Baryons as Fock states of 3, 5, ... quarks

Dmitri Diakonov\textsuperscript{1,2,3} and Victor Petrov\textsuperscript{3}

\textsuperscript{1} Thomas Jefferson National Accelerator Facility, Newport News, VA 23606, USA
\textsuperscript{2} NORDITA, Blegdamsvej 17, DK-2100 Copenhagen, Denmark
\textsuperscript{3} St. Petersburg Nuclear Physics Institute,
Gatchina, 188 300, St. Petersburg, Russia

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Abstract

We present a generating functional producing quark wave functions of all Fock states in the octet, decuplet and antidecuplet baryons in the mean field approximation, both in the rest and infinite momentum frames. In particular, for the usual octet and decuplet baryons we get the $SU(6)$-symmetric wave functions for their 3-quark component but with specific corrections from relativism and from additional quark-antiquark pairs. For the exotic antidecuplet baryons we obtain the 5-quark wave function [1].
I. INTRODUCTION

It is a great pleasure for us to write a paper in honor of our old friend Klaus Goeke who has made an enormous contribution to the development of the Chiral Quark Soliton Model (CQSM).

We have noticed from experience that many people hearing the word “soliton” immediately imagine the XIX century English gentleman racing after a solitary wave along the Thames, and are for that reason forever scared off from the model. In more recent times, another English gentleman T.H.R. Skyrme suggested that nucleons can be viewed as solitons of the pion field. This scares some people even more as few would imagine a fermion built from a bosonic field. This paper is another attempt to persuade those people that the CQSM is about quarks: we are going to present explicit quark wave functions following in particular from that model. Few people call atoms or nuclei solitons; however, they can be called solitons of the electrostatic and of the mean nuclear field, respectively. Maybe we should rename the CQSM into the Mean Field Approximation, to make it sound more traditional and less challenging the common sense.

The mean field approach to bound states is usually justified by the large number of participants. The Thomas–Fermi approximation to atoms is justified at large $Z$, and the shell model for nuclei is justified at large $A$. In baryons, the appropriate large parameter justifying the mean field approach would be the number of colors $N_c$ [2]. There are two kinds of corrections in $1/N_c$. One kind is due to the fluctuations of the chiral field about its mean-field value in a baryon. These are loop corrections and are additionally suppressed by factors of $1/(2\pi)$. With the present precision, such corrections can be ignored. The second type can be called kinematical: they are due to the rotations of a baryon, and are not suppressed additionally. Many baryon observables get such corrections, the isovector magnetic moment or the axial constant being examples [3, 4]. For example, $g_A$ of the nucleon gets a correction factor $(1+2/N_c)$ equal to $5/3$ in the real world. Such kind of corrections should be collected, if possible. When and if this is done, the CQSM or else the Mean Field Approximation for baryons becomes quite precise.

It should be noted that these anomalously large corrections arise from the imaginary part of the effective chiral action [5], and are absent e.g. in the Skyrme model. Being important in ordinary baryons, the anomalously large $1/N_c$ corrections become crucial for
exotic baryons. In the Skyrme model the exotic \((10, \frac{1}{2}^+)\) baryon antidecuplet does not exist unless one extends the parameters of the model [6]; even if this is done, the exotic \(\Theta^+\) baryon appears as a broad resonance. Meanwhile in the CQSM, the formally leading term for the \(\Theta\) width is strongly cancelled by terms non-existing in the Skyrme model [7], and this cancellation pertains at any \(N_c\) [8].

Therefore, it would be helpful to have a formalism which relies on the mean chiral field but treats the rotational corrections exactly at any \(N_c\) and at the real-world \(N_c = 3\) in particular. In fact, this was the logic always adopted in Bochum. Its philosophy has been recently reviewed in Ref. [9]. We shall apply it here to reveal the 3,5,7...-quark wave functions of the octet, decuplet and antidecuplet baryons.

A schematic view of baryons in the Mean Field Approximation is presented in Fig. 1, and it is within this construction all observables have been so far computed in the CQSM.

![FIG. 1: A schematic view of baryons in the Mean Field Approximation. There are three “valence” quarks at a discrete energy level created by the mean field, and the negative-energy Dirac continuum distorted by the mean field, as compared to the free one.](image)

![FIG. 2: Equivalent view of baryons in the same approximation, where the distorted Dirac sea is presented as quark-antiquark pairs. Their wave function is given by the quark Green function in the background mean field, at equal times.](image)

An alternative but mathematically equivalent description has been recently suggested by one of the authors and Polyakov [10]. If the mean field makes the Dirac sea less dense than in the vacuum at certain momenta, it means a hole or the presence of an antiquark with positive energy. If the Dirac sea is more dense in some partial wave and at some momenta, it means the presence of an additional negative-energy quark with the corresponding quantum
numbers. Since the total number of levels in the sea is its baryon number and is conserved whatever the background field, it implies that any distortion of the Dirac sea by the mean field creates an equal number of quarks and antiquarks or, else, quark-antiquark \((Q\bar{Q})\) pairs inside a baryon, in addition to the three valence quarks, see Fig. 2. Algebraically, the average number of \(Q\bar{Q}\) pairs is proportional to the amplitude squared of the mean field, times \(N_c\). It also depends on the quantum numbers of the baryon in question. For nucleons it appears to be somewhat less than one pair on the average. In the antidecuplet, the number of \(Q\bar{Q}\) pairs is strictly larger than one. Theoretically, at large \(N_c\) there are \(\sim N_c\) \(Q\bar{Q}\) pairs in any baryon in addition to the \(N_c\) valence quarks.

II. THE EFFECTIVE ACTION

The effective action approximating QCD at low momenta describes “constituent” quarks with the momentum dependent dynamical mass \(M(p)\) interacting with the scalar (\(\Sigma\)) and pseudoscalar (\(\Pi\)) fields such that \(\Sigma^2 + \Pi^2 = 1\) at spatial infinity. The momentum dependence \(M(p)\) serves as a formfactor of the constituent quarks and provides the effective theory with the ultraviolet cutoff. Simultaneously, it makes the theory non-local. The action is [5, 11]

\[
S_{\text{eff}} = \frac{1}{(2\pi)^8} \int \frac{d^4p d^4p'}{(2\pi)^8} \bar{\psi}(p) \left[ \gamma \left( \frac{2\pi}{(2\pi)^4} \delta^{(4)}(p - p') - \sqrt{M(p)} \left( \Sigma(p - p') + i\Pi(p - p')\gamma_5 \right) \sqrt{M(p')} \right] \psi(p'),
\]

where \(\psi, \bar{\psi}\) are quark fields carrying color, flavor and Dirac bispinor indices. In the instanton model of the QCD vacuum from where this action has been originally derived [11] the function \(M(p)\) is such that there is no real solution of the mass-shell equation \(p^2 = M(-p^2)\), therefore quarks are not observable as asymptotic states, – only their bound states. However, this is not the true confinement. Unfortunately, the instanton model’s \(M(p)\) has a cut at \(p^2 = 0\) corresponding to massless gluons left in that model. In the true confining theory there should be no such cuts. Nevertheless, such \(M(p)\) creates some kind of a soft “bag” for quarks. Contrary to the primitive bag picture which violates all principles one can think of, eq. (1) supports all general principles, like relativistic invariance and sum rules for conserved quantities.

