Introduction to DSMT for Information Fusion

Jean Dezert

ONERA/DTIM/IED
29 Av. de la Division Leclerc
92320 Châtillon, France

jean.dezert@onera.fr

Florentin Smarandache

Department of Mathematics
University of New Mexico
Gallup, NM 8730, U.S.A.

smarand@unm.edu

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Outline

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DSmT in few lines

DSmT (Dezert-Smarandache Theory) is a new alternative to Dempster-Shafer Theory (DST) which :

- 1 - proposes a new mathematical framework for information fusion
- 2 - incorporates any kinds of model (free, hybrid DSm models and/or Shafer's model) for taking into account any integrity constraints of the fusion problem
- 3 - combines uncertain, high conflicting and imprecise sources of evidence with a new rule of combination and overcomes limitations of the Dempster's rule
- 4 - is adapted to static or dynamic fusion applications represented in terms of belief functions based on the same general formalism

DSmT Foundations

Frame of the problem

$$\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}.$$

Finite set of exhaustive elements
(discrete/continuous/fuzzy/relative concepts)

Hyper-power set (Dedekind's lattice) $|D^\Theta| > |2^\Theta|$

1. $\emptyset, \theta_1, \dots, \theta_n \in D^\Theta$
2. $\forall A \in D^\Theta, B \in D^\Theta, (A \cup B) \in D^\Theta, (A \cap B) \in D^\Theta$
3. No other elements belong to D^Θ , except those, obtained by using rules 1 or 2.

Note : hyper-power set reduces to power set for the Shafer's model

Cardinalities $d(n)$ of hyper-power sets follow Dedekind's sequence and elements can be generated by isotone Boolean functions. Hyper-power set is closed under union and intersection operators but is not a Boolean algebra.

Example for $n=3$ $d(n=3)=19$

$\alpha_0 \triangleq \emptyset$	$\alpha_4 \triangleq \theta_2 \cap \theta_3$	$\alpha_8 \triangleq (\theta_1 \cap \theta_2) \cup (\theta_1 \cap \theta_3) \cup (\theta_2 \cap \theta_3)$	$\alpha_{12} \triangleq (\theta_1 \cap \theta_2) \cup \theta_3$	$\alpha_{16} \triangleq \theta_1 \cup \theta_3$
$\alpha_1 \triangleq \theta_1 \cap \theta_2 \cap \theta_3$	$\alpha_5 \triangleq (\theta_1 \cup \theta_2) \cap \theta_3$	$\alpha_9 \triangleq \theta_1$	$\alpha_{13} \triangleq (\theta_1 \cap \theta_3) \cup \theta_2$	$\alpha_{17} \triangleq \theta_2 \cup \theta_3$
$\alpha_2 \triangleq \theta_1 \cap \theta_2$	$\alpha_6 \triangleq (\theta_1 \cup \theta_3) \cap \theta_2$	$\alpha_{10} \triangleq \theta_2$	$\alpha_{14} \triangleq (\theta_2 \cap \theta_3) \cup \theta_1$	$\alpha_{18} \triangleq \theta_1 \cup \theta_2 \cup \theta_3$
$\alpha_3 \triangleq \theta_1 \cap \theta_3$	$\alpha_7 \triangleq (\theta_2 \cup \theta_3) \cap \theta_1$	$\alpha_{11} \triangleq \theta_3$	$\alpha_{15} \triangleq \theta_1 \cup \theta_2$	

DSm Models

The granularity of the model of the frame characterizes the intrinsic nature (discrete/continuous, precise/vague, absolute/relative, etc) of the concepts involved in the fusion process.

Free DSm model

Elements of the frame are vague and potentially overlapping. Free = no constraint on elements. Useful to manipulate continuous concepts having relative interpretation (where ultimate refinement is inaccessible)

Hybrid DSm model

Some elements of the frame can be exclusive and/or non existing specially for dynamic fusion applications. Hybrid = introduction of integrity constraints into the free DSm model.

