$. Then the only massless field excited on the brane is $R(\xi^\mu) = \sqrt{X^m X_m(\xi^\mu)}$. This restriction to the radial motion is consistent since, for coincident NS5-branes the background (2.1) is $SO(4)$ invariant. Since then $B$ field is in the angular directions and angular degrees of freedom are not excited, the induced $B$ field vanishes. Then the induced metric takes the form

$$G_{\mu\nu} = \eta_{\mu\nu} + \partial_\mu X^m \partial_\nu X^n G_{mn},$$  \hspace{1cm} (2.7)

so that together with inclusion of tachyon we obtain the action

$$S = -\tau_p \int d^{p+1} \xi V(T) \frac{1}{\sqrt{H}} \sqrt{-\det (\eta_{\mu\nu} + H(R) \partial_\mu R \partial_\nu R + \partial_\mu T \partial_\nu T)}.$$  \hspace{1cm} (2.8)

We can now define the new \textquotedblright tachyon \textquotedblright field $T$ via the relation

$$\frac{dT}{dR} = \sqrt{H(R)} = \left[ 1 + \frac{k}{R^2} \right].$$  \hspace{1cm} (2.9)

In terms of this variable the non-BPS D-brane effective action has the form

$$S = -\int d^{p+1} \xi V(T, T) \sqrt{-\det (\eta_{\mu\nu} + \partial_\mu T \partial_\nu T + \partial_\mu T \partial_\nu T)},$$  \hspace{1cm} (2.10)

where we have defined the potential $V(T, T)$ as

$$V(T, T) = \frac{\tau_p}{\sqrt{H(R(T))}} \frac{1}{\cosh \frac{T}{\sqrt{2}}}.$$  \hspace{1cm} (2.11)
It is interesting to analyse the asymptotic behaviour of the potential $V$. Namely, for $T \to -\infty$ we obtain

$$V = \tau_p e^{\sqrt{\frac{T}{2}}} \frac{1}{\cosh(\sqrt{\frac{T}{2}})} .$$

(2.12)

Note that for $k = 2$ and for $T \to -\infty$ the potential (2.12) takes the form

$$V = 2\tau_p e^{\sqrt{\frac{T}{2}}} \frac{1}{\sqrt{2}} .$$

(2.13)

Then one can see that non-BPS Dp-brane action (2.10) is invariant under $Z_2$ symmetry

$$Z_2 : T' = T , T' = k T .$$

(2.14)

Another enhanced $Z_2$ symmetry emerges in the limit $T \to -\infty , T \to \infty$ when (2.12) can be written as

$$V = 2\tau_p e^{\sqrt{\frac{T}{2}}} \frac{1}{\sqrt{2}} .$$

(2.15)

It is easy to see that now the action (2.10) is invariant under the transformation

$$T' = -T , T' = -T .$$

(2.16)

In other words for $k = 2$ one can find symmetry between $T$ and $T$ at asymptotic regions $T \to -\infty , T \to \pm\infty$. It is also clear that these symmetries are broken for finite values of $T , T$ and for $k \neq 2$.

3. Solution of the equation of motion

In this section we begin the general discussion of the solutions of the equations of motion of the non-BPS D-brane action (2.10) for the case of coincident $NS5$-branes.

Let us now presume that the fields representing the transverse position of D-brane, $X^m , m = 6, 7, 8, 9$ depend only on time $X^m = X^m(t)$. In this case the induced $B$ field vanishes and the induced metric takes the form

$$G_{\mu\nu} = \eta_{\mu\nu} + \delta^0_{\mu} \delta^0_{\nu} \dot{X}^m \dot{X}^m H(X^n)$$

so that we find following action for $X^m$ and $T$

$$S = -\tau_p \int d^{p+1}\xi \frac{V(T)}{\sqrt{H(X^n)}} \sqrt{\det(I + M)} ,$$

(3.2)

where $I^\mu_{\nu} = \delta^\mu_{\nu}$ is $(p + 1) \times (p + 1)$ unit matrix and where we have also defined $(p + 1) \times (p + 1)$ matrix $M^\mu_{\nu}$ as

$$M^\mu_{\nu} \equiv \eta^{\mu\kappa} \delta^0_{\nu} \delta^0_{\kappa} \dot{X}^m \dot{X}^m H(X^n) + \eta^{\mu\kappa} \partial_{\kappa} T \partial_{\nu} T .$$

(3.3)
Then the equations of motion for $X^m$ that arise from (3.2) take the form

$$\begin{align*}
- \frac{\partial_m H(X^n)}{2H(X^n)^{3/2}} V(T) \sqrt{\det(I + M)} - & \\
+ \frac{d}{dt} \left( \frac{1}{\sqrt{H(X^n)}} V(T) \dot{X}^m H(X^n) \left((I + M)^{-1}\right)_0 \sqrt{\det(I + M)} \right) + & \\
- \frac{1}{2\sqrt{H(X^n)}} V(T) \dot{X}^n \dot{X}^m \partial_m H(X^n) \left((I + M)^{-1}\right)_0 \sqrt{\det(I + M)} = 0 \\
\end{align*}$$