Turning to baryons, the mean \(\Sigma, \Pi\) field (called chiral field for short in what follows) in the full non-local formulation (1) has been found by Broniowski, Golli and Ripka [12]. It sets an example how one has to proceed in the model calculations. However, to simplify the
mathematics we shall use here a more standard approach: we shall replace the constituent quark mass by a constant and mimic the decreasing function $M(p)$ by the UV Pauli–Villars cutoff [13].

III. BARYON WAVE FUNCTION IN TERMS OF QUARK CREATION-ANNIHILATION OPERATORS

Let $a, a^\dagger(p)$ and $b, b^\dagger(p)$ be the annihilation–creation operators of quarks and antiquarks (respectively) of mass $M$, satisfying the usual anticommutator algebra \{\(a(p)a^\dagger(p')\}\} = \{\(b(p)b^\dagger(p')\}\) = \((2\pi)^3\delta^{(3)}(p - p')\) with $a, b|0> = 0$, $<0|a^\dagger, b^\dagger = 0$. For quarks, the annihilation-creation operators carry, apart from the 3-momentum $p$, also the color $\alpha$, flavor $f$ and spin $\sigma$ indices but we shall suppress them until they are explicitly needed. The Dirac sea is presented by the coherent exponent of the quark and antiquark creation operators,

\[
\text{coherent exponent} = \exp(\int (dp)(dp') a^\dagger(p) W(p, p') b^\dagger(p')) |0>,
\]

where $(dp) = d^3p/(2\pi)^3$ and $W(p_1, p_2)$ is the quark Green function at equal times in the background $\Sigma, \Pi$ fields, see Fig. 2. In the saddle point approximation these fields are replaced by the mean field:

\[
\pi(x) = n \cdot \tau P(r), \quad n = x/r, \quad \Sigma(x) = \Sigma(r).
\]

On the chiral circle $\Pi = n \cdot \tau \sin P(r)$, $\Sigma(r) = \cos P(r)$ where $P(r)$ is the profile function of the self-consistent field. We shall specify the pair wave function $W(p, p')$ below.

It is assumed that the self-consistent chiral field creates a bound-state level for quarks, whose wave function $\psi_{\text{lev}}$ satisfies the static Dirac equation with eigenenergy $E_{\text{lev}}$ [5, 14, 15]:

\[
\psi_{\text{lev}}(x) = \begin{pmatrix} e^{ij} h(r) \\ -i e^{jk} (\sigma \cdot n) j(r) \end{pmatrix}, \quad \begin{cases} h' + h M \sin P - j(M \cos P + E_{\text{lev}}) = 0, \\
 j' + 2j/r - j M \sin P - h(M \cos P - E_{\text{lev}}) = 0. \end{cases}
\]

In the non-relativistic limit ($E_{\text{lev}} \approx M$) the $L = 0$ upper component of the Dirac bispinor $h(r)$ is large while the $L = 1$ lower component $j(r)$ is small.

The valence quark part of the baryon wave function is given by the product of $N_c$ quark creation operators that fill in the discrete level:

\[
\text{valence} = \prod_{\text{color}=1}^{N_c} \int (dp) F(p) a^\dagger(p),
\]
\[ F(p) = \int (dp') \sqrt{\frac{M}{\epsilon_p}} \left( \bar{u}(p) \gamma_0 \psi_{lev}(p) (2\pi)^3 \delta(p-p') - W(p, p') \bar{v}(p') \gamma_0 \psi_{lev}(-p') \right), \]

where \( \psi_{lev}(p) \) is the Fourier transform of eq. (4). The second term in Eq. (6) is the contribution of the distorted Dirac sea to the one-quark wave function. \( u_\sigma(p) \) and \( v_\sigma(p) \) are the plane-wave Dirac bispinors projecting to the positive and negative frequencies, respectively. In the standard basis they have the form

\[
\begin{align*}
\begin{cases}
u_\sigma(p) = \left( \frac{\sqrt{\epsilon - M}}{2M} p_\sigma | s_\sigma \rangle, \\ \frac{\sqrt{\epsilon - M}}{2M} | p_\sigma \rangle | s_\sigma \rangle \end{cases}
\end{align*}
\]

where \( \epsilon = +\sqrt{p^2 + M^2} \) and \( s_\sigma \) are two 2-component spinors normalized to unity, for example,

\[
\begin{align*}
s_1 &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}, & s_2 &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}, & \sigma &= 1, 2.
\end{align*}
\]

The full baryon wave function is given by the product of the valence part (5) and the coherent exponent (2) describing the distorted Dirac sea. Symbolically, one writes the baryon wave function in terms of the quark and antiquark creation operators [10]:

\[
B[a^\dagger, b^\dagger] = \prod_{\text{color}=1}^{N_c} \int (dp) F(p) a^\dagger(p) \exp \left( \int (dp)(dp') a^\dagger(p) W(p, p') b^\dagger(p') \right) |0>.
\]

At this point one has to recall that the saddle point at the self-consistent chiral field is degenerate in global translations and global \( SU(3) \) flavor rotations [the \( SU(3) \) breaking by the strange mass can be treated as a perturbation.] Integrating over translations leads to the momentum conservation: the sum of all quarks and antiquarks momenta have to be equal to the baryon momentum. Integration over rotations \( R \) leads to the projection of the flavor state of all quarks and antiquarks onto the spin-flavor state \( B(R) \) describing a particular baryon from the \( (8, \frac{1}{2}^+) \), \( (10, \frac{3}{2}^+) \) or \( (\overline{10}, \frac{1}{2}^+) \) multiplet.

Restoring color (\( \alpha = 1, 2, 3 \)), flavor (\( f = 1, 2, 3 \)), isospin (\( j = 1, 2 \)) and spin (\( \sigma = 1, 2 \)) indices, the quark wave function inside a particular baryon \( B \) with spin projection \( k \) is given, in full glory, by

\[
\begin{align*}
\psi_k^B &= \int dR B_k^*(R) \epsilon^{\alpha_1 \alpha_2 \alpha_3} \prod_{n=1}^{3} \int (dp_n) R^{f}_{jn} F^{j\sigma\alpha_n}(p_n) a^\dagger_{\alpha_n f_n \sigma_n} (p_n) \\
&\quad \cdot \exp \left( \int (dp)(dp') a^\dagger_{\alpha f \sigma} (p) R^{f}_{j'} W^{j_\sigma f \sigma'}(p, p') R^{j_\sigma}_{f'} b^\dagger f' \sigma' (p') \right) |0>.
\end{align*}
\]
This is the “generating functional” mentioned in the Abstract. Expanding the coherent exponent to the 0th, 1st, 2nd... order one reads off the 3-, 5-, 7-... quark wave functions of a particular baryon from the octet, decuplet or antidecuplet.

To make this powerful formula fully workable, we need to give explicit expressions for the baryon rotational states $B(R)$, the valence wave function $F^j_{\sigma}(p)$ and the $Q\bar{Q}$ wave function in a baryon $W_{j\sigma'}(p, p')$.

