Detecting gravito-magnetism with rotation of polarization by a gravitational lens

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ABSTRACT

We discuss the effects of an isolated gravitational lens on the rotation of the plane of polarization of linearly polarized light rays, the so called gravitational Faraday rotation, in metric theories of gravity. By applying the thin lens approximation, we derive simple expressions for extended, rotating astrophysical systems deflecting electromagnetic radiation from background sources. Higher order corrections and the case of a Kerr black hole are also considered to show how the rotation of polarization is a purely gravito-magnetic effect. Prospects of a future detection of a gravitational Faraday rotation are examined.

Key words: black hole physics – gravitational lensing – polarization – relativity

1 INTRODUCTION

Conceivable post-Newtonian theories of gravity, such as general relativity, predict space-time curvature by mass-energy currents relative to other masses. This feature is known as intrinsic gravito-magnetism (Ciufolini & Wheeler 1995). The occurrence of several phenomena is due to gravito-magnetism. Small test-gyroscopes rotate with respect to distant stars in the vicinity of a spinning body due to its rotation. This is called the Lense-Thirring precession. Together with test particles and gyroscopes, also photons and clocks probe some gravito-magnetic phenomena. Bending and time delay of electromagnetic waves are two important phenomena predicted by theories of gravity. Gravito-magnetic effects due to the spin of an astrophysical object might be detected in gravitational lensing events (Sereno 2002, Sereno & Cardone 2002). The rotation of the lens induces particular signatures on both the position and magnification of multiple images of a background source, also perturbing the microlensing induced amplification curve (Sereno 2003b).

Another well-known gravitational effect due to space-time curvature by mass currents is the rotation of the plane of polarization for linearly polarized light rays. Under geometric optics, a light ray follows a null geodesic regardless of its polarization state and the polarization vector is parallel transported along the ray (Misner et al. 1973). Such a gravitational rotation of the plane of polarization in stationary space-times is a gravitational analogue of the electro-magnetic Faraday effect (Piran & Safrer 1985, Ishihara et al. 1988, Nouri-Zonoz 1999), i.e., the rotation that a light ray undergoes when passing through plasma in the presence of a magnetic field. The gravitational effect is known as gravitational Faraday rotation or Rytov or Skrotskii effect.

From the original investigations in Skrotskii (1957) and Rytov (1938), the effect of a gravitational field on the plane of polarization of electromagnetic waves has been addressed by several authors (Balazs 1958, Plebanski 1960, Godfrey 1970, Su & Mallett 1980, Connors et al. 1980, Piran & Safrer 1985, Ishihara et al. 1988, Nouri-Zonoz 1999, Dyer & Shaver 1992, Nouri-Zonoz 1999, Kopeikin & Mashoon 2002, Sereno 2004). The case of a weak gravitational field has deserved particular attention. In Plebanski (1960), the Maxwell’s equations in the gravitational field of an isolated physical system were solved and the rotation of the plane of polarization around the propagation vector was considered. In Kopeikin & Mashoon (2002), a formula describing the Skrotskii effect for arbitrary translational and rotational motion of gravitating bodies was derived by solving the equations of motion of a light ray in the first post-Minkowskian approximation. In Sereno (2004), the general formula for the angle of rotation of the plane of polarization of a linearly polarized electromagnetic wave in a stationary space-time was re-obtained with an heuristic approach based on Mach’s principle on dragging of inertial frames, without integrating the equations of motion.

The assumption of the existence of a dynamical space-time curvature, as opposed to a flat space-time of special relativity, is shared among different viable theories of gravity, such as general relativity, Brans-Dicke theory and the Rosen bimetric theory. Metric theories of gravity are defined such that (Will 1993, Ciufolini & Wheeler 1995): i) space-time is a Lorentzian manifold; ii) the world lines of test bodies are geodesics; iii) the equivalence principle holds in the medium strong form. The usual rules for the motion of particles and photons in a given metric still apply, but the
metric may be different from that derived from the Einstein’s field equations.