(3.4)

and the equation of motion for $T$ is

$$\begin{align*}
\frac{\delta V}{\delta T} \frac{1}{\sqrt{H}} \sqrt{\det(I + M)} - & \\
- \partial_\kappa \left( \frac{V(T)}{\sqrt{H}} \eta^{\kappa\mu} \partial_\nu ((I + M)^{-1})_\mu^{\nu} \sqrt{\det(I + M)} \right) = 0 . \\
\end{align*}$$

(3.5)

Let us now consider the case when the tachyon depends on time only. Then the matrix $(I + M)$ is equal to

$$(I + M)_\mu^{\nu} = \begin{pmatrix}
1 - \dot{X}^m \dot{X}^m H(X^n) - \dot{T}^2 & 0 \\
0 & I_{p \times p}
\end{pmatrix}$$

(3.6)

so that the equation of motion for $X^m$ takes the form

$$\frac{V(T) \partial_m H(1 - \dot{T}^2)}{H^{3/2} \sqrt{1 - H \dot{X}^n \dot{X}^n - \dot{T}^2}} = \frac{d}{dt} \left( \frac{V(T) \sqrt{H} \dot{X}^m}{\sqrt{1 - H \dot{X}^n \dot{X}^n - \dot{T}^2}} \right)$$

(3.7)

and the equation of motion for tachyon is

$$\frac{\delta V \sqrt{1 - H \dot{X}^m \dot{X}^m - \dot{T}^2}}{\sqrt{H}} + \frac{d}{dt} \left( \frac{V(T) \dot{T}}{\sqrt{H} \sqrt{1 - H \dot{X}^m \dot{X}^m - \dot{T}^2}} \right) = 0 .$$

(3.8)

Now we will consider the case when $k$ NS5-branes coincide. Then $H$ depends on $R = \sqrt{X^m X^m}$ and using $\partial_m H = \frac{dH}{dR} \partial_m R = H' \frac{X^m}{R}$ we get following equation of motion for $R$

$$\frac{V(T) X^m H'(1 - \dot{T}^2)}{2RH^{3/2} \sqrt{1 - H \dot{X}^n \dot{X}^n - T^2}} = \frac{d}{dt} \left( \frac{X^m V(T) \sqrt{H}}{\sqrt{1 - H \dot{X}^n \dot{X}^n - T^2}} \right) ,$$

(3.9)
while the equation of motion for tachyon (3.8) does not change. We see that (3.8) and (3.9) are complicated differential equations of the second order. As is well known from the study of the tachyon condensation [9, 10, 11, 12] it is more convenient to find all possible conserved charges in order to simplify the description of the system. The same procedure was performed in case of BPS Dp-brane in the NS5-brane background in [1] where the dynamics of given Dp-brane was characterised by conserved energy and angular momentum when Dp-brane moves in \((x^6, x^7)\) plane. Now the existence of these conserved charges allows to analyse the possible trajectories of Dp-brane in NS5-branes background [1]. On the other hand for non-BPS Dp-brane the situation is more involved thanks to the existence of an additional mode \(T\) on the worldvolume of a non-BPS Dp-brane. It seems to be useful to find additional conserved charge in order to be able study of the dynamics of the non-BPS Dp-brane in the fivebrane background. Unfortunately we were not able to find such a charge for general values of \(R\) and \(T\). On the other hand as we will see in the next section in some region of the tachyon and radial mode field theory space it is possible to find this new conserved charge. Then with the help of this conservation quantity we will be able to determine the time evolution \(R\) and \(T\) explicitly.

4. Another form of the tachyon effective action and new conserved charge

It turns out that in order to find additional conserved charge it is more useful to describe the non-BPS Dp-brane by the action proposed in [20] that in the flat spacetime has the form

\[
S = \tau_p \int d^{p+1} \xi e^{-\Phi_0} \frac{1}{1 + \frac{T^2}{2}} \sqrt{1 + \frac{T^2}{2} + \eta^{\mu\nu} \partial_\mu T \partial_\nu T} .
\]

(4.1)

It can be shown [20] that there exists field redefinition that maps (4.1) to (2.10). The existence of this redefinition shows that these two actions are equivalent at least at flat spacetime.