### A. Baryon rotational states

In general, baryon rotational states $B(R)$ are given by the $SU(3)$ Wigner finite-rotation matrices [16], and any particular projection can be obtained by a routine $SU(3)$ Clebsch–Gordan technique. However, in order to see the symmetries of the quark wave functions it is helpful to use explicit expressions for $B(R)$, and integrate over the Haar measure in eq. (10) explicitly.

Let us give a few examples of the baryons’ (conjugate) rotational wave functions $B^*(R)$:

- **proton**, spin projection $k$:
  \[ p_k^*(R) = \sqrt{8} \epsilon_{kl} R_1^l R_3^3, \]  \[ (11) \]

- **neutron**, spin projection $k$:
  \[ n_k^*(R) = \sqrt{8} \epsilon_{kl} R_2^l R_3^3, \]  \[ (12) \]

- **$\Delta^{++}$**, spin projection $+\frac{3}{2}$:
  \[ \Delta_{11}^{++}(R) = \sqrt{10} R_1^1 R_2^2 R_3^3, \]  \[ (13) \]

- **$\Delta^{0}$**, spin projection $+\frac{1}{2}$:
  \[ \Delta_{12}^{0+}(R) = \sqrt{10} R_1^1 R_2^1 (2 R_1^2 R_2^1 + R_2^2 R_1^1), \]  \[ (14) \]

- **$\Theta^+$**, spin projection $k$:
  \[ \Theta_k^*(R) = \sqrt{30} R_3^3 R_3^3 R_3^3, \]  \[ (15) \]

- **neutron** from $\Xi$, spin projection $k$:
  \[ n_{10, k}^*(R) = \sqrt{10} R_3^3 (2 R_1^3 R_3^k + R_3^3 R_1^k). \]  \[ (16) \]

They are normalized in such a way that for any spin projection

\[ \int dR B_{\text{spin}}^*(R) B^{\text{spin}}(R) = 1. \]  \[ (17) \]

For example, somebody is interested in the quark wave function of the $\Theta^+$. Then one has to substitute (15) into the general eq. (10) and integrate over $R$. If only three valence quarks are taken and the coherent exponent is ignored, one gets

\[ \int dR R_3^3 R_3^3 R_3^k R_1^j R_2^j R_3^3 = 0 \]  \[ (18) \]

meaning, of course, that one cannot built the exotic $\Theta^+$ from three quarks. The first non-zero Fock component of the $\Theta$ is 5Q and is obtained by expanding the $Q\bar{Q}$ exponent to the
linear order. In this case the appropriate group integral is

\[ T(\Theta)^{f_1 f_2 f_3 f_4 j_5}_{j_1 j_2 j_3 j_4 j_5} = \int dR \Theta^*_k(R) R^f_{j_1} R^f_{j_2} R^f_{j_3} R^f_{j_4} R^f_{j_5} \]

\[ \frac{\sqrt{30}}{180} \delta^{j_5}_{j_k} \left( \epsilon_{j_1 j_2} \epsilon_{j_3 j_4} \epsilon_{f_1 f_2} \epsilon_{f_3 f_4} \epsilon_{f_5 f_4} \right) + \epsilon_{j_1 j_3} \epsilon_{j_2 j_4} \epsilon_{f_1 f_3} \epsilon_{f_2 f_4} \epsilon_{f_5 f_4} \right). \]

(19)

It tells us that the antiquark in the \( \Theta \) is necessarily \( \bar{s} \) thanks to \( \delta^{j_3}_{j_5} \), and that the four quarks are \( uudd \) since in the 2-dimensional antisymmetric tensors \( \epsilon_{f_1 f_2} \) and the like, the flavor indices are \( f_1 - 4 = 1, 2 = u, d \).

**B. The \( Q\bar{Q} \) pair wave function**

As explained in Ref. [10], the pair wave function \( W^{j\sigma}_{\sigma'}(p, p') \) is expressed through the finite-time quark Green function at equal times in the external chiral field. Schematically, it is shown in Fig. 2. We define the Green function as the solution of the equation

\[ [i\partial / - M(\Sigma + i\Pi \gamma_5)]_{x_1, t_1} G(x_1, t_1|x_2, t_2) = \delta(t_1 - t_2) \delta^{(3)}(x_1 - x_2). \]

(20)

The quantity \( V = M(-1 + \Sigma + i\Pi \gamma_5) \) will be called the perturbation as due to the non-zero mean field. In what follows we shall rename \( \Sigma - 1 \rightarrow \Sigma \). For the static hedgehog chiral field lying on the chiral circle

\[ \Sigma^j_{\sigma'}(x) = (\cos P(r) - 1)\delta^j_{\sigma'}, \quad \Pi^j_{\sigma'}(x) = (n \cdot \tau)^j_{\sigma'} \sin P(r). \]

(21)

We shall need their Fourier transforms,

\[ \Sigma(q) = \int d^3x e^{-iq \cdot x} \Sigma(x), \quad \Pi(q) = \int d^3x e^{-iq \cdot x} \Pi(x), \]

(22)

where \( \Sigma(q) \) is real and even while \( \Pi(q) \) is purely imaginary and odd. In the frame where a baryon has a constant velocity \( v \) along the \( z \) axis both fields get the arguments \( (x, y, z) \rightarrow \left(x, y, \frac{z - vt}{\sqrt{1 - v^2}} \right) \). Both fields can be written through the Fourier transforms in the rest frame:

\[ \Sigma, \Pi \left(x, y, \frac{z - vt}{\sqrt{1 - v^2}} \right) = \int (dq) \exp \left( iq_x x + iq_y y + iq_z \frac{z - vt}{\sqrt{1 - v^2}} \right) \Sigma, \Pi(q). \]

(23)

One can present the Green function as a perturbation expansion in \( V = \Sigma + i\Pi \gamma_5 \):

\[ \frac{1}{i\partial / - M - V} = \frac{1}{i\partial / - M} + \frac{1}{i\partial / - M} V \frac{1}{i\partial / - M} + \frac{1}{i\partial / - M} V \frac{1}{i\partial / - M} V \frac{1}{i\partial / - M} + \ldots \]

(24)
The important point (used in the derivation of eq. (9) [10]) is that all free Green functions in this equation should be understood with the Feynman $i\epsilon$ prescription in the momentum space, meaning the shift $M - i\epsilon$ in the free propagators.

The perturbation (21) is in fact very specific: its modulus is always less than unity. If the pion field is much less than unity, the perturbation is small. If the chiral field is not small but has either low ($q \ll M$) or large ($q \gg M$) momenta, the perturbation is, effectively, also small as will become clear from the final expression for the pair wave function. Therefore, it is not a bad idea to restrict oneself to the first order in the perturbation in $V$, which we are going to do here. Keeping higher orders in $V$ has no principle difficulties but in our experience the first order is usually within 10-15% from exact (all orders) calculations.

