Correlated Quantum Memory: Manipulating Atomic Entanglement via Electromagnetically Induced Transparency

Hui Jing\textsuperscript{a}\textsuperscript{*}, Xiong-Jun Liu\textsuperscript{b}, Mo-Lin Ge\textsuperscript{b} and Ming-Sheng Zhan\textsuperscript{a}
\textsuperscript{a} State Key Laboratory of Magnetic Resonance and Atomic and Molecular Physics, Wuhan Institute of Physics and Mathematics, CAS, Wuhan 430071, P. R. China
\textsuperscript{b} Theoretical Physics Division, Nankai Institute of Mathematics, Nankai University, Tianjin 300071, P.R. China

We propose a feasible scheme of quantum state storage and manipulation via electromagnetically induced transparency (EIT) in flexibly united multi-ensembles of three-level atoms. For different atomic array configurations, one can properly steer the signal and the control lights to generate different forms of atomic entanglement within the framework of linear optics. These results opens up new possibility for the future design of quantum memory devices by using, e.g., an atomic grid.

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The remarkable demonstration of ultraslow light speed in a Bose-Einstein condensate in 1999\cite{1} have stimulated rapid advances in both experimental and theoretical works on exploring the novel mechanism and its fascinating applications of Electromagnetically Induced Transparency (EIT)\cite{2,3,4,5,6,7,8,9,10,11}. In 2000, Fleischhauer and Lukin presented a dark-state polaritons (DSPs) theory, which shows an elegant mapping-and-readout quantum memory technique by exchanging the quantum state information between the two components of DSPs, the quantized light field and the collective atomic excitations\cite{4,5}. The crucial condition of adiabatic passage for the dark states was then fully confirmed by Sun et al. by revealing the dynamical symmetry of a single EIT medium consisting of three-level atoms\cite{6}. As an extension, the quantum memory process in an atomic ensemble composed of complex N-level (N > 3) atoms was also studied\cite{7,8}, which shows more freedoms for the quantum state control.

Most recently, the storage of two entangled lights in two independent ensembles of three-level atoms was also proposed\cite{9}, motivated by building a quantum communication network in which the stored entanglement in each of and/or among the atomic nodes should be conveniently manipulated\cite{10}.

In this Letter, we present a feasible technique of correlated quantum memory via EIT mechanism in many ensembles composed of Λ-type three-level atoms, which has build-up flexible ability to generate and manipulate entangled states of atomic ensembles. This differs from previous entanglement schemes (even the EIT-based ones)\cite{9,10}, since it inserts new freedoms of manipulations between the mapping and readout processes in itself. We resolve the general quantized DSPs model of m-atomic-ensemble system and find that, for two atomic ensembles, the input probe light can be stored in either the first or the second atomic ensemble or both of them (in a correlated manner) by steering the control fields. Particularly, by preparing the initial probe light in a coherent superposition state, the entangled atomic states of two or three ensembles can be created within the framework of linear optics. Further manipulations of the atomic entanglement are manifested under two configurations for three ensembles.

This scheme may have an impact on future research to design an atomic grid as a versatile quantum memory device by, e.g., combining the EIT and an optical lattice techniques even in a chip\cite{11}.

\textsuperscript{*} Electronic address: jinghui@wipm.ac.cn
where the mixing angles $\theta$ and $\phi_j$ are defined through

$$
\tan \theta = \frac{\sqrt{g_1^2 N_1^2 \Omega_{2,1}^2 \Omega_{2,3}^2 \ldots \Omega_{m,2}^2 + g_2^2 N_1 \Omega_1^2 \Omega_{2,3}^2 \ldots \Omega_{m,2}^2 \ldots + g_m^2 N_m \Omega_1^2 \Omega_{2,3}^2 \ldots \Omega_{m,2}^2 \ldots \Omega_{m-1,1}^2}}{\Omega_1 \Omega_2 \ldots \Omega_m}
$$
and
\[
\tan \phi_{m-1} = \frac{g_m^2 \sqrt{N_m \Omega_1 \Omega_2 \cdots \Omega_{m-1}}}{\sqrt{g_m^2 N_1 \Omega_2 \cdots \Omega_m + g_m^2 N_2 \Omega_1^2 \cdots \Omega_m^2 + \cdots + g_m^2 N_{m-1} \Omega_1 \Omega_2 \cdots \Omega_{m-2} \Omega_m^2}}
\]  \tag{4}

