The mass composition of cosmic rays above $10^{17}$ eV*

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Our knowledge of the mass composition of cosmic rays is deficient at all energies above $10^{17}$ eV. Here, systematic differences between different measurements are discussed and, in particular, it is argued that there is no compelling evidence to support the common assumption that vast majority of the cosmic rays of the highest energies are protons. Our knowledge of the mass needs to be improved if we are to resolve uncertainties about the energy spectrum. Improvement is also needed for proper interpretation of data on the arrival direction distribution of cosmic ray.

1. The Scientific Motivation for Studying the Highest Energy Cosmic Rays

Since the recognition in 1966, by Greisen and by Zatsepin and Kuzmin, that protons with energies above $4 \times 10^{19}$ eV would interact with the cosmic microwave radiation, there has been great interest in measuring the spectrum, arrival direction distribution and mass composition of ultra high-energy cosmic rays (UHECR), defined as those cosmic rays having energies above $10^{19}$ eV. Specifically they pointed out that if the sources of the highest energy protons are universally distributed, there should be a steepening of the energy spectrum in the range from 4 to $10 \times 10^{19}$ eV. This feature has become known as the GZK ‘cut-off’ but the sharpness of the steepening expected depends on unknown factors such as the evolution and production spectrum of the sources. If the UHECR are mainly Fe nuclei then the spectrum is also expected to steepen, but it is harder to predict the character of this feature as the relevant diffuse infrared photon field is poorly known: steepening is expected to set in at higher energy.

2. Importance of mass composition for accurate estimates of the energy spectrum

The energy spectrum of cosmic rays has frequently been inferred from observations of signals in arrays of scintillators or water-Cherenkov detectors (see Nagano and Watson 11 for a review). In practice, the energy is derived from a measurement of the detector response at (typically) 600 m from the shower axis using the results of detailed Monte Carlo calculations. For example, in the most recent reports of spectra from the AGASA 2 and Haverah Park groups 3, the QGSJET model of high energy interactions has been adopted. Additionally, in the AGASA work, a study of the impact of different models was made and it was found, for example, using QGSJET98 in the Corsika propagation code, that the energy estimates for protons or iron nuclei differ by about 12% near $10^{20}$ eV (with protons giving the higher energy), while with SIBYLL 1.6 the difference is 19%, in the same direction and the overall energy estimates are about 5% higher. These two sources of systematic error in energy estimates—from mass and model—are essential to keep in mind when comparing data sets. As the relevant centre of mass energy at $10^{20}$ eV is well beyond that anticipated at the LHC, it is clear that the systematic effect in the model is extremely difficult to quantify.

The other method that has been used to measure primary energies is the fluorescence technique in which photons emitted by excited N$_2$ are detected. With this method, it is possible to make a calorimetric estimate of the large fraction of the primary energy that is transferred into ionisation as the shower particles cross the atmosphere. The estimate of this part of the primary energy loss is relatively model independent with, as first dis-
cussed by Linsley [4], and more recently by Song et al. [5], a correction of around 10\% being required for energy that does not go into atmospheric ionisation. The most recent journal presentation of a fluorescence spectrum by the HiRes group is one in which monocular data from two detectors are reported [6]. Note that the correction for missing energy is mass dependent, with an energy estimate about 5\% higher being made if the primaries are assumed to be Fe nuclei.

When making comparisons of spectra, a log-log plot of J (the differential intensity) against energy, \( E \), has significant advantages over the \( J E^3 \) vs. \( E \) plots commonly adopted in recent years. In particular, propagators of the latter style (including the present author) almost always ignore the fact that the error bars on such a plot should be shown as diagonal lines and that the uncertainty in energy (often justifiably omitted in the x-direction because of the bin size) should be added in quadrature to the uncertainty in intensity, as it comes in as the third power. The \( J E^3 \) vs. \( E \) representations do the data a disservice as the incorrect error assignments serve to give an erroneous impression of the level of incompatibility between the data sets. For example, the differences between the AGASA and HiRes spectra could largely be resolved, except perhaps at the very highest energies, if one or other of the energy estimates had a systematic error of only \( \sim 20 - 30\% \). Some movement towards reconciliation would arise if the primary particles were iron nuclei.

### 3. The mass of UHECR

Knowledge about the mass of primary cosmic rays at energies above \( 10^{17} \) eV is rudimentary. Different methods of measuring the mass give different answers and the conclusions are dependent, in all cases, upon the model calculations that are assumed. Results from some of the techniques that have been used are now described and the conclusions drawn reviewed. Some of these techniques will be applicable with the Pierre Auger Observatory [7].

#### 3.1. The Elongation Rate

The elongation rate is the term used to describe the rate of change of depth of shower maximum with primary energy. The term was introduced by Linsley [8] and, although his original approach has been, to some extent, superseded by the results of detailed Monte Carlo studies, the concept remains useful for organising and describing data and calculations. In figure [1] there is a summary of measurements of the depth of maximum together with predictions from a variety of model calculations reproduced from [9]. It is clear that, if certain models are correct, one might infer that the primaries above \( 10^{19} \) eV are dominantly protons but that others are indicative of a mixed composition. In particular, the QGSJET set of models (basic QGSJET01 and the 5 options discussed in [9]) and the Sibyll 2.1 model force contrary conclusions. Note that the experimental data on \( X_{\text{max}} \) do not yet extend beyond 25 EeV.

