HYBRID MECHANISMS FOR GAS/ICE GIANT PLANET FORMATION

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ABSTRACT

The effects of gas pressure gradients on the motion of solid grains in the solar nebula substantially enhances the efficiency of forming protoplanetary cores in the standard core accretion model in 'hybrid' scenarios for gas/ice giant planet formation. Such a scenario is enhanced core accretion which results from Epstein-drag induced inward radial migration of mm-sized grains and subsequent particle subdisk gravitational instability needed to build up a population of 1 km planetesimals. Solid/gas ratios can be enhanced by nearly $\sim 10^6$ over those in Minimum Mass Solar Nebula (MMSN) in the outer solar nebula (a $> 20$ AU), increasing the oligarchic core masses and decreasing formation timescales for protoplanetary cores. A $10 M_\oplus$ core can form on $\sim 10^8 - 10^7$ year timescales at 15 - 25 AU compared to $\sim 10^9$ years in the standard model, alleviating the major problem plaguing the core accretion model for gas/ice giant planet formation.

Subject headings: solar system: formation planetary systems: formation planetary systems: protoplanetary disks

1. INTRODUCTION

The main challenges facing the standard core accretion model for planet formation are finding a mechanism for building up a population of planetesimals and forming sufficiently massive proto-Uranus and proto-Neptune cores before the end of the oligarchic growth stage and before the solar nebula dissipates on $10^7$ yr timescales.

In the core accretion model, planetesimals are built up from approximately μm-sized grains to km-sized bodies by collisional sticking. Two major problems face collisionally sticking these bodies together. First, growing particles from centimeter through kilometer sizes is problematic. While many have argued for the coagulation of grains based on some requisite 'coagulation velocity' (e.g. Weidenschilling 1997) such velocities are often much higher than those below which sticking might occur based on microgravity experiments such as Blum & Muench (1993). More recently, Colwell (2003) showed that at least in some conditions cm-sized grain coagulation can occur, but this occurs for relative velocities below $\sim 12 cm/s$ and greater relative velocities are encountered in either a turbulent or laminar disk (see Youdin & Shu 2002, hereafter YS02, and references therein). An operative sticking mechanism also remains in question: both known solid state sticking forces and the bodies' self gravity are arguably too weak (YS02). Second, even if such sticking were possible, the timescales for meter-sized bodies to spiral in to the Sun (by gas drag) is short (Weidenschilling 1977), $10^5$ years at 1 AU, such that planetesimal formation in all but the outermost regions of the solar nebula might not proceed fast enough. One can circumvent this problem if a particle subdisk gravitational instability (GI) can very quickly build up a population of $1 km$ planetesimals directly (Goldreich & Ward, 1973: YS02).

However, formation timescale problems still remain for ice giants Uranus and Neptune. For a MMSN distribution, the oligarchic growth stage does not yield Neptune-mass cores (for a $> 15$ AU), so that a $15 M_\oplus$ core must be formed by colliding together sub Earth-mass oligarchs. Levison & Stewart (2001) showed that the embryos' mutual perturbations result in a large number of ejections (not mergers) of embryos in the outer solar system, so that the formation timescale for a $10 - 15 M_\oplus$ proto-Neptune core is prohibitively long. The two most widely explored attempts at a solution are positing a massive (> $6 \times$ MMSN) disk during the planet formation epoch (Goldreich et al., 2004, hereafter GO04) or that ice giant cores were gravitationally scattered to their present orbits after brisker formation in the trans-Saturnian region (Thommes et al. 2002; Thommes et al. 2003). However, these ideas, while plausible, introduce very case-specific assumptions about the initial state of the solar nebula or its dynamical history.

The purpose of this paper is to show that another mechanism for forming ice giant cores within the core accretion model exists that is more generally applicable to protoplanetary disks. Specifically, one may be able to form Uranus and Neptune-mass cores in situ and on solar nebula dissipation timescales from what is currently the most plausible mechanism for inducing particle subdisk GI necessary for planetesimal formation. In Section 2 & 3 we first review this planetesimal formation mechanism, called here the Sekiya-Youdin gravitational instability model, and then argue that it could aid ice giant formation in proceeding quickly. Next, in Section 4 we describe our recalculation of grain pileups in this model originally done by YS02 and Youdin & Chiang (hereafter YC04), develop a simple model for calculating protoplanetary core masses at the end of oligarchy and core formation timescales. In Section 5 we present our the results of our core mass and formation timescale calculations, showing how the $\Sigma_p$, solid body surface density, profile resulting from this planetesimal formation mechanism alters these values substantially. In the discussion section we put this model in a historical context, summarize it, and describe what research within this model should be done further. Finally, we include an appendix describing
the protoplanetary disk conditions necessary to allow GI.

