Charm elliptic flow from quark coalescence dynamics

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Abstract. From covariant transport theory, a significant ~ 10% light quark elliptic flow at RHIC implies an elliptic flow of similar magnitude for charm quarks, at moderately large \( p_T > 2.5 - 3 \) GeV. At lower transverse momenta, charm quark elliptic flow reduces progressively, reminiscent of the mass ordering pattern in ideal hydrodynamics. From the quark flows we predict the elliptic flow of \( D \) mesons at RHIC via quark coalescence. The large parton opacities needed to generate the light quark flow also lead to substantial ~ 40 – 50% secondary charm production.

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1. Introduction

Elliptic flow \( v_2 \equiv \langle \cos(2\phi) \rangle \), the second Fourier moment of the azimuthal momentum distribution, is an important experimental probe that provides information about the opacity [2, 3, 4, 5] of the nuclear medium created in noncentral \( A + A \) reactions. Heavy flavor elliptic flow is especially interesting because it can tell to what degree heavy quarks interact and thermalize. Though heavy quarks experience the same partonic environment as light partons, their large mass is expected to hamper equilibration, at least at RHIC energies \( \sqrt{s_{NN}} \sim 100 – 200 \) GeV. Studying the breakdown of equilibrium for the most abundant charm quarks may shed more light on the origin of the remarkable thermalization apparent in the light sector [6, 7, 8, 9].

In principle the experimentally more accessible heavy flavor spectra can also constrain dynamical scenarios. However, surprisingly, current (indirect) measurements [10] from \( Au + Au \) at RHIC are compatible with both perturbative production without charm rescattering and complete charm equilibration [11]. Charm hadron, e.g., \( D \) meson, elliptic flow will provide a more decisive experimental test [12].

Though several schematic scenarios have been explored [13, 14], up to now there is no quantitative calculation of charm elliptic flow in heavy ion collisions at RHIC. The aim of this study is to make predictions for \( Au + Au \) at \( \sqrt{s_{NN}} = 200 \) GeV based on covariant parton transport theory [2, 3, 15, 16, 17, 18, 19, 20]. That approach successfully describes the magnitude and saturation of \( v_2(p_T) \) in the light sector, but requires initial parton densities and/or cross sections that are much larger than perturbative QCD estimates [3]. Large opacities are also indicated by pion interferometry data [15]. Nevertheless, the origin of such opaque conditions is a puzzle.

One scenario that alleviates the opacity problem is hadronization via quark coalescence [21, 22, 23, 24, 25, 3, 19]. In the coalescence process mesons form...
from a constituent quark and antiquark, while (anti)baryons from three (anti)quarks. Because comoving constituents are strongly favored, elliptic flow can be amplified \[22\] \[13\] \[19\] \[26\] \[27\], reducing the opacities needed \[5\] to explain the data. Utilizing the formulas in \[13\], we present predictions for the elliptic flow of \(D\) and \(D_s\) mesons at RHIC.

2. Covariant parton transport theory

We consider here an inelastic extension of the Lorentz-covariant parton transport theory in Refs. \[2\] \[3\] \[13\] \[16\] \[18\], in which the on-shell parton phase space densities \(\{f_i(x, \tilde{p})\}\) evolve with elastic \(2\to 2\) and inelastic \(2\to 2\) rates as

\[
p_t^i \partial_t f_{i,i} = \frac{1}{16\pi^2} \sum_{jkt} \int \int \int \left( f_{3,k} f_{4,t} \frac{g_j g_k}{g_k g_t} - f_{1,i} f_{2,j} \right) |\tilde{M}^{ij}_{12-34}|^2 \delta^4(p_1 + p_2 - p_3 - p_4) + S_i(x, \tilde{p}) \, .
\]

(1)

\(|\tilde{M}|^2\) is the polarization averaged scattering matrix element squared, the integrals are shorthands for \(\int \equiv \int d^3p_a/(2E_a)\), \(g_i\) is the number of internal degrees of freedom for species \(i\), while \(f_{a,i} \equiv f_i(x, \tilde{p}_a)\). The source functions \(\{S_i(x, \tilde{p})\}\) specify the initial conditions.