In the first order in the external field $V$ the Green function is, according to eq. (24),

$$G^{(1)}(x_1, t_1|x_2, t_2) = \int_0^T dt \int d^3z \, G^{(0)}(x_1, t_1|z, t) \, V(z, t) \, G^{(0)}(z, t|x_2, t_2). \quad (25)$$

Here $T$ is the “observation time” during which the external chiral field exists; it should be put to infinity to obtain the ground-state baryon with given quantum numbers. We can write it further in the momentum representation:

$$G^{(1)}(x_1, t_1|x_2, t_2) = \int (d\mathbf{q}_1)(d\mathbf{q}_1) \, \exp(-i\mathbf{q}_1 \cdot \mathbf{x}_1 - i\mathbf{q}_2 \cdot \mathbf{x}_2) \, G^{(1)}(\mathbf{q}_1, t_1|\mathbf{q}_2, t_2),$$

$$G^{(1)}(\mathbf{q}_1, t_1|\mathbf{q}_2, t_2) = \int \frac{d\omega_1 d\omega_2}{(2\pi)^2} \int_0^T dt' \int d^3z \int (d\mathbf{q}) \, \exp \left[ i\omega_1 \mathbf{q}_1 \mathbf{z} + i\omega_2 \mathbf{q}_2 \mathbf{z} + i(\mathbf{q}_1 \cdot \mathbf{z} - \mathbf{q}_2 \cdot \mathbf{z}) \frac{1}{q_1 - M + i\epsilon} \right] \frac{1}{q_2 - M + i\epsilon} \quad (26)$$

where $q_{1,2\mu} = (\omega_{1,2}, \mathbf{q}_{1,2}), \quad \delta_{1,2} = \omega_{1,2}\gamma_0 - \mathbf{q}_{1,2} \cdot \gamma.$

The definition of the (conjugate) pair wave function is [10]

$$W_{c\bar{c}\sigma'}(\mathbf{p}', \mathbf{p}) = -i \sqrt{\frac{e \epsilon'}{M^2}} \left[ \bar{G}^{\sigma'}(\mathbf{p}') \mathcal{G}(\mathbf{p}, 0|\mathbf{p}', 0)u_{\sigma}(\mathbf{p}) \right] \quad (27)$$

with the plane-wave bispinors $u, v$ defined in eq. (7). One has to integrate eq. (26) over $\omega_{1,2}$ and the intermediate point $(\mathbf{z}, t')$ where the perturbation $V$ acts. Because of the Feynman “$M - i\epsilon$” rule, one closes the integration contour in $\omega_1$ in the lower semiplane and finds the contribution of the pole $\omega_1 = \epsilon = \sqrt{p^2 + M^2}$. Integration over $\omega_2$ is closed in the upper semiplane with $\omega_2 = -\epsilon' = -\sqrt{p'^2 + M^2}$. This is an important although natural result: the $Q\bar{Q}$ pair has a positive-energy antiquark and necessarily a negative-energy quark. The physical interpretation, in terms of the level density of the Dirac sea, has been given in the Introduction.
Integration over \( d^3z \) leads to the 3-momentum conservation, \( q_\perp = -(p + p')_\perp \), \( q_z = -(p_z + p'_z)/\sqrt{1 - v'^2} \). Integration over the intermediate time \( t' \) gives the energy denominator 
\[-i/\epsilon + \epsilon' - i0 - (p_z + p'_z)v \]. Finally, one has to use the Dirac equation for the plane-wave bispinors: 
\( (M - p')\sigma(p) = 0, \bar{\sigma}'(p')(M + p') = 0 \). As a result one obtains

\[
W_{ejj'}^{\sigma\sigma'}(p', p) = \sqrt{\frac{M^2}{\epsilon\epsilon'}} \sqrt{\frac{1 - v'^2}{\epsilon + \epsilon'}} \left[ \bar{\psi}'(p')V(-p - p')\psi(p) \right].
\] (28)

Its explicit form in the baryon rest frame has been given in Ref. [18]. In the infinite momentum frame (IMF) one has to take the limit \( v \to 1 \). The momentum of the baryon with mass \( M \) is

\[
P = \frac{Mv}{\sqrt{1 - v^2}}, \quad \text{hence} \quad v = \frac{P}{\sqrt{P^2 + M^2}} \approx 1 - \frac{M^2}{2P^2}.
\] (29)

The quark and the antiquark of the \( Q\bar{Q} \) pair have the 4-momenta

\[
p_\mu = \left( zP + \frac{p_{\perp}^2 + M^2}{2zP}, P_\perp, zP \right), \quad p'_\mu = \left( z'P + \frac{p^{'\perp}_2 + M^2}{2z'P}, P'_\perp, z'P \right),
\] (30)

hence the energy denominator is

\[
\frac{\sqrt{1 - v'^2}}{\epsilon + \epsilon' - (p_z + p'_z)v} = \frac{M}{P} \frac{2zz'P}{Z}, \quad Z \equiv M^2zz'(z + z') + z(p^2_{\perp} + M^2) + z'(p^{'}_{\perp} + M^2).
\] (31)

In the infinite momentum frame it is convenient to rescale the annihilation-creation operators, \( a^{IMF}_\sigma(z, p_\perp) = \sqrt{P/2\pi} a_\sigma(p) \) and similarly for \( a^\dagger, b, b^\dagger \), where the subscript \( \sigma = 1, 2 \) refers now to the \( \pm \) helicity states. The new operators satisfy the anticommutation relations

\[
\{a^{\alpha_1 f_1 \sigma_1}(z_1, p_{\perp 1}), a^{\alpha_2 f_2 \sigma_2}(z_2, p_{\perp 2})\} = \delta_{\sigma_2}^{\sigma_1} \delta_{f_2}^{f_1} \delta_{\alpha_2}^{\alpha_1} \delta(z_1 - z_2) (2\pi)^2 \delta(2)_p \delta(2)_q (p_{\perp 1} - p_{\perp 2})
\] (32)

and similarly for the new \( b, b^\dagger \). The use of the rescaled operators requests rescaling \( W_{ejj'}^{\sigma\sigma'} \) by a factor of \( P/(2\pi) \). Taking the \( v \to 1 \) limit in the bispinors (7) one gets finally [10] [17]

\[
W_{ejj'}^{\sigma\sigma'}(z, p_\perp; z', p'_\perp) = \frac{M^2}{2\pi Z} \left\{ \Sigma_j^j(q) [M(z' - z)\sigma_3 + Q_\perp \cdot \sigma_1]^{\sigma'}_{\sigma} + i \Pi_j^j(q) [-M(z' + z)1 - i\epsilon_{\alpha\beta}Q_{\perp \alpha} \sigma_\perp \beta]^{\sigma'}_{\sigma} \right\},
\] (33)

\[
q = ((p + p')_\perp, (z + z')M), \quad Q_{\perp \alpha} = zp^{'}_{\perp \alpha} - z'p_{\perp \alpha}, \quad \alpha, \beta = 1, 2.
\]

The non-primed indices refer here to the quark and the primed ones to the antiquark. Note that we have written here the conjugate pair wave function. To obtain the pair wave function actually used in eq. (10), one has to take the hermitian conjugate of \( W_e \), namely replace \( (j \leftrightarrow j'), (\sigma \leftrightarrow \sigma') \) and change the sign of the last \( i\epsilon_{\alpha\beta} \) term in eq. (33):

\[
W_{j'j''}^{\sigma\sigma'}(z, p_\perp; z', p'_\perp) = \frac{M^2}{2\pi Z} \left\{ \Sigma_j^{j'}(q) [M(z' - z)\sigma_3 + Q_\perp \cdot \sigma_1]^{\sigma'}_{\sigma} + i \Pi_j^{j'}(q) [-M(z' + z)1 + i\epsilon_{\alpha\beta}Q_{\perp \alpha} \sigma_\perp \beta]^{\sigma'}_{\sigma} \right\},
\] (34)
where, again, the primed indices refer to the antiquark.

