CP sensitive observables in chargino production and decay into a $W$ boson

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Abstract

We study CP sensitive observables in chargino production in electron-positron collisions with subsequent two-body decay of one chargino into a $W$ boson. We identify the CP odd elements of the $W$ boson density matrix and propose CP sensitive triple-product asymmetries of the chargino decay products. We calculate the density-matrix elements, the CP asymmetries and the cross sections in the Minimal Supersymmetric Standard Model with complex parameters $\mu$ and $M_1$ for an $e^+e^-$ linear collider with $\sqrt{s} = 800$ GeV and longitudinally polarized beams. The asymmetries can reach 7% and we discuss the feasibility of measuring these asymmetries.

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1 Introduction

In the Minimal Supersymmetric Standard Model (MSSM) [1] several supersymmetric (SUSY) parameters can be complex. In the chargino sector of the MSSM this is the Higgsino mass parameter $\mu = |\mu| e^{i \phi_\mu}$, and in the neutralino sector of the MSSM also the $U(1)$ gaugino mass parameter $M_1 = |M_1| e^{i \phi_{M_1}}$ can have a physical phase [2]. Usually it is claimed that these phases, in particular $\phi_\mu$, have to be small [3, 4], due to the experimental upper bounds of the electric dipole moments (EDMs) of electron and neutron. For example in the constrained MSSM $|\phi_\mu|$ has to be smaller than $\pi/10$ [4] for a supersymmetric (SUSY) particle spectrum of the order a few 100 GeV. However, the EDM restrictions may be less stringent if cancellations among the different SUSY contributions occur [3]. The restrictions may disappear if also lepton flavor violating terms are included [5]. Thus, the restrictions on the phases are very model dependent and independent measurements are desirable. The study of chargino production at an $e^+e^-$ linear collider [6] will play an important role. By measurements of the chargino masses and cross sections, a method has been developed in [7, 8] to determine $\cos \phi_\mu$, in addition to the parameters $M_2, |\mu|$ and $\tan \beta$. However, also the sign of $\phi_\mu$ has to be determined unambiguously by using CP sensitive observables. One such observable is the chargino polarization perpendicular to the production plane [7, 8]. At tree level, this polarization leads to triple-product asymmetries [9, 10, 11, 12]. For chargino production and subsequent two-body decay of one chargino into a sneutrino, such an asymmetry can be as large as 30% [13] and it will allow us to constrain $\phi_\mu$. In the present work we will study chargino production and decay into a $W$ boson. We will show that, due to the spin correlations between the chargino and the $W$ boson, also an asymmetry is obtained which is sensitive to $\phi_{M_1}$.

We study chargino production

$$e^+ + e^- \rightarrow \tilde{\chi}_i^+ (p_{\chi_i^+}, \lambda_i) + \tilde{\chi}_j^- (p_{\chi_j^-}, \lambda_j),$$

with longitudinally polarized beams and the subsequent two-body decay

$$\tilde{\chi}_i^+ \rightarrow \tilde{\chi}_n^0 (p_{\chi_n^0}, \lambda_n) + W^+ (p_W, \lambda_k),$$

where $p$ and $\lambda$ denote momentum and helicity. We define the triple product

$$T_I = \mathbf{p}_{e^-} \cdot (\mathbf{p}_{\chi_i^+} \times \mathbf{p}_W)$$

and the T odd asymmetry

$$A^T_I = \frac{\sigma(T_I > 0) - \sigma(T_I < 0)}{\sigma(T_I > 0) + \sigma(T_I < 0)}$$

with $\sigma$ the cross section of chargino production (1) and decay (2). The asymmetry $A^T_I$ is sensitive to the CP violating phase $\phi_\mu$. In this context it is interesting to note that asymmetries vanish if they correspond to a triple product which contains a transverse polarization vector of the $e^+$ and $e^-$ beams [7, 14].
In order to probe also the phase $\varphi_{M_1}$, which enters in the chargino decay process (2), we consider the subsequent hadronic decay of the $W$ boson

$$W^+ \to c + \bar{s}. \quad (5)$$

The correlations between the $\tilde{\chi}_i^+$ polarization and the $W$ boson polarization lead to CP sensitive elements of the $W$ boson density matrix, which we will identify and discuss in detail. With the triple product

$$T_{II} = p_{e^-} \cdot (p_c \times p_s), \quad (6)$$

which includes the momenta of the $W$ decay products and thus probes the $W$ polarization, we define a second T odd asymmetry

$$A_{II}^T = \frac{\sigma(T_{II} > 0) - \sigma(T_{II} < 0)}{\sigma(T_{II} > 0) + \sigma(T_{II} < 0)}. \quad (7)$$

Here, $\sigma$ is the cross section of production (1) and decay of the chargino (2) followed by that of the $W$ boson (5). Owing to the spin correlations, $A_{II}^T$ has CP sensitive contributions from $\varphi_\mu$ due to the chargino production process (1) and contributions due to $\varphi_\mu$ and $\varphi_{M_1}$ from the chargino decay process (2). We treat the decay (5) as Standard Model process.

The T odd asymmetries $A_{II}^T$ and $A_{II}^T$ have also absorptive contributions from s-channel resonances or final-state interactions, which do not signal CP violation. In order to eliminate these contributions, we study the two CP odd asymmetries

$$A_I = \frac{1}{2}(A_I^T - \bar{A}_I^T), \quad A_{II} = \frac{1}{2}(A_{II}^T - \bar{A}_{II}^T), \quad (8)$$

where $\bar{A}_{II}^T$ are the CP conjugated asymmetries for the processes $e^+e^- \to \tilde{\chi}_i^-\tilde{\chi}_j^+; \tilde{\chi}_i^- \to W^-\tilde{\chi}_n^0$ and $e^+e^- \to \tilde{\chi}_i^-\tilde{\chi}_j^+; \tilde{\chi}_i^- \to W^-\tilde{\chi}_n^0; W^- \to \bar{c}s$, respectively.

In Section 2 we give our definitions and formalism used, and obtain the analytical formulae for the differential cross section and the $W$ boson density matrix. In Section 3 we discuss general properties of the asymmetries. We present numerical results in Section 4 and Section 5 gives a summary and conclusions.

### 2 Definitions and formalism

We give the analytical formulae for the differential cross section of chargino production (1) with longitudinally polarized beams and the subsequent decay chain of one of the charginos (2) followed by the decay of the $W$ boson

$$W^+ \to f' \bar{f}, \quad (9)$$

which may be leptonic, $f' = \nu_{\ell}, \bar{f} = \bar{\ell}$ with $\ell = e, \mu, \tau$ or hadronic, $f' = q_u, \bar{f} = \bar{q}_d$ with $q_u = u, d$ and $q_d = c, s$. For a schematic picture of the chargino production and decay process see Fig. 1. In the following we will derive the $W$ boson spin-density matrix and relate it to the asymmetries $A_{II}^T$ (4) and $A_{II}^T$ (7).
2.1 Lagrangian and couplings

The MSSM interaction Lagrangians relevant for our study are [1, 15]:

\begin{align}
\mathcal{L}_{Z^0\ell\ell} &= -\frac{g}{\cos \theta_W} Z_{\mu} \bar{\ell} \gamma^\mu [L_{\ell} P_L + R_{\ell} P_R] \ell, \\
\mathcal{L}_{\gamma \tilde{\chi}_i^+ \tilde{\chi}_i^-} &= -e A_{\mu} \tilde{\chi}_i^+ \gamma^\mu \delta_{ij}, \quad e > 0, \\
\mathcal{L}_{\tilde{\ell} \tilde{\nu} \tilde{\chi}_i^+} &= -g U_{\lambda i} \tilde{\chi}_i^+ P_L \bar{\nu} \ell - g V_{\lambda i} \tilde{\chi}_i^+ P_L \ell \tilde{\nu} + \text{h.c.}, \\
\mathcal{L}_{W^- \tilde{\chi}_i^+ \tilde{\chi}_k^0} &= g W_{\mu} \tilde{\chi}_k^0 \gamma^\mu [O_{ki}^L P_L + O_{ki}^R P_R] \tilde{\chi}_i^+ + \text{h.c.},
\end{align}

with the couplings:

\begin{align}
L_{\ell} &= T_{3\ell} - e_{\ell} \sin^2 \theta_W, \quad R_{\ell} = -e_{\ell} \sin \theta_W, \\
O_{ij}^L &= -V_{i1} V_{j1}^\ast - \frac{1}{2} V_{i2} V_{j2}^\ast + \delta_{ij} \sin^2 \theta_W, \\
O_{ij}^R &= -U_{i1}^\ast U_{j1} - \frac{1}{2} U_{i2}^\ast U_{j2} + \delta_{ij} \sin^2 \theta_W, \\
O_{ki}^L &= -1/\sqrt{2} \left( \cos \beta N_{k4} - \sin \beta N_{k3} \right) V_{i2} + \left( \sin \theta_W N_{k1} + \cos \theta_W N_{k2} \right) V_{i1}^\ast, \\
O_{ki}^R &= +1/\sqrt{2} \left( \sin \beta N_{k4} + \cos \beta N_{k3} \right) U_{i2} + \left( \sin \theta_W N_{k1} + \cos \theta_W N_{k2} \right) U_{i1}^\ast,
\end{align}

with \( i,j = 1,2 \) and \( k = 1,\ldots,4 \). Here \( P_{L,R} = \frac{1}{2} (1 \mp \gamma_5) \), \( g \) is the weak coupling constant \( (g = e/\sin \theta_W) \), and \( e_\ell \) and \( T_{3\ell} \) denote the charge and the third component of the weak isospin of the lepton \( \ell \). Furthermore, \( \tan \beta = \frac{v_2}{v_1} \) where \( v_{1,2} \) are the vacuum expectation values of the two neutral Higgs fields. The chargino-mass eigenstates \( \tilde{\chi}_i^\pm = (\chi_i^+ / \chi_i^-) \) are defined by \( \chi_i^+ = V_{i1} w^+ + V_{i2} h^+ \) and \( \chi_i^- = U_{j1} w^- + U_{j2} h^- \) with \( w^\pm \) and \( h^\pm \) the two-component spinor fields of the W-eino and the charged Higgsinos, respectively.