Though, in principle, (1) could be generalized for bosons and fermions, or inelastic \(3 \leftrightarrow 2\) processes \[17\] \[20\]. no practical algorithm yet exists (for opacities at RHIC) to handle the new nonlinearities such extensions introduce. We therefore limit our study to quadratic dependence of the collision integral on \(f\).

We apply (1) to a system of massless gluons \((g = 16)\), massless light \((u,d)\) and strange quarks/antiquarks, and charm quarks/antiquarks \((g = 6)\) with mass \(M_c = 1.2\) GeV. All elastic and inelastic \(2 \to 2\) QCD processes were taken into account: \(gg \to gg\), \(gg \to q\bar{q}\), \(gq \to q\bar{q}\), \(q\bar{q} \to q\bar{q}\), \(q\bar{q} \to gg\), \(gg \to q\bar{q}\), and \(q\bar{q} \to q\bar{q}\). The matrix elements for massive quarks were taken from \[28\]. As in Refs. \[2\] \[13\], only the most divergent parts of the matrix elements were considered, regulated using a Debye mass of \(\mu_D = 0.7\) GeV.

Thus, for all elastic scatterings, \(d\sigma/dt \sim 1/t^2 \to 1/(t - \mu_D)^2\), and for \(gg \to q\bar{q}\) with massless quarks, \(d\sigma/dt \sim 1/(ut) \to 1/(u - \mu_D^2)(t - \mu_D^2)\). For charm, \(q\bar{q} \to c\bar{c}\) to first approximation can be considered isotropic, while \(gg \to c\bar{c}\) can be well approximated with the angular dependence \(d\sigma/dt \sim 1/[1 - (1 - 4M_c^2/s) \cos^2 \theta_{cm}]^2\).

The total cross section for \(gg \to gg\) was taken to be constant, neglecting its weak logarithmic energy dependence. Its value fixes the total cross sections for all other elastic channels:

\[
\sigma_{gg \to gg} = (4/9)\sigma_{gg \to gg} \quad , \quad \sigma_{gg \to q\bar{q}} = (4/9)^2\sigma_{gg \to gg} \, .
\]

(2)

It also determines the inelastic total cross sections, which on the other hand, decrease at high energy. For processes with only massless partons, the energy dependence is through the ratio \(r = \mu_D^2/s\), while for charm, \(r\) and \(R \equiv M_c^2/s\) both play a role \[28\]:

\[
\begin{align*}
\sigma_{gg \to q\bar{q}} &= \frac{2r}{27} \left[ 1 + r \right] \ln \left[ 1 + \left( \frac{1}{r} \right) \right] \sigma_{gg \to gg} \quad , \quad \sigma_{q\bar{q} \to q\bar{q}} = \frac{16r}{243} \sigma_{gg \to gg} \\
\sigma_{gg \to c\bar{c}} &= \frac{2r}{27} \Theta(1 - 4R) \left[ (1 + 4R + R^2) \ln \left( \frac{1 + \sqrt{1 - 4R}}{1 - \sqrt{1 - 4R}} - (7 + 3R) \sqrt{1 - 4R} \right) \right] \sigma_{gg \to gg} \\
\sigma_{q\bar{q} \to c\bar{c}} &= \frac{16r}{243} \Theta(1 - 4R)(1 + 2R) \sqrt{1 - 4R} \sigma_{gg \to gg}
\end{align*}
\]

(3)
In order to generate sufficient elliptic flow at RHIC, we take \( \sigma_{gg \rightarrow gg} = 10 \text{ mb} \), about three times the perturbative QCD estimate. As discussed in \cite{5}, this choice approximates contributions from \( 2 \leftrightarrow 3 \) inelastic processes \cite{17}, and also relies on the amplification of elliptic flow during hadronization via the coalescence process.