Eq. (34) gives the wave function of the additional $Q\bar{Q}$ pairs in a baryon in the infinite momentum frame. The corresponding expression for the rest frame is presented in Ref. [18]. Eq. (33) is the conjugate wave function needed to evaluate matrix elements for baryon observables. The indices $j, j' = 1, 2$ are the isospin indices (to be rotated by the $SU(3)$ flavor matrices $R$ in eq. (10)) and $\sigma, \sigma' = 1, 2$ are the quark and antiquark helicity states. The annihilation-creation operators in eq. (10) are now understood to be normalized by the condition (32), and the integrals over momenta there are understood as $\int dz \int d^2p_\perp/(2\pi)^2$.

C. Discrete-level wave function

As seen from eq. (6), the discrete-level wave function $F^{j\sigma}(p) = F^{j\sigma}_{\text{lev}}(p) + F^{j\sigma}_{\text{sea}}(p)$ consists of two pieces: one is directly the wave function of the valence level, the other is related to the change of the number of quarks at the discrete level as due to the presence of the Dirac sea; it is a relativistic effect and can be ignored in the non-relativistic limit, together with the lower $L=1$ component $j(r)$ of the level wave function. Indeed, in the baryon rest frame the evaluation of the first term in eq. (6) gives

$$F^{j\sigma}_{\text{lev}}(p) = e^{j\sigma} \left( \sqrt{\frac{E_{\text{lev}} + M}{2E_{\text{lev}}}} h(p) + \sqrt{\frac{E_{\text{lev}} - M}{2E_{\text{lev}}}} j(p) \right),$$

(35)

where $h, j(p)$ are the Fourier transforms of the valence wave functions (4):

$$h(p) = \int d^3x e^{-ip\cdot x} h(r) = 4\pi \int dr r^2 \frac{\sin pr}{pr} h(r),$$

(36)

$$j^a(p) = \int d^3x e^{-ip\cdot x} (-i\gamma^a) j(r) = \frac{p^a}{|p|} j(p), \quad j(p) = \frac{4\pi}{p^2} \int dr (pr \cos pr - \sin pr) j(r).$$

(37)

One sees that the second term in eq. (35) is double-suppressed in the non-relativistic limit $E_{\text{lev}} \approx M$: first, owing to the kinematical factor, second, since in this limit the $L=1$ wave $j(r)$ is much less than the $L=0$ wave $h(r)$.

In the infinite momentum frame the evaluation of the bispinors $\bar{u}, \bar{v}$ from eq. (7) produces [10] [19]

$$F^{j\sigma}_{\text{lev}}(z, p_\perp) = \sqrt{\frac{M}{2\pi}} \left[ \epsilon^{j\sigma} h(p) + (p_z 1 + i\epsilon_{\alpha\beta}p_\perp \sigma_{\perp\beta}) \sigma^\alpha, \epsilon^{j\sigma'} j(p) \right]_{p_z = zM}$$

(38)

Similarly, the evaluation of the “sea” part of the discrete-level wave function gives

$$F^{j\sigma}_{\text{sea}}(z, p_\perp) = -\sqrt{\frac{M}{2\pi}} \int dz \frac{d^2p_\perp}{(2\pi)^2} W^{j\sigma}(p, p') \epsilon^{j\sigma''} \left[ (\sigma_3)_{\sigma'}^{\sigma''} h(p') - (\sigma \cdot p')_{\sigma''} j(p') \right] \right]_{p_z = zM}$$

(39)
where the pair wave function (34) has to be used. The conjugate functions are hermitian conjugate.

We have thus fully determined all quantities entering the master eq. (10) for the 3, 5, 7... Fock components of baryons’ wave functions.

IV. EXAMPLES OF BARYON WAVE FUNCTIONS IN THE NON-RELATIVISTIC LIMIT

If the coherent exponent with $Q\bar{Q}$ pairs is ignored, one gets from the general Eq. (10) the 3-quark Fock component of the octet and decuplet baryons. It depends on the quark “coordinates”: the position in space ($r$), the color ($\alpha$), the flavor ($f$) and the spin ($\sigma$), and also on the baryon spin projection $k$. For example, the neutron $3Q$ wave function turns out to be

$$\langle n \uparrow \rangle f_1 f_2 f_3, \sigma_1 \sigma_2 \sigma_3 (r_1, r_2, r_3) = \epsilon^{f_1 f_2} \epsilon^{\sigma_1 \sigma_2} \delta^{\sigma_3}_k h(r_1) h(r_2) h(r_3)$$

$$+ \text{permutations of } 1, 2, 3,$$

(40)

times the antisymmetric $\epsilon^{\alpha_1 \alpha_2 \alpha_3}$ in color. It is better known in the form

$$|n \uparrow \rangle = 2 d \uparrow (r_1) d \uparrow (r_2) u \downarrow (r_3) - d \downarrow (r_1) u \uparrow (r_2) d \downarrow (r_3) - u \uparrow (r_1) d \downarrow (r_2) d \uparrow (r_3)$$

$$+ \text{permutations of } r_1, r_2, r_3,$$

(41)

which is the well-known non-relativistic $SU(6)$ wave function of the nucleon! In Ref. [10] the corresponding $SU(6)$ function in the infinite momentum frame was obtained.

Similarly, the $3Q$ component of the $\Delta^0$ baryon with spin projection $1/2$, whose wave function may be compared with that of the neutron, is

$$|\Delta^0 \uparrow \rangle f_1 f_2 f_3, \sigma_1 \sigma_2 \sigma_3 (r_1, r_2, r_3) = \left( \delta^{f_1 f_2}_1 \delta^{f_3}_2 + \delta^{f_1 f_2}_2 \delta^{f_3}_1 + \delta^{f_1 f_2}_2 \delta^{f_3}_1 \right)$$

$$\cdot \left( \delta^{\sigma_1}_1 \delta^{\sigma_2}_2 \delta^{\sigma_3}_1 + \delta^{\sigma_1}_2 \delta^{\sigma_2}_1 \delta^{\sigma_3}_2 + \delta^{\sigma_1}_2 \delta^{\sigma_2}_1 \delta^{\sigma_3}_1 \right) h(r_1) h(r_2) h(r_3)$$

(42)

which can be presented also as a familiar $SU(6)$ wave function

$$|\Delta^0 \uparrow \rangle = u \uparrow (r_1) d \uparrow (r_2) d \downarrow (r_3) + d \downarrow (r_1) u \uparrow (r_2) d \downarrow (r_3) + d \uparrow (r_1) d \uparrow (r_2) u \downarrow (r_3)$$

$$+ \text{permutations of } r_1, r_2, r_3.$$

(43)
There are, of course, relativistic corrections to these SU(6)-symmetric formulae, arising from i) exact treatment of the discrete level, eqs. (38,39), and ii) additional $QQ$ pairs described by eq. (34). Both effects are not small.