2. PLANETESIMAL FORMATION FROM SEKIYA-YOUDIN GI

Unless one assumes a hitherto unknown efficient sticking mechanism for growing 1 cm - 1 km bodies one likely has to posit some sort of particle layer GI to build up km-sized planetesimals. Planetesimal formation by GI, as developed by Goldreich & Ward (1973), was long considered problematic, though. The chief reason for this was that as particles settle towards the disk midplane the disk becomes sufficiently vertically stratified that Kelvin-Helmholtz turbulence develops. Turbulent eddies resulting from the Kelvin-Helmholtz instability induce particle random velocities that are too high to allow GI, and equilibrium vertical profiles for particles result in spatial densities that are at least an order of magnitude too low (Weidenschilling 1980; Cuzzi, Dobrovolskis, & Champney 1995; Weidenschilling 1995).

Sekiya et al. (1998), however, found that if the ratio of dust to gas surface densities was sufficiently enhanced over standard solar values, turbulence induced by Kelvin-Helmholtz vertical shear will not be sufficient to stir all the solids: the rest will be able to precipitate towards the disk midplane and induce GI. YS02 and YC04 provide a likely mechanism for achieving this enhancement by pileup of grains from gas drag-induced migration, avoiding the main difficulty with GI from Goldreich & Ward (1973). We now describe this mechanism following YS02 and YC04.

Because gas is more sensitive to pressure gradients, it orbits at slightly sub-Keplerian velocities where the relative velocity difference between gas and dust is proportional to \( \eta = -(\partial P_g/\partial \ln a)/2 \rho g v_k^2 \sim (c_g/v_k)^2 \). The dust then experiences a headwind, and thus a drag force. For a stopping time of a particle moving relative to the gas of \( \tau_{\text{stop}} = \rho_p s/\rho g c_g \), and gas and particle densities of \( \rho_g \) and \( \rho_p \) respectively and particle sizes \( s \), the inward particle flux through the disk is \( f = f_{\text{ep}} + f_{\text{turb}} \), migration due to Epstein drag is \( f_{\text{ep}} = \rho_p (2/\pi)^2 v_g \eta \tau_{\text{stop}} \rho \) and turbulent stresses is \( f_{\text{turb}} \sim \rho_p v_g \eta \tau_{\text{stop}} \rho \) (whose effect is small: see YC04). This inward particle flux from Epstein drag-induced migration results in grain pileups if the flux decreases with decreasing stellocentric distance \( a \). Explicitly, whether such a condition is satisfied depends on the radial profiles for solid and gas surface densities \( \Sigma_p \) and \( \Sigma_g \) as well as the temperature profile.

In a MMSN model, one starts with solid body gas profiles of \( \Sigma_g \propto a^{-7/2} \) and \( \Sigma_p \propto a^{-3} \) and a temperature profile of \( T \propto a^{-q/2} \). The migration rate due to Epstein drag is

\[
v_a \sim v_{\text{ep,ind}} = \frac{\rho_p}{\rho} \eta \tau_{\text{stop}} O^2 a.
\]  

As \( v_a \) depends on \( \eta \) and \( \tau_{\text{stop}} \), which in turn depend on \( T \) and \( \Sigma_g \), Isolating the radial power law dependence of \( T \) and \( \Sigma_g \) on \( a \) one finds that the drift rate dependence goes as \( v_a \propto a^{d/2} \), where

\[
d = p - q + 1/2. \tag{2}
\]

The mass accretion rate is \( \propto \Sigma_a v_a \propto a^E \) where \( E = d- n+1 \) or \( E = 3/2 + p - n - q \) for grain migration in the Epstein drag regime. If \( E > 0 \) initially then the accretion rate increases with stellocentric distance, resulting in particle pileups and thus \( \Sigma_p \) enhancement. Epstein drag then results in grain pileups from the particle flux’s sensitivity to the gas density and temperature profiles.

Once enhancements of \( \Sigma_p/\Sigma_g \) from migration are sufficiently high such that the solid particle surface density, \( \Sigma_p \), rises above some critical value \( \Sigma_{pc} \), then particle sub-disk GI (and thus planetesimal formation) commences from dust in excess of \( \Sigma_{pc} \). Epstein drag-induced migration brings about particle layer GI because pileup of solid grains from migration can raise \( \Sigma_p \) above \( \Sigma_{pc} \), overcoming Kelvin-Helmholtz shear instabilities. Particle layer gravitational instabilities are allowed once the solid to gas ratio is enhanced by \( 5 - 20 \times \) (see YC04). Garaud & Lin (2004) find a similar criteria for GI using a more sophisticated two-fluid treatment.

3. ENHANCED EFFICIENCY OF PROTOPLANET CORE ACCRETION FROM SEKIYA-YOUDIN GI

YS02 and YC04 showed how Epstein drag-induced grain migration can procure planetesimal formation but didn’t explore the effect that such migration has on the efficiency of planet formation. Epstein drag-induced migration of grains, as shown by YS02 & YC04, yields substantially different solid body surface density profiles from a MMSN profile, enhancing \( \Sigma_p \). Since the criteria for GI (and planetesimal formation) is a \( 5 - 20x \) enhancement of \( \Sigma_p \), the mass of planetesimals available form which to form protoplanets should greatly increase within the disk subdisk as the mass from which such planetesimals is formed is locally enhanced by importing material from the outer solar nebula via migration.