The parton initial conditions for \( Au + Au \) at \( \sqrt{s} = 200A \text{ GeV} \) at RHIC with \( b = 8 \text{ fm} \) (\( \approx 30\% \) central) were the same as in \cite{19}, except that initial charm production was, of course, included. Leading order pQCD minijet three-momentum distributions were used (with a \( K \)-factor of 2, GRV98LO PDFs, and \( Q^2 = p_T^2 \), while \( Q^2 = \hat{s} \) for charm). The low-\( p_T \) divergence in the light-parton jet cross sections was regulated via a smooth extrapolation below \( p_T < 2 \text{ GeV} \) to yield a total parton \( dN(b=0)/dy = 2000 \) at midrapidity. This choice is motivated by the observed \( dN_{ch}/dy \sim 600 \) and the expectation that hadronization is dominated by quark coalescence \cite{22, 23, 24, 5, 19}. The transverse density distribution was proportional to the binary collision distribution for two Woods-Saxon distributions, therefore \( dN_{\text{parton}}(b=8 \text{ fm})/dy \approx 500 \). Perfect \( \eta = y \) correlation was assumed.

The transport solutions were obtained via Molnar’s Parton Cascade algorithm \cite{29} (MPC), which employs the parton subdivision technique \cite{30} to maintain Lorentz covariance and causality.

### 3. Results for charm quarks

Because of the inelastic channels, \cite{1} is suitable for studying flavor equilibration at RHIC (with the limitation that the total parton number is fixed). Results for the initial and final rapidity distributions of each parton species are shown in the left and right panels of Fig. 1, respectively. The initial condition is dominated by gluons, roughly half of which fuse to \( q - \bar{q} \) pairs. This fills in the dip in the light (\( u, d \)) quark distributions near midrapidity, doubles the light antiquark distributions (\( \bar{u}, \bar{d} \)), enhances strangeness about five-fold, and increases charm by 40 – 50%. Flavor changing processes involving only quarks \( q_i \bar{q}_i \rightarrow q_j \bar{q}_j \) influence the flavor chemistry by only 10 – 15%.

![Figure 1](image-url). Initial (left panel) and final (right panel) parton rapidity distributions in \( Au + Au \) at \( \sqrt{s_{NN}} = 200 \text{ GeV} \) with \( b = 8 \text{ fm} \) at RHIC, computed from the transport model MPC \cite{29}.

These results show a high degree of light flavor and strangeness equilibration at
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RHIC. The final light antiquark ($\bar{u}, \bar{d}$) and strange quark distributions ($s, \bar{s}$) are nearly equal, though are still below the light quark ($u, d$) yields. The baryon number distribution, which is one-third of the $u + d - (\bar{u} + \bar{d})$ difference, does not vanish even at midrapidity, showing that a baryon free region is not realized at RHIC energies.

On the other hand, the elliptic flow evolution is driven mainly by elastic scatterings. Scatterings off gluons $gX \to gX$ are the most relevant because gluons are most abundant and also have larger cross sections (2). Figure 2a shows the elliptic flow of partons at freezeout as a function of $p_T$. For light partons, $v_2(p_T)$ reaches $\approx 10\%$, in quantitative agreement with [3]. However, instead of saturation between $p_T \sim 1 - 6$ GeV, this calculation shows a turnover and slow decrease of $v_2$ at high $p_T$. The reason for the difference is that Ref. [3] considered thermal initial conditions, while here the initial spectra are not thermal. The slight difference in Fig. 2a between strange and light quark $v_2$ is due to the flavor dependence of the initial minijet distributions. Because of the larger gluon cross sections (2), the gluon $v_2$ exceeds the quark flows.

