Deuteron Compton Scattering: A Random Walk

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Abstract. In this sketch, some recent developments in Compton scattering off the deuteron are reviewed. The strong energy-dependence of the scalar magnetic dipole polarisability $\beta_{M1}$ turns out to be crucial to understand the data from Saskatoon at 94 MeV. Chiral Effective Field Theory is used to extract the static iso-scalar dipole polarisabilities as $\bar{\alpha}_{s} = 12.6 \pm 1.4_{\text{stat}} \pm 1.0_{\text{wavefu}}$ and $\bar{\beta}_{s}^{*} = 2.3 \pm 1.7_{\text{stat}} \pm 0.8_{\text{wavefu}}$, in units of $10^{-4}$ fm$^3$. Therefore, proton and neutron polarisabilities are identical within error bars. For details and a better list of references, consult e.g. Refs. \cite{1, 2}.

A PROBLEM WITH DEUTERON COMPTON SCATTERING

As free neutrons can only rarely be used in experiments, their properties are usually extracted from data taken on few-nucleon systems by subtracting nuclear binding effects. Take the polarisabilities: The photon field displaces the charged constituents of the neutron, inducing a non-vanishing multipole moment. Polarisabilities are a measure for the polarisation induced, i.e. for the global stiffness of the neutron against an electromagnetic field. They are canonically parameterised starting from the most general interaction between the nucleon $N$ and an electro-magnetic field of non-zero energy $\omega$:

$$2\pi N \left[ \alpha_{E1}(\omega) \bar{E}^2 + \beta_{M1}(\omega) \bar{B}^2 + \gamma_{E1E1}(\omega) \bar{\sigma} \cdot (\bar{E} \times \dot{\bar{E}}) + \gamma_{M1M1}(\omega) \bar{\sigma} \cdot (\bar{B} \times \dot{\bar{B}}) - 2\gamma_{M1E2}(\omega) \sigma_i B_j E_{ij} + 2\gamma_{E1M2}(\omega) \sigma_i E_j B_{ij} + \ldots \right] N \quad (1)$$

Here, the electric or magnetic $(X, Y = E, M)$ photon undergoes a transition $Xl \rightarrow Yl'$ of definite multipolarity $l, l' = l \pm \{0, 1\}; T_{ij} := \frac{1}{2}(\partial_i T_j + \partial_j T_i)$. There are six dipole polarisabilities: two spin-independent ones ($\alpha_{E1}(\omega), \beta_{M1}(\omega)$) for electric and magnetic dipole transitions which do not couple to the nucleon spin; and in the spin sector, two diagonal (“pure”) spin-polarisabilities ($\gamma_{E1E1}(\omega), \gamma_{M1M1}(\omega)$), and two off-diagonal (“mixed”) spin-polarisabilities, $\gamma_{E1M2}(\omega)$ and $\gamma_{M1E2}(\omega)$. These spin-polarisabilities are particularly interesting, as they parameterise the response of the nucleon spin to the photon field, having no classical analogon. In addition, there are higher ones like quadrupole and octupole polarisabilities, with negligible contributions.

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Little is known about the nucleon polarisabilities, and albeit these nucleon structure
effects have been known for many decades, most experiments have focused on just two
numbers, namely the static electric and magnetic polarisabilities $\bar{\alpha} := \alpha_E(\omega = 0)$ and
$\bar{\beta} := \beta_M(\omega = 0)$. For the proton, the generally accepted values are $\bar{\alpha}_p \approx 12$, $\bar{\beta}_p \approx 2$, with error bars of about $1^3$. For the neutron, different types of experiments report a range of values $\bar{\alpha}_n \in [-4; 19]:$ Coulomb scattering of neutrons off lead, or deuteron Compton-scattering with and without breakup, see [2] for a list. So, does the neutron and proton react similarly under deformations ($\bar{\alpha}_p \approx \bar{\alpha}_n$, $\bar{\beta}_p \approx \bar{\beta}_n$) or not?