Special hybrid model: Shafer's model

All exhaustive elements of the frame are known to be truly exclusive (i.e. a refinement is accessible)

Constraints are represented by the characteristic non-emptiness function $\Phi(A)$ for all A in hyper-power set.

Parts have vague boundaries

Parts have precise boundaries

Generalized belief functions

Generalized basic belief assignment

$$m(.) : D^{\Theta} \rightarrow [0, 1] \quad m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \in D^{\Theta}} m(A) = 1$$

Generalized belief/credibility function

$$\text{Bel}(A) = \sum_{\substack{B \subseteq A \\ B \in D^{\Theta}}} m(B)$$

Generalized plausibility function

$$\text{Pl}(A) = \sum_{\substack{B \cap A \neq \emptyset \\ B \in D^{\Theta}}} m(B)$$

Generalized Pignistic Transformation (GPT)

GPT allows to build a subjective probability measure $P\{.\}$ over hyper-power set to help the decision making under uncertainty (but other issues are possible).

$$\forall A \in D^{\Theta} \quad P\{A\} = \sum_{X \in D^{\Theta}} \frac{c_{\mathcal{M}}(X \cap A)}{c_{\mathcal{M}}(X)} m(X)$$

$$\text{Bel}(A) \leq P\{A\} \leq \text{Pl}(A)$$

General DSm rule of combination

For any DSm model, the fusion of k independent equally* reliable sources of evidences is done by

$$m_{\mathcal{M}(\Theta)}(A) \triangleq \phi(A) \left[S_1(A) + S_2(A) + S_3(A) \right] \quad (I)$$

$$\begin{aligned} S_1(A) &\triangleq \sum_{\substack{X_1, \dots, X_k \in D^\Theta \\ (X_1 \cap \dots \cap X_k) = A}} \prod_{i=1}^k m_i(X_i) & S_3(A) &\triangleq \sum_{\substack{X_1, X_2, \dots, X_k \in D^\Theta \\ (X_1 \cup X_2 \cup \dots \cup X_k) = A \\ (X_1 \cap X_2 \cap \dots \cap X_k) \in \emptyset}} \prod_{i=1}^k m_i(X_i) \\ S_2(A) &\triangleq \sum_{\substack{X_1, X_2, \dots, X_k \in \emptyset \\ [\mathcal{U} = A] \vee [(\mathcal{U} \in \emptyset) \wedge (A = I_t)]}} \prod_{i=1}^k m_i(X_i) \end{aligned}$$

General DSm rule does not require normalization and is not equivalent to Dempster's rule.

Special case: Classic DSm rule for free DSm model This corresponds to conjunctive consensus.

$$m_{\mathcal{M}^f(\Theta)}(A) = S_1(A) \triangleq \sum_{\substack{X_1, \dots, X_k \in D^\Theta \\ (X_1 \cap \dots \cap X_k) = A}} \prod_{i=1}^k m_i(X_i)$$

* otherwise discounting techniques of the sources must be applied at first

On the associativity of DSm rules

The general/hybrid DSm rule of combination

The general DSm rule of combination is commutative but is in general not associative.

$$\underbrace{[m_1 \oplus m_2 \oplus m_3](.)}_{\text{Optimal Fusion}} \neq \underbrace{[(m_1 \oplus m_2) \oplus m_3](.)}_{\text{Suboptimal fusion}} \neq \underbrace{[m_1 \oplus (m_2 \oplus m_3)](.)}_{\text{Suboptimal fusion}} \neq \underbrace{[m_2 \oplus (m_1 \oplus m_3)](.)}_{\text{Suboptimal fusion}}$$

This is normal. To preserve optimality and coherence of the fusion result, all the sources have to be combined together at same fusion level (centralized fusion), not sequentially. Sequential/decentralized fusion is only suboptimal since part of information is lost during intermediate fusion steps.

Special case : The classic DSm rule of combination (for free DSm model)

Classic DSm rule (i.e. conjunctive consensus) over hyper-power set is commutative and associative.