Since \([\hat{d}, \hat{d}^\dagger] = 1, [\hat{V}, \hat{d}] = 0\), we can obtain the general atomic dark states as \(|D_\nu = [\nu!]^{-1/2} (\hat{d}^\dagger)^\nu |0\rangle\), where \(|0\rangle = |b^{(1)}, b^{(2)}, \ldots, b^{(m)}\rangle_{\text{atom}} \otimes |0\rangle_{\text{photon}}\) denotes the vacuum state of the probe field and \(|b^{(\sigma)} = |b_1, b_2, \ldots, b_{N_\sigma}\rangle\) is the collective ground state of the \(\sigma\)-th atomic ensemble.

In order to get more insights on the quantum state control of this system, we first consider the special case of two atomic ensembles and the system takes the form
\[
|D_n\rangle = \sum_{k=0}^{n} \sum_{j=1}^{n-k} \sqrt{\frac{n!}{k! j! (j)!}} (\cos \theta)^k (\sin \theta)^{n-k} (\sin \phi_1)^j (\cos \phi_1)^j |c^{(1)}_j, c^{(2)}_j\rangle_{\text{spin}} \otimes |k\rangle_{\text{photon}},
\]  \tag{5}

where \(l = n - k - j\). We will show that not only the quantum memory process still can be revealed in this quite general system but also for the novel build-up ability to generate and manipulate the atomic entanglement in a highly extensible style. In fact, if the initial total state of the quantized field and atomic ensembles is prepared in \(|\Psi_0\rangle = \sum_n P_n(\alpha_0)|b^{(1)}, b^{(2)}\rangle_{\text{atom}} \otimes |n\rangle_{\text{photon}}\), where \(P_n(\alpha_0) = \frac{n!}{\sqrt{n!}} e^{-|\alpha_0|^2/2}\) is the probability of distribution function, then the mixing angle \(\theta\) is rotated from 0 to \(\pi/2\) while keeping the ratio \(\Omega_1/\Omega_2\) by, e.g., an acoustic-optical modulator (AOM) \([12]\) and switching off them adiabatically (\(\lim_{\Omega_1, \Omega_2 \to 0} [g_2 \sqrt{N_2} \Omega_1 / g_1 \sqrt{N_1} \Omega_2]\)), the total system evolves into
\[
|\Psi_e\rangle = |\alpha_1\rangle_{\text{spin} 1} \otimes |\alpha_2\rangle_{\text{spin} 2} \otimes |0\rangle_{\text{photon}}
\]  \tag{6}

where \(\alpha_1 = \alpha_0 \cos \phi\) and \(\alpha_2 = \alpha_0 \sin \phi\). Clearly, the injected optical state can be converted into the atomic coherences via manipulating two control fields. Particularly, i) if \(\phi = 0\), the injected light is fully stored in the first ensemble (\(\alpha_2 = 0\)); ii) if \(\phi = \pi/2\), the input pulse is now stored in the second (\(\alpha_1 = 0\)). This mechanism can be extended to any non-classical or entangled state of the input light.

The important issue of entanglement generation in the macroscopic atomic ensembles hardly can be overestimated due to its practical applications in quantum information processing \([11, 12]\). For the present scheme, if the injected quantized field is in a coherent superposition state, e.g., for the initial state \(|\Psi_0\rangle^\pm = \frac{1}{\sqrt{N_{\pm}(\alpha_0)} (|\alpha_0\rangle \pm - |\alpha_0\rangle)_{\text{photon}} \otimes |b^{(1)}, b^{(2)}\rangle_{\text{atom}}\) (here \(N_{\pm}(\alpha_0)\) is a normalized factor \([12]\)), a two-ensemble entangled state would be created as \((|\Psi_0\rangle^\pm \rightarrow |\Psi_e\rangle^\pm)\)
\[
|\Psi_e\rangle^\pm = \frac{1}{\sqrt{N_{\pm}(\alpha_0)}} |0\rangle_{\text{photon}} \otimes (|\alpha_1, \alpha_2\rangle \pm | - \alpha_1, - \alpha_2\rangle)_{\text{spin}},
\]  \tag{7}