As the masses of post-oligarchic cores as well as protoplanet formation and inelastic collisional damping and collision timescales depend strongly on \( \Sigma_p \) a seemingly simple enhancement of \( \Sigma_p \) by about an order of magnitude could have very important implications for the efficiency of planet formation in regions with such enhancement. The effect wasn’t obvious from YS02 and YC04 since both papers did not include solid enhancement from water ice condensation in their initial \( \Sigma_p \) profiles. For example, model H in YC04 then triggers GI at an outer boundary of only \( \approx 2 \) AU while model A triggers GI at an inner boundary with an enhancement of only \( \approx 1.5 - 2 \times \Sigma_{p,MMSN} \) at 15-25 AU when one compares the resulting \( \Sigma_p \) profile with an MMSN profile including the water ice enhancement. However, if one includes water ice condensation in \( \Sigma_p, \text{ initi} \) then pileup should proceed faster further away from the Sun. Planetesimal formation could then be triggered earlier and the resulting \( \Sigma_p \) profile could influence planet formation efficiency in the outer solar system (10-30 AU).

4. CALCULATIONS

We then are motivated to redo the YS02 calculations (equations 22-30) for the radial migration of mm-sized grains but include the solid enhancement for \( a > 3 \) AU due to the water ice condensation. We also estimate core masses expected at the end of the oligarchic growth stage as well as formation timescales for 10\( M_\oplus \) cores.

4.1. Recalculation of Grain Radial Drift/Pileup

Following YS02 we assume a population of uniformly-sized particles comprising the solid body column \( \Sigma_p \). Its
time-dependent profile is
\[ \frac{\partial \Sigma_p}{\partial t} = v_a \frac{\partial \Sigma_p}{\partial a} + \frac{\Sigma_p}{a} \frac{\partial}{\partial a} (a v_a), \]
which has an analytical solution:
\[ \Sigma_p(a, t) = \Sigma_o a^{-d-1} a_t^{d+1-n}(a, t) \]
where \(d = 1/2p - q \) and \(a_t(a, t)\) is the initial location of a particle now at a after time given by Eq. 26 of YS02:
\[ a_t(a, t) = a(1 - (d - 1) \frac{v_a}{a})^{-1/(d-1)}. \]

4.2. Oligarchic Core Masses and Core Formation Timescales

We then estimate oligarchic core masses and core formation timescales in the standard MMSN model and the model modified for migration of mm-sized grains. Oligarchic growth ends in once oligarchs' velocity excitations from mutual viscous stirring overtakes their damping rates which result from dynamical friction with smaller planetesimals. This condition is met once the "surface density" of protoplanets, \(\Sigma_M = M_{\text{pro}}/2\pi a \Delta a \) where \(\Delta a\) is the feeding zone size, becomes comparable to the residual surface density of smaller bodies such that \(\Sigma_M \sim \Sigma_m(t)\) (GO04). Assuming that little mass is yet lost from the system this condition can be related to the initial surface density of solids:
\[ \Sigma_M + \Sigma_m(t) = \Sigma_p \] (6)
Solving for \(M_{\text{pro}}\), then yields the protoplanet mass at the end of oligarchy (\(M_{\text{olig}}\)):
\[ M_{\text{olig}} \sim 0.04(b/10)^{1.5}(\Sigma_p/10 \text{g/cm}^{-2})^{1.5}(a/1 \text{AU})^3 M_\oplus. \]

After oligarchy ends dynamical interactions between oligarchs results in ejection of many of them, not mergers (Levison & Stewart 2001).

For calculating the core formation timescale we generally follow Ida & Lin (2004) and Kokubo & Ida (2002) with some modifications depending on the size of accreted planetesimals gleaned from Rafikov (2004). From Kokubo & Ida (2002), the mass accretion rate of a protoplanetary core depends on \(\Sigma_p\), the mass of the core at time \(t\), the core's size \(r_p\), its stellocentric distance \(a\), and the velocity dispersion of accreted planetesimals:
\[ \dot{M} = C \pi \Sigma_p \frac{2GM r_p}{<e^2>^{1/2} a^2 \Omega}. \]

where \(<e^2>^{1/2}\) is the rms eccentricity of planetesimals and \(C\) is a factor of order unity. The planetesimals' rms eccentricity is found by equating the viscous stirring and gas drag timescales \(T_{\text{vs}} = T_{\text{gas}}\) (Kokubo & Ida (2002)). The formation timescale is then given as \(\tau_{\text{c,acc}} = \frac{\dot{M}}{M}\). The approximate formation timescale for a core is then given by Ida & Lin (2004):
\[ \tau_{\text{c,acc}} \sim 1.2 \times 10^5 (10 \text{g/cm}^{-2}/\Sigma_p)(2400 \text{g/cm}^{-2}/\Sigma_g)^{0.4} \times (a/1 \text{AU})^{-0.6}(M_r/M_\oplus)^{1/3}(m/10^{18} \text{g})^{2/15} \text{years}, \]