It is also useful to rewrite (4.1) into the more suggestive form

\[
S = -\tau_p \int d^{p+1} \xi e^{-\Phi_0} \frac{1}{\sqrt{1 + \frac{T^2}{2}}} \sqrt{- \det(\eta_{\mu\nu} + (1 + \frac{T^2}{2})^{-1} \partial_\mu T \partial_\nu T)} =
\]

\[
= -\tau_p \int d^{p+1} \xi \sqrt{F} e^{-\Phi_0} \sqrt{- \det(\eta_{\mu\nu} + F \partial_\mu T \partial_\nu T)} , F = \frac{1}{1 + \frac{T^2}{2}} .
\]

(4.2)

As a next step we will presume that the non-BPS Dp-brane (1.2) is valid in the nontrivial closed string background as well. Then for general spacetime metric and dilaton (1.2) can be written as

\[
S = -\tau_p \int d^{p+1} \xi e^{-\Phi - \Phi_0} \sqrt{F} \sqrt{- \det(G_{\mu\nu} + F \partial_\mu T \partial_\nu T)} .
\]

(4.3)
If we consider the non-BPS Dp-brane in the NS5-brane background with the worldvolume parallel with the worldvolume of fivebranes and use the static gauge then the action (4.3) reduces into

\[ S = -\tau_p \int d^{p+1}\xi \frac{\sqrt{F}}{\sqrt{H}} \left[ -\det(\eta_{\mu\nu} + H \partial_\mu X^m \partial_\nu X^m + F \partial_\mu T \partial_\nu T) \right]. \]  

(4.4)

If we now forget about the origin of the tachyon then we can consider the action (4.4) as the DBI action for Dp-brane embedded in nontrivial eleven dimensional spacetime where the additional coordinate is labelled by \( T \) and where the metric and dilaton are given as

\[ ds^2 = dx_\mu dx^\mu + H(x^\mu)dx^m dx^m + F(T)dT^2, \]

\[ e^{2(\Phi - \Phi_0)} = \frac{H(x^n)}{F(T)}. \]  

(4.5)

It would be certainly very nice to have such a geometrical interpretation of the tachyon on the worldvolume of unstable D-brane, especially in the context of the recent paper [3]. Of course more work is needed to prove whether this conjecture is true.

Let us now consider the radial motion of D-brane in the background of \( k \) coincident NS-branes. By \( SO(4) \) rotation-symmetry of the problem, we can consider the moving of non-BPS Dp-brane in two dimensional plane, say the \( (x^6, x^7) \). Then it is useful to introduce polar coordinates

\[ X^6 = R \cos \theta , \quad X^7 = R \sin \theta. \]  

(4.6)

At the moment let us now presume that \( \theta \) is zero which, as we will see, corresponds to the zero angular momentum. Then the action (4.4) takes the form

\[ S = -V_p \tau_p \int dt \frac{1}{\sqrt{1 + \frac{T^2}{2} \sqrt{1 + \frac{k}{R^2}}} \left[ 1 - \left(1 + \frac{k}{R^2}\right)\dot{R}^2 - \left(1 + \frac{T^2}{2}\right)^{-1}\dot{T}^2 \right]. \]  

(4.7)

When \( T^2/2 \gg 1 \) and \( k/R^2 \gg 1 \) the action (4.4) simplifies considerably

\[ S = -V_p \tau_p \int dt \frac{1}{\sqrt{\frac{T^2}{2} \frac{k}{R^2}}} \sqrt{1 - \frac{k}{R^2} \dot{R}^2 - \frac{2}{T^2} \dot{T}^2}. \]  

(4.8)

It is easy to see that (4.8) is invariant under the transformation

\[ T' = \lambda T , \quad R' = \lambda R, \]  

(4.9)

where \( \lambda \) is constant. As usual this symmetry implies an existence of the conserved charge. In fact, for \( \lambda = 1 + \epsilon \), \( \epsilon \ll 1 \) we get

\[ \delta T = \epsilon T \, , \, \delta R = \epsilon R \]  

(4.10)
Since the action is invariant for constant $\epsilon$ then for $\epsilon = \epsilon(t)$ its variation should be proportional to the time derivation of $\epsilon$ \(^4\)

\[ \delta S = \int dt J \dot{\epsilon} = - \int dt \dot{J} \epsilon. \] (4.11)

For fields obeying equation of motion $\delta S = 0$ and we obtain the conserved charge $\dot{J} = 0$. In the case of the variations (4.10) the corresponding conserved charge is

\[ J = V_p \tau_p \sqrt{\frac{2 R}{k T}} \left( \frac{k \dot{R}}{R} + \frac{2 \dot{T}}{T} \right) \frac{1}{\sqrt{1 - \frac{k}{k R^2} R^2 - \frac{2}{T^2} T^2}}. \] (4.12)

The second conserved charge that will be useful for the study of the dynamics of the non-BPS Dp-brane is the energy

\[ E = P \dot{R} + \Pi \dot{T} - L = V_p \tau_p \sqrt{\frac{2 R}{k T}} \frac{1}{\sqrt{1 - \frac{k}{k R^2} R^2 - \frac{2}{T^2} T^2}}, \] (4.13)

where

\[ P = \frac{\delta L}{\delta \dot{R}} = V_p k \tau_p \sqrt{\frac{2}{k T R}} \frac{\dot{R}}{\sqrt{1 - \frac{k}{k R^2} R^2 - \frac{2}{T^2} T^2}}, \]

\[ \Pi = \frac{\delta L}{\delta \dot{T}} = 2 V_p \tau_p \sqrt{\frac{2}{k T^3}} \frac{\dot{T}}{\sqrt{1 - \frac{k}{k R^2} R^2 - \frac{2}{T^2} T^2}}. \] (4.14)