The $5Q$ component of a baryon is obtained when one expands the coherent exponent to the linear order and then projects it into the concrete baryon in question. In general, the $5Q$ wave functions look rather complicated as they depend on five quark “coordinates”, including their coordinates proper, spin, flavor and color. We do not write explicitly the color degrees of freedom but always imply that the $(1,2,3)$ quarks of the level are antisymmetric in color while the quark-antiquark pair $(4,5)$ is a color singlet, as it follows from eq. (10). For example, the $5Q$ component of the neutron has the wave function

$$\begin{align}
(n>k)_{f_5,\sigma_5}^{f_1 f_2 f_3 f_4,\sigma_1 \sigma_2 \sigma_3 \sigma_4} (r_1, r_2, r_3, r_4, r_5) &= F^{j_1 \sigma_1} (r_1) F^{j_2 \sigma_2} (r_2) F^{j_3 \sigma_3} (r_3) W_{j_5 \sigma_5}^{j_4 \sigma_4} (r_4, r_5) \\
\cdot \{ &\epsilon^{f_1 f_2} \epsilon_{j_1 j_2} \left[ \epsilon^{f_3 f_4} \delta_{j_3} \left( 4 \delta_{j_4} \delta_{j_4} - \delta_{j_4} \delta_{j_4} \right) + \epsilon^{f_3 f_4} \delta_{j_3} \left( 4 \delta_{j_4} \delta_{j_4} - \delta_{j_4} \delta_{j_4} \right) \right] \\
+ &\epsilon^{f_1 f_2} \epsilon_{j_1 j_4} \left[ \delta_{j_2} \delta_{j_4} \delta_{j_3} \left( 4 \delta_{j_4} \delta_{j_4} - \delta_{j_4} \delta_{j_4} \right) + \delta_{j_2} \delta_{j_4} \left( 4 \delta_{j_4} \delta_{j_4} - \delta_{j_4} \delta_{j_4} \right) \right] \} \epsilon^{k' k} \\
+ &\text{permutations of } (1,2,3)
\end{align}$$

where the pair wave function $W$ in the rest frame can be found in Ref. [18]. Indices 1-3 refer to quarks at the discrete level, 4 refers to the quark in the additional pair, and 5 refers to the antiquark in the pair. Terms of the type of $\delta_{f_5}^{\alpha}$ mean the flavor-symmetric combination $ss + uu + dd$. We have not invented how to present it in a more compact form.

Turning to the exotic baryons from the $\left( \bar{\Pi}, \frac{1}{2}^+ \right)$, projecting the three quarks from the discreet level on the $\Theta$ rotational function (15) gives an identical zero (see eq. (18)), in accordance with the fact that the $\Theta$ cannot be made of 3 quarks. The non-zero projection is achieved when one expands the coherent exponent at least to the linear order. One gets then from eq. (19) the $5Q$ component of the $\Theta$ wave function :

$$\begin{align}
|\Theta^+_{f_5,\sigma_5}^{f_1 f_2 f_3 f_4,\sigma_1 \sigma_2 \sigma_3 \sigma_4} (r_1, \ldots r_5) &= \epsilon^{f_1 f_2} \epsilon_{j_1 j_2} \delta_{j_3} \epsilon^{\sigma_1 \sigma_2} \\
\cdot h(r_1) h(r_2) h(r_3) W^{\sigma_3 \sigma_4} (r_4, r_5) + \text{permutations of } 1,2,3
\end{align}$$

The color structure of the antidecuplet wave function is $\epsilon^{\alpha_1 \alpha_2 \alpha_3 \delta_{\alpha_4}}$. Indices 1 to 4 refer to quarks and index 5 refers to the antiquark, in this case $s$ owing to $\delta_{f_5}^s$. The quark flavor indices are $f_{1-4} = 1, 2 = u, d$. Naturally, we have obtained $\Theta^+ = uudds$. Since in the CQSM the functions $h(r_{1,2,3})$ and $W(r_4, r_5)$ are known, eq. (45) gives the complete color, flavor, spin and space 5-quark wave function of the $\Theta^+$ in its rest frame. The structure $\epsilon^{f_1 f_2} \epsilon^{\sigma_1 \sigma_2}$
clearly shows that there is a pair of \textit{ud} quarks in the spin and isospin zero combination, exactly as in the nucleon, eq. (40). However, it does not mean that there are prominent scalar isoscalar diquarks either in the nucleon or in the \( \Theta \): that would require their spatial correlation which, as we see, is absent in the mean field approximation.

The \( Q\bar{Q} \) pair wave function \( W \) is a combination of four partial waves with different permutation symmetries. The partial waves depend separately on the coordinates \( r_{4,5} \) measured from the baryon center of mass. More explicit formulae can be found in Ref. [18]. It would be interesting to compare eq. (45) with the wave functions obtained in non-relativistic dynamical models or discussed in that framework [20].

Unfortunately, the meaning of the baryon wave function in the rest frame is unclear since it has fundamental flaws, especially in the world with the spontaneous chiral symmetry breaking and light pions, as we have had several chances to explain [9, 21]. In particular, a rotation of the chiral phase is a zero-energy vacuum rearrangement in the chiral limit, however it can be decomposed into a large number of \( Q\bar{Q} \) pairs. To find the true number of \( Q\bar{Q} \) pairs in a hadron, one has to separate its proper structure from that of the vacuum. To this end one is forced to consider hadrons in the infinite momentum frame where the vacuum \( Q\bar{Q} \) pairs with an arbitrary high momenta are nevertheless separated from those belonging to a hadron and thus having an infinite momentum. Therefore, in the IMF (and only there) the number of \( Q\bar{Q} \) pairs in a hadron has a precise mathematical meaning, and the Fock states are well defined.

V. THREE QUARKS: NORMALIZATION, VECTOR AND AXIAL CHARGES

The normalization of a baryon wave function in the second-quantization representation (10) is found from
\[
N_B = \frac{1}{2} \delta^k_l <\Psi^B \Psi^B_\dagger > .
\] (46)
The annihilation operators in \( \Psi^B \) must be dragged to the right where they ultimately nullify the vacuum state \( |0\rangle \) and the creation operators from \( \Psi^B_\dagger \) should be dragged to the left where they ultimately nullify the vacuum state \( <0| \). The result is non-zero owing to the anticommutation relations (32) or the “contractions” of the operators.