![Figure 1: Schematic picture of the chargino production and decay process.](image-url)
complex unitary $2 \times 2$ matrices $U_{mn}$ and $V_{mn}$ diagonalize the chargino mass matrix $X_{\alpha\beta}$, $U^*_{ma}X_{\alpha\beta}V_{nb}^{-1} = m_{\chi_n^+}\delta_{mn}$, with $m_{\chi_n^+} > 0$. The complex unitary $4 \times 4$ matrix $N_{ij}$ diagonalizes the neutral gaugino-Higgsino mass matrix $Y_{\alpha\beta}$, $N_{ia}Y_{\alpha\beta}N_{\beta k}^t = m_{\tilde{\chi}_i^0}\delta_{ik}$, with $m_{\tilde{\chi}_i^0} > 0$, in the neutralino basis $\tilde{\gamma}, \tilde{Z}, \tilde{H}_a^0, \tilde{H}_b^0$.

### 2.2 Helicity amplitudes

The helicity amplitudes $T^\lambda_{P,\lambda}$ for the production process are given in [15]. Those for the chargino decay (2) are

$$T^\lambda_{D_1,\lambda_i} = ig\bar{u}(p_{\chi_i^0}, \lambda_i)\gamma^\mu[O^L_{\mu}\mathcal{P}_L + O^R_{\mu}\mathcal{P}_R]u(p_{\chi_i^+}, \lambda_i)\epsilon_{\mu}^{\lambda_k}$$

(20)

and those for the $W$ decay (9) are

$$T^\lambda_{D_2,\lambda_k} = ig\sqrt{2}\bar{u}(p_{f'}, \lambda_{f'})\gamma^\mu\mathcal{P}_Lv(p_f, \lambda_f)\epsilon_{\mu}^{\lambda_k}.$$  

(21)

The $W$ polarization vectors $\epsilon_{\mu}^{\lambda_k}, \lambda_k = 0, \pm 1$, are given in Appendix A. The amplitude for the whole process (1), (2), (9) is

$$\mathcal{T} = \Delta(\tilde{\chi}_i^+)\Delta(W^+) \sum_{\lambda_i,\lambda_k} T^\lambda_{P,\lambda_j} T^\lambda_{D_1,\lambda_i} T^\lambda_{D_2,\lambda_k}.$$  

(22)

with the chargino propagator $\Delta(\tilde{\chi}_i^+) = i/[2\tilde{\chi}_i^+ - m_{\chi_i^+}^2 + im_{\chi_i^+}\Gamma_{\chi_i^+}]$ and the $W$ boson propagator $\Delta(W^+) = i/[2p_W^2 - m_W^2 + im_W\Gamma_W]$.

### 2.3 Cross section

For the calculation of the cross section for the combined process of chargino production (1) and the subsequent two-body decays (2), (9) of $\tilde{\chi}_i^+$ we use the same spin-density matrix formalism as in [15, 16]. The (unnormalized) spin-density matrix of the $W$ boson

$$\rho_P(W^+)_{\lambda_k'\lambda_i'} = |\Delta(\tilde{\chi}_i^+)|^2 \sum_{\lambda_i,\lambda_k'} \rho_P(\tilde{\chi}_i^+)_{\lambda_i',\lambda_k'} \rho_D(\tilde{\chi}_i^+)_{\lambda_k'\lambda_i'};$$

(23)

is composed of the spin-density production matrix

$$\rho_P(\tilde{\chi}_i^+)_{\lambda_i',\lambda_i'} = \sum_{\lambda_j} T^\lambda_{P,\lambda_j} T^\lambda_{P,\lambda_j'}*$$

(24)

and the decay matrix of the chargino

$$\rho_D(\tilde{\chi}_i^+)_{\lambda_k'\lambda_i'} = \sum_{\lambda_i} T^\lambda_{D_1,\lambda_k} T^\lambda_{D_1,\lambda_i} T^\lambda_{D_2,\lambda_k'}*.$$  

(25)

With the decay matrix for the $W$ decay

$$\rho_D(W^+)_{\lambda_k'} = \sum_{\lambda_j,\lambda_k} T^\lambda_{D_2,\lambda_k} T^\lambda_{D_2,\lambda_k'}*$$

(26)
the amplitude squared for the complete process $e^+e^- \rightarrow \tilde{\chi}_i^+\tilde{\chi}_j^-; \tilde{\chi}_i^+ \rightarrow W^+\tilde{\chi}_n^0; W^+ \rightarrow f\bar{f}$ can now be written

$$|T|^2 = |\Delta(W^+)|^2 \sum_{\lambda_k, \lambda'_k} \rho_p(W^+)_{\lambda_k} \rho_{D_2}(W^+)_{\lambda'_k|\lambda_k}.$$  

(27)

The differential cross section is then given by

$$d\sigma = \frac{1}{2s}|T|^2 d\text{Lips}(s, p_{\chi_i^+}, p_{\chi_n^0}, p_f, p_{\bar{f}}),$$  

(28)

where $d\text{Lips}(s, p_{\chi_i^+}, p_{\chi_n^0}, p_f, p_{\bar{f}})$ is the Lorentz invariant phase-space element, see (B.1) of Appendix B. More details concerning kinematics and phase space can be found in Appendices A and B.

For the polarization of the decaying chargino $\tilde{\chi}_i^+$ with momentum $p_{\chi_i^+}$ we introduce three space-like spin vectors $s_{\chi_i^+}^a, a = 1, 2, 3$, which together with $p_{\chi_i^+}/m_{\chi_i^+}$ form an orthonormal set with $s_{\chi_i^+}^a \cdot s_{\chi_i^+}^b = -\delta^{ab}$, $s_{\chi_i^+}^a \cdot p_{\chi_i^+} = 0$. Then the (unnormalized) chargino density matrix can be expanded in terms of the Pauli matrices $\sigma^a, a = 1, 2, 3$:

$$\rho_p(\tilde{\chi}_i^+)_{\lambda_k \lambda'_k} = 2(\delta_{\lambda_k \lambda'_k} P + \sigma^a_{\lambda_k \lambda'_k} \Sigma_p^a),$$  

(29)

where we sum over $a$. With our choice of the spin vectors $s_{\chi_i^+}^a$, given in Appendix A, $\Sigma_p^3/P$ is the longitudinal polarization of chargino $\tilde{\chi}_i^+$, $\Sigma_p^1/P$ is the transverse polarization in the production plane and $\Sigma_p^2/P$ is the polarization perpendicular to the production plane. We give in Appendix D the analytical formulae for $P$ and $\Sigma_p^a$ in the laboratory system.

To describe the polarization states of the $W$ boson, we introduce a set of spin vectors $t_W^c, c = 1, 2, 3$, and choose polarization vectors $\varepsilon^\lambda_{\mu \nu}, \lambda = 0, \pm 1$, given in Appendix A. Then we obtain for the decay matrix

$$\rho_{D_1}(\tilde{\chi}_i^+/\chi_n^0)_{\lambda_k \lambda'_k} = (\delta_{\lambda_k \lambda'_k} D_1^{\mu \nu} + \sigma^a_{\lambda_k \lambda'_k} \Sigma_p^a)\varepsilon^\lambda_{\mu \nu} \varepsilon_{\nu \lambda'_{k}^*},$$  

(30)

and

$$\rho_{D_2}(W^+)_{\lambda_k \lambda'_k} = D_2^{\mu \nu} \varepsilon^\lambda_{\mu \nu} \varepsilon_{\nu \lambda'_{k}^*},$$  

(31)

with:

$$D_1^{\mu \nu} = g^2(|O_R^\chi_i|^2 + |O_L^\chi_i|^2)[2p_{\chi_i^+}^\mu p_{\chi_i^+}^\nu - (p_{\chi_i^+}^\mu p_W^\nu + p_{\chi_i^+}^\nu p_W^\mu) - \frac{1}{2}(m_{\chi_i^+}^2 + m_{\chi_n^0}^2 - m_W^2)g^{\mu \nu}]
+ 2g^2 Re(O_R^\chi_i O_L^\chi_i) m_{\chi_i^+} m_{\chi_n^0} g^{\mu \nu} \epsilon^{\mu \nu \alpha \beta} s_{\chi_i^+}^a p_{\chi_i^+ a} p_{W \beta},$$  

$$\Sigma_p^a \epsilon^{\mu \nu \alpha \beta} = 
+ 2g^2 Re(O_R^\chi_i O_L^\chi_i) m_{\chi_i^+} m_{\chi_n^0} g^{\mu \nu} \epsilon^{\mu \nu \alpha \beta} s_{\chi_i^+}^a p_{\chi_i^+ a} p_{W \beta}
+ 2g^2 Im(O_R^\chi_i O_L^\chi_i) m_{\chi_n^0} (s_{\chi_i^+}^a p_{\chi_i^+}^\nu - s_{\chi_i^+}^a p_{\chi_i^+}^\mu; (\epsilon_{0123} = 1),$$  

