HYDRODYNAMICAL SIMULATIONS OF CONVECTION-RELATED STELLAR MICRO-VARIABILITY

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Abstract

We used a series of CO5BOLD hydrodynamical model atmospheres covering stellar objects from white dwarfs to red giants to derive theoretical estimates of the photometric and photocentric stellar variability in wavelength-integrated light across the Hertzsprung-Russell diagram. We validated our models against solar measurements from the SOHO/VIRGO instrument. Within our set of models we find a systematic increase of the photometric as well as photocentric variability — which turn out to be closely connected — with decreasing surface gravity. The estimated absolute levels of the photocentric variability do not affect astrometric observations on a precision level expected to be achieved by the GAIA mission — with the exception of close-by giants. The case of supergiants remains to be investigated. In view of the ongoing debate about the photometric non-detection of p-modes in Procyon by the Canadian MOST satellite we remark that we obtain a factor of ≈ 3 in amplitude between the granular background “noise” in the Sun and Procyon. This statement refers to a particular representation of temporal power spectra as discussed in Sect. 5.

Key words: variability, photometry, astrometry, granulation, hydrodynamics, simulation, oscillations, exoplanets

1. Introduction

The presence of a small-scale time-dependent granulation pattern on the surfaces of late-type stars (see Fig. 1) leads to low-level temporal fluctuations in a star’s total luminosity, apparent position given by its photocenter, (see Fig. 2), and spectroscopically measured radial velocity. During recent years this kind of micro-variability got into the focus of research since it forms an important noise source in searches for solar-like oscillations and exoplanets. This is particularly the case for already operational or upcoming satellite mission like MOST, COROT, and KEPLER; granulation induced micro-variability might also become a limiting factor for precision astrometry missions like SIM and GAIA (Hatzes 2002, Green et al. 2003, Aigrain et al. 2004, Matthews et al. 2004). We performed radiation-hydrodynamics simulations for stellar atmospheres to derive theoretical estimates of the convection-related photometric and photocentric variability across the Hertzsprung-Russell diagram. To our knowledge the work of Trampedach et al. (1998) is the only previous example of such a theoretical effort who studied the brightness and radial velocity variability in the Sun, α Cen A, and Procyon A.

2. CO5BOLD rad.-hydrodynamics simulations

We used the radiation-hydrodynamics code CO5BOLD (for further information about the code and applications see Freytag et al. 2002 and Wedemeyer et al. 2004) to construct a series of 3D Cartesian “local-box” model atmospheres. Table 1 summarises the model properties. We calculated three solar models which differ in numerical details. They are used as basic reference and for validating our models by comparison with SOHO/VIRGO observational data. The two models “Procyon A” and “ξ Hydrae” have parameters close to the actual parameters of the stars they are named after. The other models are not intended to represent particular stars but are generic spanning a large range primarily in surface gravity. Two models are intended to investigate effects of metallicity. Note, that the variation in effective temperature among the models...
is not very large, i.e. the set of models does not fully cover the parameter space. While not apparent from the table significant effort was invested to follow the evolution of the models over long periods of time to ensure statistically representative results.

3. From simulation boxes to stars

The CO5BOLD simulations provide a statistically representative, rectangular patch (or “tile”) of the emergent radiation field and its temporal evolution. In order to derive disk-integrated, observable quantities we have to extrapolate this information to the whole visible stellar hemisphere. To this end we envision the stellar surface being tiled by a possibly large number of simulation patches. Being interested in integrated quantities we can ignore the spatial structure within in a patch and only need to consider averages of the emergent intensity $I_m$ over the patch surface as a function of time and inclination $\tilde{\mu}_m = \cos(\theta_m)$. Due to limitations of computing resources the discretisation of the radiation field in solid angle is rather coarse, and usually we have only a total number of inclinations $M$ (see Tab. 1) of two or three available.

Figure 2 shows an example of a resulting time series with $M = 3$. Clearly visible is the effect of limb-darkening and residual intensity fluctuations after averaging over the patch’s surface. The crucial assumption now is that the size of a patch is large enough so that its emission can be considered as statistically independent of all other patches tiling the surface. Moreover, we assume that all patches share the same statistics given by the statistics of the simulated patch. With these assumptions, Ludwig (2004) derived expressions for the statistics of disk-integrated observables which we summarise below.