Since $J$ and $E$ contain an overall volume factor $V_p$ it is more natural to work with the densities (which we label with the same letters $E, J$) when we strip of the volume factors. From (1.12) and (4.13) we can easily find the relation between $R$ and $T$

\[ \frac{J}{E} = \frac{k \dot{R}}{R} + \frac{2 \dot{T}}{T} = k \frac{d \ln R}{dt} + 2 \frac{d \ln T}{dt} = \frac{d}{dt} \left( \ln R^k + \ln T^2 \right) \]

\[ \frac{J}{E} = \frac{d \ln R^k T^2}{dt} \Rightarrow R^k T^2 = C^2 e^{\frac{J}{2E} t} \Rightarrow T = C e^{\frac{J}{2E} t} R^{-k/2}, \] (4.15)

where the integration constant $C$ will be determined below using the value of the tachyon $T_0$ at time $t_0 = 0$. Using (1.15) we also get

\[ \dot{T} = \left( \frac{J}{2E} - \frac{k \dot{R}}{2R} \right) T. \] (4.16)

\(^4\)For more detailed discussion of conserved charges and Noether method in the context of string theory, see [27, 28].
Now if we insert (4.16) together with (4.15) into (4.13) we obtain differential equation with parameters $E$ and $J$ that determines the time evolution of $R$. Let us now consider the situation when $J = 0$. Then (4.15) implies

$$\dot{T} = -\frac{k}{2} \frac{\dot{R}}{R} T. \quad (4.17)$$

Inserting (4.17) into (4.13) we obtain the differential equation for $R$

$$\dot{R}^{2} = \frac{1}{(k + \frac{k^2}{2})} \left( R^{2} - \frac{2\tau_{P}^{2}}{kE^2C^2} R^{4+k} \right) \quad (4.18)$$

that has the solution

$$\frac{t}{\sqrt{k + \frac{k^2}{2}}} = \pm \frac{2}{2 + k} \arctanh \sqrt{1 - \frac{2\tau_{P}^{2}}{kE^2C^2} R^{2+k} + C_{0}}. \quad (4.19)$$

If we demand that at $t = 0$ the non-BPS D-brane is in its turning point we obtain that $C_{0}$ is equal to zero and consequently

$$\frac{1}{R} = \left( \frac{2\tau_{P}^{2}}{kE^2C^2} \right)^{\frac{1}{2+k}} \left( \cosh \left( \frac{(2 + k)t}{2\sqrt{k + k^2/2}} \right) \right)^{\frac{k}{2+k}}. \quad (4.20)$$

We see that the time dependence of the radial coordinate is different from the case of BPS Dp-brane studied in [1]. In fact, for $t \to \infty$ we get

$$R \sim e^{-\frac{t}{\sqrt{k + \frac{k^2}{2}}}} \text{ (non-BPS D-brane)}; R \sim e^{-\frac{t}{\sqrt{k}}} \text{ (BPS D-brane)}. \quad (4.21)$$

Physical explanation of this difference is as follows. The effective tension of non-BPS D-brane lowers in the process of the tachyon condensation. Consequently the gravitative attraction from the NS5-brane becomes weaker than in the case of BPS D-brane and hence non-BPS D-brane approaches NS-brane slowly than BPS D-brane.

In order to determine the time dependence of $T$ we use (4.17) that together with (4.20) gives

$$T = CR^{-k/2} = C \left( \cosh \left( \frac{(2 + k)t}{2\sqrt{k + k^2/2}} \right) \right)^{\frac{k}{2+k}} \left( \frac{2\tau_{P}^{2}}{kE^2C^2} \right)^{\frac{k}{2(k+1)}}. \quad (4.22)$$

If we demand that at $t = 0$ the tachyon is equal to $T_0 \gg 1$ then using (4.22) we can find relation between $T_0$, energy $E$ and the integration constant $C$

$$C = T_0^{\frac{2+k}{k}} \left( \frac{kE^2}{2\tau_{P}^{2}} \right)^{\frac{1}{2+k}}. \quad (4.23)$$
It is also interesting to study the time dependence of the components of the stress energy tensor. To begin with let us replace the flat worldvolume metric $\eta_{\mu\nu}$ with the general one $g_{\mu\nu}$ in the action (4.4). Then we get

$$S = -\tau_p \int d^{p+1}\xi \sqrt{-g} \sqrt{G/\Omega} \chi \left( \delta^\mu_\nu + H g^{\mu\kappa} \partial_\kappa X^m \partial_\nu X^m + F g^{\mu\kappa} \partial_\kappa T \partial_\nu T \right). \quad (4.24)$$

Since the action has the form $S = -\int d^{p+1}\xi \sqrt{-g} L$ then the stress energy tensor defined as $T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}}$ is equal to

$$T_{\mu\nu} = -g_{\mu\nu} L + 2 \frac{\delta L}{\delta g_{\mu\nu}}. \quad (4.25)$$