(33)
and

\[ D_{2}^{\mu\nu} = g^{2}(-2\rho_{f}^{\mu}\rho_{f}^{\nu} + \rho_{W}^{\mu}\rho_{W}^{\nu} + \rho_{f}^{\mu}\rho_{W}^{\nu} - \frac{1}{2}m_{W}^{2}g^{\mu\nu}) \epsilon^{\mu\nu\alpha\beta}\rho_{W\alpha}\rho_{f\beta}. \]  

(34)

where here, and in the following, the signs in parenthesis hold for the charge conjugated processes, here \( \bar{\chi}_{i} \rightarrow W^{-}\chi_{n}^{0} \) and \( W^{-} \rightarrow \bar{f} f \), respectively. In (30) and (31) we use the expansion [17]:

\[ \varepsilon_{\mu}^{\lambda_{k}^{*}}\varepsilon_{\nu}^{\lambda_{k}} = \frac{1}{3}\delta_{\lambda_{k}^{*}\lambda_{k}}I_{\mu\nu} - \frac{i}{2m_{W}}\epsilon_{\mu\nu\rho\sigma}\rho_{W}^{\rho}\epsilon_{\omega}^{\sigma}(J^{c})^{\lambda_{k}^{*}\lambda_{k}} - \frac{1}{2}t_{W}^{d}t_{W}^{d}(J^{cd})^{\lambda_{k}^{*}\lambda_{k}}, \]  

(35)

summed over \( c, d \), and \( \epsilon_{0123} = 1 \). \( J^{c}, c = 1, 2, 3, \) are the 3 \( \times \) 3 spin-1 matrices with \( [J^{c}, J^{d}] = i\epsilon_{cde}J^{e} \). The matrices

\[ J^{cd} = J^{c}J^{d} + J^{d}J^{c} - \frac{1}{3}\delta^{cd}, \]  

(36)

with \( J^{11} + J^{22} + J^{33} = 0 \), are the components of a symmetric, traceless tensor. An explicit form of \( J^{c} \) and \( J^{cd} \) is given in Appendix C. The completeness relation of the polarization vectors

\[ \sum_{\lambda_{k}}\varepsilon_{\mu}^{\lambda_{k}^{*}}\varepsilon_{\nu}^{\lambda_{k}} = -g_{\mu\nu} + \frac{P_{W\mu}P_{W\nu}}{m_{W}^{2}} \]  

(37)

is guaranteed by

\[ I_{\mu\nu} = -g_{\mu\nu} + \frac{P_{W\mu}P_{W\nu}}{m_{W}^{2}}. \]  

(38)

The second term of (35) describes the vector polarization and the third term describes the tensor polarization of the \( W \) boson. The decay matrices can be expanded in terms of the spin matrices \( J^{c} \) and \( J^{cd} \). The first term of the decay matrix \( \rho_{D_{1}}, (30) \), which is independent of the chargino polarization, then is

\[ D_{1}^{\mu\nu}\varepsilon_{\mu}^{\lambda_{k}^{*}}\varepsilon_{\nu}^{\lambda_{k}} = D_{1}\delta_{\lambda_{k}^{*}\lambda_{k}} + cD_{1}(J^{c})^{\lambda_{k}^{*}\lambda_{k}} + cdD_{1}(J^{cd})^{\lambda_{k}^{*}\lambda_{k}}, \]  

(39)

summed over \( c, d \), with

\[ D_{1} = \frac{1}{6}g^{2}(|O_{n1}^{R}|^{2} + |O_{n1}^{L}|^{2})[m_{\chi_{1}^{+}}^{2} + m_{\chi_{0}^{0}}^{2} - 2m_{W}^{2} + \frac{(m_{\chi_{1}^{+}}^{2} - m_{\chi_{0}^{0}}^{2})^{2}}{m_{W}^{2}}] \]

\[-2g^{2}Re(O_{n1}^{R}O_{n1}^{L})m_{\chi_{1}^{+}}m_{\chi_{0}^{0}}, \]  

(40)

\[ cD_{1} = (+)g^{2}(|O_{n1}^{R}|^{2} - |O_{n1}^{L}|^{2})m_{W}(t_{W}^{c} \cdot p_{\chi_{1}^{+}}), \]  

(41)

\[ cdD_{1} = -g^{2}(|O_{n1}^{R}|^{2} + |O_{n1}^{L}|^{2})[(t_{W}^{c} \cdot p_{\chi_{1}^{+}})(t_{W}^{d} \cdot p_{\chi_{1}^{+}}) + \frac{1}{4}(m_{\chi_{1}^{+}}^{2} + m_{\chi_{0}^{0}}^{2} - m_{W}^{2})\delta^{cd}] \]

\[ +g^{2}Re(O_{n1}^{R}O_{n1}^{L})m_{\chi_{1}^{+}}m_{\chi_{0}^{0}}\delta^{cd}. \]  

(42)

As a consequence of the completeness relation (37), the diagonal coefficients are linearly dependent

\[ 11D_{1} + 22D_{1} + 33D_{1} = -\frac{3}{2}D_{1}. \]  

(43)
For large chargino momentum $p_{\chi^+}$, the $W$ boson will mainly be emitted in the direction of $p_{\chi^+}$, i.e. $\hat{p}_{\chi^+} \approx \hat{p}_W$, with $\hat{p} = p/|p|$. Therefore, for high energies we have $(t_W^1 \cdot p_{\chi^+}) \approx 0$ in (42), resulting in $\Sigma^1 \approx \Sigma^{22}$.

For the second term of $\rho_{D_1}$ (30), which depends on the polarization of the decaying chargino, we obtain

\[
\Sigma^a_{D_1} \epsilon^{\lambda_4 \kappa_4} = \Sigma^a_{D_1} (J^c)^{\lambda_4 \kappa_4} + c \Sigma^a_{D_1} (J)^{\lambda_4 \kappa_4},
\]

summed over $c$, with

\[
\Sigma^a_{D_1} = \left[ (-\frac{2}{3}) g^2 (|O_{\nu_1}|^2 - |O_{\nu_1'}|^2) m_{\chi^+} \left( s_{\chi^+} \cdot p_W \right) \left( \frac{m^2_{\chi^+} - m^2_{\chi^0} - 1}{2m_W^2} \right) \right],
\]

\[
c \Sigma^a_{D_1} = \frac{g^2}{m_W^2} \left( |O_{\nu_1}|^2 + |O_{\nu_1'}|^2 \right) m_{\chi^+} \left[ \left( t_{\nu_1} \cdot p_{\chi^+} \right) \left( s_{\chi^+} \cdot p_W \right) + \frac{1}{2} \left( t_{\nu_1} \cdot s_{\chi^+} \right) \left( m^2_{\chi_0} - m^2_{\chi^+} + m_W^2 \right) \right]
\]

\[= \frac{2g^2}{m_W^2} \text{Re}(O_{\nu_1} O_{\nu_1'}) m_{\chi^+} \left[ \left( t_{\nu_1} \cdot p_{\chi^+} \right) \left( s_{\chi^+} \cdot p_W \right) + \frac{1}{2} \left( t_{\nu_1} \cdot s_{\chi^+} \right) \left( m^2_{\chi_0} - m^2_{\chi^+} + m_W^2 \right) \right]
\]

\[= \frac{2g^2}{m_W^2} \text{Im}(O_{\nu_1} O_{\nu_1'}) m_{\chi^+} \left( \epsilon_{\mu \rho \sigma} s_{\chi^+} p_{\nu_1}^\mu p_{\nu_1'}^\rho t_{\nu_1} t_{\nu_1'}^\sigma \right),
\]

\[
c \Sigma^a_{D_1} = \left[ (-\frac{1}{3}) g^2 (|O_{\nu_1}|^2 - |O_{\nu_1'}|^2) m_{\chi^+} \right]
\]

\[\left[ \left( s_{\chi^+} \cdot p_W \right) \delta^{\cd} - \left( t_{\nu_1} \cdot p_{\chi^+} \right) \left( t_{\nu_1} \cdot s_{\chi^+} \right) - \left( t_{\nu_1} \cdot t_{\nu_1'} \right) \left( t_{\nu_1} \cdot s_{\chi^+} \right) \right].
\]

A similar expansion for the $W$ decay matrix (31), results in

\[
\rho_{D_2} (W^+) \lambda_{\chi^+} = \Sigma^a_{D_2} (J^c)^{\lambda_{\chi^+}} + c \Sigma^a_{D_2} (J)^{\lambda_{\chi^+}},
\]

where we sum over $c$, with

\[
D_2 = \frac{1}{3} g^2 m_W^2,
\]

\[
c D_2 = \left[ (-\frac{1}{3}) g^2 m_W^2 \left( t_{\nu_1} \cdot p_{\nu_1} \right) \right],
\]

\[
c \Sigma^a_{D_2} = \left[ g^2 \left( t_{\nu_1} \cdot p_{\nu_1} \right) - \frac{1}{2} m_W^2 \delta^{\cd} \right],
\]

The diagonal coefficients are linearly dependent

\[
\Sigma^a_{D_2} + \Sigma^{12} D_2 + \Sigma^{33} D_2 = -\frac{3}{2} D_2.
\]