where \(m\) is the mean accreted planetesimal mass for a protoplanet in a core feeding zone size of \(10R_{\text{Hill}}\) (\(b = 10\)). The rate of accretion of \(\sim 100m - 10 km\) planetesimals can be described by the above equation and accretion is then said to be 'dispersion dominated'. However, if the accreted planetesimals are small enough their relative velocities to cores are set by differential shear within the Hill radius of the core. In this case, the approach velocity of a planetesimal to the core is \(\sim \Omega R_H\) where \(R_H\) is the core's Hill radius, and the vertical component of the planetesimal's velocity, \(v_z\), small. For a small enough \(v_z\) the planetesimal disk becomes very thin and the embryo can accrete the entire vertical column of planetesimals (Rafikov 2004). This regime of accretion is called 'shear-dominated accretion' as the shear from the planetesimal disk around a core sets that core’s velocity. Rafikov (2004) then shows that the formation timescale for protoplanetary cores accreting shear-dominated planetesimals then is shorter. We rewrite the timescale here, generalizing it to variables \(\Sigma_p\), \(\rho_p\), \(M_r\), and \(\chi\):
\[ \tau_{\text{c,acc}} \sim 2.36 \times 10^4 \chi^{-1}(10 \text{g/cm}^{-2}/\Sigma_p) \times (M_r/M_\oplus)^{1/3}(\rho_p/1 \text{g/cm}^{-3})^{1/6} \text{years}, \]

where \(\chi\) is the fraction of planetesimals which are shear dominated and \(\rho_p\) is their mass density. These isolated body equations for \(M_{\text{olig}}\) and \(\tau_{\text{c,acc}}\) should be valid as long as growth occurs prior to the end of oligarchy and on timescales comparable to or less than the complete dissipation of the gas disk.

4.3. Size of Accreted Planetesimals

The size of accreted planetesimals is likely to be smaller than the \(\sim 1 km\) bodies formed after GI. Kilometer-sized objects inelastically collide and fragment if their collisional velocities exceed \(Q_D\), the energy per gram needed to release half of the planetesimal’s mass upon collision. For a 300 m projectile hitting a 1 km target \(Q_D \sim 4 \times 10^4\text{erg/g}\) (Leinhardt & Richardson 2002), where \(Q_D\) decreases for mass ratios approaching unity: low \(Q_D\)’s are consistent with the standard low internal strength, 'rubble pile' model for icy km-sized bodies such as comets (e.g. Ashbaugh & Benz 1996). The random velocity needed to disrupt a 1 km planetesimal is \(\sim 10m/s\) for a comparably-sized projectile of \(s = 500m\). This velocity is comparable to \(v_H\) at 30 AU. The velocity of collisions actually experienced between \(\sim 1 km\)-sized planetesimals near a protoplanetary core is set by the Hill velocity, \(v_H\), because their random velocities are set by scattering encounters with the core (GO04, Kenyon & Bromley 2004, and Rafikov 2004). Furthermore, random velocities after a scattering encounter are likely to exceed \(v_H\) (see Rafikov 2004, equations 23, 27, 42, and 46) unless the bodies already have \(s << 1 km\). This means that 1 km planetesimals scattered by protoplanetary cores should fragment upon collision with one another, an outcome confirmed by numerical simulations of planetesimal interactions around cores (e.g. Kenyon & Bromley 2004).

This is important because, as was alluded to before, the size of planetesimals very fundamentally affects core accretion rates. Specifically, for small planetesimals accretion can be much more rapid when planetesimal accretion occurs in the aforementioned 'shear-dominated' regime. We now describe the conditions for such accretion as they are given in Rafikov (2004).

Whether planetesimal accretion is shear or dispersion dominated depends largely on values for a fiducial
mass, $M_f$ and a fiducial size, $r_f$ which relate the planetesimal mass and sizes to planetesimal velocities induced after embryo-planetesimal scattering events. In particular, $M_f = M_c r_f^3 \sim 8 \times 10^{-4} (a/1\text{AU})^{3/2} g$ and $r_f = \frac{a^2 \Omega_m}{\rho_c s} \sim 120 (a/1\text{AU})^{-1/4} m$. Also important is a size, $r_s$, where the Reynolds number of the gas, $Re$, is $Re = \frac{\rho v a}{\mu}$, corresponding to changes in the drag coefficient experienced by a planetesimal moving through gas and where the planetesimal’s velocity dispersion equals its velocity difference with the gas: $v = \delta v_g$ (Rafikov 2004). This size is given as

$$r_s = \frac{\lambda R_{eq} \Omega a}{3 \rho_s} \approx 2.5(a/1\text{AU})^{5/2} m,$$  \hspace{1cm} (11)

where $\lambda \sim 1.25(a/1\text{AU})^{11/4} \text{cm}$, the molecular mean free path ($\Sigma_g$) is reduced by a factor of 1.25 compared to Rafikov (2004) affecting $r_f$, $\lambda$, and $r_s$.