Statistics of the photometric variability. We characterise the photometric variability by the temporal power spectrum of the relative fluctuations of the observable flux $f$. The expectation value of a frequency component denoted by $\hat{f}(\nu)$ is then given by

$$\langle \hat{f} \hat{f}^* \rangle = N^{-1} \sum_{m=1}^{M} w_m \tilde{\mu}_m^2 \langle \hat{I}_m \hat{I}_m^* \rangle \left( \sum_{m=1}^{M} w_m \tilde{\mu}_m \langle I_m \rangle \right)^2.$$  (1)

Angular brackets denote expectation values, an asterisk the conjugate complex. The sums are discrete analogs of integrals over one half of the total solid angle where $w_m$ is the integration weight. The numerator expresses that a frequency component of the resulting power spectrum is a $\tilde{\mu}_m^2$ weighted integral of the frequency components of the individual intensity power spectra. The denominator is essentially the square of the average observable flux. The spectral power density scales inversely proportional to the number of patches $N$ tiling the visible hemisphere which is given by

$$NA = 2\pi R^2,$$  (2)

where $A$ is the patch surface area and $R$ is the stellar radius. $R$ is assumed for the star the atmosphere model is associated with. In this paper we used plausible but to some extend arbitrary values for the stellar radii. For possible later adjustments of the radius one should keep in mind that the power of the photometric fluctuations scales as $R^{-2}$. The power spectra presented in this paper are given as spectral density normalised so that the integral between zero and the Nyquist frequency corresponds to the variance of the original signal.

Statistics of the photocentric displacement. We characterise the absolute photocentric displacement along an ar-

Figure 2. A statistical realisation of the motion of the photocenter in the plane of the sky based on data from the red giant simulation. The time between two diamonds amounts to $10^5$ s.

Figure 3. Intensity time series for the three inclinations of model Sun1 showing residual fluctuations and overall limb-darkening.
Table 1. CO$^5$BOLD radiation-hydrodynamics model atmospheres: “Model” is a model’s nickname used in this paper, $T_{\text{eff}}$ the effective temperature, log $g$ the gravitational acceleration, $R$ an assumed stellar radius (not intrinsic to the simulation proper), $l$ the linear horizontal size of the computational box, $\tau_c$ the sound crossing time over $H^p_{\text{surf}}$, $H^p_{\text{surf}}$ the pressure scale height at Rosseland optical depth unity, $\delta I_{\text{rms}}$ the relative spatial white light intensity contrast, $N_{\text{obm}}$ the number of equivalent frequency points considered in the solution of the radiative transfer equation (RTE), $M$ the number of inclinations used in the discretization of the RTE, “Modelcode” an internal identifier of the model sequence.

<table>
<thead>
<tr>
<th>Model</th>
<th>$T_{\text{eff}}$ [K]</th>
<th>log $g$ [cm/s$^2$]</th>
<th>Radius $R$ [Mm]</th>
<th>Box size $l$ [Mm]</th>
<th>$\tau_c$ [s]</th>
<th>$H^p_{\text{surf}}$ [Mm]</th>
<th>$\delta I_{\text{rms}}$</th>
<th>$N_{\text{obm}}$</th>
<th>$M$</th>
<th>Modelcode</th>
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<tr>
<td>Sun1</td>
<td>5780</td>
<td>4.44</td>
<td>696</td>
<td>5.6</td>
<td>17.8</td>
<td>0.141</td>
<td>0.172</td>
<td>5</td>
<td>3</td>
<td>d3gt57g44n53</td>
</tr>
<tr>
<td>Sun2</td>
<td>5740</td>
<td>4.44</td>
<td>696</td>
<td>18.0</td>
<td>17.8</td>
<td>0.141</td>
<td>0.156</td>
<td>1</td>
<td>2</td>
<td>d3gt57g44s56</td>
</tr>
<tr>
<td>Sun3</td>
<td>5760</td>
<td>4.44</td>
<td>696</td>
<td>11.2</td>
<td>17.8</td>
<td>0.141</td>
<td>0.176</td>
<td>5</td>
<td>3</td>
<td>gt57g44n67</td>
</tr>
<tr>
<td>White Dwarf</td>
<td>12000</td>
<td>8.00</td>
<td>8.9</td>
<td>7.5 [km]</td>
<td>10.6 [ms]</td>
<td>133 [ms]</td>
<td>0.173</td>
<td>1</td>
<td>2</td>
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<tr>
<td>[M/H] = 0.0</td>
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<td>4.44</td>
<td>696</td>
<td>4.85</td>
<td>16.9</td>
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<td>[M/H] = -2.0</td>
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<td>4.44</td>
<td>696</td>
<td>4.85</td>
<td>17.2</td>
<td>0.118</td>
<td>0.047</td>
<td>6</td>
<td>3</td>
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<tr>
<td>Procyon A</td>
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<td>4.00</td>
<td>1460</td>
<td>29.54</td>
<td>49.6</td>
<td>0.391</td>
<td>0.212</td>
<td>1</td>
<td>2</td>
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<tr>
<td>ξ Hydrea</td>
<td>4880</td>
<td>2.94</td>
<td>7340</td>
<td>147.5</td>
<td>513</td>
<td>3.74</td>
<td>0.180</td>
<td>1</td>
<td>2</td>
<td>d3gt50g29n01</td>
</tr>
<tr>
<td>Cepheid</td>
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<td>2.00</td>
<td>21000</td>
<td>2125</td>
<td>4310</td>
<td>30.2</td>
<td>0.185</td>
<td>1</td>
<td>2</td>
<td>d3t50g20mm00n2</td>
</tr>
<tr>
<td>Red Giant</td>
<td>3680</td>
<td>1.00</td>
<td>66300</td>
<td>15750</td>
<td>40700</td>
<td>242</td>
<td>0.220</td>
<td>1</td>
<td>2</td>
<td>d3t38g10mm00n2</td>
</tr>
</tbody>
</table>

The similarity in the power spectra has the con-
Figure 4. Comparison of power spectra of disk-integrated photometric fluctuations between three solar simulations and observational solar data from SOHO/VIRGO. Note the steep decline in power in the range of the p-mode frequencies.