![Figure 2. Final parton elliptic flows (left panel) and D meson elliptic flow (right panel) as a function of $p_T$ in Au + Au at $\sqrt{s_{NN}} = 200$ GeV with $b = 8$ fm at RHIC, computed using the transport model MPC [29]. The D meson $v_2$ was obtained from the parton flows via the quark coalescence formula in [13]. The dashed lines correspond to the two extreme scenarios in [13] (see text). For hadronization via independent fragmentation, charm hadron $v_2$ is approximately the same as the charm quark $v_2$.](image)

At low $p_T \lesssim 2$ GeV charm quark elliptic flow is suppressed relative to the $v_2$ of light partons, confirming expectations that heavy quark momenta are more difficult to randomize. This behavior is reminiscent of the reduction of elliptic flow for heavy particles found in ideal hydrodynamics [7, 31, 8, 9]. In contrast, at high $p_T > 3$ GeV, charm quark $v_2$ is about the same as the light quark $v_2$. At large momenta, the heavy-light difference should disappear because the particle mass cannot play a role as $E/M \to \infty$. Remarkably, the transport solutions show that most of the difference disappears already by $p_T \sim 2.5 - 3$ GeV, i.e., when $p_T/M_c \sim$ a few.

4. Charm hadron elliptic flow

From the elliptic flow of partons, one can predict the hadron elliptic flows using a suitable hadronization model. Here we are interested in the elliptic flow of the $D$ and $D_s$, which will likely be the first charm hadrons measured with sufficient statistics at
high $p_T$ at RHIC.

A simple hadronization model is independent fragmentation, in which each parton fragments independently to hadrons. In the collinear approximation commonly employed, hadron momenta are related to the parton momentum via $E_{\text{had}} = z E_{\text{parton}}$, where $z$ is distributed according to the fragmentation function $D(z)$. Because for heavy quarks $D(z)$ strongly peaks around $z \approx 1$, charm hadron elliptic flow is essentially identical to the charm quark $v_2$ shown in Fig. 2a.

On the other hand, there are strong indications [33, 34, 35] that at intermediate $2 \lesssim p_T \lesssim 5$ GeV hadronization may dominantly occur via quark coalescence [21, 23, 22, 5, 25]. In the quark coalescence approach, a constituent quark and an antiquark that are close in phasespace can combine to form a meson, $\alpha\beta \rightarrow M$, while three constituent (anti)quarks can form an (anti)baryon, $\alpha\beta\gamma \rightarrow B$.

In the simplest (but most successful) variant of the model [22, 5, 13], hadron elliptic flow is approximately the sum of constituent flows

$$v_{2,B}(x, p_T) \approx \sum_{i=\alpha,\beta,\gamma} v_{2,i}(x, p_T,i), \quad v_{2,M}(x, p_T) \approx \sum_{i=\alpha,\beta} v_{2,i}(x, p_T,i),$$

with corrections $O(\{v_{2,i}\}^3)$ that are small in our case. Here $p_T,i = p_T$, and the hadron momentum is shared roughly in proportion to constituent masses [13] For hadrons composed of $u, d$, and $s$ quarks, the sharing is approximately equal, while for $D$ mesons or the $\Lambda_c$, the heavy quark carries most of the momentum.

Figure 2b shows $D$ and $D_s$ elliptic flow as a function of $p_T$ given by [4], for constituent mass ratios $m_u, d : m_c = 1 : 5$, and $m_s : m_c = 1 : 3$. The results are compared to two extreme scenarios [13]: i) zero charm $v_2$, i.e., no rescatterings at all (lower dashed lines); and ii) $v_{2,\text{charm}}(p_T) = v_{2,\text{light}}(p_T)$ (upper dashed lines), which is equivalent to the assumption that $D$ flows the same way as all light mesons $v_{2}^D(p_T) \approx v_{2}^N(p_T) \approx v_{2}^\pi(p_T)$. Neither of the two extremes applies in general. At high $p_T > 2.5 - 3$ GeV, $v_{2,c} \approx v_{2,q}$ and hence the situation agrees with scenario ii), while at low $p_T$ it is closer to i) because of the suppression in the charm quark $v_2$. Thus, $D$ meson elliptic flow is predicted to rise smoothly at low $p_T$, and to saturate at $\approx 50\%$ higher $p_T$ than the pion and kaon $v_2$ but at the same magnitude.