DST and Dempster's rule of combination

Assumption: Shafer's model (close world assumption + refinement)

Basic belief assignment (bba):

$$m(.) : 2^\Theta \rightarrow [0, 1]$$

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \in 2^\Theta} m(A) = 1$$

Credibility:

$$\text{Bel}(A) = \sum_{B \in 2^\Theta, B \subseteq A} m(B)$$

Plausibility:

$$\text{Pl}(A) = \sum_{B \in 2^\Theta, B \cap A \neq \emptyset} m(B)$$

Dempster's rule: Fusion of 2 independent equally reliable sources with bba m_1 and m_2

$$m(\emptyset) = 0 \quad \text{and} \quad \forall A \neq \emptyset, m(A) = \frac{1}{1 - k_{12}} \sum_{\substack{X, Y \in 2^\Theta \\ X \cap Y = A}} m_1(X) m_2(Y)$$

$$\text{Degree of conflict: } k_{12} \triangleq \sum_{\substack{X, Y \in 2^\Theta \\ X \cap Y = \emptyset}} m_1(X) m_2(Y)$$

Dempster's rule generates counter-intuitive results when conflict becomes high and is even not defined (0/0) when $k_{12}=1$ (sources in full contradiction). Problem arises not only from reliability of sources but is due also to the limited knowledge or experience of sources/experts to build their bba and their own interpretation of elements of the frame.

Classical issues:

- 1 - choose some heuristic thresholding techniques on k_{12} to accept fusion result
- 2 - modify the discounting factors
- 3 - Adopt one of existing alternative rules proposed in literature (disjunctive, Yager, Dubois & Prade, Smets open-world approach)

But how to prevent fusion troubles with the Dempster's rule beforehand ?

-> Switch for a new competitive theory to eliminate inherent and potential troubles

Main existing alternatives to Dempster's rule

Assumption: Shafer's model

Yager's rule:

[Yager 1983]

$$\begin{cases} m_Y(\emptyset) = 0 \\ m_Y(A) = \sum_{\substack{X, Y \in 2^\Theta \\ X \cap Y = A}} m_1(X)m_2(Y) & \forall A \in 2^\Theta, A \neq \emptyset, A \neq \Theta \\ m_Y(\Theta) = m_1(\Theta)m_2(\Theta) + \sum_{\substack{X, Y \in 2^\Theta \\ X \cap Y = \emptyset}} m_1(X)m_2(Y) & \text{when } A = \Theta \end{cases}$$

Disjunctive rule:

[Dubois & Prade 1986]

$$m_{\cup}(\emptyset) = 0 \quad \text{and} \quad \forall A \neq \emptyset, m_{\cup}(A) = \sum_{\substack{X, Y \in 2^\Theta \\ X \cup Y = A}} m_1(X)m_2(Y)$$

Dubois & Prade's (hybrid) rule:

[Dubois & Prade 1988]

$$\begin{cases} m_{DP}(\emptyset) = 0 \\ m_{DP}(A) = \sum_{\substack{X, Y \in 2^\Theta \\ X \cap Y = A \\ X \cap Y \neq \emptyset}} m_1(X)m_2(Y) + \sum_{\substack{X, Y \in 2^\Theta \\ X \cup Y = A \\ X \cap Y = \emptyset}} m_1(X)m_2(Y) & \forall A \neq \emptyset \end{cases}$$

Assumption: Open-world

One allows positive bba on empty set

Smets' rule: [Smets 1994]

Smets' rule is the non-normalized version of the Dempster's rule.

$$\begin{cases} m_S(\emptyset) \equiv k_{12} = \sum_{\substack{X, Y \in 2^\Theta \\ X \cap Y = \emptyset}} m_1(X)m_2(Y) \\ m_S(A) = \sum_{\substack{X, Y \in 2^\Theta \\ X \cap Y = A}} m_1(X)m_2(Y) & \forall A \neq \emptyset \end{cases}$$

Note : A general unified formulation exists for all rules involving conjunctive consensus (Dempster's, Smets', Yager's, Dubois & Prade's, etc). [Lefevre et al. 2002]