After insertion of the flat spacetime metric and using the fact that the worldvolume fields are the time dependent only we get

$$T_{00} = \tau_p \sqrt{F} \frac{1}{\sqrt{H} \sqrt{1 - H \dot{X}_m \dot{X}^m - F T^2}},$$
$$T_{ij} = -\delta_{ij} \tau_p \sqrt{F} \sqrt{1 - H \dot{X}_m \dot{X}^m - F T^2}. \quad (4.26)$$

that for large $T$ and small $R$ take the forms

$$T_{00} = \tau_p \sqrt{\frac{2R}{k_T}} \frac{1}{\sqrt{1 - k \dot{R}^2 / R^2 - 2T^2 / T^2}},$$
$$T_{ij} = -\delta_{ij} \tau_p \sqrt{\frac{2R}{k_T} \sqrt{1 - k \dot{R}^2 / R^2 - 2T^2 / T^2}}. \quad (4.27)$$

Firstly we know that $T_{00} = E$ is constant. On the other hand the spatial components $T_{ij}$ are equal to

$$T_{ij} = -\delta_{ij} \frac{\tau_p 2R}{T} \sqrt{1 - (k + \frac{k^2}{2}) \frac{\dot{R}^2}{R^2} - e^{-\frac{k^{1/2}}{2\sqrt{k}}} \sqrt{t}} \to 0. \quad (4.28)$$

We see that as in the case of the BPS D-brane \(^\square\) the pressure goes exponentially to zero at asymptotic future. This asymptotic behaviour is also in agreement with the rolling tachyon description \([9, 10, 11, 12]\).

Now we will briefly discuss the case of nonzero $J$. We can think about this as follows. Inserting (4.16) into (4.13) we obtain the equation

$$(k + \frac{k^2}{2}) \dot{R}^2 = R^2 (1 - \frac{J^2}{2E^2}) + \frac{Jk \dot{R}R}{2E} - \frac{2\tau_p^2 R^2}{kE^2 C^2} R^{4+k}. \quad (4.29)$$
It is very difficult to find general solution of this equation so that we restrict ourselves to the case of large $t$. Then by presumption $R \ll 1$ and we can neglect the term $R^4 + k$ in (4.29). If we now presume the time dependence of $R$ for $t \to \infty$ is $R \sim e^{\beta t}$ then we get from (4.29) the quadratic equation for $\beta$

\[
(k + \frac{k^2}{2})\beta^2 - \frac{Jk}{2E} \beta \left(1 - \frac{J^2}{2E^2}\right) = 0 \Rightarrow \beta = \frac{1}{(k + \frac{k^2}{2})} \left(-\sqrt{k + \frac{k^2}{2}} \left(1 - \frac{J^2}{2E^2}\right) + \frac{Jk}{2E}\right). \tag{4.30}
\]

We see that (4.30) is equal to $-\frac{1}{\sqrt{k + \frac{k^2}{2}}}$ for $J = 0$ with agreement with the scaling (4.21). More precisely, let us presume that $\frac{J}{E} \ll 1$. Then $\beta$ given in (4.30) can be written as

\[
\beta \approx -\frac{1}{\sqrt{k + \frac{k^2}{2}}} + \frac{Jk}{E(k + \frac{k^2}{2})}. \tag{4.31}
\]

As in the case of the vanishing charge $J$, $T_{00}$ is energy density equal to $E$ while the spatial components of the stress energy tensor have asymptotic form

\[
T_{ij} \approx -\delta_{ij} e^{\left(-\frac{2+k}{\sqrt{k + \frac{k^2}{2}} - \frac{(2+k)J}{2E(k + \frac{k^2}{2})}}\right) t}. \tag{4.32}
\]

We again see that the pressure vanishes at far future. This result confirms the presumption that the time evolution of the system at nonzero $J$ has similar form as in the case of $J = 0$.

Finally we will consider the case of general $\theta$. Now the action (4.8) is equal to

\[
S = -V_p \tau_p \int dt \frac{1}{\sqrt{\frac{T^2 - k}{R^2}}} \sqrt{1 - \frac{k}{R^2} \left(\hat{R}^2 + \hat{\theta}^2 R^2\right) - \frac{2}{T^2} \hat{T}^2}. \tag{4.33}
\]

It is again easy to see that this action is invariant under scaling $T' = \lambda T$, $R' = \lambda R$. The corresponding conserved charge is equal to

\[
J = V_p \tau_p \sqrt{\frac{2R}{kT}} \left(\frac{k\hat{R}}{R} + \frac{2\hat{T}}{T}\right) \sqrt{1 - \frac{k}{R^2}(\hat{R}^2 + R^2\hat{\theta}^2) - \frac{2}{T^2} \hat{T}^2}. \tag{4.34}
\]