The above results ignore fragmentation contributions and therefore are not valid at very high $p_T$. For light quarks, the region of validity was estimated [36] to be $p_T \lesssim 4 - 5$ GeV. For charm, the window is smaller because: i) while for light quarks coalescence “amplifies” transverse momenta by factor two to three, for charm the hadron $p_T$ is only 20–30% larger than the charm quark $p_T$; ii) fragmentation is harder for charm quarks than for lighter partons, therefore charm hadron spectra from fragmentation do not fall as steeply at high $p_T$; and iii) radiative energy loss (jet quenching) is expected to be smaller for charm quarks [37, 38] than for light partons, therefore the fragmentation contribution is less suppressed at high $p_T$. Thus, it is not likely that the coalescence results are valid above $p_T \sim 3 - 4$ GeV.

Finally we emphasize that, despite the success of the simple quark coalescence models, severe problems arise when spacetime inhomogeneities or dynamical aspects are considered [19, 20]. For example, surface-like emission of high-$p_T$ particles (as seen from covariant transport theory) results in large, spatially nonuniform local momentum anisotropies, for which the scaling [4] requires a high degree of lucky cancellations to occur [20].

Furthermore, in the dynamical coalescence approach of Ref. [19], elliptic flow scaling [4] is violated for light quarks. That model combines covariant particle
transport theory with the coalescence formula [39] by Gyulassy, Frankel and Remler that is applicable to diffuse 4D freezeout distributions in spacetime. Constituents without a coalescence partner are assumed to fragment independently. Because a significant fraction of constituents has no partner near enough in phasespace to coalesce with, elliptic flow is reduced relative to the flow scaling expectation.

The same problem would be present for charm quarks in that approach, as illustrated in Fig. 3. The fraction of light and charm constituents that hadronize via coalescence as opposed to fragmentation are shown as a function of $p_T$. Above $p_T > 3$ GeV, less than 25% of charm quarks comes from coalescence in the dynamical approach, resulting in a reduction of $D$ meson elliptic flow by almost a half above $p_T \approx 3.5$ GeV.

![Figure 3. Fraction of charm quarks that hadronize via coalescence as opposed to fragmentation, as a function of $p_T$ in $Au+Au$ at $\sqrt{s_{NN}} = 200$ GeV with $b=8$ fm at RHIC. Computed in a dynamical coalescence approach [19] using the transport model MPC [29] and the Gyulassy-Frankel-Remler coalescence formula [39].](image)

5. Conclusions

This study is the first calculation of charm elliptic flow in $Au + Au$ at $\sqrt{s_{NN}} \sim 200$ GeV at RHIC from covariant parton transport theory. Both elastic $2 \rightarrow 2$ and inelastic $2 \rightarrow 2$ parton interactions were included using MPC [29]. We show that parton opacities needed to generate a $\approx 10\%$ light quark elliptic flow also imply a charm quark elliptic flow of similar magnitude, at moderately large $p_T > 2.5 - 3$ GeV. At lower transverse momenta charm $v_2$ is suppressed, analogously to the mass dependence of elliptic flow found in ideal hydrodynamics. Based on hadronization via quark coalescence, we predict that the elliptic flows of $D$ and $D_s$ saturate at the same magnitude $\approx 20\%$ as pion and kaon $v_2$ but at $p_T \approx 3$ GeV, i.e., 50% higher than the “meson value” of 2 GeV. In contrast, from hadronization via independent fragmentation, charm hadron elliptic flow is only $\approx 10\%$ (same as charm quark $v_2$).

We emphasize that the above quark coalescence prediction relies on the approximate additivity of elliptic flow, which agrees well with observations in the light sector but is quite problematic to preserve in a dynamical coalescence approach [19, 26]. In any case, above $p_T \sim 3-4$ GeV, fragmentation contributions are expected to dominate and reduce $D$ and $D_s$ $v_2$ to $\approx 10\%$. 
Finally, the large charm elliptic flow is accompanied by significant flavor equilibration at RHIC, with an about five-fold enhancement of strangeness and a 40 – 50% secondary production of charm.

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References