A comparison among general relativity and other viable theories of gravity can be led with suitable tests on the basis of higher-order effects, such as intrinsic gravito-magnetism. To date, results from laser-ranged satellites provide the only demonstration of the gravito-magnetic field. In 1995-2002, the Lense-Thirring precession, due to the Earth spin, was measured by studying the orbital perturbations of LAGEOS and LAGEOS II satellites (Ciufolini & Pavlis 1998). Its experimental value agrees with about 20% accuracy with the prediction of general relativity. The NASA’s Gravity Probe-B satellite should improve this measurement to an accuracy of 1%.

In this paper, we discuss the gravitational Faraday rotation by an isolated rotating gravitational deflector in the standard framework of gravitational lensing (Petters et al. 2001; Schneider et al. 1992), when the source of radiation and the observer are remote from the source of the gravitational field (lens). As known, thanks to some usual approximations, the effect of the spin of the deflector on the lensing potential can be treated in a quite simple way (Sereno 2002). The lens equation can be modified to consider some observable quantities, like position and amplification of images, are perturbed by the rotation (Sereno & Cardone 2002; Sereno 2003b). Here, we want to extend such a formalism to the Skrotskii effect and to consider some extended lenses of astrophysical interest. Standard assumptions allow us to compute higher-order approximation terms in the calculation, so that a general treatment of the Skrotskii effect can be performed in metric theories of gravity.

The paper is as follows. In Sec. 2 the weak-field, slow motion approximation in metric theories of gravity is introduced and the weak field limit of the gravitational Faraday rotation is performed. In Section 3 we derive some simple formulæ, using the thin lens approximation, to treat systems of astrophysical interest. Higher order correction to the Skrotskii effect are discussed in Sec. 4; then, the case of light rays propagating in the vacuum region outside a Kerr black hole is considered. Section 5 contains some final considerations.

2 THE WEAK FIELD LIMIT

Standard hypotheses of gravitational lensing (Petters et al. 2001; Schneider et al. 1992) assume that the gravitational lens is localized in a very small region of the sky and its lensing effect is weak. In Sereno (2003a), an approximate metric element generated by an isolated mass distribution was written in the weak field regime and slow motion approximation, up to the post-post-Newtonian order, and with non-diagonal components accounting for effects of gravity by currents of mass. In an asymptotically Cartesian coordinate system, the metric \( g_{ij} \) can be expressed as

\[ ds^2 \simeq \left( 1 + 2 \frac{\phi}{c^2} + O(\epsilon^4) \right) c^2 dt^2 - \left( 1 - 2 \gamma \frac{\phi}{c^2} + O(\epsilon^4) \right) \delta_{ij} dx^i dx^j. \]

where \( \epsilon \ll 1 \) denotes the order of approximation. In the above metric element, \( \gamma \) is a standard coefficient of the post-Newtonian parametrized expansion of the metric tensor (Ciufolini & Wheeler 1995; Will 1993), measuring space curvature produced by mass. In general relativity, it is \( \gamma = 1 \); in the Brans-Dicke theory, \( \gamma = \frac{1+\omega}{2} \). \( \mu \) is a non standard parameter which quantifies the contribution to the space-time curvature of the mass-energy currents. It measures the strength of the intrinsic gravito-magnetic field (Ciufolini & Wheeler 1995). In general relativity, \( \mu = 1 \); in Newtonian theory, \( \mu = 0 \). Additional terms in a parametrized expansion of the metric element can also be considered (Matzner et al. 1981; Kichler & Matzner 1982). Preferred frame effects, violations of conservation of four momentum, preferred location effects are not considered in our approximate metric element.

In Eq. 1, \( \phi \) is the Newtonian potential,

\[ \phi(t, x) \simeq -G \int_{\mathbb{R}^3} \frac{\rho(t, \mathbf{x})}{|\mathbf{x} - \mathbf{x}'|} d^3 x', \]

where \( G \) is the gravitational Newton constant and \( \rho \) is the mass density of the deflector; \( \phi/c^2 \) is order \( \sim O(\epsilon^2) \).

\[ \nabla \left( \frac{3}{v} \right) \equiv \nabla + O(\epsilon^2) \] is a vector potential taking into account the gravito-magnetic field produced by mass currents. To the lowest order of approximation,

\[ V_i(t, x) \simeq -G \int_{\mathbb{R}^3} \frac{\rho \epsilon_i}{|\mathbf{x} - \mathbf{x}'|} d^3 x', \]

where \( v \) is the velocity field of the mass elements of the deflector \(^2\).