and the entanglement of atomic coherences \([13]\)
\[E^\pm(\alpha_1, \alpha_2) = - \text{tr}(\rho^\pm_{\alpha_1} \ln \rho^\pm_{\alpha_1})\], with the reduced density matrix \(\rho^\pm_{\alpha_1} = \text{tr}_{\alpha_2, \text{atom}}(|\Psi_e\rangle \langle \Psi_e|)^\pm\), can be controlled by the two control fields. In particular, if we start from an initial state \(|\Psi_0\rangle^-\) and choose \(\phi = \pi/4\), we then obtain an EPR-type entangle state: \(|(|+\rangle - | - \rangle)_{\text{spin}} / \sqrt{2}, \text{where} |\pm\rangle = (|\alpha_0 / \sqrt{2}\rangle \pm | - \alpha_0 / \sqrt{2}\rangle)_{\text{spin}} / \sqrt{N(\alpha_0 / \sqrt{2})}\). This process may be viewed as a simple linear optical circuit which transforms a standard basis to an entangled one \([13]\). Following the method developed in Ref. \([8]\) it is straightforward also here to confirm the condition of adiabatic evolution and therefore the the robustness of the system \([11, 12]\). In addition, as a noticeable analogy, it deserves further explorations by recalling our scheme of generating two entangled \(lights\) from an initial optical superposition state via a \(single\) four-state atomic medium \([12]\).
These remarkable properties can be readily extended to the general m-atomic-ensembles case. For an interesting example, we consider the case of m = 3 in which the dynamical symmetry is governed by the Lie group so(4) ⊗ su(2)⊗ b_0 (see also Fig.1a). Now, if the probe light is in a coherent superposition state, e.g., |Ψ_0⟩ = \frac{1}{\sqrt{N_{0±}}}((|α_0⟩ ± |β_0⟩)_{\text{photon}} ⊗ |b^{(1)}, b^{(2)}, b^{(3)}⟩_{\text{atom}} with a normalized factor N_{0±} = 2 ± 2e^{-|α_0 - β_0|^2/2}, the achieved entangled state between three ensembles reads (|Ψ_0⟩ ± |Ψ_e⟩ ±)

|Ψ_e⟩ = \frac{1}{\sqrt{N_{0±}}}(|0⟩_{\text{photon}} ⊗ (|α_1, α_2, α_3⟩ ± |β_1, β_2, β_3⟩)_{\text{spin}}\right).

where \(α_1 = cos φ_1 cos φ_2 α_0, α_2 = sin φ_1 cos φ_2 α_0, α_3 = sin φ_2 α_0\) and \(β_1 = cos φ_1 cos φ_2 β_0, β_2 = sin φ_1 cos φ_2 β_0, β_3 = sin φ_2 β_0\). If we choose \(φ = π/4\) and \(φ = tan^{-1}(\sqrt{2}/2)\), we can get a "GHZ-like" entangled state: (|α, α, α⟩ ± |β, β, β⟩)_{\text{spin}}/\sqrt{N_{0±}}, where \(α = α_0/\sqrt{3}\), \(β = β_0/\sqrt{3}\). Notably, with the orthogonal basis (±) \(= (|α⟩ ± |β⟩)_{\text{spin}}\) (for a normalized factor), this state also can be put into: \(Φ_{123}(±) = ξ(±)|±⟩|±⟩ ± ζ(±)|W_{±}⟩\), where \(ξ(±)\) and \(ζ(±)\) are the normalized factors, and \(|W_{±}⟩ = |±⟩|±⟩ + |±⟩|±⟩ + |±⟩|±⟩|±⟩|±⟩ + |±⟩|±⟩|±⟩|±⟩|±⟩ ± is the W-state [14]. It is fascinating to see that the two-ensemble state is still entangled after reducing the third one, and the general feature of coherent entanglement oscillations of Ramsey fringes for a highly entangled array also should be observed [12].