For the 3Q Fock component of a baryon, there are 3! possible (and equivalent) contractions, and the ensuing contraction in color indices gives another factor of 3! = \( \epsilon^{\alpha_1\alpha_2\alpha_3} \epsilon_{\alpha_1\alpha_2\alpha_3} \).
Flavor projecting to a baryon with specific quantum numbers gives a tensor

\[
T^{J_1 J_2 J_3}_{j_1 j_2 j_3 k} = \int dR B_k^*(R) R^1_{j_1} R^2_{j_2} R^3_{j_3}
\]

and a hermitian conjugate for the conjugate wave function. Hence the normalization of the 3Q component is

\[
\mathcal{N}^{(3)} = \frac{(6 \cdot 6)}{2} \delta_{kk} T^{f_1 f_2 f_3}_{j_1 j_2 j_3} T^{l_1 l_2 l_3}_{f_1 f_2 f_3} \int dz_{1,2,3} \int d^2 p_{1,2,3} \frac{1}{(2\pi)^3} \delta(z_1 + z_2 + z_3 - 1)
\cdot (2\pi)^2 \delta((p_1 + p_2 + p_3)_\perp) F^{j_1 \sigma_1}(p_1) F^{j_2 \sigma_2}(p_2) F^{j_3 \sigma_3}(p_3) F_{l_1 \sigma_1}(p_1) F_{l_2 \sigma_2}(p_2) F_{l_3 \sigma_3}(p_3)
\]

where \( F^{j_\sigma}(z, p_\perp) \) are the level wave functions (38,39). In the non-relativistic limit \( F^{j_\sigma}(p) F^{l_\sigma}_{l_\sigma}(p) = \delta^j_l h^2(p) \). Therefore in this simple case the normalization is the full contraction of the two \( T \) tensors, times an integral over momenta which can be performed numerically once the level wave function \( h(p) \) is known. Since the normalization of this function is arbitrary one can always choose it such that \( \mathcal{N}^{(3)} = 1 \) for all baryons possessing a 3Q component.

A typical physical observable is a matrix element of some operator (which should be written down in terms of the quark annihilation-creation operators \( a, b, a^\dagger, b^\dagger \) sandwiched between initial and final baryon wave functions (10). We shall consider as examples the operators of the vector and axial charges which can be written through the annihilation-creation operators as

\[
\begin{align*}
\left\{ \begin{array}{c} Q \\ Q_5 \end{array} \right\} &= \int d^3 x \bar{\psi}_e J^e_h \\
&= \int dz \frac{d^2 p_\perp}{(2\pi)^2} \left[a^\dagger_{e\sigma}(z, p_\perp)a^{b\rho}(z, p_\perp) J^e_h \left\{ \begin{array}{c} \delta^\pi_{\rho} \\ (-\sigma)^{\pi}_{\rho} \end{array} \right\} \right]\ \\
&- b^\dagger^{b\rho}(z, p_\perp)b_{e\pi}(z, p_\perp) J^e_h \left\{ \begin{array}{c} \delta^\pi_{\rho} \\ (-\sigma)^{\pi}_{\rho} \end{array} \right\}
\end{align*}
\]

where \( J^e_h \) is the flavor contents of the charge, and \( \pi, \rho = 1, 2 \) are helicity states. For example, if we consider the \( \rho^+ = \bar{d}u \) current which annihilates \( u \) quarks and creates \( d \) quarks and annihilates \( \bar{d} \) antiquarks and creates \( \bar{u} \) ones, the flavor currents in eq. (49) are \( J^e_h(\rho^+) = \delta^2_{\rho} \delta^1_h \). Notice that there are no \( a^\dagger b^\dagger \) or \( ab \) terms in the charges. This is a great advantage of the IMF where the number of \( Q\bar{Q} \) pairs is not changed by the current. Hence there will be only diagonal transitions between Fock components with equal numbers of pairs.

In the matrix elements between the 3Q components the \( b^\dagger b \) part of the current is passive as there are no antiquarks. The \( a^\dagger a \) part is a sum over colors. As in the normalization, one
gets the factor $6 \cdot 6$ from all contractions. Let it be the third quark whose charge is measured: there is a factor of 3 from three quarks to which the charge operator can be applied, see Fig. 3. Denoting for short $\int (dp_{1-3})$ the integrals over momenta with the conservation $\delta$-functions as in eq. (48) we arrive at the following expression for the matrix element of the vector charge:

$$V^{(3)} = \frac{(6 \cdot 6 \cdot 3)}{2} \delta^{k}_{l} T(1)^{f_{1}f_{2}f_{3}} T(2)^{l_{1}l_{2}l_{3},l} \int (dp_{1-3}) \cdot \left[ F^{j_{1}\sigma_{1}}(p_{1}) F^{j_{2}\sigma_{2}}(p_{2}) F^{j_{3}\sigma_{3}}(p_{3}) \right] \left[ F_{l_{1}\sigma_{1}}^{\dagger}(p_{1}) F_{l_{2}\sigma_{2}}^{\dagger}(p_{2}) F_{l_{3}\sigma_{3}}^{\dagger}(p_{3}) \right] \left[ \delta_{\sigma_{3}}^{\tau_{3}} J_{f_{3}}^{\tau_{3}} \right].$$

(50)

One can easily check using eqs. (11,12) that, say, for the $p \rightarrow n\rho^{+}$ transition, the above vector charge gives exactly the same expression as for the normalization (48). Therefore, the $g_{V}$ of this transition is unity, as it should be for the conserved vector current.

For the axial transition, one replaces averaging over baryon spin by $\frac{1}{2}(\sigma_{3})^{k}_{l}$, and the axial charge operator is now $(-\sigma_{3})^{\tau}_{\sigma_{3}}$ instead of $\delta^{\tau}_{\sigma_{3}}$, see eq. (49). All the rest is the same as in eq. (50):

$$A^{(3)} = \frac{(6 \cdot 6 \cdot 3)}{2} (-\sigma_{3})^{k}_{l} T(1)^{f_{1}f_{2}f_{3}} T(2)^{l_{1}l_{2}l_{3},l} \int (dp_{1-3}) \cdot \left[ F^{j_{1}\sigma_{1}}(p_{1}) F^{j_{2}\sigma_{2}}(p_{2}) F^{j_{3}\sigma_{3}}(p_{3}) \right] \left[ F_{l_{1}\sigma_{1}}^{\dagger}(p_{1}) F_{l_{2}\sigma_{2}}^{\dagger}(p_{2}) F_{l_{3}\sigma_{3}}^{\dagger}(p_{3}) \right] \left[ (-\sigma_{3})^{\tau}_{\sigma_{3}} J_{f_{3}}^{\tau_{3}} \right].$$

(51)

The result, however, is now different as the axial charge is not conserved. For example, for the $p \rightarrow n\pi^{+}$ transition one gets the expression identical to that for the normalization but with the factor $5/3$. It means that we have obtained in the non-relativistic limit for the $3Q$ component of the nucleon $g_{A}^{(3)}(N) = 5/3$. It is the well-known result of the non-relativistic...
quark model. However, it is modified by the relativistic corrections to the valence quark wave functions (38,39) and by the 5Q component of the nucleon.