We assume that, during the time light rays take to traverse the lens, the potentials in Eqs. 2, 3 vary negligibly little. Then, the lens can be treated as stationary. In Eqs. 4, 5, we have neglected the retardation (Schneider et al. 1992).

The polarization vector is dragged along by the rotation of the inertial frames. Once we have introduced the notation

\[ h \equiv g_{00}, A_i \equiv \frac{-g_{0i}}{g_{00}} \]

the net angle of rotation around the unit tangent three-vector \( \hat{\mathbf{k}} \) along the path between the source and the observer, in a stationary space-time, reads (Nourt-Zonoz 1999; Sereno 2004)

\[ \Omega_{\text{sk}} = -\frac{1}{2} \int_{\text{source}}^{\text{observer}} \sqrt{h} \nabla \times \mathbf{A} \cdot d\mathbf{x}, \]

where \( d\mathbf{x} = \hat{\mathbf{k}} dl_{\mathbf{p}} \) and \( dl_{\mathbf{p}} \) is the spatial distance in terms of the spatial metric \( \gamma_{ij} \) (Landau & Lifshits 1983).

\[ dl_{\mathbf{p}} \equiv \left( -g_{ij} + \frac{g_{0i} g_{0j}}{g_{00}} \right) dx^i dx^j \equiv \gamma_{ij} dx^i dx^j. \]

Operations on three-vectors are defined in the three-dimensional space with metric \( \gamma_{ij} \).

In Dyer & Shiver (1992), it was shown, using symmetry arguments in a relativistic context, that, for astrophysically interesting cases of both observer and source at large distances from the lens, a nonshperical, nonrotating lens cannot cause a net rotation of the polarization vector. This argument is confirmed by Eq. 5: the Skrotskii effect is due to mass-currents, i.e., it is a purely gravito-magnetic phenomenon. An example can be found from Eq. 3, the order of approximation is determined by off-diagonal components of the metric. The main term is order \( \sim O(\epsilon^2) \). To determine a gravito-magnetic effect to the second leading order, i.e., to \( O(\epsilon^4) \), we need to consider terms in \( g_{00} \) and \( g_{0i} \) up to \( O(\epsilon^2) \). Higher order terms would give contributions \( \sim O(\epsilon^4) \times \nabla \sim O(\epsilon^7) \) to the Faraday rotation. \( \Omega_{\text{sk}} \) is at best

\(^2\) Bold symbols denote spatial vectors. The contravariant components of spatial three-vectors are equal to the spatial components of the corresponding four-vectors.
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3 THE THIN LENS APPROXIMATION

In almost all astrophysical systems, the gravitational lens can be treated as geometrically thin. The extent of the lens in the direction of the incoming ray is small compared to both the distances between lens and observer and lens and source, so that the maximal deviation of the actual ray from a light path propagating through unperturbed space-time is small compared to the length scale on which the gravitational field changes. In this picture, gravitational effects are quite local to the lens vicinity.

The lens plane is transverse to the incoming, unperturbed light ray direction. The position vector in the lens plane is the spatial vector $\xi \equiv \{x^3, x^2, 0\}$. We introduce the coordinate $l$ along the $x^3$-axis, so that the lens plane corresponds to $l = 0$. The Born approximation assumes that rays of electromagnetic radiation propagate along straight lines. The integration along the line of sight (l.o.s.) is accurate enough to evaluate the main contribution to the Skrotskii effect. To this order, we will employ the unperturbed Minkowski metric $\eta_{\alpha\beta} = (1, -1, -1, -1)$ for operations on vectors.