As shown in equations (42) and (46) of Rafikov (2004), the velocity of a planetesimal relative to the Hill velocity, $v_H$, depends on the core mass, $M_c$, as well as on $M_f$, $\lambda$, $r_f$, and $r_s$. If $v \leq v_H$ then shear-dominated accretion sets in, and thus we can describe the boundary between dispersion and shear-dominated accretion as it depends on these parameters. For the Stokes drag regime the condition for shear-dominated accretion is

$$M_c \leq \left( \frac{r_f}{r_s} \right)^3 \left( \frac{f}{s} \right)^6$$  \hspace{1cm} (12)

and for the Epstein drag regime it is

$$M_c \leq \left( \frac{r_f}{\lambda} \right)^3 \left( \frac{f}{s} \right)^3$$  \hspace{1cm} (13)

Thus, after some time of planetesimal accretion, the planetesimals must collide frequently enough such that the population is ground down, keeping $s$ small, so that shear-dominated accretion may persist.

### 4.4. Model Assumptions and Initial Conditions

For the radial migration calculations we begin with the following distributions of solids and gas. We set $\Sigma_o = \epsilon \times 10 \text{ gcm}^{-2}$ where $\epsilon = 1$ interior to 3 AU, and $\epsilon = 4.2$ exterior. The initial gas $&$ solid surface densities are given as $\Sigma_g = 2400 \times (a/1\text{AU})^{-p} \text{ gcm}^{-2}$ and $\Sigma_r = \Sigma_g \times \epsilon$ where $p = 2$ and temperature profile given as $T = 280 \times (a/1\text{AU})^{-q} K$ where $p \times q = 1.5$, and $q = 0.5$, corresponding to Model H in YS02. The disk initially extends out to 250 AU in this model, though the outer edge of the solid component of the disk moves inward with time. For this model, the drift rate can be given as

$$v_d \sim 3(a/1\text{AU})^{1.5} \frac{10 \rho_g s}{g \text{ cm}^{-2}} \frac{1 \text{ AU}}{10^9 \text{ yr}}.$$  \hspace{1cm} (14)

We assume that the grains are comparable in size to the largest chondrules ($s = 1 \text{ mm}$) to make direct comparisons for the same run in YS02. We do calculations for the formation timescale of a 10 $M_\oplus$ core, one with the mean size of accreted planetesimals, $s$, set at 1 km, another with $s = 100m$ to take into account fragmentation, and $s = 10m$ to further explore shear-dominated accretion. The density, $\rho_p$, is set to $3 \text{ gcm}^{-3}$, representing an upper limit for planetesimals and ice-rock cores.

### 5. RESULTS

The solid particle migration run was stopped once one part of the disk had the requisite enhancement to induce a particle layer gravitational instability according to YS02. Figure 1 shows the time evolution of $\Sigma_g(t)$ from an initial MMSN profile to the onset of particle subdisk GI. When the bump from water ice condensation is included in the surface density profile the time for the onset of GI is $\sim 10\times$ shorter and occurs much further out at $\sim 27$ AU than in the same model in YS02 and YC04, yielding $\Sigma_g(t)/\Sigma_g(0) \sim 6 \sim 8 \times$ from 20-25 AU and $\sim 25\times$ at the outer edge of the particle subdisk ($26.7 AU$) before GI is initiated at $t = 4.25 \times 10^4$ yr. In the ‘enhanced core accretion’ scenario, the disk contains far more material from which to form protoplanets in precisely the regions where such formation has been most problematic for the standard core accretion model. As the figure shows, this occurs because the initial distribution in $\Sigma_g$ is now ‘squeezed’ into a smaller surface area bounded roughly by the ice giant planet forming region due to Epstein drag-induced migration.

Figure 2 shows the masses of oligarchs for standard and migration-enhanced core accretion models (labeled Sekiya-Youdin migration). For the latter, the oligarchic mass for Jupiter at $5.2 AU$ is $3.4 M_\oplus$, $21.1 M_\oplus$ for Uranus at $20 AU$ and $87 M_\oplus$ for Neptune at $25 AU$, values much greater than in the standard model, implying that Uranus and Neptune could have accreted most, if not all, of their mass ($15 \sim 17 M_\oplus$) by the end of the oligarchic growth phase. The masses of embryos at the end of oligarchy are large enough that additional, post-oligarchic growth can be negligible. Dynamical friction exerted by smaller bodies during oligarchic growth should dampen the embryos orbital eccentricities inhibiting ejection. Substantial post-oligarchic growth by coalescing is then unnecessary since cores have already grown sufficiently massive during oligarchy. Extra protoplanetary mass in the outer solar nebula is ejected after oligarchy ends (GO04).

Figure 3 compares formation timescales for a 10 $M_\oplus$ protoplanet core for the standard core accretion model and the model utilizing the particle subdisk GI from Sekiya and Youdin’s work to set $\Sigma_p$. For the standard MMSN surface density profile the formation timescale for a 10 $M_\oplus$ proto-Neptune core is $4.4 \times 10^8$ and $1.75 \times 10^8$ years at 25 AU for $s = 1 km$ and $s = 100 m$ respectively. Since $\tau_{ac,acc} > \tau_{dissip}$ and $M_{ac} \sim 1 < 10^8 M_\oplus$, the MMSN model would likely fail to produce Neptune-like planets on requisite timescales. However, for the enhanced model the formation timescales are $5.5 \times 10^7$ and $2.2 \times 10^7$ years at 25 AU for $s = 1 km$ and $s = 100 m$. It then could be possible to form proto-Neptune nearly in situ on a timescale less than the disk dissipation timescale (10-30 Myr) for $s \leq 100 m$.