5. Trends in the Photometric Variability

Figure 5 shows a comparison of the brightness fluctuations for a sequence of models spanning a range between DA white dwarfs with convective outer envelopes and red giants. The power spectral density $P_{\nu}$ is plotted as $\nu P_{\nu}$ which makes it independent of the unit in which the frequency is measured. This facilitates the intercomparison among the models where we scaled the frequency with the sound crossing time over a pressure scale height at the surface ($\tau_c$). As evident from Fig. 5, all spectra show a similar shape, and are essentially located in the same scaled frequency range. We find a systematic increase of the photometric variability towards giants. Note, that in fact the square root of the power is plotted in Fig. 5. I.e. our models predict an increase in the amplitude by a factor of $\approx 1000$ between the white dwarf and red giant model.

Our result is in marked contrast to the modelling of Trampedach et al. (1998) who did not find an increase in the granular photometric signal comparing the Sun and Procyon A. However, they pointed out that their time series might have been not long enough to provide sufficient statistics and coverage of lower frequencies. In view of the ongoing debate about the photometric non-detection of p-modes in Procyon by the Canadian MOST satellite Matthews et al. (2003) Christensen-Dalsgaard & Kjeldsen (2004) we stress that we obtain a factor of $\approx 3$ in amplitude between the background signal in the Sun and Procyon. This refers to a comparison in a representation like Fig. 5 where differences in power can be essentially described by uniform vertical shift. In a representation in absolute frequencies the power ratio would be frequency-dependent.

By comparing models of similar effective temperature and surface gravity, but different metallicity we find a decrease of the brightness fluctuations with decreasing metallicity, see Fig. 6. This is in line with expectation that the higher densities encountered at optical depth unity in metal poor models (due to lower overall opacity) leads to smaller convective fluctuations and consequently smaller brightness fluctuations.

Figure 6. Spectral power density of the brightness fluctuations for two models around $T_{\text{eff}} = 5000$ K of solar (filled) and 1/100 solar metallicity (dotted). To facilitate comparison the x-axis includes the same range of frequencies as the one in Fig. 5.
6. Trends in the Photocentric Displacement

We find an almost perfectly linear relationship between the standard deviation of the photometric displacement and the surface gravity of the models (see Fig. 7). One has to keep in mind, however, that there will be some scatter around this line when considering atmospheres of markedly different effective temperature or chemical composition from the ones considered here. The diamond depicting the metal poor model already indicates this.

Instead of the basic atmospheric parameters ($T_{\text{eff}}, \log g$, chemical composition), two parameters more closely related to the convection pattern are perhaps physically better suited to describe the functional behaviour of the photocentric displacement — namely the relative spatial intensity contrast $\delta I_{\text{rms}}$ and the typical size of a convective cell. The size of granular cells is related to the pressure scale height at optical depth unity $H_p^{\text{surf}}$ [Freytag 2001]. Figure 8 depicts the outcome of a test of a relation between photocentric displacement and the product $\delta I_{\text{rms}} H_p^{\text{surf}}$: indeed, all models including the metal poor one now follow the same trend. This also indicates that the proportionality to $g^{-1}$ seen in Fig. 7 is primarily reflecting the systematic increase of granular cell size with decreasing gravitational acceleration.

For our most extreme giant model with $\log g = 1$ we find a standard deviation of the photocentric position of $3 \cdot 10^{-4}$ AU or 0.3 mas/D [pc] where $D$ denotes the distance. On a precision level expected to be reached by the GAIA astrometry such a variability does only affect the achievable precision for close-by giants. The situation for supergiants remains to be investigated; a linear extrapolation of the curves shown in Figs. 7 and 8 towards lower surface gravity is unlikely to give a reliable estimate of their photocentric variability since sphericity effects render convection in supergiants markedly different [Freytag et al. 2002] from convection in Cartesian geometry as presented here.

7. Concluding Remarks

We found systematic changes of the convection-induced fluctuations in brightness and photocentric displacement with stellar parameters. The result is a nice example for an application of hydrodynamical model atmospheres (in this respect see Ludwig & Kucinskas, this volume). We would like to emphasise again that we have been considering the contribution of granulation to the stellar variability only. At highest temporal frequencies it is the dominant contributor to the variability. However, at lower frequencies variability related to magnetic activity, spottedness, and rotation dominates, and the level of the fluctuations is likely to be significantly larger than the convective contribution.

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