Let us define the weighted average of a quantity $Q$ along the line of sight,

$$\langle Q \rangle_{\text{l.o.s.}}(\xi) \equiv \frac{\int Q(\xi, l) \, dl}{\Sigma(\xi)},$$

where $\Sigma$ is the surface mass density of the deflector,

$$\Sigma(\xi) \equiv \int \rho(\xi, l) \, dl.$$  

The leading contribution to the Skrotskii effect can be written as

$$\Omega_{\text{Sk}} = \frac{4\mu}{c^3} \int \frac{\Sigma(\xi)(L_{\text{l.o.s.}}(\xi, l), V_{\xi, 1,0} \cdot \xi^{'}) \, d^2\xi^{'}}{|\xi - \xi'|}.$$  

where $L_{\text{l.o.s.}}$ is the angular momentum, per unit mass, of a mass element at $\xi$ with respect a photon with impact parameter $\xi$.

$$\langle L_{\text{l.o.s.}} \rangle_{\text{l.o.s.}} = \langle v^1 \rangle_{\text{l.o.s.}}(\xi^2 - \xi^{'2}) - \langle v^2 \rangle_{\text{l.o.s.}}(\xi^3 - \xi^{'3});$$

$v^1$ and $v^2$ are the components of $v$ along the $\xi^1$- and the $\xi^2$-axes, respectively.

The angular momentum of the lens can determine a direction of rotation for the polarization vector. When its projection along the line of sight is not null, a handness is introduced in the system and some rotation can be caused. Once the deflector is oriented such that $\langle L_{\text{l.o.s.}} \rangle_{\text{l.o.s.}} = 0$, the photon moves like in an equatorial plane and a mirror symmetry about the lens plane holds. Such a symmetry cancels out the effect [Dyer & Shafer 1992].

3.1 Spherical lenses

In most of the astrophysical situations, lenses can be approximated with nearly spherical mass density profiles. Let us consider spherical deflector in rigid rotation (constant angular velocity $\omega$), when $v = \omega \times x$. We limit to a slow rotation so that the deformation caused by rotation is negligible and the body has a spherical symmetry.

Taking the centre of the source as the spatial origin of a background inertial frame, the weighted average of the rotational velocity along the line of sight turns out to be

$$\langle \omega \rangle_{\text{l.o.s.}} = \{\xi^2 \omega^1 - \xi^1 \omega^2, -\xi^2 \omega^1, \xi^1 \omega_{\text{l.o.s.}}\}. $$  

We can, now, evaluate the integral in Eq. (15), the polarization vector of a linearly polarized light ray with impact parameter $|\xi|$ is rotated by

$$\Omega_{\text{Sk}} = \frac{4\pi G\mu}{c^3} \omega_{\text{l.o.s.}} M_{\text{Cyl}}(>|\xi|)$$

where $M_{\text{Cyl}}(|\xi|)$ is the projected lens mass outside $|\xi|$,

$$M_{\text{Cyl}}(>|\xi|) = 2\pi \int_{|\xi|}^{+\infty} \Sigma(\xi) \xi' \, d\xi'.$$
The gravitational Faraday rotation is proportional to the mass outside $|\xi|$ and can be significant for lenses with slowly decreasing mass density. $\Omega_{\text{Sk}}$ can be compared to the contribution to bending of light due to the gravito-magnetic field, $\Omega_{\text{GRM}}$ (see Eqs. (16) in Sereno & Cardone 2002). Typically, $\Omega_{\text{Sk}}$ and $\Omega_{\text{GRM}}$ are of the same order. Contrary to the Faraday effect, the gravito-magnetic contribution to the deflection angle depends only on the component of the angular momentum in the lens plane (Sereno 2002). Whereas the components of the angular momentum along the $\xi$-axis, related to the momentum of inertia of the mass within $\xi$ about a central axis, contribute only to $\Omega_{\text{GRM}}$ (Sereno & Cardone 2002), the contribution to the mass outside the impact parameter appears in both $\Omega_{\text{Sk}}$ and $\Omega_{\text{GRM}}$.