We have also the conserved energy

\[
E = V_p \tau_p \sqrt{\frac{2R}{kT}} \frac{1}{\sqrt{1 - \frac{k}{R^2}(\hat{R}^2 + R^2\hat{\theta}^2) - \frac{2}{T^2} \hat{T}^2}}. \tag{4.35}
\]
One can also see that the action (4.33) is invariant under the transformation $\theta' = \theta + \epsilon$, where $\epsilon$ is constant. The corresponding conserved charge is angular momentum and it is equal to

$$L = V_p \tau_p \sqrt{\frac{2}{kT}} \frac{k\dot{\theta}}{\sqrt{1 - \frac{k}{R^2}(R^2 + \dot{R}^2)} - \frac{2}{\tau_p} T^2}.$$  

(4.36)

Now we can solve this system of equations for fixed $E, L, J$. For simplicity we restrict ourselves to the case $J = 0$ which allows us to express $T$ as function of $R$ exactly in the same way as in the case of $J = 0$ given in (4.17). Inserting this expression into (4.36) we can express $\dot{\theta}$ as function of $R,
\dot{R}$

$$\dot{\theta}^2 = \frac{L^2 \left(1 - \left(k + \frac{k^2}{2} \right) \frac{\dot{R}^2}{R^2}\right)}{\left(L^2 k + \frac{2\tau_p^2 k R^{2+k}}{C^2}\right)}.$$  

(4.37)

If we insert this result into (4.37) and using (4.17) we obtain the differential equation for $R$

$$(k + \frac{k^2}{2}) \ddot{R} = \frac{(kE^2 - C^2 L^2)}{kE^2} R^2 - \frac{2\tau_p^2}{kE^2 C^2} R^{4+k}$$

(4.38)

with the general solution

$$\frac{t}{\sqrt{kE \sqrt{k + k^2/2}}} = \mp \frac{2}{(2 + k) \sqrt{kE^2 - C^2 L^2}} \arctanh \sqrt{1 - \frac{2\tau_p^2}{kE^2 C^2 - C^4 L^2} R^{2+k} + C_0}.$$  

(4.39)

If we demand that at $t = 0$ non-BPS Dp-brane is in its turning point we get $C_0 = 0$. Then we finally obtain

$$\frac{1}{R^{2+k}} = \frac{2\tau_p^2}{kE^2 C^2(1 - \frac{C^2 L^2}{kE^2})} \cosh \left(\frac{2 + k}{2\sqrt{k + k^2}} \sqrt{1 - \frac{C^2 L^2}{kE^2} t}\right).$$  

(4.40)

We see that nonzero angular momentum slows down the exponential decrease of $R$ as $t \to \infty$. This is the same behaviour as was observed in case of BPS Dp-brane in [1]. Since

$$\frac{L}{E} = k\dot{\theta}$$  

(4.41)

one can easily find time dependence of $\theta$

$$\theta = \frac{L}{E k} t.$$  

(4.42)

---

5Now $E, J, L$ mean corresponding densities.
The solution (4.40), (4.42) and the relation $T = CR^{-k/2}$ describes the non-BPS D-brane that during the worldvolume tachyon condensation spirals towards the origin, circling around it an infinite number of times in the process.

In this section we have studied the time dependent tachyon condensation on non-BPS Dp-brane in the $NS5$-brane background in the regime where $k/R^2 \gg 1$ and $T^2 \gg 1$. In the next section we will consider the second example of the tachyon condensation when the tachyon is spatial dependent while the radial coordinate depends on time only.

5. Spatial dependent tachyon

In this section we will analyse the case when the tachyon depends on one spatial coordinate, say $\xi^1 = x$. We will study this problem using the non-BPS Dp-brane action given in (3.2). For this configuration the matrix $I + M$ takes the form

$$\begin{pmatrix}
1 - \dot{X}^m \dot{X}^m H & 0 & 0 \\
0 & 1 + T'^2 & 0 \\
0 & 0 & I_{(p-1) \times (p-1)}
\end{pmatrix}, \quad (5.1)$$

where $T' \equiv \frac{dT}{dx}$. Consequently the action (3.2) simplifies as

$$S = - \int dt \mathcal{L} = -V_{p-1} \int dt dx \frac{V(T)}{\sqrt{H}} \sqrt{\text{det}(I + M)} = \frac{\sqrt{H} \dot{X}^m}{1 - P} \left[ \frac{\partial_m H}{2H^{3/2} \sqrt{1 - H \dot{X}^n \dot{X}^n}} + \frac{d}{dt} \left( \frac{\sqrt{H} \dot{X}^m}{\sqrt{1 - H \dot{X}^n \dot{X}^n}} \right) \right] = 0$$

using the fact that $T$ does not depend on $t$. The equation of motion for the tachyon is

$$\frac{\sqrt{1 - H \dot{X}^n \dot{X}^n}}{\sqrt{H}} \left[ \frac{\delta V}{\delta T} \sqrt{1 + T'^2} - \frac{d}{dx} \left( \frac{V(T)T'}{\sqrt{1 + T'^2}} \right) \right] = 0 \quad , (5.4)$$

where now we have used the fact that $X^m$ are functions of $t$ only. From (5.3) and (5.4) we see that tachyon and scalar modes decouple and can be studied separately.
The solution considering the spatial dependent tachyon is well known [18, 19, 23, 24, 25]. In particular, for \( V = \frac{\tau_p}{\cosh(\sqrt{2}T)} \) the solution of (5.4) takes the form

\[
\sinh \frac{T}{\sqrt{2}} = \sqrt{\frac{\tau_p^2}{K^2} - 1} \sin \frac{x}{\sqrt{2}},
\]