Comments on refinement

$$\Theta = \{\theta_1 = \text{Small}, \theta_2 = \text{Tall}\}.$$

$$m_1(\theta_1) = 0.4 \quad m_1(\theta_2) = 0.5 \quad m_1(\theta_1 \cup \theta_2) = 0.1$$

$$m_2(\theta_1) = 0.6 \quad m_2(\theta_2) = 0.2 \quad m_2(\theta_1 \cup \theta_2) = 0.2$$

Case 1: Assumption = Shafer's model holds

Dempster's rule yields:

$$k_{12} = m_1(\theta_1)m_2(\theta_2) + m_2(\theta_1)m_1(\theta_2) = 0.38$$

$$m(\emptyset) = 0 \quad m(\theta_1) = \frac{0.38}{1 - 0.38} = 0.613 \quad m(\theta_2) = \frac{0.22}{1 - 0.38} = 0.355 \quad m(\theta_1 \cup \theta_2) = \frac{0.02}{1 - 0.38} = 0.032$$

General/hybrid DSm rule yields: DSm rule is not equivalent to Dempster's rule

$$m(\emptyset) = 0 \quad m(\theta_1) = 0.38 \quad m(\theta_2) = 0.22 \quad m(\theta_1 \cup \theta_2) = 0.02 + 0.38 = 0.40$$

For this simple 2D static fusion problem, the general DSm rule coincides with Yager's and Dubois & Prade's rules of combination.

Question : What to do if one does not accept the previous Shafer's model (because of the continuity of elements of the frame and their relative interpretation) ?

Comments on refinement (cont'd)

Case 2: Assumption = Shafer's model does not hold (one accepts the free DSm model)

Classical DSm rule yields directly:

$$m(\emptyset) = 0 \quad m(\theta_1 \cap \theta_2) = 0.38 \quad m(\theta_1) = 0.38 \quad m(\theta_2) = 0.22 \quad m(\theta_1 \cup \theta_2) = 0.02$$

Starting with same bba as inputs and assume now that a refined frame exists in order to apply Dempster's rule, i.e.

$$\theta_1 \cap \theta_2 = \theta'_2$$

$$\theta_2 = \theta'_2 \cup \theta'_3$$

$$\theta_1 = \theta'_1 \cup \theta'_2$$

$$\Theta_{ref} \triangleq \{\theta'_1 = \text{Small}', \theta'_2 \triangleq \text{Medium}, \theta'_3 = \text{Tall}'\}$$

$$m'_1(\theta'_1 \cup \theta'_2) = 0.4 \quad m'_1(\theta'_2 \cup \theta'_3) = 0.5 \quad m'_1(\theta'_1 \cup \theta'_2 \cup \theta'_3) = 0.1$$

$$m'_2(\theta'_1 \cup \theta'_2) = 0.6 \quad m'_2(\theta'_2 \cup \theta'_3) = 0.2 \quad m'_2(\theta'_1 \cup \theta'_2 \cup \theta'_3) = 0.2$$

Dempster's rule on the refined frame yields: (there is now no conflicting mass)

$$m(\emptyset) = 0$$

$$m(\theta'_2) = m'_1(\theta'_1 \cup \theta'_2)m'_2(\theta'_2 \cup \theta'_3) + m'_2(\theta'_1 \cup \theta'_2)m'_1(\theta'_2 \cup \theta'_3) = 0.38$$

$$m(\theta'_1 \cup \theta'_2) = m'_1(\theta'_1 \cup \theta'_2)m'_2(\theta'_1 \cup \theta'_2) + m'_1(\theta'_1 \cup \theta'_2 \cup \theta'_3)m'_2(\theta'_1 \cup \theta'_2) + m'_2(\theta'_1 \cup \theta'_2 \cup \theta'_3)m'_1(\theta'_1 \cup \theta'_2) = 0.38$$

$$m(\theta'_2 \cup \theta'_3) = m'_1(\theta'_2 \cup \theta'_3)m'_2(\theta'_2 \cup \theta'_3) + m'_1(\theta'_1 \cup \theta'_2 \cup \theta'_3)m'_2(\theta'_2 \cup \theta'_3) + m'_2(\theta'_1 \cup \theta'_2 \cup \theta'_3)m'_1(\theta'_2 \cup \theta'_3) = 0.22$$

$$m(\theta'_1 \cup \theta'_2 \cup \theta'_3) = m'_1(\theta'_1 \cup \theta'_2 \cup \theta'_3)m'_2(\theta'_1 \cup \theta'_2 \cup \theta'_3) = 0.02$$