Inserting the density matrices (29) and (30) into (23) leads to:

\[
\rho_P (W^+) = \left[ |\Delta(\chi^+)^2| \right] \left[ (P D_1 + \Sigma^a \Sigma^a_{D_1}) \delta^{\lambda_{\chi^+}} + (P c D_1 + \Sigma^a \Sigma^a_{D_1}) (J^c)^{\lambda_{\chi^+}} + (P c D_1 + \Sigma^a \Sigma^a_{D_1}) (J)^{\lambda_{\chi^+}} \right],
\]

summed over $a$, with

\[
\left| T \right|^2 = \left| \Delta(\chi^+)^2 \right| \left| \Delta(W^+) \right|^2 \left[ 3(P D_1 + \Sigma^a \Sigma^a_{D_1}) D_2 + 2(P c D_1 + \Sigma^a \Sigma^a_{D_1}) c D_2 \right]
\]

\[+ 4 \left[ (P c D_1 + \Sigma^a \Sigma^a_{D_1}) c D_2 - \frac{1}{3} (P c c D_1 + \Sigma^a \Sigma^a_{D_1}) d D_2 \right],
\]

summed over $a$, with
2.4 Density matrix of the W boson

The mean polarization of the W bosons in the laboratory system is given by the $3 \times 3$ density matrix $< \rho(W^+) >$, obtained by integrating (53) over the Lorentz invariant phase space element $d\text{Lips}(s, p_{\lambda_j^a}, p_{\lambda_k^a}, p_W)$ see (B.1), and normalizing by the trace:

$$< \rho(W^+) > = \frac{\int \rho_P(W^+) d\text{Lips}}{\int \text{Tr}\{\rho_P(W^+)\} d\text{Lips}} = \frac{1}{3} \delta_{\lambda \lambda'} + V_c (J^c)^{\lambda \lambda'} + T_{cd} (J^{cd})^{\lambda \lambda'}, \quad (55)$$

summed over $c$, $d$. The vector and tensor coefficients $V_c$ and $T_{cd}$ are given by:

$$V_c = \frac{\int |\Delta(\tilde{\chi}_i^+)|^2 (P^e D_1 + \Sigma_{P^a}^e \Sigma_{D_1}) d\text{Lips}}{3 \int |\Delta(\tilde{\chi}_i^+)|^2 P D_1 d\text{Lips}}, \quad (56)$$

$$T_{cd} = T_{dc} = \frac{\int |\Delta(\tilde{\chi}_i^+)|^2 (P^{cd} D_1 + \Sigma_{P^a}^{cd} \Sigma_{D_1}) d\text{Lips}}{3 \int |\Delta(\tilde{\chi}_i^+)|^2 P D_1 d\text{Lips}}, \quad (57)$$

with sum over $a$. The density matrix in the circular polarization basis (A.11) is given by

$$< \rho(W^+) > = \frac{1}{2} - V_3 + T_{33}, \quad (58)$$

$$< \rho(W^+) > = -2T_{33}, \quad (59)$$

$$< \rho(W^+) > = \frac{1}{\sqrt{2}} (V_1 + iV_2) - \sqrt{2} (T_{13} + iT_{23}), \quad (60)$$

$$< \rho(W^+) > = T_{11} - T_{22} + 2iT_{12}, \quad (61)$$

$$< \rho(W^+) > = \frac{1}{\sqrt{2}} (V_1 + iV_2) + \sqrt{2} (T_{13} + iT_{23}), \quad (62)$$

where we have used $T_{11} + T_{22} + T_{33} = -\frac{1}{2}$.

3 T odd asymmetries

From (53) we obtain for asymmetry $A_I^T$ (4):

$$A_I^T = \frac{\int \text{Sign}[T_I] \text{Tr}\{\rho_P(W^+)\} d\text{Lips}}{\int \text{Tr}\{\rho_P(W^+)\} d\text{Lips}} = \frac{\int |\Delta(\tilde{\chi}_i^+)|^2 \text{Sign}[T_I] \Sigma_{P^a}^2 \Sigma_{D_1}^2 d\text{Lips}}{\int |\Delta(\tilde{\chi}_i^+)|^2 P D_1 d\text{Lips}}, \quad (63)$$

with $d\text{Lips}(s, p_{\lambda_j^a}, p_{\lambda_k^a}, p_W)$ and $d\text{Lips}(s_{\lambda_j^a}, p_{\lambda_k^a}, p_{W^a})$, given in (B.1). In the numerator of (63), only the spin correlations $\Sigma_{P^a}^2 \Sigma_{D_1}^2$ perpendicular to the production plane remain, since only this term contains the the triple product $T_I = p_{e^-} \cdot (p_{\chi_i^a} \times p_W)$. In the denominator only the term $P D_1$ remains, and all spin correlations vanish due to the integration over the complete phase space. Note that $A_I^T \propto \Sigma_{D_1}^2 \propto (|O_{m1}^R|^2 - |O_{m1}^L|^2)$, see (45), and thus $A_I^T$ may be reduced for $|O_{m1}^R| \approx |O_{m1}^L|$. Moreover $A_I^T$ will be small for $m_{\chi_i}^2 - m_{\lambda_k}^2 \approx 2m_W^2$.

For the asymmetry $A_{II}^T$ (7), we obtain from (54):

$$A_{II}^T = \frac{\int \text{Sign}[T_{II}] |T| d\text{Lips}}{\int |T| d\text{Lips}} = \frac{\int |\Delta(\tilde{\chi}_i^+)|^2 |\Delta(W^+)|^2 \text{Sign}[T_{II}] \Sigma_{P^a} \Sigma_{D_1} D_2 d\text{Lips}}{\int |\Delta(\tilde{\chi}_i^+)|^2 |\Delta(W^+)|^2 3P D_1 D_2 d\text{Lips}}, \quad (64)$$
We study the dependence of the numerical results on the MSSM parameters and the density matrix \( \rho(W^+) \), defined in (B.1). In the numerator only the vector part of \(|T|^2\) remains because only the vector part contains the triple product \( T_{II} = p_e \cdot (p_e \times p_a) \). In the denominator the vector and tensor parts of \(|T|^2\) vanish due to phase space integration. Owing to the correlations between the \( \tilde{\chi}_i^+ \) and the \( W \) boson polarization, \( \Sigma^{a}_{D1} \Sigma^{a}_{D2} \), there are contributions to the asymmetry \( A_{II}^T \) from the chargino production process (1), and/or from the chargino decay process (2). The contribution from the production is given by the term with \( a = 2 \) in (64) and it is proportional to the transverse polarization of the chargino perpendicular to the production plane, \( \Sigma^{2}_{D2} \) (29). For \( e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^- \) we have \( \Sigma^{2}_{D} = 0 \). The contributions from the decay, which are the terms with \( a = 1, 3 \) in (64), are proportional to

\[
\sum_{D1}^{a} c D_2 \supset -2g^4 m_{\chi_n} Im(O_{ni}^{R} O_{ni}^{L}) (t^c_{W} \cdot p_f) \epsilon_{\mu \nu \rho \sigma} s_{\chi_i^+}^{a} p_{\chi_j^+}^{\nu} p_{\chi_i^+}^{\rho} t_{\chi_i^+}^{\sigma},
\]

which contains the \( \epsilon \)-tensor, see the last term of (46). Thus \( A_{II}^T \) can be enhanced (reduced) if the contributions from production and decay have the same (opposite) sign. Note that the contributions from the decay would vanish for a two-body decay of the chargino into a scalar particle instead of a \( W \) boson.

The relative statistical error of \( A_{II}^T \) is \( \delta A_{II}^T = \frac{\Delta A_{II}^T}{|A_{II}^T|} = 1/(|A_{II}^T|\sqrt{N}) \), where \( N = L \cdot \sigma \) is the number of events for the integrated luminosity \( L \) and the cross section \( \sigma = \sigma_p(e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-) \times BR(\tilde{\chi}_i^+ \to W^+\chi_n^0) \). For the CP asymmetry \( A_I \) (8), we have \( \Delta A_I = \Delta A_{II}^T / \sqrt{2} \). The statistical significance, with which the asymmetry can be measured, is then given by \( S_I = |A_I|\sqrt{2L \cdot \sigma} \). A similar result is obtained for the asymmetry \( A_{II} \) with \( S_{II} = |A_{II}|\sqrt{2L \cdot \sigma} \) and the cross section \( \sigma = \sigma_p(e^+e^- \to \tilde{\chi}_i^+\tilde{\chi}_j^-) \times BR(\tilde{\chi}_i^+ \to W^+\chi_n^0) \times BR(W^+ \to c\bar{s}) \). Note that in order to measure \( A_I \) the momentum of \( \tilde{\chi}_i^+ \), i.e. the production plane, has to be kinematically reconstructed. This could be accomplished by measuring the decay of the other chargino \( \tilde{\chi}_j^- \), if the masses of the charginos and the masses of their decay products are known. For the measurement of \( A_{II} \), the flavors of the quarks and the \( \tilde{s} \) have to be distinguished, which will be possible by flavor tagging of the \( c \)-quark [18, 19]. In principle, for the decay \( W \to u \bar{d} \) also an asymmetry similar to \( A_{II} \) can be considered, if it is possible to distinguish between the \( u \) and \( d \) jet, for instance, by measuring the average charge. Also it is clear that detailed Monte Carlo studies taking into account background and detector simulations are necessary to predict the expected accuracies. However, this is beyond the scope of the present work.