Let us consider an isothermal sphere (IS). The IS is a density profile widely used to model systems on very different scales, from galaxy haloes to clusters of galaxies. The surface mass density is

$$\Sigma_{\text{IS}} = \frac{\sigma^2}{2c^2} \sqrt{|\xi|^2 + \xi^2} .$$

(20)

where $\sigma$ is the velocity dispersion and $\xi$ is a finite core radius. Since the total mass is divergent, we introduce a cut-off radius $R \gg |\xi|$. The gravitational Faraday rotation is

$$\Omega_{\text{Sk}}(|\xi|) = \frac{8\pi G \mu}{c^2} \sigma^2 \omega \omega_{\text{L.o.s.}} \left( \sqrt{R^2 + \xi^2} - \sqrt{|\xi|^2 + \xi^2} \right) .$$

(21)

In particular, in the inner regions ($|\xi| \ll R$), the above equation reduces to

$$\Omega_{\text{Sk}}(|\xi| \ll R) = \frac{8\pi G \mu}{c^2} \sigma^2 \omega \omega_{\text{L.o.s.}} R .$$

(22)

In the case of lensing of distant quasars by foreground galaxies, images may form inside the galaxy radius. We can model a typical galaxy as a singular ($\xi = 0$) IS with $\sigma_v \sim 200$ km s$^{-1}$, $R \sim 20$ Kpc and $J \sim 0.1 M_\odot$ Kpc$^2$s$^{-1}$, as derived from numerical simulations Vetriska et al. 2002. From Eq. (22), we find $\Omega_{\text{Sk}} \sim 6$ milliarcsec.

Let us consider $\Omega_{\text{GRM}}$ for an isothermal sphere. It is (Sereno & Cardone 2002).

$$\Omega_{\text{GRM}}(|\xi| \ll R) = \frac{8\pi G \mu}{c^2} \sigma^4 \omega \omega_{\text{L.o.s.}} \left( \sqrt{|\xi|^2 + \xi^2} \right) .$$

(23)

$\Omega_{\text{Sk}}$ and $\Omega_{\text{GRM}}$ are of the same order but depend on different components of the angular momentum of the lens.

4 HIGHER ORDER CORRECTIONS

To determine higher order terms in the gravitational Faraday rotation, it is enough to consider only the monopole term in the gravitoelectric field but we must go over the dipole moment in the gravitomagnetic potential. We want now discuss higher order terms in a rotating black hole.

4.1 The Kerr black hole

The Faraday effect in a Kerr metric, in the weak field limit, has been already discussed by several authors (Ishihara et al. 1988 Nouri-Zonoz 1999). The right order of approximation in the calculation has been ascertained, but a disagreement on the numerical value still persists. Now, we want to apply our formalism to light rays passing through the vacuum region outside a rotating black hole.

For a source at rest at the origin of the coordinates, assuming that the polar axis of the coordinate system coincides with the rotation axis, the weak field limit of the Kerr metric in Boyer-Lindquist coordinates reads

$$ds^2 \approx ds_{\text{Sch}}^2 - \frac{4 \mu G J}{c^2 r^2} \sin^2 \theta (r d\phi)(cdt),$$

(24)

where $ds_{\text{Sch}}^2$ is the line element for a spherically symmetric solution and where terms of quadratic and higher order in the angular momentum $J$ have been neglected.

The metric in Eq. (23) can be expressed in an equivalent isotropic form by introducing a new radius variable, $\rho$, such as

$$r \approx \rho \left( 1 + \frac{GM}{c^2 \rho} + \mathcal{O}(\epsilon^4) \right) ,$$

(25)

where $M$ is the total mass of the black hole. Substituting Eq. (25) and introducing quasi-Minkovskian coordinates related to $(\rho, \theta, \phi)$ by the usual transformation rules, we get the weak field limit of the Kerr metric in the isotropic form. By arbitrarily orientating the angular momentum $J$, we get

$$ds^2 \approx \left( 1 - \frac{2GM}{c^2 \rho} \right) (cdt)^2 - \left( 1 + \frac{2\gamma GM}{c^2 \rho} \right) \delta_{ij} dx^i dx^j - \frac{8\mu}{c^4} (V_i dx^i) (cdt)$$