Furthermore, a formation timescale of 20 Myr at 25 AU is most likely an overestimate as, using arguments from Rafikov (2004), protoplanets here should grow a large fraction of their mass while the accreted planetesimals are in the fast, shear-dominated regime. Initially one starts with 1-10 km planetesimals where the initial growth rate is fast. However, once the core mass increases enough, planetesimal accretion becomes dispersion dominated and growth occurs much more slowly.
To get back to faster, shear-dominated accretion, the initial population of planetesimals must drop in size enough such that $s < 1 \text{km}$. Quantitatively, we refer back to the equations (11) through (13). At 25 AU $M_f$ and $r_f$, are $\sim 10^{27}$ g and $\sim 0.43 m$ respectively and $r_e$ is $\sim 7.8 km$. Then, in the Stokes regime, dispersion-dominated accretion persists for $M_e \geq 3.8 \times 10^{19} g$ and $3.8 \times 10^{25} g$ for $s = 1 \text{km}$ and 100$m$ the latter still not an appreciable fraction of Neptune's current mass.

However, the $\sim 7 - 9r$ enhancement resulting from migration and pileup of mm-sized grains and subsequent GI helps to bring one back into the shear-dominated accretion regime earlier and persist there for longer times by the following argument. This enhancement drives down the collision timescale by nearly an order of magnitude as the collision rate is proportional to the number density of planetesimals and thus $\Sigma_p$ (e.g. Wetherill & Stewart 1993, equation A1). This increases the dominance of collisions and collisional damping over gas damping and inspiral of solids (the latter timescale is longer than gas damping by a factor of $1/(1.6 \times 10^{-3} R_{\text{Hab}}^{1/2}) \sim 125$). If $\Sigma_p$ is enhanced we should expect more frequent collisions early on. As the collisions tend to be disruptive at impact speeds of $v \geq v_H$, an enhancement in $\Sigma_p$ should result in planetesimal swarms of smaller mean sizes.

The smaller sizes translate into planetesimal dynamics occurring in different drag regimes and longer times spent in shear dominated planetesimal accretion. Specifically, at 25 AU, planetesimals are in the Epstein regime for $s \leq \lambda \sim 88 m$. As this is slightly below an order of magnitude reduction in the size of the initial swarm and collision rates between planetesimals are greater for enhanced $\Sigma_p$, it is likely that an appreciable fraction of accreted solids are in the Epstein drag regime and this regime is reached earlier than when a standard MMSN $\Sigma_p$ distribution is used. One stays in the shear-dominated Epstein drag regime as long as $M_e \leq 5 \times 10^{26} g$ and $6 \times 10^{28} g$ for $s = 50 m$ and 10m, the latter being comparable to the present rock/ice mass of Neptune. In the context of planetesimal accretion this means that cores can accrete planetesimals in the fast shear-dominated regime through a larger fraction of their total mass. This has a drastic effect on the core formation timescale. For shear-dominated accretion, we then find the formation timescale for a 10$M_J$ core at 25 AU to be $\tau \sim 2.26 \times 10^7 \chi^{-1} \text{yr} = 2.26 \times 10^8 \text{yr}$ if 10% of planetesimals are shear dominated (shown in Figure 3) and $2.26 \times 10^5 \text{yr}$ if all of them are: both values are less than the solar nebula dissipation timescale, $\tau_{\text{dissip}} \sim 10^7 \text{yr}$.

Formation timescales in the outer solar nebula are then sufficiently reduced and conditions are more likely to be met such that accelerated protoplanet growth persists through a sufficient fraction of proto-Neptune’s final mass.

6. DISCUSSION

The general idea in this paper that an enhancement of solid material in the Uranus & Neptune regions of the solar nebula during the first 10 Myr is what led to rapid formation of ice giants is not new. Various incarnations of this idea have appeared in the last twenty years (e.g. Lissauer 1987; Pollack et al.1996; GO04) and even when such an enhancement was not considered the primary factor in forming Neptune on gas dissipation timescales, $\tau_{\text{dissip}}$, it often plays a supporting role in increasing a proto-Neptune’s feeding zone (e.g. Bryden et al. 2000). These papers, however, usually assume such an enhancement of solids over MMSN values results solely from the disk mass itself being much greater than MMSN values. This paper relaxes that requirement, assuming a $\Sigma_p$ and initial $\Sigma_p$ profile comparable to MMSN values (total disk mass here is $\sim 0.05 M_J$ spread over 250 AU, out to 30 AU it is $\sim 0.02 M_J$), with very large enhancements of the solids achieved from grain migration and GI. Thus, in this model it may be possible to get the same benefit you have from an initially massive disk (e.g. more solids from which to form planets, higher planetesimal collision rates, etc.) without the ‘cost’ of having that disk mass be unreasonably large.