## 4 Numerical results

We study the dependence of \( A_I, A_{II} \) (8), and the density matrix \( \rho(W^+) \) (55), on the MSSM parameters \( \mu = |\mu| e^{i \phi} \), \( M_1 = |M_1| e^{i \varphi_{M_1}} \), \( \tan \beta \) and the universal scalar mass parameter \( m_0 \). We will allow \( \varphi_{M_1} \in [\pi, -\pi] \), however take into account \( |\varphi_{\mu}| \leq 0.1 \pi \) in some of the plots, as suggested from the EDM analyses [3, 4]. In order to show the full phase dependence of the asymmetries, however, we will relax the EDM restrictions in some of the examples studied.
The feasibility of measuring the asymmetries depends also on the cross sections \( \sigma = \sigma_P(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^-) \times \text{BR}(\tilde{\chi}_1^+ \rightarrow W^+\tilde{\chi}_1^0) \times \text{BR}(W^+ \rightarrow c\bar{s}) \), which we will discuss in our scenarios. We choose a center of mass energy of \( \sqrt{s} = 800 \text{ GeV} \) and longitudinally polarized beams with \( (P_L, P_+) = (-0.8, +0.6) \). This choice enhances sneutrino exchange in the chargino production process, which results in larger cross sections and asymmetries. For the calculation of the branching ratios \( \text{BR}(\tilde{\chi}_1^+ \rightarrow W^+\tilde{\chi}_1^0) \) and widths \( \Gamma_{\tilde{\chi}_1^+} \), we include the two-body decays:

\[
\tilde{\chi}_1^+ \rightarrow W^+\tilde{\chi}_1^0, \quad \tilde{\nu}_1^+\nu_\tau, \quad \tilde{\nu}_1^+\tilde{\nu}_\tau,
\]

and neglect three-body decays. For the \( W \) boson decay we take the experimental value \( \text{BR}(W^+ \rightarrow c\bar{s}) = 0.31 \) [20]. In order to reduce the number of parameters, we assume the relation \( |M_I| = 5/3 M_2 \tan^2 \theta_W \) and use the renormalization group equations [21] for the slepton and sneutrino masses, \( m_{\tilde{e}_L}^2 = m_0^2 + 0.79 M_2^2 + m_0^2 \cos 2\beta(-1/2 + \sin^2 \theta_W) \) and \( m_{\tilde{\nu}_\mu}^2 = m_0^2 + 0.79 M_2^2 + m_0^2/2 \cos 2\beta \). In the stau sector [22], we fix the trilinear scalar coupling parameter to \( A_\tau = 250 \text{ GeV} \).

### 4.1 Production of \( \tilde{\chi}_1^+ \tilde{\chi}_1^- \)

For the production \( e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^- \) of a pair of charginos the polarization perpendicular to the production plane vanishes, and thus \( A_I = 0 \). However, \( A_{II} \) need not to be zero and is sensitive to \( \varphi_\mu \) and \( \varphi_{M_1} \), because this asymmetry has contributions from the chargino decay process. For \( (\varphi_{M_1}, \varphi_\mu) = (0.5\pi, 0) \) we show in Fig. 2a the \( |\mu|-M_2 \) dependence of \( A_{II} \), which can reach values of 5%-7% for \( M_2 \gtrsim 400 \text{ GeV} \). We also studied the \( \varphi_\mu \) dependence of \( A_{II} \) in the \( |\mu|-M_2 \) plane. For \( \varphi_{M_1} = 0, \varphi_\mu = 0.1\pi(0.5\pi) \) and the other parameters as given in the caption of Fig. 2, we find \( |A_{II}| < 2\% (7\%) \).

In Fig. 2b we show the contour lines of the cross section \( \sigma = \sigma_P(e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-) \times \text{BR}(\tilde{\chi}_1^+ \rightarrow W^+\tilde{\chi}_1^0) \times \text{BR}(W^+ \rightarrow c\bar{s}) \) in the \( |\mu|-M_2 \) plane for \( (\varphi_{M_1}, \varphi_\mu) = (0.5\pi, 0) \). The production cross section \( \sigma_P(e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-) \) reaches more than 400 fb. For our choice of \( m_0 = 300 \text{ GeV} \), \( \tilde{\chi}_1^+ \rightarrow W^+\tilde{\chi}_1^0 \) is the only allowed two-body decay channel.

In Fig. 3a we plot the contour lines of \( A_{II} \) for \( |\mu| = 350 \text{ GeV} \) and \( M_2 = 400 \text{ GeV} \) in the \( \varphi_\mu-\varphi_{M_1} \) plane. Fig. 3a shows that \( A_{II} \) is essentially depending on the sum \( \varphi_\mu + \varphi_{M_1} \). However, maximal phases of \( \varphi_{M_1} = \pm 0.5\pi \) and \( \varphi_\mu = \pm 0.5\pi \) do not lead to the highest values of \( |A_{II}| \geq 6\% \), which are reached for \( (\varphi_{M_1}, \varphi_\mu) \approx (\pm 0.8\pi, \pm 0.6\pi) \). The reason for this is that the spin-correlation terms \( \Sigma_p \Sigma_0 \Sigma_0 \Sigma_2 \) in the numerator of \( A_{II} \) (64) are products of CP odd and CP even factors. The CP odd (CP even) factors have a sine-like (cosine-like) phase dependence. Therefore, the maximum of the CP asymmetry \( A_{II} \) may be shifted to smaller or larger values of the phases. In the \( \varphi_\mu-\varphi_{M_1} \) region shown in Fig. 3a the cross section \( \sigma = \sigma_P(e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-) \times \text{BR}(\tilde{\chi}_1^+ \rightarrow W^+\tilde{\chi}_1^0) \times \text{BR}(W^+ \rightarrow c\bar{s}) \), with \( \text{BR}(\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 W^+) = 1 \), does not depend on \( \varphi_{M_1} \) and ranges between 74 fb for \( \varphi_\mu = 0 \) and 66 fb for \( \varphi_\mu = \pi \).

In Fig. 3b we show the contour lines of the significance \( S_{II} = |A_{II}|\sqrt{2\mathcal{L} \cdot \sigma} \), defined in Section 3. For \( \mathcal{L} = 500 \text{ fb}^{-1} \) and for e.g. \( (\varphi_{M_1}, \varphi_\mu) \approx (\pi, 0.1\pi) \) we have \( S_{II} \approx 8 \) and thus \( A_{II} \) could be measured even for small \( \varphi_\mu \).
Figure 2: Contour lines of the asymmetry $A_{II}$ (2a) and $\sigma = \sigma_P(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^-) \times BR(\tilde{\chi}_1^+ \rightarrow W^+\tilde{\chi}_1^0) \times BR(W^+ \rightarrow cs)$ (2b), in the $|\mu|-M_2$ plane for $(\varphi_{M_1}, \varphi_{\mu}) = (0.5\pi, 0)$, $\tan\beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV and $(P_e^-, P_e^+) = (-0.8, 0.6)$. The area A is kinematically forbidden by $m_{\tilde{\chi}_1^+} + m_{\tilde{\chi}_1^-} > \sqrt{s}$, and the area B by $m_{W} + m_{\tilde{\chi}_1^0} > m_{\tilde{\chi}_1^+}$. The gray area is excluded by $m_{\tilde{\chi}_1^\pm} < 104$ GeV.

In Figs. 4a,b we show the $\tan\beta-m_0$ dependence of $A_{II}$ and $\sigma$ for $(\varphi_{M_1}, \varphi_{\mu}) = (0.7\pi, 0)$. The asymmetry is rather insensitive to $m_0$, and shows strong dependence on $\tan\beta$ and decreases with increasing $\tan\beta \gtrsim 2$. The production cross section $\sigma_P(e^+e^- \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^-)$ increases with increasing $m_0$ and decreasing $\tan\beta$. For $m_0 \lesssim 200$ GeV, the branching ratio $BR(\tilde{\chi}_1^+ \rightarrow W^+\tilde{\chi}_1^0) < 1$, since the decay channels of $\tilde{\chi}_1^\pm$ into sleptons and/or sneutrinos are open.