Thus the Dempster's rule reduces to Classical DSm rule but with the necessity of existence and justification of possible refinement. It introduces useless complexity w.r.t the DSm formalism.

Static and dynamic Fusion

Based on the same general formalism DSMT is well adapted to deal with:

1- static fusion applications (i.e. the frame and model do not change with time).

2- dynamic fusion applications (i.e. the frame and/or model vary with time) with the use of the proper characteristic non-emptiness function.

3 - Fusion over different frames by min. specificity extension [Smets 1993]

Simple example of dynamic fusion (testimony problem)

$$\Theta(t_l) \triangleq \{\theta_1 \equiv \text{young}, \theta_2 \equiv \text{old}, \theta_3 \equiv \text{white hairs}\}$$

Witnesses
reports

$$\begin{cases} m_1(\theta_1) = 0.5 & m_1(\theta_3) = 0.5 \\ m_2(\theta_2) = 0.5 & m_2(\theta_3) = 0.5 \end{cases}$$

Applying the general DSMT rule (based on free DSMT model), one gets:

$$m_{\mathcal{M}^f(\Theta(t_l))}(\theta_1 \cap \theta_2) = 0.25 \quad m_{\mathcal{M}^f(\Theta(t_l))}(\theta_1 \cap \theta_3) = 0.25 \quad m_{\mathcal{M}^f(\Theta(t_l))}(\theta_2 \cap \theta_3) = 0.25 \quad m_{\mathcal{M}^f(\Theta(t_l))}(\theta_3) = 0.25$$

If one learns that young people don't have white hairs, one introduces this integrity constraint in the model, by setting:

$$\Phi(\theta_1 \cap \theta_3) \stackrel{\mathcal{M}}{=} 0$$

Applying the general DSMT rule, one gets now:

$$m_{\mathcal{M}(\Theta(t_{l+1}))}(\theta_3) = 0.25 \quad m_{\mathcal{M}(\Theta(t_{l+1}))}(\theta_1 \cap \theta_2) = 0.25 \quad m_{\mathcal{M}(\Theta(t_{l+1}))}(\theta_2 \cap \theta_3) = 0.25 \quad m_{\mathcal{M}(\Theta(t_{l+1}))}(\theta_1 \cup \theta_3) = 0.25$$

DST vs. DSMT - Examples

Frame

$$\Theta = \{\theta_1, \theta_2, \theta_3\},$$

Inputs

$$\begin{aligned} m_1(\theta_1) &= 1 - \epsilon_1 & m_1(\theta_2) &= 0 & m_1(\theta_3) &= \epsilon_1 \\ m_2(\theta_1) &= 0 & m_2(\theta_2) &= 1 - \epsilon_2 & m_2(\theta_3) &= \epsilon_2 \end{aligned}$$

Dempster's rule

(Shafer's model)

$$m(\theta_3) = \frac{e_1 e_2}{(1 - e_1) \cdot 0 + 0 \cdot (1 - e_2) + e_1 e_2} = 1$$

Other masses are zero.
Counter-intuitive result

When $0 < e_1 < 1$ and $0 < e_2 < 1$, Dempster's rule provides in this specific case same result whatever the values of e_1 and e_2 are !!! Dempster's rule is mathematically not defined when $e_1 = e_2 = 0$. It provides only a coherent and trivial solution when $e_1 = e_2 = 1$.