That a global radial redistribution of solids influences some part of the planet formation process has also been suggested previously. Specifically, Stepinski & Valageas (1997) and Kornet et al. (2001) posited that gas drag in a turbulent disk result in solid surface density distributions that differ from the gas surface density, providing enhancements in the inner regions of the solar nebula when compared to their initial distributions. These papers, though, use a very simple treatment for grain coagulation for $1cm - 10 km$ bodies, the size regime where a hierarchical sticking hypothesis seems to be highly problematic. The effect any solid body redistribution has on the efficiency of post-planetesimal growth wasn’t explored: rather, at least for Kornet et al. (2001) emphasis was placed on how different initial conditions for the disk mass and radial extent explain the diversity of planetary systems via their differences in grain redistribution. It appears that grain redistribution can lead to not only a diversity of system architectures but changes in the efficiency of planet formation.

The way in which planetesimal formation, particularly that of the GI model for planetesimal formation, influences the efficiency of planet formation has been hitherto undereported and warrants further investigation. A comparison between analytical estimates of core formation timescales and numerical calculations assuming a $\Sigma_p$ distribution set by dust migration models such as YSO2 should be made. Furthermore, incorporating the ‘shear-dominated’ planetesimal accretion model into such calculations is needed as many papers making pronouncements about the efficiency of protoplanetary core formation (e.g. Thommes et al. 2003) adopt a prescription for the mass accretion rate similar to that for ‘dispersion-dominated accretion’ in this paper, where a planetesimal’s rms equilibrium eccentricity is set by balancing viscous stirring by the protoplanet with gas drag from the solar nebula. As argued here and especially in Rafikov (2004), if the planetesimals are small then accretion might proceed more rapidly.

If one considers the effect that Epstein drag-induced migration has on the distribution of solids, the formation of cores in the outer solar nebula occurs on far shorter than the standard core accretion model. Assuming an initially massive disk or requiring substantial gravitational scattering of proto-Uranus and proto-Neptune cores, while still plausible, may be unnecessary for forming Uranus and Neptune in $\leq 10^8 \text{yr}$. Then ‘hybrid’ mechanisms such as this one resulting from Sekiya-Youdin particle subdisk GI could
eliminate core formation timescale problems in the outer solar system. Furthermore, the mechanism outlined in this scenario is generic if planetesimal formation proceeds by particle subdisk gravitational instability as described by YS02 & YC04, operating regardless of whether others do. Gas pressure gradients then play an indispensable role in forming the cores of gas/ice giant planets quickly, as they do in the rival disk instability model (see Boss et al. 2002, Haghighipour & Boss 2003), though in a slightly more indirect (and less obvious) way.

The author would like to thank Andrew Youdin, Scott Kenyon, and Richard Nelson for extremely fruitful discussions and Nader Haghighipour, Brad Hansen, and the anonymous referee for suggestions that strengthened the arguments presented in this paper. This research was supported in part by the NASA Astrobiology Institute.

APPENDIX

DISK CONDITIONS FOR GRAIN SETTLING/GI

It is important to ask whether the conditions for particle subdisk gravitational instability can be met. Specifically, the main requirement for GI is that the disk must be sufficiently passive to initially allow the dust layer to collapse to a thickness set by a balance between settling and Kelvin-Helmholtz shear. If viscous stirring is strong enough to prevent this amount of settling then the entire GI mechanism for forming planetesimals becomes highly problematic. We show here that the assumption that GI can occur is at least reasonable.

We first analyze previous attempts at constraining the equilibrium dust layer scale height achieved by balancing settling with diffusivity generated by Richardson turbulence. YC04 calculated this criteria to be \( \alpha \leq 10^{-7}(a/1AU)^{1+p-q} \) or \( \alpha \leq 10^{-7}(a/1AU)^{2} \) for a MMSN model. The coefficient \( \alpha \) must then be less than \( \sim 6.3 \times 10^{-5} \) for \( a = 25 \) AU. More stringent requirements can be found from Dubrulle et al. (1995). From their equations 36 and 37, an equilibrium distribution of solids is written in terms of the ratio between the dust and gas scale heights (\( h \) and \( H \)), the friction (stopping) time \( \tau_{\text{stop}} \), and \( \alpha \):

\[
\frac{h}{H} \sim \sqrt{\frac{\alpha}{\Omega \tau_{\text{stop}}}}.
\]  

The criteria for GI in terms of volume density is \( \rho_{\text{cr}} \sim M_{\odot}/a^{3} \) or \( \sim 2.4 \times 10^{-11} \text{gcm}^{-3} \) or about a factor of \( 1000 \times \) increase over its solar abundance (and \( \sim 100 \times \) the gas density). We then can rewrite the above condition as (\( \Sigma_{d}/100\Sigma_{g})^{2} \sim 10^{-6} \sim \alpha/(\tau_{\text{stop}}\Omega) \). As \( \tau_{\text{stop}} \Omega \sim 10^{-4}(a/1AU)^{1.5} \) the criteria then becomes \( \alpha \leq 10^{-8} \) for grain sedimentation to a requisite thin dust subdisk. While estimates for \( \alpha \) from molecular viscosity are small enough (\( \sim 10^{-12} \)), values for \( \alpha \) needed to explain mass accretion rates onto T Tauri stars range from \( 10^{-2} \) to \( 10^{-4} \).