VI. FIVE QUARKS: NORMALIZATION, VECTOR AND AXIAL CHARGES

Already in the normalization of the 5Q Fock component of a baryon there are two types of contributions: direct and exchange ones, see Fig. 4. In the former, one contracts $a^\dagger$ from the pair wave function with an $a$ in the conjugate pair, and all the valence operators are contracted with each other. There are 6 such possibilities, and the contraction in color gives a factor $3 \cdot 6$, all in all 108. In the exchange contributions, one contracts $a^\dagger$ from the pair with one of the three $a$’s from the valence level. Further on, $a$ from the conjugate pair is contracted with one of the three $a^\dagger$’s from the valence level. There are 18 such possibilities but the contraction in color gives now only a factor of 6. Therefore for the exchange contractions we also get a factor of 108 but with an overall negative sign as one has to anticommute fermion operators to get the exchange terms. As a result we obtain the following general expression for the normalization of the 5Q Fock component:

$$\mathcal{N}^{(5)} = \frac{108}{2} \int (dp_{1-5}) \delta_k^k T_{j_1 j_2 j_3 j_4 f_5, k} T_{f_1 f_2 g_3 g_4 f_5, l}$$

$$\cdot F^{j_1 \sigma_1}(p_1) F^{j_2 \sigma_2}(p_2) F^{j_3 \sigma_3}(p_3) W^{j_4 \sigma_4}(p_4, p_5) F^{\dagger}_{l_1 \sigma_1}(p_1) F^{\dagger}_{l_2 \sigma_2}(p_2)$$

$$\cdot \left[ F^{\dagger}_{l_3 \sigma_3}(p_3) W^{l_5 \sigma_5}_{c_4 \sigma_4}(p_4, p_5) \delta_{f_3}^{g_3} \delta_{f_4}^{g_4} - F^{\dagger}_{l_5 \sigma_5}(p_4) W^{l_5 \sigma_5}_{c_4 \sigma_4}(p_3, p_5) \delta_{f_3}^{g_3} \delta_{f_4}^{g_4} \right]. \tag{52}$$

The flavor tensor here is the group integral projecting the 5Q state onto a particular baryon:

$$T_{j_1 j_2 j_3 j_4 f_5, k} = \int dR B^*_k(R) R^{f_1}_{j_1} R^{f_2}_{j_2} R^{f_3}_{j_3} R^{f_4}_{j_4} R^{f_5}_{f_5} \cdot \tag{53}$$

The ratio of the normalizations $\mathcal{N}^{(5)}/\mathcal{N}^{(3)}$ gives the probability to find a 5Q component in a mainly 3Q baryon. It depends on the mean field inside a baryon through the pair wave function $W$ (and is quadratic in the mean field), and on the particular baryon through its spin-flavor contents $T$.

For the vector and axial transitions there are three basic contributions: one when the charge of the antiquark is measured, the second when the charge operator acts on the quark from the pair, and the third when it acts on one of the three valence quarks. These three types are further divided into the direct and exchange contributions (Figs. 5,6). We write below only the direct contributions.
The vector transition:

\[
V^{(5)\text{direct}} = \frac{108}{2} \int (dp_{1-5}) \delta^k_1 T(1)f_1 f_2 f_3 f_4 j_s \ T(2)f_1 \bar{f}_2 \bar{f}_3 \bar{f}_4 \bar{g}_5, l
\]

\[
\cdot F^{j_1 \sigma_1} (p_1) F^{j_2 \sigma_2} (p_2) F^{j_3 \sigma_3} (p_3) W^{j_4 \sigma_4} (p_4, p_5) F^{j_5 \sigma_5} (p_1) \bar{F}^{j_2 \sigma_2} (p_2) \bar{F}^{j_3 \sigma_3} (p_3) W^{l_4 \tau_4} (p_4, p_5)
\]

\[
\cdot \left[ -\delta^9_{f_3} \delta^9_{f_4} J^5_f \delta^5_{\tau_3} \delta^8_{\tau_4} \delta^5_{\tau_5} + \delta^9_{j_3} J^5_{g_4} \delta^5_{g_5} \delta^8_{\tau_3} \delta^4_{\tau_4} \delta^5_{\tau_5} + 3J^5_{f_4} \delta^9_{f_5} \delta^5_{f_6} \delta^5_{\tau_3} \delta^8_{\tau_4} \delta^5_{\tau_5} \right] .
\]

The axial transition:

\[
A^{(5)\text{direct}} = \frac{108}{2} \int (dp_{1-5}) (\sigma_3)^k_1 T(1)f_1 f_2 f_3 f_4 j_s \ T(2)f_1 \bar{f}_2 \bar{f}_3 \bar{f}_4 \bar{g}_5, l
\]

\[
\cdot F^{j_1 \sigma_1} (p_1) F^{j_2 \sigma_2} (p_2) F^{j_3 \sigma_3} (p_3) W^{j_4 \sigma_4} (p_4, p_5) F^{j_5 \sigma_5} (p_1) \bar{F}^{j_2 \sigma_2} (p_2) \bar{F}^{j_3 \sigma_3} (p_3) W^{l_4 \tau_4} (p_4, p_5)
\]

\[
\cdot \left[ \delta^9_{f_3} J^5_{f_4} \delta^9_{g_4} \delta^5_{g_5} \delta^8_{\tau_3} \delta^4_{\tau_4} \delta^5_{\tau_5} - \delta^9_{f_3} J^5_{f_4} \delta^5_{g_5} \delta^8_{\tau_3} \delta^4_{\tau_4} \delta^5_{\tau_5} - 3J^5_{f_4} \delta^9_{f_5} \delta^5_{f_6} \delta^5_{\tau_3} \delta^8_{\tau_4} \delta^5_{\tau_5} \right] ,
\]

where \( J^5_f \) is the flavor content of the current defined in the previous section.

Applications of these general formulæ to physically interesting cases, for example for the calculation of the \( \Theta^+ \) width, will be presented in a subsequent publication.

VII. CONCLUSIONS

We have presented a technique allowing to write down explicitly the quark wave functions of the octet, decuplet and antidecuplet baryons, in the mean field approximation. Having patience (and space) one can write down the 19-quark component of the proton or the 7-quark component of the exotic \( \Xi^{-} \). This technique is mathematically equivalent to the
“valence quarks plus Dirac continuum” method exploited previously, but brings the mean field approach even closer to the language of the quark wave functions used by many people. We have shown that the standard $SU(6)$ wave functions are easily reproduced for the octet and decuplet baryons, if one assumes the non-relativistic limit. However, we have given explicit formulae for the relativistic corrections to the $3Q$ wave function, and also explicitly and for the first time, the $5Q$ wave function of the nucleon and of the exotic $\Theta^+$. There seems to be a broad field of applications. One has been already started in Ref. [10] and involves distribution amplitudes, exclusive processes, parton distributions and the like. The other, probably even more broad, is for low energies. One can compute any kind of transition amplitudes between baryons, including effects of $SU(3)$ symmetry violation, mixing between multiplets and the widths.

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[17] The present pair wave function is that of Ref. [10] multiplied by a constant matrix $\sigma_3$.
[19] This expression differs from that of Ref. [10] by a multiplication by the matrix $\sigma_3$. Also, $h(p), j(p)$ used here are those used in Ref. [10] divided by $p^2$. This comment refers also to eq. (39).