(5.5)

where \( K \) is integration constant. The corresponding energy density is

\[
\rho(x) = V(T)\sqrt{1 + T'^2} = \frac{V^2(T)}{K} = \frac{\tau_p^2}{K} \frac{1}{1 + \left(\frac{\tau_p^2}{K^2} - 1\right) \sin^2 \frac{x}{\sqrt{2}}}. \tag{5.6}
\]

The interpretation of this solution is usually in terms of array of kinks and antikinks. Then by integrating the energy density over a half period of the solution we can find energy of the kink

\[
T_{p-1} \equiv \int_{\sqrt{2}\pi/2}^{-\sqrt{2}\pi/2} dx \rho(x) = \pi \sqrt{2}\tau_p \tag{5.7}
\]

This is nothing but the tension of the BPS kink identified as D(p-1)-brane. Therefore the tachyon solution may be interpreted as representing an array of D(p-1)-branes and D(p-1)-antibranes in the background of \( k \) NS5-branes. The fact that we consider \( X^m \)'s independent on \( x \) implies that \( X^m \)'s parametrise the collective motion of the configuration of D(p-1)-branes and D(p-1)-antibranes.

Now let as return to the study of the dynamics of \( X^m \), where we will specialise to the case of coincident fivebranes. As in the previous subsection we will demand that \( H \) only depends on \( R = \sqrt{X^n X^n} \) so that the equation of motion (5.3) takes the form

\[
\frac{d}{dt} \left( \frac{\dot{X}^m\sqrt{H}}{\sqrt{1 - H\dot{X}^n\dot{X}^n}} \right) = \frac{H'X^m}{2RH^{3/2}\sqrt{1 - H\dot{X}^n\dot{X}^n}}, \tag{5.8}
\]

where now \( H' \equiv \frac{dH}{dR} \). In order to solve this equation we must specify the initial data \( X^m(t = 0) \) and \( X^m(t = 0) \). As was argued in [3] these two vectors define plane in the transverse \( R^4 \) that can be by \( SO(4) \) rotation taken to the \((x^6, x^7)\) plane. Then the motion will remain in this plane for all time. More precisely, the angular momentum for the motion in \((x^6, x^7)\) plane can be determined through the Noether method with the result

\[
L = V_{p-1} \int dx V(T)\sqrt{1 + T'^2} \frac{\sqrt{H}(X^6\dot{X}^7 - \dot{X}^6X^7)}{\sqrt{1 - H\dot{X}^n\dot{X}^n}} \tag{5.9}
\]

In the same way we can determine the conserved energy from the requirement of the invariance of the action with respect to the time translation

\[
E = V_{p-1} \int dx \left( P_m\dot{X}^m - \mathcal{L} \right) = V_{p-1} \int dx \frac{V(T)\sqrt{1 + T'^2}}{\sqrt{H}\sqrt{1 - H\dot{X}^n\dot{X}^n}} \tag{5.10}
\]
using
\[
P_m(x) = \frac{\delta L}{\delta X^m} = V_{p-1} \frac{V(T) \sqrt{H} \dot{X}^m \sqrt{1 + T'^2(x)}}{\sqrt{1 - H \dot{X}^n \dot{X}^n}} \tag{5.11}
\]

We again introduce the coordinate
\[
X^6 = R \cos \theta , \quad X^7 = R \sin \theta \tag{5.12}
\]
so that the conserved energy and momentum are equal to
\[
E = V_{p-1} \int dx \frac{V(T) \sqrt{1 + T'^2}}{\sqrt{H} \sqrt{1 - H (\dot{R}^2 - R^2 \dot{\theta}^2)}} ,
\]
\[
L = V_{p-1} \int dx \frac{V(T) \sqrt{1 + T'^2} R \dot{\theta}}{\sqrt{H} \sqrt{1 - H (\dot{R}^2 + R^2 \dot{\theta}^2)}} .
\tag{5.13}
\]