Thus at first glance it seems as though conditions for GI include an implausibly low disk viscosity. However, as pointed out by YC04, turbulence need not be isotropic, and when \( \alpha \) is resolved into components the criteria radially is given as \( \alpha_{r} \leq \Omega_{\text{stop}} \sim 10^{-4}(a/1AU)^{3/2} \) (from their equation B2). The condition at 25 AU is then that \( \alpha_{r} \leq 1.25 \times 10^{-2} \) which is reasonable. The effective \( \alpha_{r} \) may differ substantially from the \( \alpha_{z} \) if the viscosity source is more effective at mixing radially than vertically. An example of a ratio between horizontal and vertical viscosities of order \( 10^{4} \) is cited by YC04 for Earth’s oceans and atmosphere.

A stronger argument can probably be made based on investigating disk structure models in terms of whether a particular region is or is not viscously evolving, especially considering the effect of dust grains on levels of disk viscosity. The disk may spend a substantial fraction of its lifetime accreting through only thin surface layers sandwiched between a quiescent, magnetically dead layer containing a large fraction of the total column of the disk material in so called ‘layered disk’ models (Gammie 1996). The anomalous viscosity then presumably tapers off and disappears beyond a certain gas column density into the disk a result consistent with MHD simulations of layered disks from Fleming & Stone (2003). Fromang et al. (2002) shows that for \( \alpha = 10^{-2} \) and \( M \sim 10^{-8} M_{\odot}/yr \) the outer radius of the dead zone can be as large as \( 100 \) AU and occupy 90 \% of the vertical column density of the disk for metal fractions, \( x_{M} \), slightly below cosmic abundances. For \( \alpha = 10^{-3} \) a dead zone exists for all values of the disk metal fraction, not just subsolar values. Furthermore, these calculations were done assuming that the grains do not affect the magnetic coupling to the disk, and the authors specifically assumed that settling of the grains to the disk midplane had already occurred: thus their ability to inhibit coupling of magnetic field lines to the gas would be irrelevant except for a vanishingly thin dust layer at the midplane. Fleming & Stone (2003) also suggest that including the effects of grains would inhibit the ability of magnetic field lines to couple to the gas.

In absence of this assumption, Fromang et al. (2002) suggests that because grains are effective at ‘scavenging’ charge they may induce a magnetically dead state within the subdisk they are contained, preventing turbulence from the anomalous source of viscosity from stirring them up and thus allowing sedimentation to the disk midplane if vertical mixing isn’t particularly strong. This intuition now seems to have been confirmed numerically. Recently, Nelson et al. (2005, in prep) has investigated the effect that dust grains have on the coupling of magnetic field lines to the gas in the disk: thus their effect on the level of MRI turbulence. This work suggests that dust grains are particularly efficient at scavenging charge in the disk, and their inclusion leads to dead zone \( \Sigma_{g} \)'s that are much larger than those in Fromang et al. (2002). They find that in order to drive MRI turbulence throughout the disk cross section at some distance, one needs to reduce the number density of dust grains by a factor of \( 10^{8} \). Accretion through the entire vertical column of the disk would then seem to presume that substantial grain growth and settling had already occurred. Clearly, one
For this model (equivalent to Model H in YS02 & YC04), Epstein drag experienced by mm-sized grains causes them to migrate, pile up, and induce particle subdisk GI to form planetesimals. This leaves a $\sim 8$-$9x$ enhancement in the outer regions of the particle disk after $\sim 4.25 \times 10^4$ yr, yielding a $\Sigma_p$ differing significantly from the standard MMSN profile.

cannot at the moment make a strong case against the existence of conditions necessary for GI.

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Fig. 2.— Protoplanetary core masses at the end of oligarchic growth for the standard core accretion model and the 'hybrid', migration-enhanced model. \( M_{\text{diss}} \) is greater than the predicted solid body masses for Uranus and Neptune at roughly their present orbits in the latter: all of their growth could conceivably occur during oligarchy.

Fig. 3.— Formation timescales for 10\( M_\oplus \) protoplanetary cores for 1km and 100m mean accreted planetesimal sizes. The enhancement of \( \Sigma_p \) after particle subdisk GI due to Epstein drag-induced migration reduces the core formation timescale by nearly 10x for dispersion-dominated planetesimals. 10m-sized planetesimals drop into the shear-dominated regime for \( a > 12.5 \) AU and dominate protoplanet accretion at these distances such that \( \tau_{\text{acc}} \approx 10^6 \) yr to the edge of the planetesimal disk if 10% of them are 10m. Thus, for \( s < 100 \) m proto-Neptune can form on timescales \( \tau_{\text{acc}} \approx \tau_{\text{dissip}} \) (the lifetime of the solar nebula) or less, even at 25 AU.