In Fig. 5a we show the $\varphi_{\mu}$ dependence of the vector ($V_i$) and tensor ($T_{ij}$) elements of the density matrix $<\rho(W^+)>$ for $\varphi_{M_1} = \pi$, see (56) and (57). In Fig. 5b we show their dependence on $\varphi_{M_1}$ for $\varphi_{\mu} = 0$. In both figures, the element $V_2$ is CP odd, while $T_{13}, T_{11}, T_{22}$ and $V_1, V_3$ show a CP even behavior. As discussed in Section 2.3, the tensor elements $T_{11}$ and $T_{22}$ are almost equal and have the same order of magnitude as $V_1$ and $V_3$, whereas the other elements $T_{12}, |T_{23}| < 10^{-5}$ are small. In the CP conserving case $(\varphi_{M_1}, \varphi_{\mu}) = (0, 0)$ and $M_2 = 400$ GeV, $|\mu| = 350$ GeV, $\tan\beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV, $(P_e^-, P_e^+) = (-0.8, 0.6)$ the density matrix reads:

$$<\rho(W^+)> = \begin{pmatrix}
<\rho^{-+}> & <\rho^{-0}> & <\rho^{+-}>
<\rho^{0-}> & <\rho^{00}> & <\rho^{0+}>
<\rho^{+0}> & <\rho^{++}>
\end{pmatrix} = \begin{pmatrix}
0.200 & -0.010 & -0.001 \\
-0.010 & 0.487 & 0.137 \\
-0.001 & 0.137 & 0.313
\end{pmatrix}.$$ (67)

In the CP violating case, e.g. for $(\varphi_{M_1}, \varphi_{\mu}) = (0.7\pi, 0)$ and the other parameters as above,
Figure 3: Contour lines of and the asymmetry $A_{II}$ (3a) and the statistical significance $S_{II}$ (3b) for $e^+e^- \rightarrow \tilde{\chi}_1^+\bar{\chi}_1^-$; $\tilde{\chi}_1^+ \rightarrow W^+\tilde{\chi}_1^0$; $W^+ \rightarrow cs$, in the $\varphi_{\mu}$-$\varphi_{M_1}$ plane for $|\mu| = 350$ GeV, $M_2 = 400$ GeV, $\tan \beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV, $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$ and $L = 500$ fb$^{-1}$. In the gray shaded area of Fig. 3b we have $S_{II} < 5$.

the density matrix has imaginary parts due to a non-vanishing $V_2$:

$$< \rho(W^+) > = \begin{pmatrix}
0.219 & -0.010 + 0.025i & 0.002 \\
-0.010 - 0.025i & 0.405 & 0.171 + 0.025i \\
0.002 & 0.171 - 0.025i & 0.376
\end{pmatrix} \tag{68}$$

Imaginary parts of the density matrix are thus an indication of CP violation.

4.2 Production of $\tilde{\chi}_1^+\tilde{\chi}_2^-$

For the production of an unequal pair of charginos, $e^+e^- \rightarrow \tilde{\chi}_1^+\bar{\chi}_2^-$, their polarization perpendicular to the production plane is sensitive to the phase $\varphi_{\mu}$, which leads to a non-vanishing asymmetry $A_I$ (63). We will study the decay of the lighter chargino $\tilde{\chi}_1^+ \rightarrow W^+\tilde{\chi}_1^0$ for $|M_2| = 250$ GeV and $\varphi_{M_1} = 0$, we show in Fig. 6a the $|\mu|$-$\varphi_{\mu}$ dependence of $A_I$, which attains values up to 4%. Note that $A_I$ is not maximal for $\varphi_{\mu} = 0.5\pi$, but is rather sensitive for phases in the regions $\varphi_{\mu} \in [0.7\pi, \pi]$ and $\varphi_{\mu} \in [-0.7\pi, -\pi]$. As mentioned before, values of $\varphi_{\mu}$ close to the CP conserving points $\varphi_{\mu} = 0, \pm \pi$ are suggested by EDM analyses. For $\varphi_{\mu} = 0.9\pi$ and $|\mu| = 350$ GeV the statistical significance is $S_I = |A_I|\sqrt{2L \cdot \sigma} \approx 1.5$ with $L = 500$ fb$^{-1}$. Thus $A_I$ could be measured at a confidence level larger than 68% ($S_I = 1$).

In Fig. 6b we show contour lines of the corresponding cross section $\sigma = \sigma_P(e^+e^- \rightarrow \tilde{\chi}_1^+\bar{\chi}_2^-) \times BR(\tilde{\chi}_1^+ \rightarrow W^+\tilde{\chi}_1^0)$ in the $|\mu|$-$\varphi_{\mu}$ plane for the parameters as above. The cross
section shows a CP even behavior, which has been used in [7, 8, 14] to constrain \( \cos \varphi_\mu \).

In our scenario we have considered the decay of the lighter chargino \( \tilde{\chi}_1^+ \rightarrow W^+ \tilde{\chi}_1^0 \) since for our choice \( m_0 = 300 \text{ GeV} \) we have \( \text{BR}(\tilde{\chi}_1^+ \rightarrow W^+ \tilde{\chi}_1^0) = 1 \). For the decay of \( \tilde{\chi}_2^+ \), one would have to take into account also the decays into the Z boson and the lightest neutral Higgs boson, which would reduce \( \text{BR}(\tilde{\chi}_2^+ \rightarrow W^+ \tilde{\chi}_1^0) \approx 0.2 \).

The asymmetry \( A_{II} \) is also sensitive to the phase \( \varphi_{M_1} \). We show the \( \varphi_\mu \)-\( \varphi_{M_1} \) dependence of \( A_{II} \), choosing the parameters as above, in Fig. 7a. In Fig. 7b we show the contour lines of the significance \( S_{II} = |A_{II}| \sqrt{2L \cdot \sigma} \) for \( L = 500 \text{ fb}^{-1} \). For \( (\varphi_{M_1}, \varphi_\mu) \approx (\pi, 0.1\pi) \) we have \( S_{II} \approx 2.4 \) and thus \( A_{II} \) could be accessible even for small phases by using polarized beams.

### 5 Summary and conclusions

We have proposed and analyzed CP sensitive observables in chargino production, \( e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_j^- \), with subsequent two-body decay, \( \tilde{\chi}_i^+ \rightarrow W^+ \chi_n^0 \). We have defined the CP asymmetry \( A_I \) of the triple product \( p_{e^-} \cdot (p_{\tilde{\chi}_i^+} \times p_W) \). In the MSSM with complex parameters \( \mu \) and \( M_1 \), we have shown that \( A_I \) can reach 4\% and that even for \( \varphi_\mu \approx 0.9\pi \) the asymmetry could be accessible in the process \( e^+e^- \rightarrow \tilde{\chi}_i^+ \tilde{\chi}_2^- \). Further we have analyzed the CP sensitive density-matrix elements of the W boson. The phase \( \varphi_{M_1} \) enters in the decay \( \tilde{\chi}_i^+ \rightarrow W^+ \chi_n^0 \) due to correlations of the chargino and the W boson spins, which can be
Figure 5: Dependence of Vector ($V_i$) and tensor ($T_{ij}$) elements of the $W^+$ density matrix $<\rho(W^+)>$ on $\varphi_\mu$ (5a) and on $\varphi_M$ (5b), for $e^+e^-\rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-$; $\tilde{\chi}_1^+ \rightarrow W^+\tilde{\chi}_1^0$, for $|\mu| = 350$ GeV, $M_2 = 400$ GeV, $\tan \beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$.

By analyzing the statistical errors of $A_I$ and $A_{II}$ we found that the phases $\varphi_\mu$ and $\varphi_M$ could be strongly constrained in future $e^+e^-$ collider experiments in the 800 GeV range with high luminosity and longitudinally polarized beams.

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Appendix

A Coordinate frame and spin vectors

We choose a coordinate frame in the laboratory system such that the momentum of the chargino $\tilde{\chi}_j^-$ points in the z-direction (in our definitions we follow closely [15]).
Figure 6: Contour lines of the asymmetry $A_I$ (6a) and $\sigma = \sigma_P(e^+e^- \to \tilde{\chi}_1^+\tilde{\chi}_1^-) \times \text{BR} (\tilde{\chi}_1^+ \to W^+\tilde{\chi}_1^0)$ (6b), in the $|\mu| - \varphi_\mu$ plane for $\varphi_{M_1} = 0$, $M_2 = 250$ GeV, $\tan \beta = 5$, $m_0 = 300$ GeV, $\sqrt{s} = 800$ GeV and $(P_{e^-}, P_{e^+}) = (-0.8, 0.6)$. The area A is kinematically forbidden by $m_{\chi_2^+} + m_{\chi_1^-} > \sqrt{s}$, the area B by $m_W + m_{\chi_1^0} > m_{\chi_1^+}$. The gray area is excluded by $m_{\chi_1^\pm} < 104$ GeV.

The scattering angle is $\theta_1 \angle (\mathbf{p}_{e^-}, \mathbf{p}_{\chi_1^-})$ and the azimuth $\phi$ can be chosen zero. The momenta are:

\[ p_{\mu}^{\mu} = E_b(1, -\sin \theta, 0, \cos \theta), \quad p_{e^+}^{\mu} = E_b(1, \sin \theta, 0, -\cos \theta), \]
\[ p_{\chi_1^+}^{\mu} = (E_{\chi_1^+}, 0, 0, -q), \quad p_{\chi_1^-}^{\mu} = (E_{\chi_1^-}, 0, 0, q), \]  
(A.1)

with the beam energy $E_b = \sqrt{s}/2$ and

\[ E_{\chi_1^+} = \frac{s + m_{\chi_1^+}^2 - m_{\chi_1^-}^2}{2\sqrt{s}}, \quad E_{\chi_1^-} = \frac{s + m_{\chi_1^-}^2 - m_{\chi_1^+}^2}{2\sqrt{s}}, \quad q = \frac{\lambda^2(s, m_{\chi_1^+}^2, m_{\chi_1^-}^2)}{2\sqrt{s}}, \]
(A.3)

where $\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz)$. For the description of the polarization of chargino $\tilde{\chi}_1^\pm$ we choose three spin vectors:

\[ s_{\chi_1^\pm}^{1,\mu} = (0, -1, 0, 0), \quad s_{\chi_1^\pm}^{2,\mu} = (0, 0, 1, 0), \quad s_{\chi_1^\pm}^{3,\mu} = \frac{1}{m_{\chi_1^\pm}}(q, 0, 0, -E_{\chi_1^\pm}). \]
(A.4)

