Bell’s inequality tests: from photons to $B$–mesons

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Abstract

We analyse the recent claim that a violation of a Bell’s inequality has been observed in the $B$–meson system [A. Go, *Journal of Modern Optics* **51** (2004) 991]. The results of this experiment are a convincing proof of quantum entanglement in $B$–meson pairs similar to that shown by polarization entangled photon pairs. However, we conclude that the tested inequality is not a genuine Bell’s inequality and thus cannot discriminate between quantum mechanics and local realistic approaches.

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A recent paper [1] claims that a clear violation of a Bell’s inequality [2,3] has been observed using particle–antiparticle correlations in semileptonic B–meson decays. The B–meson pairs were produced via \( \Upsilon(4S) \to B^0\bar{B}^0 \) decays at Belle with a wave function which is formally identical to that of polarization entangled photon pairs. The experiment [1] is the first attempt to perform a Bell’s inequality test with high–energy particles —rather than with the more conventional photons in optical tests— and thus deserves special attention [4]. The purpose of this Letter is to show that the results of Ref. [1], which conclusively prove the entangled behaviour of such heavy–meson pairs, cannot be considered a genuine Bell–test, i.e. a test discriminating between local realism (LR) and quantum mechanics (QM).

Just after the \( \Upsilon(4S) \) decay \((t = 0)\), the state of the \( B^0\bar{B}^0 \) pair is

\[
|\Upsilon(0)\rangle = \frac{1}{\sqrt{2}} \left[ |B^0\rangle_l |\bar{B}^0\rangle_r - |B^0\rangle_r |\bar{B}^0\rangle_l \right] = \frac{1}{\sqrt{2}} \left[ |B_L\rangle_l |B_H\rangle_r - |B_H\rangle_l |B_L\rangle_r \right],
\]

where \( l \) and \( r \) denote the ‘left’ and ‘right’ directions of motion of the two separating mesons and (small) \( CP \)–violating effects have been (safely [5]) neglected in the last equality. This approximation implies

\[
|B_L\rangle = \{ |B^0\rangle + |\bar{B}^0\rangle \}/\sqrt{2} \quad \text{and} \quad |B_H\rangle = \{ |B^0\rangle - |\bar{B}^0\rangle \}/\sqrt{2}.
\]

Note the strong similarity between the state (1) and the singlet state used in two–photon polarization Bell–tests. According to QM, the time evolution in free space of the light– and heavy–mass eigenstates, \( B_L \) and \( B_H \), is given by

\[
|B_{L,H}\rangle \to e^{-i\Gamma_{L,H}t} e^{-\frac{i}{2}} \Gamma_{L,H} t |B_{L,H}\rangle,
\]

where \( \Gamma_{L,H} \) account for the common \( B_L \) and \( B_H \) decay rates. Experimentally one has \( \Gamma_L \simeq \Gamma_H = 1/\tau_B \) and \( \tau_B = (1.536 \pm 0.014) \times 10^{-12} \) s [1,5]. The evolution of the initial state (1) up to left– and right–side times \( t_l \) and \( t_r \) is then

\[
|\Upsilon(t_l; t_r)\rangle = \frac{1}{2\sqrt{2}} e^{-\frac{t_l + t_r}{2\tau_B}} \left\{ (1 - e^{i(\Delta m(t_l - t_r))}) \left[ |B_l\rangle_l |\bar{B}_l\rangle_r - |\bar{B}_l\rangle_l |B_l\rangle_r \right] \right.

+ \left. (1 + e^{i(\Delta m(t_l - t_r))}) \left[ |B_r\rangle_l |\bar{B}_r\rangle_r - |\bar{B}_r\rangle_l |B_l\rangle_r \right] \right\},
\]

where \( \Delta m \equiv m_H - m_L \neq 0 \) induces flavour or \( B^0 - \bar{B}^0 \) oscillations in time. These oscillations are crucial for a Bell–test and, formally, they play the same role as the different orientations of polarization analysers in photonic experiments (see Refs. [6,7]). Eq. (3) leads to the same– and opposite–flavour joint detection probabilities

\[
P_{B^0\bar{B}^0}(t_l; t_r) = P_{\bar{B}^0B^0}(t_l; t_r) = \frac{1}{4} e^{-\frac{t_l + t_r}{\tau_B}} \left[ 1 - \cos(\Delta m \Delta t) \right],
\]

\[
P_{B^0\bar{B}^0}(t_l; t_r) = P_{\bar{B}^0B^0}(t_l; t_r) = \frac{1}{4} e^{-\frac{t_l + t_r}{\tau_B}} \left[ 1 + \cos(\Delta m \Delta t) \right],
\]

with \( \Delta t \equiv t_l - t_r \). These \( P_{B_iB_j}(t_l; t_r) \)‘s, with \( B_{i,r} = B^0, \bar{B}^0 \), are the QM predictions for the joint probability measurements performed by two hypothetical flavour detectors inserted along the left and right beams at ‘time–of–flight distances’ \( t_l \) and \( t_r \).
But no such detectors are available and the flavour (either $B^0$ or $\bar{B}^0$) of each member of the $B$–meson pair has to be identified by observing its decay modes. The various decay products $f = D^*(2010)^- l^+ \nu_l$, $D^– \pi^+$, . . . , which are forbidden for a $\bar{B}^0$, unambiguously come from a $B^0$, while the opposite is true for the respective charge conjugated modes $\bar{f} = D^*(2010)^+ l^– \nu_l$, $D^+ \pi^–$ . . . , ($l^\pm = e^\pm, \mu^\pm$). The corresponding partial decay widths satisfy $\Gamma_{B^0 \rightarrow f} = \Gamma_{\bar{B}^0 \rightarrow \bar{f}}$ [5]. Experimentally, one counts the number of joint $B$–meson decay events into the distinct decay modes $f_{i,r}$ and in the appropriate time intervals $[t_{i,r}, t_{i,r} + dt_{i,r}]$; then the joint decay probabilities $P_{f_{i,r}, t_{i,r}}$ are obtained after dividing these numbers by the total number of initial $B^0\bar{B}^0$ pairs. Finally, the corresponding joint decay rates $\Gamma_{f_{i,r}}(t_{i,r})$ are derived as:

$$\Gamma_{f_{i,r}}(t_{i,r}) \equiv \frac{dP_{f_{i,r}, t_{i,r}}}{dt} = P_{B_{i}B_{r}}(t_{i,r}) \Gamma_{B_{i} \rightarrow f_{i}} \Gamma_{B_{r} \rightarrow f_{r}},$$

from which the joint probabilities $P_{B_{i}B_{r}}(t_{i,r})$ immediately follow. The data from Ref. [1], where the only detected modes were $B^0 \rightarrow D^*(2010)^- l^+ \nu_l$ and $\bar{B}^0 \rightarrow D^*(2010)^+ l^– \nu_l$ occurring with a 5.4% probability each, are found to be in good agreement with the QM predictions in Eq. (4). This is a convincing proof of the entanglement between the members of each $B$–meson pair and thus suggests the possibility of interesting Bell–tests.

To this end, one proceeds as in optical experiments and defines the normalized correlation function [1]

$$E_R(\Delta t) \equiv \frac{P_{B^0\bar{B}^0}(\Delta t) + P_{B^0\bar{B}^0}(\Delta t) - P_{B^0\bar{B}^0}(\Delta t) - P_{B^0\bar{B}^0}(\Delta t)}{P_{B^0\bar{B}^0}(\Delta t) + P_{B^0\bar{B}^0}(\Delta t) + P_{B^0\bar{B}^0}(\Delta t) + P_{B^0\bar{B}^0}(\Delta t)} = -\cos(\Delta m \Delta t),$$

which turns out to depend on $\Delta t \equiv t_i - t_r$, but not on the specific decay modes detected in a given experiment. In conventional Bell–tests one then derives from LR an inequality which has to be necessarily satisfied by these correlation functions. Ref. [1] makes use of the Clauser, Horne, Shimony and Holt (CHSH) version of the Bell inequality [3]

$$S(\Delta t) = |3E_R(\Delta t) - E_R(3\Delta t)| \leq 2,$$

which turns out to be clearly violated by the data below $\Delta t \approx 1.7 \tau_B \approx 2.62$ ps. The maximal violation [$S_{\text{max}} = 2.73 \pm 0.19$, occurring when $\Delta t = (2 \pm 0.5)$ ps] is more than three $\sigma$’s above the limit in Eq. (7) and compatible with the maximal violation predicted by QM [$S_{\text{max}} = 2\sqrt{2}$]. The existence of QM ‘non–local’ correlations is certainly proved, but, is this really a proof against LR theories as claimed in Ref. [1]?

The conventional and most convincing procedure to disclaim such a conclusion consists in constructing a local model of hidden variables which agrees with the quantum mechanical predictions and thus with the experimental data of Ref. [1]. In the present case, this is easily achieved by simply adapting an original argument introduced by Kasday [8] in another context. Each $B^0\bar{B}^0$ pair is assumed to be produced at $t = 0$ with a set of hidden variables $\{t_i, f_t, f_r\}$ deterministically specifying $ab\ ovo$ the future decay times and decay modes of its two members. Different $B$–meson pairs are indeed supposed to be produced with
a probability distribution coinciding precisely with the joint decay probability $P_{f_l,f_r}(t_l; t_r)$ entering Eq. (5). Note that the conventional normalization in the hidden variable space, $\int d\lambda \rho(\lambda) = 1$, is now similarly given by

$$\Sigma_{f_l,f_r} \int dt_l \int dt_r \Gamma_{f_l,f_r}(t_l; t_r) = 1,$$

(8)

where the time integrals extend from 0 to $\infty$ and the sum to all $B^0$ and $\bar{B}^0$ decay modes (Ref. [1] considered only a single decay mode, which was consistently normalized according to Eq. (6)). Note also that our proposed hidden variable distribution function $P_{f_l,f_r}(t_l; t_r)$ reproduces the successful QM description of all the measurements in Ref. [1]. More importantly, our ad hoc LR model also violates the inequality (7) measured there. This proves that the inequality tested in Ref. [1] is not a genuine Bell–inequality, which, by definition, has to be satisfied in any LR approach. A similar criticism applies to the inequalities derived in Ref. [9] for entangled $K^0\bar{K}^0$ pairs.

Experiments performed with freely propagating meson–antimeson pairs, whose flavour is measured through the identification of their spontaneous decay modes, are not suitable for Bell–tests. The main reason is that they can be run in a single experimental setup, not involving alternative settings [10]. Such experiments then admit a successful QM description of the meson decay modes and decay times which can be directly used as an equally successful probability distribution of hidden variables in a LR approach. In other words, when deriving a Bell’s inequality from LR, one has to argue that measurement outcomes on one side cannot be affected by the experimental setting chosen to measure on the other side. Such counterfactual considerations [3,11] do not apply to experiments like that of Ref. [1] and the tested inequality is useless in order to discriminate between QM and LR. For this important purpose, other experiments involving alternative measurement settings are required. Some genuine Bell’s inequality tests for the $K^0\bar{K}^0$ system, exploiting the neutral–kaon regeneration effects, have been proposed by various authors [7,12], but their applicability to the extremely short–lived $B$–mesons seems to offer serious difficulties.

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