Testing quantum superpositions of the gravitational field with Bose-Einstein condensates

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Abstract

We consider the gravity field of a Bose-Einstein condensate in a quantum superposition. The gravity field then is also in a quantum superposition which is in principle observable. Hence we have “quantum gravity” far away from the so-called Planck scale.
The existence of macroscopically distinguishable superpositions in Bose-Einstein condensates (BEC) have been discussed by several authors [1–3]. Such a superposition may be achieved as the ground state of two interacting BECs in a double well potential [1], or by a continuous quantum measurement on a condensate trapped in such a potential [2].

We now consider the gravitational field of a BEC (Bose-Einstein condensate) of total mass $M$ in such a double well potential and we scatter a beam of particles of mass $m$ in the middle of the double potential well. (We assume that the potential that affects the BEC does not effect these particles, or that we know how to correct for its effect in our calculations).

In order to avoid decoherence, the density matrix of the scattered particles must be factorized in the form: $\rho = \rho_{\text{int}} \otimes \rho_{\text{CM}}$ where $\rho_{\text{int}}$ stands for the density matrix of the internal degrees of freedom, and $\rho_{\text{CM}}$ for the density matrix of the external degrees of freedom.

Let us treat classically the interaction of the center of mass of the scattered particles with the gravitational field of the BEC. Suppose that we know for certain that the BEC condensate is localized within the left well or the right well (this may happen, for example, in the case of an infinite barrier in the double potential well). The scattered particles will be deflected from their course due to the gravitational attraction between them and the BEC. Here we also assume that all other interactions between the scattered particles and the BEC either do not exist or are negligible with respect to the gravitational interaction (which for itself is very weak). We shall set up our axis system so that the double potential well lies on the x axis (the maximum of the potential barrier is at $x = 0$), and the scattered particles initial trajectory is along the y axis, with $x = 0$, so that their initial momentum is $p_0 = (0, p_y, 0)$. The distances between the two minima in the potential will be $a$, and the particle trajectories are such that without the gravitational interaction they would pass at a distance $a$ within each minima. In the crudest, “undergraduate” approximation, the scattered particles, will be under the influence of a gravitational force $GmM/a^2$ during a time interval of $a/v_y = amp_y$. The scattered particles, initially with $p_x = 0$ will have, after the interaction, a non zero $x$ component for their momentum $\pm GMm^2/ap_y$, where $+$ is for
the case in which the BEC is in the right well, and — if it was on the left one. The deflection angle, in each case will be small and can be written as \( \theta \approx \pm p_x/p_y \) = \( \frac{GMm^2}{ap_y^2} \) = \( \frac{GM}{av_y^2} \).

Now assume that the potential barrier between the two wells is finite and the BEC condensate is in a symmetric state \( \psi_S = \psi_L + \psi_R \) where \( \psi_R \) and \( \psi_L \) are functions that are localized in the right and left wells respectively. What is the result of scattering the particles from the gravitational field of this state? After the scattering, the state of the center of mass of the particles will be entangled with the state of the BEC and in a superposition of states, one being a result of a deflection from a BEC in the right well, and the other being the result of deflection from a BEC in the left well. The state of the system can be written in the following manner:

\[
\varphi(r) = A_1 \psi_L \otimes \exp\left(i/\hbar (p_x, p_y, 0) \cdot r\right) + A_2 \psi_R \otimes \exp\left(i/\hbar (-p_x, p_y, 0) \cdot r\right)
\]  

(1)

Here it is essential that the initial momentum spread \( \Delta p \) of the BEC be much larger than the momentum kick due to the interaction.

We do not record the state of the BEC during the experiment, or in other words we are tracing over the BEC states. If \( \psi_L \) and \( \psi_R \) were orthogonal, tracing over the BEC state would prevent us from seeing any interference fringes. The situation is different if they are not orthogonal. Let \( \xi = \langle \psi_L | \psi_R \rangle \). The density matrix of the center of mass of the scattered particles after tracing over the BEC states is

\[
\rho(r, r') = |A_1|^2 \exp[i/\hbar (p_x, p_y, 0) \cdot (r - r')] \\
+ |A_2|^2 \exp[i/\hbar (-p_x, p_y, 0) \cdot (r - r')] \\
+ 2\text{Re}\{A_1 A_2^* \xi \exp i/\hbar [(x, p_y, 0) \cdot r] \}
\]

(2)

Assuming \( A_1 = A_2 \), it is easy to see that \( \rho(r, r) \) will be maximal when \( \cos(p_x \bar{x}/\hbar) = 1 \), or \( p_x \bar{x}/\hbar = 2\pi m \), so the distance between fringes will be

\[
\Delta x = 2\pi \hbar/p_x = (hav_y)/(GMm)
\]

(3)
Let us try to see the parameters needed in such an experiment in order for it to be feasible. As we shall soon see, the distance between fringes tends to be very large, so let us choose parameters as to make it as small as possible (while difficult, it does not violate any law of physics). A BEC usually contains up to $10^7$ Rb atoms, which give a mass of about $10^{-18}$ Kg. For the scattered particles, let us take big particles (grains) with a mass of about a nanogram, or $10^{-12}$ Kg. The constants $h$ and $G$ have values

$$h \approx 6 \times 10^{-34} \text{ Joule sec}$$

$$G \approx 6 \times 10^{-11} \text{ Kg}^{-1} \text{ m}^3 \text{ sec}^{-2}$$

We need to get the product $av_y$ as small as possible. The distance between the minima in the double potential well cannot be much smaller than about 1 micron, or $10^{-6}$ meters. The question now is, how slow can the incoming particles be and still scatter coherently only from the gravitational field of the BEC? Introducing all the above numbers, we have

$$\Delta x = 10 v_y \text{ [meters]} \quad (4)$$

So in order to achieve a distance between fringes that is on the order, say, of cm, so that the experiment will be feasible we need the incoming particles speed to be of the order $10^{-3}$ meters/sec. These are slow speeds, but nevertheless, there is no physical law that prevents them. We can also try to take heavier particles for the scattered particles. We can write:

$$\Delta x = 10^{-11} (v_y/m) \text{ [meters]}. \quad (5)$$

The ratio $v_y/m$ will have to be of the orders of $10^9$ meters/(kg sec).
REFERENCES