Classic DSMT rule (free DSMT model)

$$m(\theta_3) = e_1 e_2 \quad m(\theta_1 \cap \theta_2) = (1 - e_1)(1 - e_2) \quad m(\theta_1 \cap \theta_3) = (1 - e_1)e_2 \quad m(\theta_2 \cap \theta_3) = (1 - e_2)e_1$$

General DSMT rule (one adopts the Shafer's model)

$$m(\theta_3) = e_1 e_2 \quad m(\theta_1 \cup \theta_2) = (1 - e_1)(1 - e_2) \quad m(\theta_1 \cup \theta_3) = (1 - e_1)e_2 \quad m(\theta_2 \cup \theta_3) = (1 - e_2)e_1$$

DSMT provides coherent result depending on e_1 and e_2 .
 e_1 and e_2 can take any values in $[0,1]$

DST vs. DS_mT - Examples (cont'd)

Frame

$$\Theta = \{\theta_1, \theta_2, \theta_3, \theta_4\}$$

Inputs

$$\begin{array}{ll} m_1(\theta_1) = 0.99 & m_1(\theta_3 \cup \theta_4) = 0.01 \\ m_2(\theta_2) = 0.98 & m_2(\theta_3 \cup \theta_4) = 0.02 \end{array}$$

Dempster's rule

(Shafer's model)

$$m(\theta_3 \cup \theta_4) = \frac{(0.01 \cdot 0.02)}{(0 + 0 + 0 + 0 + 0.01 \cdot 0.02)} = 1$$

Other masses are zero.
Counter-intuitive result

Classic DS_m rule (free DS_m model)

$$m(\theta_1 \cap \theta_2) = 0.9702 \quad m(\theta_1 \cap (\theta_3 \cup \theta_4)) = 0.0198 \quad m(\theta_2 \cap (\theta_3 \cup \theta_4)) = 0.0098 \quad m(\theta_3 \cup \theta_4) = 0.0002$$

General DS_m rule (one adopts the Shafer's model)

$$m(\theta_1 \cup \theta_2) = 0.9702 \quad m(\theta_1 \cup \theta_3 \cup \theta_4) = 0.0198 \quad m(\theta_2 \cup \theta_3 \cup \theta_4) = 0.0098 \quad m(\theta_3 \cup \theta_4) = 0.0002$$

DS_mT still provides a coherent result

There exist several infinite classes of fusion problems where the Dempster's rule fails to provide coherent results (or at least counter-intuitive results)

[Dezert/Smarandache/Khosnevisan 2004]

Comparisons of fusion rules

Frame

$$\Theta = \{\theta_1, \theta_2, \theta_3\},$$

set of a priori exclusive and exhaustive murders

Inputs

$$\begin{array}{llll} m_1(\theta_1) = 0.1 & m_1(\theta_2) = 0.4 & m_1(\theta_3) = 0.2 & m_1(\theta_1 \cup \theta_2) = 0.3 \\ m_2(\theta_1) = 0.5 & m_2(\theta_2) = 0.1 & m_2(\theta_3) = 0.3 & m_2(\theta_1 \cup \theta_2) = 0.1 \end{array}$$

original witnesses reports

Consider the Dynamic fusion problem with non-existential constraint

New info.: The third suspect provides a strong alibi

$$\theta_3 \stackrel{\mathcal{M}}{=} \emptyset \quad (\text{extra integrity constraint})$$

Classic DS_m rule (free DS_m model)

$$\begin{array}{llll} m(\theta_1) = 0.21 & m(\theta_2) = 0.11 & m(\theta_3) = 0.06 & m(\theta_1 \cup \theta_2) = 0.03 \\ m(\theta_1 \cap \theta_2) = 0.21 & m(\theta_1 \cap \theta_3) = 0.13 & m(\theta_2 \cap \theta_3) = 0.14 & m(\theta_3 \cap (\theta_1 \cup \theta_2)) = 0.11 \end{array}$$

General DS_m rule (hybrid DS_m model = Shafer's model + extra constraint)

The conflicting mass to transfer is then

$$k_{12} = 0.06 + 0.21 + 0.13 + 0.14 + 0.11 = 0.65$$

Fusion result

$$m(\emptyset) = 0 \quad m(\theta_1) = 0.34 \quad m(\theta_2) = 0.25 \quad m(\theta_1 \cup \theta_2) = 0.41$$