Expanded illustration.—Finally we prove that the novel characteristics in the above scheme can be flexibly extended to other different atomic configurations. As a concrete example, we consider the three ensembles of Λ type atoms with such an atomic array (see Fig.1b): one probe light beam is injected to interact with the atoms of the first and second ensembles with the coupling constants \(g_1\) and \(g_1′\), while another beam interacts with the first and third ones with the coupling \(g_2\) and \(g_2′\), and the three classical control fields couple the transitions |c⟩ → |a⟩ with time-dependent Rabi-frequencies \(Ω_σ(t)\) (\(σ = 1, 2, 3\)) [12]. For simplicity, we still consider the single-mode probe lights; the generalizations to the multi-mode case is straightforward. The interaction Hamiltonian of the total system now is

\[\hat{V} = g_1\sqrt{N_1}\hat{a}_1\hat{A}_1 + g_1′\sqrt{N_2}\hat{a}_2\hat{A}_2 + g_2\sqrt{N_1}\hat{a}_1\hat{A}_2 + g_2′\sqrt{N_3}\hat{a}_2\hat{A}_3 + \Omega_1(t)\hat{T}_1^+ + \Omega_2(t)\hat{T}_2^+ + \Omega_3(t)\hat{T}_3^+ + h.c.\]

and the operators here are in the same definitions as the above. The dynamical symmetry of this system is governed by the Lie algebra so(4) ⊗ su(2)⊗ b_0 and now the DSPs operator is given by

\[\hat{d} = \hat{d}_1 + \hat{d}_2,\]

where \(\hat{d}_1 = cos θ_1\hat{a}_1 - sin θ_1 cos φ_1\hat{C}_1 - sin θ_1 sin φ_1\hat{C}_2, \hat{d}_2 = cos θ_2\hat{a}_2 - sin θ_2 cos φ_2\hat{C}_1 - sin θ_2 sin φ_2\hat{C}_3\) and the mixing angles \(θ_1, 2\) and \(φ_1, 2\) are defined just as Eq. (3)-(4), e.g., tan \(φ_1 = g_1′\sqrt{N_2}\Omega_1/(g_1\sqrt{N_1}\Omega_2)\) and tan \(φ_2 = g_2′\sqrt{N_3}\Omega_1/(g_2\sqrt{N_2}\Omega_2)\). It can be readily verified that [\(\hat{d}, \hat{d}^\dagger\)] = 1 and [\(\hat{d}, \hat{V}\)] = 0. Now we consider the initial state of the system with two entangled probe lights: |Ψ_0⟩ = \frac{1}{\sqrt{N_{0±}}}(|α_0⟩|α_0⟩ ± |β_0⟩)_{\text{photon}} ⊗ |b^{(1)}, b^{(2)}, b^{(3)}⟩_{\text{atom}} (N_{0±} is a normalized factor), then it turns out that the atomic entangled state of the three ensembles as Eq. (8) can be obtained again by properly steering the control fields, but with different parameters (η = α, β): η_1 = (cos φ_1 + cos φ_2)α_0, η_2 = sin φ_1α_0 and η_3 = sin φ_2α_0. In particular, if φ_1 = φ_2 = π/2 or the Rabi frequency Ω_1 is kept much smaller than both Ω_2 and Ω_3 during the process that the three control fields are turned off, we can arrive at the reduced two-ensemble entanglement of the three atomic ensembles

\[Φ_{\text{atom}}(±) = \frac{1}{\sqrt{N_{0±}}}|b^{(1)}⟩⊗(|α_0⟩|α_0⟩ ± |β_0⟩)_{\text{spin}}.\]
The entanglement of two probe lights are fully transferred into that of the second and third ensembles! Clearly, this way to create the atomic entanglement differs itself from previous schemes [5, 10, 11].

In conclusion, we have proposed a feasible scheme to achieve correlated quantum memory of photons among many atomic ensembles composed of identical three-level atoms. The essential feature of the present scheme is that both the storage style and the quantum entanglement form can be controlled just using the "standard" EIT procedure in a "single" memory node within an actual quantum network. Our method is based on a general quantized DSPs model of the united system of \( m \)-atomic-ensembles from which we find that, for the special case of two atomic ensembles, the input quantized light can be stored in either the first or the second ensemble or both of them (in a correlated manner) by steering the external control fields. In particular, by preparing an initial probe light in a coherent superposition state through, e.g., a beam-splitter-based catalysis technique [16], an atomic entanglement of two ensembles can be created within the framework of linear optics. The interesting analogy with the EIT process in a single four-state atomic ensemble and the validity of adiabatic passage conditions are also confirmed. The manipulations of atomic entangled state are explored for the case of three ensembles under two different configurations, which can be flexibly extended to more complex structures like a tree or square array (see e.g., Fig.1c). This may have a strong impact on future research in designing new versatile quantum memory devices by using an atomic grid based on an EIT-type experiment.

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