As deuteron Compton scattering $d\gamma \to d\gamma$ should provide a clean way to extract the
iso-scalar polarisabilities $\bar{\alpha}_s := 1/2(\bar{\alpha}_p + \bar{\alpha}_n)$ and $\bar{\beta}_s$ in complete analogy to determinations of the proton polarisabilities, experiments were performed in Urbana [3] at $\omega = 49$ and 69 MeV, in Saskatoon (SAL) [4] at 94 MeV, and in Lund [5] at 55 and 66 MeV. While the low-energy extractions are consistent with small iso-vectorial polarisabilities, the SAL data lead to conflicting analyses, see Fig. 1: The original publication [4] gave $\bar{\alpha}_s = 8.8 \pm 1.0$, employing the well-known Baldin sum rule for the static nucleon polarisabilities, $\bar{\alpha}_s + \bar{\beta}_s = 14.5 \pm 0.6$. Without it, Levchuk and L’vov obtained $\bar{\alpha}_s = 11 \pm 2, \bar{\beta}_s = 7 \pm 2$ [6]; and recently, Beane et al. found $\bar{\alpha}_s = 13 \pm 4, \bar{\beta}_s = -2 \pm 3$ [7].

The high-energy extraction being very sensitive to the polarisabilities (see right panel
in Fig. 1), this seems discouraging news. Can embedding the neutron into a nucleus lead
to the discrepancy? Of course, two-body contributions from meson exchange currents and
wave-function dependence must be subtracted from data with minimal theoretical
prejudice and an estimate of the theoretical uncertainties. Chiral Effective Field Theory ($\chi$EFT), the low-energy variant of QCD, provides just that: As extension of Chiral Perturbation Theory to the few-nucleon system, it contains only those degrees of freedom which are observed at the typical energy of the process, interacting in all ways allowed by the underlying symmetries of QCD. A power counting allows for results of finite, systematically improvable accuracy, and thus for an error-estimate. Figure 2 lists

\footnote{The scalar dipole polarisabilities are usually measured in units of $10^{-4}$ fm$^3$: The nucleon is quite stiff.}
the contributions to Compton scattering off the deuteron to next-to-leading order. The calculation is parameter-free, except for the nucleon polarisabilities. Note that the two-nucleon contribution does not contain contributions from the \( \Delta(1232) \)-resonance in the intermediate state at this order, as the deuteron is an iso-scalar target. Also, the nucleon- and nuclear-structure contributions clearly separate at this (and the next) order.

As an aside, Phillips showed recently by example of the electro-magnetic form factors of the deuteron that this separation allows also to judge the quality of \( \chiEFT \) in the deuteron only [10]. While it is well-known that the iso-scalar nucleon form factors are ill-described in \( \chiEFT \), replacing their contributions by the experimental parameterisations but keeping the two-body currents from \( \chiEFT \) brings the deuteron form factors in very good agreement with the measurements even at momentum transfers of \( \sim 0.7 \) GeV. It is thus the one-body sector in this process which needs improvement, not the two-body part. We will come to the same conclusion in Compton scattering. End aside.

Beane et al. [7, 9] use in the extraction mentioned above state-of-the-art deuteron wave-functions and a meson-exchange kernel derived from Chiral Perturbation Theory. And yet, their static polarisabilities from the SAL-data still disagree with those from lower energies. Is this a failure of \( \chiEFT \), just like the potential-model approach fails?

**ENTER DYNAMICAL POLARISABILITIES**

We argue in Ref. [2] that the discrepancy is dissolved once the full energy-dependence of the polarisabilities is taken into account, including all degrees of freedom at low energies inside the nucleon: Polarisabilities depend on the photon energy \( \omega \) because different polarisation mechanisms react quite differently to real photon fields of non-zero frequency. Therefore, these *energy-dependent* or *dynamical polarisabilities* contain detailed information about dispersive effects, caused by internal relaxation, baryonic resonances and mesonic production thresholds. At present, various theoretical frameworks are able to provide a consistent, qualitative picture for the leading static polarisabilities. Their dynamical origin is however only properly revealed by their energy-dependence. A rigorous definition of the dynamical polarisabilities starts instead of \( \Pi \) from a multipole-
decomposition of the $T$-matrix of real Compton scattering; purists consider Ref. [1]. It turns out that all polarisabilities beyond the dipole ones are so far invisible in observables. This is why they were sacrificed to brevity in the expressions above.

Dynamical polarisabilities are a concept complementary to generalised polarisabilities of the nucleon. The latter probe the nucleon in virtual Compton scattering, i.e. with an incoming photon of non-zero virtuality, and possibly provide information about the spatial distribution of charges and magnetism inside the nucleon. Their extraction is however notoriously difficult. Dynamical polarisabilities on the other hand test the global response of the internal nucleonic degrees of freedom to a real photon of non-zero energy and answer the question which internal degrees of freedom govern the structure of the nucleon at low energies. Like all quantities defined by multipole-decompositions, they do not contain more or less information than the corresponding Compton scattering amplitudes, but the facts are more readily accessible and easier to interpret.