Smets' rule

$$m_S(\emptyset) = 0.65 \quad m_S(\theta_1) = 0.21 \quad m_S(\theta_2) = 0.11 \quad m_S(\theta_1 \cup \theta_2) = 0.03$$

Comparisons of fusion rules (cont'd)

Yager's rule

$$m_Y(\emptyset) = 0 \quad m_Y(\theta_1) = 0.21 \quad m_Y(\theta_2) = 0.11 \quad m_Y(\theta_1 \cup \theta_2) = 0.03 + k_{12} = 0.03 + 0.65 = 0.68$$

Dempster's rule

$$m_{DS}(\emptyset) = 0 \quad m_{DS}(\theta_1) = \frac{0.21}{1 - 0.65} = 0.60 \quad m_{DS}(\theta_2) = \frac{0.11}{1 - 0.65} \approx 0.314 \quad m_{DS}(\theta_1 \cup \theta_2) = \frac{0.03}{1 - 0.65} \approx 0.086$$

Dubois & Prade's rule

If one adds up masses one gets $0.94 < 1$

$$m_{DP}(\emptyset) = 0$$

$$m_{DP}(\theta_1) = [m_1(\theta_1)m_2(\theta_1) + m_1(\theta_1)m_2(\theta_1 \cup \theta_2) + m_2(\theta_1)m_1(\theta_1 \cup \theta_2)] + [m_1(\theta_1)m_2(\theta_3) + m_2(\theta_1)m_1(\theta_3)] = 0.34$$

$$m_{DP}(\theta_2) = [m_1(\theta_2)m_2(\theta_2) + m_1(\theta_2)m_2(\theta_1 \cup \theta_2) + m_2(\theta_2)m_1(\theta_1 \cup \theta_2)] + [m_1(\theta_2)m_2(\theta_3) + m_2(\theta_2)m_1(\theta_3)] = 0.25$$

$$m_{DP}(\theta_1 \cup \theta_2) = [m_1(\theta_1 \cup \theta_2)m_2(\theta_1 \cup \theta_2)] + [m_1(\theta_1 \cup \theta_2)m_2(\theta_3) + m_2(\theta_1 \cup \theta_2)m_1(\theta_3)] + [m_1(\theta_1)m_2(\theta_2) + m_2(\theta_1)m_1(\theta_2)] = 0.35$$

Dubois & Prade's rule doesn't work for dynamic fusion problems when a singleton or an union of singletons becomes empty. This problem is fixed by the sum S_2 in the general DSm rule of combination.

When there is no non-existential constraint, the general DSm rule is consistent with Dubois & Prade's rule.

Extension of DS_m rule for imprecise beliefs

Operations on sets

Addition

Subtraction

Multiplication

$$S_1 \boxplus S_2 = S_2 \boxplus S_1 \triangleq \{x \mid x = s_1 + s_2, s_1 \in S_1, s_2 \in S_2\}$$

$$S_1 \boxminus S_2 \triangleq \{x \mid x = s_1 - s_2, s_1 \in S_1, s_2 \in S_2\}$$

$$S_1 \boxtimes S_2 \triangleq \{x \mid x = s_1 \cdot s_2, s_1 \in S_1, s_2 \in S_2\}$$

Inputs

Imprecise admissible generalized basic belief assignments $m^I(\cdot)$ of the form

$$m^I(A) = [a_1, b_1] \cup \dots \cup [a_m, b_m] \cup (c_1, d_1) \cup \dots \cup (c_n, d_n) \cup (e_1, f_1) \cup \dots \cup (e_p, f_p) \cup [g_1, h_1] \cup \dots \cup [g_q, h_q] \cup \{A_1, \dots, A_r\}$$

where all the bounds or elements involved into $m^I(A)$ belong to $[0, 1]$

Extended general DS_m rule

$$m_{\mathcal{M}(\Theta)}^I(A) \triangleq \phi(A) \boxtimes [S_1^I(A) \boxplus S_2^I(A) \boxplus S_3^I(A)]$$