Together with $p_{\chi_1^\pm}^{\mu}/m_{\chi_1^\pm}$ they form an orthonormal set. For the two-body decay $\tilde{\chi}_1^+ \to W^+\tilde{\chi}_n^0$ the decay angle $\theta_1 \angle (\mathbf{p}_{\chi_1^+}, \mathbf{p}_W)$ is constrained by $\sin \theta_1^\text{max} = q^0/q$ for $q > q^0$, where
The spin vectors $t_{W}^{c}$, $c = 1, 2, 3$, of the $W$ boson in the laboratory system are chosen as

$$t_{W}^{1,\mu} = \begin{pmatrix} 0, & P_{W}^{2} \times P_{W}^{3} \\ P_{W}^{2} \times P_{W}^{3} \end{pmatrix} \mu,$$

$$t_{W}^{2,\mu} = \begin{pmatrix} 0, & P_{W}^{1} \times P_{W}^{3} \\ P_{W}^{1} \times P_{W}^{3} \end{pmatrix} \mu,$$

$$t_{W}^{3,\mu} = \frac{1}{m_{W}} \left( |P_{W}|, E_{W} P_{W} \right).$$

The momenta in the laboratory system are

$$p_{W}^{\pm} = \left( E_{W}^{\pm}, -|p_{W}| \sin \theta_{1} \cos \phi_{1}, |p_{W}| \sin \theta_{1} \sin \phi_{1}, -|p_{W}| \cos \theta_{1} \right),$$

$$p_{f}^{\mu} = \left( E_{f}, -|p_{f}| \sin \theta_{2} \cos \phi_{2}, |p_{f}| \sin \theta_{2} \sin \phi_{2}, -|p_{f}| \cos \theta_{2} \right),$$

$$E_{f}^{\mu} = |p_{f}| = \frac{m_{W}^{2}}{2(E_{W}^{+} - |p_{W}| \cos \theta_{D_{2}})},$$

with $\theta_{D_{2}} = \theta_{2}$ and the decay angle $\theta_{D_{2}} = \theta_{D_{2}}(p_{W}, p_{f})$ given by:

$$\cos \theta_{D_{2}} = \cos \theta_{1} \cos \theta_{2} + \sin \theta_{1} \sin \theta_{2} \cos(\phi_{2} - \phi_{1}).$$

The $q^{0}$ is the chargino momentum if the $W$ boson is produced at rest. In this case there are two solutions

$$|p_{W}^{\pm}| = \frac{(m_{\tilde{\chi}_{1}^{\pm}}^{2} + m_{W}^{2} - m_{\tilde{\chi}_{1}^{0}}^{2})q \cos \theta_{1} \pm E_{\chi_{1}^{\pm}} \sqrt{\lambda(m_{\tilde{\chi}_{1}^{\pm}}^{2}, m_{W}^{2}, m_{\tilde{\chi}_{1}^{0}}^{2}) - 4q^{2} m_{W}^{2}(1 - \cos^{2} \theta_{1})}}{2q^{2}(1 - \cos^{2} \theta_{1}) + 2m_{\tilde{\chi}_{1}^{\pm}}^{2}}$$

(A.5)

Figure 7: Contour lines of the asymmetry $A_{I I}$ (7a) and the statistical significance $S_{I I}$ (7b) for $e^{+}e^{-} \to \tilde{\chi}_{1}^{\pm} \tilde{\chi}_{1}^{0}$; $\tilde{\chi}_{1}^{\pm} \to W^{\pm} \tilde{\chi}_{1}^{0}$; $W^{\pm} \to c\bar{s}$, in the $\varphi_{\mu} - \varphi_{M_{1}}$ plane for $|\mu| = 350$ GeV, $M_{2} = 250$ GeV, $\tan \beta = 5$, $m_{0} = 300$ GeV, $\sqrt{s} = 800$ GeV, $(P_{e^{-}}, P_{e^{+}}) = (-0.8, 0.6)$ and $L = 500$ fb$^{-1}$. In the gray shaded area of Fig. 7b we have $S_{I I} < 1$. 

If $q^{0} > q$, $\theta_{1}$ is not constrained and there is only the physical solution $|p_{W}^{+}|$ left.

The spin vectors $t_{W}^{c}$, $c = 1, 2, 3$, of the $W$ boson in the laboratory system are chosen as

$$t_{W}^{1,\mu} = \begin{pmatrix} 0, & P_{W}^{2} \times P_{W}^{3} \\ P_{W}^{2} \times P_{W}^{3} \end{pmatrix} \mu,$$

$$t_{W}^{2,\mu} = \begin{pmatrix} 0, & P_{W}^{1} \times P_{W}^{3} \\ P_{W}^{1} \times P_{W}^{3} \end{pmatrix} \mu,$$

$$t_{W}^{3,\mu} = \frac{1}{m_{W}} \left( |P_{W}|, E_{W} P_{W} \right).$$

(A.10)
The spin vectors and $p_W^2/m_W$ form an orthonormal set. The polarization vectors $\varepsilon^\lambda_k$ for helicities $\lambda_k = -1, 0, +1$ of the W boson are defined by:

$$
\varepsilon^- = \frac{1}{\sqrt{2}}(t_W^1 - it_W^2); \quad \varepsilon^0 = t_W^3; \quad \varepsilon^+ = -\frac{1}{\sqrt{2}}(t_W^1 + it_W^2).
$$

(A.11)

## B Phase space

The Lorentz invariant phase space element for the chargino production (1) and the decay chain (2)-(9) can be decomposed into the two-body phase space elements:

$$
dLips(s, p_{\chi_i^+}, p_{\chi_j^-}, p_f) = \frac{1}{(2\pi)^2} dLips(s, p_{\chi_i^+}, p_{\chi_j^-}) \cdot ds_{\chi_i^+} \sum_{\pm} dLips(s_{\chi_i^+}, p_{\chi_i^0}, p_W^\pm) \cdot ds_W \cdot dLips(s_W, p_f, p_f),
$$

(B.1)

$$
dLips(s, p_{\chi_i^+}, p_{\chi_j^-}) = \frac{q}{8\pi \sqrt{s}} \sin \theta \, d\theta,
$$

(B.2)

$$
dLips(s_{\chi_i^+}, p_{\chi_i^0}, p_W^\pm) = \frac{1}{2(2\pi)^2} \frac{|p_W^\pm|^2}{2|E_W^2| q \cos \theta_1 - E_{\chi_i^+}^2} \cdot d\Omega_1,
$$

(B.3)

$$
dLips(s_W, p_f, p_f) = \frac{1}{2(2\pi)^2} \frac{|p_f|^2}{m_W^2} \cdot d\Omega_2,
$$

(B.4)

with $s_{\chi_i^+} = p_{\chi_i^+}^2$, $s_W = p_W^2$ and $d\Omega_i = \sin \theta_i \, d\theta_i \, d\phi_i$. We use the narrow width approximation for the propagators: $\int |\Delta(\chi_i^+)|^2 \cdot ds_{\chi_i^+} = \frac{\pi \Gamma_{\chi_i^+}}{m_{\chi_i^+}}$. $\int |\Delta(W)|^2 \cdot ds_W = \frac{\pi \Gamma_{W}}{m_W \Gamma_{W}}$. The approximation is justified for $(\Gamma_{\chi_i^+}/m_{\chi_i^+})^2 \ll 1$, which holds in our case with $\Gamma_{\chi_i^+} \lesssim \mathcal{O}(1\text{GeV})$.

## C Spin matrices

In the basis (A.11) the spin matrices $J^c$ and the tensor components $J^{cd}$ are [12]:

$$
J^1 = \begin{pmatrix}
0 & 1/\sqrt{2} & 0 \\
1/\sqrt{2} & 0 & 1/\sqrt{2} \\
0 & 1/\sqrt{2} & 0
\end{pmatrix}, \quad
J^2 = \begin{pmatrix}
-1/\sqrt{2} & 0 & i/\sqrt{2} \\
i/\sqrt{2} & 0 & -i/\sqrt{2} \\
0 & -1/\sqrt{2} & 0
\end{pmatrix}, \quad
J^3 = \begin{pmatrix}
-1 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix},
$$

(C.1)

$$
J^{11} = \begin{pmatrix}
-1/3 & 0 & 1 \\
0 & 2/3 & 0 \\
1/3 & 0 & -1/3
\end{pmatrix}, \quad
J^{22} = \begin{pmatrix}
-1/3 & 0 & -1 \\
0 & 2/3 & 0 \\
1/3 & 0 & -1/3
\end{pmatrix}, \quad
J^{33} = \begin{pmatrix}
2/3 & 0 & 0 \\
0 & -1/3 & 0 \\
0 & 0 & 2/3
\end{pmatrix},
$$

(C.2)

$$
J^{12} = \begin{pmatrix}
0 & 0 & i \\
0 & 0 & 0 \\
-i & 0 & 0
\end{pmatrix}, \quad
J^{23} = \begin{pmatrix}
0 & -i/\sqrt{2} & 0 \\
i/\sqrt{2} & 0 & i/\sqrt{2} \\
0 & -i/\sqrt{2} & 0
\end{pmatrix}, \quad
J^{13} = \begin{pmatrix}
0 & -1/\sqrt{2} & 0 \\
1/\sqrt{2} & 0 & -1/\sqrt{2} \\
0 & 0 & 1/\sqrt{2}
\end{pmatrix},
$$

(C.3)
D Chargino production matrices

We give the analytical formulae for \( P, \Sigma_p^1, \Sigma_p^2, \Sigma_p^3 \) of the chargino production matrix \( \rho_P(\tilde{\chi}_i^+) = 2(\delta_{\lambda\lambda'}P + \sigma^a_{\lambda\lambda'}\Sigma_p^a) \) (29), in the laboratory system. Covariant expressions for these functions can be found in [15].