We have argued above that the tachyon condensation has physical interpretation as an array of D(p-1)-branes and D(p-1)-antibranes. In the same way we know that \( R, \theta \) do not depend on \( x \) so that it is natural to write
\[
E = (N + \overline{N}) E_{p-1} , \quad L = (N + \overline{N}) L_{p-1} , \tag{5.14}
\]
where
\[
(N + \overline{N}) = \frac{\int_{-\infty}^{\infty} dx \rho(x)}{\int_{-\sqrt{2\pi}/2}^{\sqrt{2\pi}/2} dx \rho(x)} \tag{5.15}
\]
is number of D(p-1)-branes and D(p-1)-antibranes. Then it is natural to introduce an energy and angular momentum densities per single D(p-1)-brane or D(p-1)-antibrane through the relations
\[
e \equiv \frac{E \int_{-\sqrt{2\pi}/2}^{\sqrt{2\pi}/2} dx \rho(x)}{V_{p-1} \int dx \rho(x)} , \quad l \equiv \frac{L \int_{-\sqrt{2\pi}/2}^{\sqrt{2\pi}/2} dx \rho(x)}{V_{p-1} \int dx \rho(x)} \tag{5.16}
\]
that using (5.13) are equal to
\[
e = \frac{T_{p-1}}{\sqrt{H} \sqrt{1 - H (\dot{R}^2 + R^2 \dot{\theta}^2)}} ,
\]
\[
l = \frac{T_{p-1} R^2 \dot{\theta}^2}{\sqrt{H} \sqrt{1 - H (\dot{R}^2 + R^2 \dot{\theta}^2)}} , \tag{5.17}
\]
where now \( T_{p-1} \) is tension of BPS D(p-1)-brane or D(p-1)-antibrane. Using the second equation in (5.17) we find
\[
\dot{\theta}^2 = \frac{l^2 (1 - H \dot{R}^2)}{\frac{T_{p-1}}{H} R^4 + l^2 R^2} \tag{5.18}
\]
and hence we get

\[ \dot{R}^2 = \frac{1}{H} - \frac{1}{e^2 H^2} \left( T_{p-1}^2 + \frac{l^2}{R^2} \right) \] (5.19)

Let us consider the case of vanishing momentum \( l = 0 \). Then (5.18) implies that \( \theta \) is constant and the equation for radial coordinate becomes

\[ \dot{R}^2 = \frac{1}{H} - \frac{T_{p-1}^2}{e^2 H^2} . \] (5.20)

If now the trajectory remains in the small \( R \) region, such that \( R \ll \sqrt{k} \) for all times which happens for \( T_{p-1}/e \ll 1 \) we can solve for the trajectory \( R(t) \) exactly since in this case \( H(R) = k/R^2 \) and we get

\[ \dot{R}^2 = \frac{R^2}{k} - \frac{T_{p-1}^2 R^2}{e^2 k^2} \] (5.21)

with the solution

\[ \frac{1}{R} = \frac{T_{p-1}}{e \sqrt{k}} \cosh \frac{t}{\sqrt{k}} . \] (5.22)

In summary, the spatial dependent tachyon condensation on the non-BPS Dp-brane leads to the array of D(p-1)-branes and D(p-1)-antibranes. If we do not worry about stability of this configuration then this problem reduces to the description of the motion of D(p-1)-brane or D(p-1)-antibrane in the background of \( k \) NS5-branes.

6. Conclusion

This paper was devoted to the study of the dynamics of the non-BPS Dp-brane in the background of \( k \) NS5-branes, where we have proceed in the similar way as in the case of BPS D-brane that was studied in [1]. In other words, we have analysed the dynamics of the non-BPS Dp-brane using the effective field theory description that is based on the DBI-like tachyon effective action. We have considered two possible cases of the tachyon condensation. In the first case, which was time dependent tachyon condensation, we have shown that in the approximation of large tachyon potential and in the region very close to the \( k \) NS5 coincident branes a non-BPS Dp-brane approaches the worldvolume of fivebranes with exponentially growing tachyon and exponentially decreasing radial mode where the factor in the exponential function was different from the factor given in the case of BPS Dp-brane. We have explained this difference as a result of the tachyon condensation on the worldvolume of non-BPS Dp-brane that effectively reduces the tension of D-brane. We have also shown that the pressure exponentially vanishes with the agreement with the paper [1]. We would like to stress that these results were obtained thanks to the existence of an additional conserved charge whose existence follows from the enhanced symmetry of the tachyon effective action for large \( T \) and small \( R \).
At this place however we must stress one important point considering the validity of the effective field theory description of the non-BPS D-brane. The DBI-like tachyon effective action was not directly obtained from the first principles of string theory, even if its success in the description of some aspects of the tachyon condensation is intriguing. On the other hand it was recently stressed in [22] that it is not completely clear how to interpret the action (2.4). It is also well known that there exist other form of the tachyon effective actions [29, 30, 31, 32, 33] that are generally valid in different regions of the tachyon field theory space. It is commonly believed that these different tachyon effective actions are related by complicated field redefinition. It would be very interesting to see whether these different tachyon effective actions can be studied in the nontrivial background as well. Another problem that deserves to be studied is the analysis of the dynamics of the non-BPS D-brane in the higher dimensional D-brane background. The situation is now more involved thanks to the nontrivial Ramond-Ramond fields that couple to the worldvolume of non-BPS D-brane [34, 35]. We hope to return to this problem in future.

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References