A simple 2D example

$A \in D^\Theta$	$m_1^I(A)$	$m_2^I(A)$
θ_1	$[0.1, 0.2] \cup \{0.3\}$	$[0.4, 0.5]$
θ_2	$(0.4, 0.6) \cup [0.7, 0.8]$	$[0, 0.4] \cup \{0.5, 0.6\}$

Inputs

Fusion on free
DS_m model

$A \in D^\Theta$	$m^I(A) = [m_1^I \oplus m_2^I](A)$
θ_1	$[0.04, 0.10] \cup [0.12, 0.15]$
θ_2	$[0, 0.40] \cup [0.42, 0.48]$
$\theta_1 \cap \theta_2$	$(0.16, 0.58]$
$\theta_1 \cup \theta_2$	0

Fusion on
Shafer's model

$A \in D^\Theta$	$m_{\mathcal{M}}^I(A) = [m_1^I \oplus m_2^I](A)$
θ_1	$[0.04, 0.10] \cup [0.12, 0.15]$
θ_2	$[0, 0.40] \cup [0.42, 0.48]$
$\theta_1 \cap \theta_2 \stackrel{\mathcal{M}}{=} \emptyset$	0
$\theta_1 \cup \theta_2$	$(0.16, 0.58]$

Summary of the DSm Fusion Principle

Practical limitations of DSMT

Complexity of the general DSm rule w.r.t. all previous existing rules of combination. This is the price to pay

DSMT is useful for problems of small dimension but becomes intractable for problems of large dimension in the worst case (i.e. when the core of bba coincides with hyper-power set) since the cardinality of hyper-power set increases rapidly (the cardinality however remains much smaller than the cardinality of the power set of the refined frame, when refinement is possible)

$$(|2^\Theta| = 2^n) < (|D^\Theta| = d(n)) < (|2^{\Theta_{ref}}| = 2^{2^n-1})$$

$ \Theta = n$	$ 2^\Theta = 2^n$	$ D^\Theta = d(n)$	$ 2^{\Theta_{ref}} = 2^{2^n-1}$
2	4	5	$2^3 = 8$
3	8	19	$2^7 = 128$
4	16	167	$2^{15} = 32768$
5	32	7580	$2^{31} = 2147483648$

There is no consensus for decision-making in DSMT framework (same problem exists within DST framework). Many criteria can actually be adopted (GPT is only an issue).

■ Covered soils widespread
 ■ Covered soils well-protection
 ■ Bare soils widespread

0 200 km

SPOT + IRS-LISS III images + GIS + agricultural experts

Optimization of disparate DSN architecture to minimize power consumption and optimize multitarget detection and classification (typical application of dynamic fusion).

Sonar amplitude measurements + fuzzification interface + DSmT

Multitarget tracking with kinematics and attribute measurements

Evaluation of neutrosophic logic and DSmT to cope with the ontological and epistemic problems of situation analysis and awareness

Analysis and comparison of Bayesian, Shaferian reasonings w.r.t. DSmt within weighted contradicting rules-based systems.

Conclusions

- DSMT proposes new mathematical foundations for information fusion and is a new competitive alternative to DST with following specificities:
 - It can work beyond the limits of the applicability of the DST
 - It takes into account the intrinsic nature/granularity of information
 - It works on any models (including Shafer's model)
 - It provides a new fusion rule able to deal with static or dynamic fusion problems expressed in terms of belief functions and based on the same general formalism
 - It allows to combine uncertain, highly conflicting and imprecise information provided by independent sources of evidence
 - Reliability (when known) of sources can be easily taken into account
 - It is a natural extension of previous works done by Yager, Dubois & Prade , Smets and others to circumvent DST limitations.
- Potentially, DSMT can be applied in most fusion applications where DST has already been used so far, but it can also cover a wider class of applications because of its new appealing specificities.

Reference: “Advances and Applications of DSMT for Information Fusion (Collected works)”, American Research Press, Rehoboth, 418 p., 2004.