To identify the microscopically dominant low-energy degrees of freedom inside the nucleon in a model-independent way, we employ again $\chi$EFT. The contributions at leading order (LO) are listed in Fig. 3: (1) Photons couple to the charged pion cloud around the nucleon and around the $\Delta$, seen in a characteristic cusp at the one-pion production threshold. (2) It is well known that the excitation of the lowest nuclear resonance, the $\Delta(1232)$, as intermediate state by the strongly paramagnetic $\gamma N \Delta M1 \rightarrow M1$ transition leads to a para-magnetic contribution to the static magnetic dipole polarisability $\bar{\beta}_\Delta = +[7 \ldots 13]$. A characteristic resonance shape should occur, like predicted by the Lorentz model of polarisabilities in classical electrodynamics. (3) As the observed static value $\bar{\beta}_p \approx 2$ is smaller by a factor of 5, another strongly diamagnetic component must exist. We subsume this short-distance Physics which is not generated by the pion or $\Delta$ into two low-energy coefficients $\delta \alpha$, $\delta \beta$, which are energy-independent. While naïve power-counting sees them suppressed by one order, experiment tells us otherwise. According to $\chi$EFT, the proton and neutron polarisabilities are furthermore very similar, iso-vectorial effects being of higher order in the power counting.

![Diagram](image)

**FIGURE 3.** The nucleon polarisabilities at $O(\varepsilon^3)$ in $\chi$EFT, left to right: pion cloud around the nucleon and $\Delta$ excitations; short-distance effects. Permutations and crossed diagrams not shown. From Ref. [1].

With the two parameters fixed by matching to Compton scattering [1], the energy-dependence of all polarisabilities is predicted, see Fig. 4 We compare with a result from dispersion theory, in which the energy-dependent effects are sub-sumed into integrals over experimental input from a different kinematical régime, namely photo-absorption cross-section $\gamma N \rightarrow X$. Its major source of error is the uncertainty in modelling the dispersive integral above the two-pion production threshold.

The pronounced pion-cusp in $\alpha_{E1}^s(\omega)$ is quantitatively reproduced already at leading order. The dipole spin-polarisabilities are predictions, three of them being completely independent of the parameter-determination, and are well-matched [1]. No genuinely new low-energy degrees of freedom inside the nucleon are missing. Most notably however
is the strong energy-dependence induced into $\beta_{M1}^s(\omega)$ even below the pion-production threshold by the unique signature of the $\Delta$ resonance: At $\omega \approx 90$ MeV, $\beta_{M1}^s$ is about 3 units larger than its static value, rendering the traditional approximation of $\beta_{M1}^s(\omega)$ as “static-plus-small-slope” $\tilde{\beta} + \omega^2 \tilde{\beta}_N$ inadequate. It also reveals the good quantitative agreement between the measured value of $\tilde{\beta}^p$ and the prediction in a $\chi$EFT without explicit $\Delta$: The contribution from the pion-cloud alone is not dispersive enough to explain the energy-dependence of $\beta_{M1}^s$. In $\chi$EFT without explicit $\Delta$, its part is played by short-distance contributions which show only a slow energy-dependence.

FIGURE 4. Energy-dependence of the spin-independent dipole polarisabilities $\alpha_{E1}^s$ (left) and $\beta_{M1}^s$ (right), predicted by Dispersion Theory (solid) and $\chi$EFT at $O(\epsilon^3)$ with (long dashed; band from fit errors), and without (short dashed) explicit $\Delta$. $\omega_\pi$: one-pion production threshold. From Ref. [1].

ISO-SCALAR POLARISABILITIES FROM THE DEUTERON

Fitting in $\chi$EFT with $\Delta$ the two short-distance parameters $\delta \alpha$, $\delta \beta$ to deuteron Compton scattering data above 60 MeV (Fig. 5), one finds for the static values:

unconstrained: $\bar{\alpha}^s = 12.6 \pm 1.4_{\text{stat}} \pm 1.0_{\text{wavefu}}$, $\bar{\beta}^s = 2.3 \pm 1.7_{\text{stat}} \pm 0.8_{\text{wavefu}}$

with Baldin: $\bar{\alpha}^s = 12.4 \pm 0.8_{\text{stat}} \pm 0.8_{\text{wavefu}}$, $\bar{\beta}^s = 2.1 \mp 0.8_{\text{stat}} \pm 0.7_{\text{wavefu}}$ (2)

The Baldin sum rule $\bar{\alpha}^s + \bar{\beta}^s = 14.5 \pm 0.6$ is already well reproduced by the unconstrained fit. Comparing with the static proton polarisabilities determined by the same method in [1], $\bar{\alpha}^p = 11.0 \pm 1.4_{\text{stat}} \pm 0.4_{\text{sys}}$, $\bar{\beta}^p = 2.8 \mp 1.4_{\text{stat}} \pm 0.4_{\text{sys}}$, we see that the proton and neutron polarisabilities are indeed identical within the statistical uncertainty.