(26)

where $x = \sqrt{\delta_{ij} x^i x^j}$ and

$$V \approx \frac{3}{2} \nabla \times V \approx - \frac{G J}{2 \rho} x .$$

(27)

In the above equations and in what follows, operations on vectors are performed using $\eta_{\alpha \beta}$. Eqs. (24) 26 are also valid for large distances from an isolated physical system. Since,

$$\nabla \times V (x) = \frac{G J - 3(J \times x)}{2 x^3} ,$$

(28)

it is easy to verify that, to order $\mathcal{O}(\epsilon^3)$, there is no gravitational Faraday rotation Plebanski 1960, Kopeikin & Mashoon 2002, Godfrey 1970. Let us consider higher order terms.

The unit wave-vector, to order $\mathcal{O}(\epsilon^2)$ reads

$$k_\gamma (i) = \left( 1 + \gamma \right) \left\{ - \frac{R_{\text{Sch}}}{2} \left[ 1 + \frac{l}{\sqrt{l^2 + |\xi|^2}} \right] \frac{\xi_1^4}{|\xi|^2} ,

(29)

\begin{align*}
- \frac{R_{\text{Sch}}}{2} \left[ 1 + \frac{l}{\sqrt{l^2 + |\xi|^2}} \right] \frac{\xi_2^2}{|\xi|^2} \frac{\phi}{\xi^2} .
\end{align*}$$

(30)

where $R_{\text{Sch}} \equiv 2GM/c^2$ is the Schwarzschild radius of the lens. The deflected path to order $G$ is actually the path in a spherically symmetric metric. To this order, the photon path lies in the plane through source, lens and observer.

After lengthy but straightforward calculations, we can perform the integrations in Eq. (11). We find\(^3\)

$$\Omega_{\text{Sk}} \Omega_{\text{GRM}} \sim - \frac{\pi}{4} \frac{G^2 M J_{1,\text{L.o.s.}}}{c^5 \xi^3} + \mathcal{O}(\epsilon^3) .$$

(31)

\(^3\) Our $J_{1,\text{L.o.s.}}$ corresponds to $-am \cos \theta_0$ in Ishihara et al. 1988 and Nouri-Zonoz 1999.
As already known ([Ishihara et al. 1988][Nouri-Zonoz 1999]), the net rotation is of order $O(\epsilon^2)$.

Only the projection of the angular momentum along the line-of-sight enters the effect. In Nouri-Zonoz [1999], the integration is performed along the unperturbed path. However, contributions from the deflected path cancel out. The parameter $\gamma$, which measures space curvature produced by mass, does not enter the effect.

5 CONCLUSIONS

We have discussed the theory of the gravitational Faraday rotation in the weak field limit in a metric theory of gravity. Due to a gravito-magnetic field, a net rotation of the polarization plane can occur. Unlike the phenomena of bending of light and time delay of electromagnetic waves, gravito-magnetism appears in the leading term of the rotation angle of the plane of polarization.

Useful formulae have been derived for relevant astrophysical systems. By applying the thin lens approximation, we obtained quite compact expressions for extended spinning gravitational lenses. The gravitational Faraday increases for lenses with a slowly decreasing mass density. The effect of a gravito-magnetic deflection of the light beam.

The Skrotskii effect turns out to be negligible with respect to today observational uncertainties. Anyway, high quality data in total flux density, percentage polarization and polarization position angle at radio frequencies already exist for multiple images of some gravitational lensing systems, like B0218+357 ([Biggs et al. 1999]). Since the impressive development of technological capabilities, it is not a far hypothesis to obtain observational evidences in a next future.

Furthermore, polarization measurements of the cosmic microwave background radiation should probe very subtle effects, such as primordial gravity wave signals or a remnant from the reionization epoch. A full understanding of the whole foreground contamination turns out to be crucial in data analysis. So, a proper modeling and quantification of the gravitational Faraday effect is necessary.

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