D.1 Chargino production

The coefficient \( P \) is independent of the chargino polarization. It can be composed into the different contributions from the production channels:

\[
P = P(\gamma\gamma) + P(\gamma Z) + P(\gamma\tilde{\nu}) + P(Z\tilde{\nu}) + P(Z\tilde{\nu}) + P(\tilde{\nu}\tilde{\nu})
\]

which read

\[
P(\gamma\gamma) = \delta_{ij}2e^2|\Delta(\gamma)|^2(c_L + c_R)E_0^2(E_{\chi_i^+}E_{\chi_j^-} + m_{\chi_i^+}m_{\chi_j^-} + q^2\cos^2 \theta),
\]

\[
P(\gamma Z) = \delta_{ij}2e^2g^2E_0^2\text{Re}\left\{ \Delta(\gamma)\Delta(Z)^* \left[ (L_ee_L - R_ee_R)(O_{ij}^{L*} - O_{ij}^{R})2E_0q\cos \theta + (L_ee_L + R_ee_R)(O_{ij}^{L*} + O_{ij}^{R})(E_{\chi_i^+}E_{\chi_j^-} + m_{\chi_i^+}m_{\chi_j^-} + q^2\cos^2 \theta) \right] \right\},
\]

\[
P(\gamma\tilde{\nu}) = \delta_{ij}e^2g^2E_0^2c_L\text{Re}\left\{ V_{i1}V_{j1}\Delta(\gamma)\Delta(\tilde{\nu})^* \right\} \times \left( E_{\chi_i^+}E_{\chi_j^-} + m_{\chi_i^+}m_{\chi_j^-} - 2E_0q\cos \theta + q^2\cos^2 \theta, \right)
\]

\[
P(Z\tilde{\nu}) = \frac{g^4}{\cos^2 \theta_W} - L_ee_L E_0^2\text{Re}\left\{ V_{i1}^*V_{j1}\Delta(Z)\Delta(\tilde{\nu})^* \times \left[ O_{ij}^{L}(E_{\chi_i^+}E_{\chi_j^-} - 2E_0q\cos \theta + q^2\cos^2 \theta) + O_{ij}^{R}m_{\chi_i^+}m_{\chi_j^-} \right] \right\},
\]

\[
P(\tilde{\nu}\tilde{\nu}) = \frac{g^4}{4}c_L|V_{i1}|^2|V_{j1}|^2|\Delta(\tilde{\nu})|^2E_0^2(E_{\chi_i^+}E_{\chi_j^-} - 2E_0q\cos \theta + q^2\cos^2 \theta).
\]

The propagators are defined by:

\[
\Delta(\gamma) = \frac{i}{p_\gamma^2}, \quad \Delta(Z) = \frac{i}{p_Z^2 - m_Z^2 + im_Z p_Z}, \quad \Delta(\tilde{\nu}) = \frac{i}{p_\tilde{\nu}^2 - m_{\tilde{\nu}}^2}.
\]

The longitudinal beam polarizations are included in the weighting factors

\[
c_L = (1 - P_{e^-})(1 + P_{e^+}), \quad c_R = (1 + P_{e^-})(1 - P_{e^+}).
\]
D.2 Chargino polarization

The coefficients $\Sigma^a_P$, which describe the polarization of the chargino $\tilde{\chi}^+_i$, decompose into:

$$
\Sigma^a_P = \Sigma^a_P(\gamma\gamma) + \Sigma^a_P(\gamma Z) + \Sigma^a_P(\gamma \tilde{\nu}) + \Sigma^a_P(Z Z) + \Sigma^a_P(Z \tilde{\nu}) + \Sigma^a_P(\tilde{\nu} \tilde{\nu}).
$$

The contributions to the transverse $\tilde{\chi}^+_i$ polarization in the production plane are:

$$
\Sigma^1_P(\gamma\gamma) = \delta_{ij} 2e^2 \Delta(\gamma)^2 (c_R - c_L) E^2_b \sin\theta \left( m_{\chi^+_i} E_{\chi^-_j} + m_{\chi^-_j} E_{\chi^+_i} \right),
$$

$$
\Sigma^1_P(\gamma Z) = \delta_{ij} 2e^2 \frac{g^2}{\cos^2(\theta_W)} E^2_b \sin\theta \text{Re}\left\{ \Delta(\gamma) \Delta(Z)^* \left[ -(L_c c_L + R_c c_R) (O^R_{ij} - O^L_{ij}) m_{\chi^+_i} q \cos\theta 
\right.ight.

$$

$\left. \left. + (R_c - L_c c_L) (O^L_{ij} + O^R_{ij}) (m_{\chi^+_i} E_{\chi^-_j} + m_{\chi^-_j} E_{\chi^+_i}) \right]\right\},
$$

$$
\Sigma^1_P(\gamma \tilde{\nu}) = -\delta_{ij} e^2 g^2 c_L E^2_b \sin\theta \text{Re}\left\{ V^*_i V_j \Delta(\gamma) \Delta(\tilde{\nu})^* \times \right\}

$$

$\left. \left[ m_{\chi^+_i} (E_{\chi^-_j} - q \cos\theta) + m_{\chi^-_j} E_{\chi^+_i} \right] \right\},
$$

$$
\Sigma^1_P(Z Z) = \frac{g^4}{\cos^4(\theta_W)} \Delta(Z)^2 E^2_b \sin\theta \text{Re}\left\{ V^*_i V_{1j} \Delta(Z) \Delta(\tilde{\nu})^* \times \right\}

$$

$\left. \left[ (O^L_{ij} m_{\chi^+_i} - q \cos\theta) + (O^R_{ij} m_{\chi^-_j} E_{\chi^+_i}) \right] \right\},
$$

$$
\Sigma^1_P(\tilde{\nu} \tilde{\nu}) = -\frac{g^4}{4} c_L |V_{i1}|^2 |V_{j1}|^2 |\Delta(\tilde{\nu})|^2 E^2_b \sin\theta \text{Im}\left\{ m_{\chi^+_i} (E_{\chi^-_j} - q \cos\theta) \right\}.
$$
\[
\Sigma_\nu^3(\gamma Z) = \delta_{ij} \frac{e^2 g^2}{\cos^2 \theta_W} E_b^2 \text{Re} \left\{ \Delta(\gamma) \Delta(Z)^* \right\} \\
\left[ (L_e c_L - R_e c_R)(O_{ij}^{LR} + O_{ij}^{LR}) (q^2 + E_{\chi_i} + E_{\chi_j} + m_{\chi_i} + m_{\chi_j}) \cos \theta \\
+(L_e c_L + R_e c_R)(O_{ij}^{LR} - O_{ij}^{LR}) q(E_{\chi_j} + E_{\chi_i} \cos^2 \theta) \right] \right\}, \quad (D.23)
\]

\[
\Sigma_\nu^3(\gamma \tilde{\nu}) = -\delta_{ij} e^2 g^2 c_L E_b^2 \text{Re} \left\{ V_{i1}^* V_{j1} \Delta(\gamma) \Delta(\tilde{\nu})^* \right\} \times \\
[q E_{\chi_j} - (q^2 + E_{\chi_i} + E_{\chi_j}) \cos \theta + q E_{\chi_i} \cos^2 \theta - m_{\chi_i} + m_{\chi_j} \cos \theta], \quad (D.24)
\]

\[
\Sigma_{Z \nu}^3(\gamma Z) = \frac{g^4}{\cos^4 \theta_W} (L_e c_L + R_e c_R)^2 \text{Re} \left\{ O_{ij}^{LR} \right\} m_{\chi_i} + m_{\chi_j} \cos \theta \\
+(L_e c_L - R_e c_R)^2 \text{Re} \left\{ O_{ij}^{LR} \right\} m_{\chi_i} + m_{\chi_j} \cos \theta \\
+(L_e c_L - R_e c_R)^2 \text{Re} \left\{ O_{ij}^{LR} \right\} \left( q^2 + E_{\chi_i} + E_{\chi_j} \cos \theta \right), \quad (D.25)
\]

\[
\Sigma_{Z \tilde{\nu}}^3(\gamma \tilde{\nu}) = \frac{g^4}{\cos^4 \theta_W} L_e c_L E_b^2 \text{Re} \left\{ V_{i1}^* V_{j1} \Delta(Z) \Delta(\tilde{\nu})^* \right\} \frac{|O_{ij}^{LR}| m_{\chi_i} + m_{\chi_j} \cos \theta}{m_{\chi_i} + m_{\chi_j}} \\
-\frac{g^4}{4 c_L} q E_{\chi_j} - (q^2 + E_{\chi_i} + E_{\chi_j}) \cos \theta + q E_{\chi_i} \cos^2 \theta \right\}, \quad (D.26)
\]

\[
\Sigma_\tilde{\nu}^3(\tilde{\nu} \tilde{\nu}) = -\frac{g^4}{4 c_L} |V_{i1}|^2 |V_{j1}|^2 \Delta(\tilde{\nu})^2 E_b^2 \times \\
[q E_{\chi_j} - (q^2 + E_{\chi_i} + E_{\chi_j}) \cos \theta + q E_{\chi_i} \cos^2 \theta]. \quad (D.27)
\]

References