Thus, the alleged discrepancy between extractions from the SAL data and experiments at lower energies is resolved. Figure 6 shows that the dispersion originating in excitations of the $\Delta$ is indeed pivotal to reproduce the shape of the data at 94 MeV in particular at back-angles: The calculations by Beane et al. [7, 9] use the same deuteron wavefunctions and meson-exchange currents, but subsume in Ref. [7] all $\Delta$-effects into short-distance operators which enter only at higher order and are only weakly dispersive. They therefore have to exclude the two SAL-points at large angles from their analysis.

FIGURE 6. Comparison between $\chi$EFT with explicit $\Delta$ (solid) and without explicit $\Delta$ (dashed: $O(p^3)$, parameter-free; dotted: $O(p^4)$, best fit). From Ref. [2] with the help of Ref. [7].

CONCLUDING WORDS

To finish the story, work is under way on a few more chapters, for example:

(i) As the extracted numbers (2) suggest, the cross-sections depend on the deuteron wave-function used on the 10%-level. The main reason is certainly that the electromagnetic currents used are not tailored to the potential; the deviation from consistent currents is however a higher-order effect (and also numerically small).

(ii) The nucleon response in the resonance region is probed at higher energies, where the non-zero width of the $\Delta$ and higher-order effects from the pion-cloud become crucial.

(iii) The proposed analysis of Compton scattering via a multipole decomposition at fixed energies [11,11] will not only provide better data on the neutron polarisabilities. It will also further our knowledge on the spin-polarisabilities, and hence on the spin-structure of the nucleon. Double-polarised, high-accuracy experiments provide a new avenue to extract the energy-dependence of the six dipole polarisabilities per nucleon, both spin-independent and spin-dependent [11,11]. What we need is more data: For example, with only 29 (un-polarised) points for the deuteron in a small energy range of $\omega \in [49; 94]$ MeV and error bars on the order of 15%, experiments can improve the situation substantially. A (certainly incomplete) list of planned or approved experiments...
at photon energies below 300 MeV shows the concerted effort in this field: polarised photons on polarised deuterons and $^3$He at TUNL/HIγS; tagged protons at S-DALINAC; polarised photons on polarised protons at MAMI; and deuteron targets at MAXlab.

(iv) Why are the data at 49 and 55 MeV not included in our analysis? In contradiction to the high-energy data, it is well known, see e.g. [8], that the correct Thomson limit puts a severe constraint on Compton scattering at low energies. The $\chi$EFT-power-counting of Fig. 2 is not tailored to the low-energy end and must be modified to produce the Thomson limit on the deuteron. This problem is partially circumvented in Ref. [7], and a full treatment is in its finishing stages [14].

(v) At lower energies, the pion-exchange terms can be integrated out, and one arrives at the “pion-less” EFT of QCD. Not only is this version computationally considerably less involved than the pion-ful version $\chi$EFT; it also has the advantage that the Thomson limit is recovered trivially. While Compton scattering becomes the less sensitive to the polarisabilities the lower the energy, a window exists between about 25 and 50 MeV where this variant can aide high-accuracy experiments e.g. at HIγS to extract the static polarisabilities in a model-independent way. Recently, Chen et al. demonstrated that due to the large iso-vectorial magnetic moment, the vector amplitudes in $d\gamma$-scattering are anomalously enhanced. Correcting a previous calculation by Rupak and Grießhammer [12], they found that the existing data at 49 and 55 MeV are well in agreement with the values given above, finding $\hat{\alpha}^s = 12 \pm 1.5$, $\hat{\beta}^s = 5 \pm 2$; see Ref. [13] for details.

Enlightening insight into the electro-magnetic structure of the nucleon has already been gained from merging Compton scattering off light nuclei, $\chi$EFT and energy-dependent or dynamical polarisabilities; and a host of experimental activities is going to add to them in the coming years.

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REFERENCES