![Figure 1. The depth of maximum, as predicted using various models, compared with measurements. The predictions of the five modifications of QGSJET, discussed in [9], from which this diagram is taken, lie below the dashed line that indicates the predictions of QGSJET01.](image-url)
3.2. Fluctuations in Depth of Maximum

Further insight is expected to come from the magnitude of the fluctuation of the position of depth of maximum, $X_{\text{max}}$. If a group of showers is selected within a narrow range of energies, then fluctuations about the mean of $X_{\text{max}}$ (or in a parameter that is closely related to $X_{\text{max}}$ such as the steepness of the lateral distribution function or the spread of muon densities) are expected to be larger for protons than for iron nuclei. A recent study applying this idea to $X_{\text{max}}$ has been reported by the HiRes group \cite{10} for 553 events above 10 EeV.

In \cite{10} it is argued that the fluctuations are so large that a large fraction of light nuclei must be present in the primary beam (figure 2): this conclusion is independent of whether Sibyll or QGSJET models are used. Clearly an understanding of the tails on the high-$X_{\text{max}}$ side of the histograms is crucial to the conclusion that they are best described by ‘a predominantly light composition’. Of the 553 events in the data set, detailed atmospheric information is available for 419: the rest (134) occurred during ‘good weather’ and were analysed assuming average atmospheric conditions. This assumption may not represent reality, as it is possible that the atmosphere deviates from the standard conditions from night to night and even during a night of observation. This view is strengthened by the results of balloon flights made from Malargüe \cite{11}, which have shown that the atmosphere changes in a significant way both diurnally and seasonally. If a standard atmosphere is used, some of the fluctuations observed in $X_{\text{max}}$ may be attributed incorrectly to shower, rather than to atmospheric, variations. It would be instructive to see the distributions in $X_{\text{max}}$ for the two weather groups.

Questions about the interpretation are also raised when the Monte Carlo analysis of uncertainties in $X_{\text{max}}$ and the fluctuations in $X_{\text{max}}$ are considered (Perrone \cite{12}). He finds that at distances beyond 20 km, there are significant systematic shifts in the $X_{\text{max}}$ values derived and in the spread of the $X_{\text{max}}$ values. At 20 km, $X_{\text{max}}$ is estimated to be 60 g cm$^{-2}$ deeper in the atmosphere, on average, than reality and the fluctuations in $X_{\text{max}}$ for iron nuclei are considerable with $\sigma \sim 100$ g cm$^{-2}$. These factors act in such a way as to suggest that the elongation rate reported by HiRes (and presumably also by Fly’s Eye) may have been systematically over-estimated and that the fluctuations in $X_{\text{max}}$ are not due entirely to protons. Thus, it may be premature to draw conclusions about the presence of light nuclei from the analyses of fluctuations as presented so far.

A further issue of some concern is the quality of events selected for the HiRes analysis. In \cite{10} it is stated that events were selected when $\chi^2$ per degree of freedom for the fit of the longitudinal development was less than 20. This is a rather loose cut. Furthermore, the uncertainties in $X_{\text{max}}$ required to be less than 200 g cm$^{-2}$ for the fit made using both Eyes (as compared with 400 g cm$^{-2}$ for each Eye). It is hard to reconcile these numbers with the resolution of 30 g cm$^{-2}$ claimed for the measurements from Monte Carlo studies and it seems reasonable to question whether all of the events in the tails in the data of figure 2 arise from the presence of light primary particles.

3.3. Mass from muon density measurements

It is well known that a shower produced by an iron nucleus will contain a greater fraction of muons at the observation level than a shower of the same energy created by a proton primary. Many efforts to derive the mass spectrum of cosmic rays have been attempted using this fact over the full range of air shower observations. However, although the differences are predicted to be relatively large (on average there are $\sim 70\%$ more muons in an iron event than a proton event), there are large fluctuations and, again, there are differences between what is predicted by particular models. Thus, the QGSJET model set predicts more muons than the Sibyll family, the difference arising from different predictions as to the pion multiplicities produced in nucleon-nucleus and pion-nucleus collisions that in turn arise from differences in the assumptions about the parton distribution within the nucleon \cite{13}. In a contribution to these proceedings, Shinosaki has described the data on muons signals from the AGASA array. There are 129 events above $10^{19}$ eV, of which 19 have energies greater than $3 \times 10^{19}$
Figure 2. Presentation in [10] of the distribution of the measured values of $X_{max}$ for the energy intervals 1 to 2.5 EeV and 2.5 to 25 EeV. The higher energy range is in the lower plots and the data (solid lines) are compared with the QGSJET model (dashed) and the Sibyll model (dotted).

eV. Measurements of muon densities at distances between 800 and 1600 m were used to derive the muon density at 1000 m with an average accuracy of 40%. This quantity is compared with the predictions of model calculations. The difference between the proton and iron predictions is small, especially when fluctuations are considered. The AGASA group conclude that above $10^{19}$ eV the fraction of Fe nuclei is $<40\%$ at the 90% confidence level. In my view, the 5 events above $10^{20}$ eV for which such measurements are possible, are fitted as well by iron nuclei as by protons. Further, the conclusions are sensitive to the model used: as the Sibyll model predicts fewer muons than the QGSJET model, higher iron fractions would have been inferred had that model been adopted. At this meeting, Ostapchenko has discussed changes to the QGSJET model that will reduce the number of muons expected for a proton primary (see also Engel [14]). The magnitude of the effect is not yet clear but it is in such a direction as to raise the fraction of iron nuclei at the energies in question.

At lower energies, there are muon data from the Akeno array and from AGASA [15]. Different analyses have been made of these. The AGASA group claim that the measurements are consistent with a mass composition that is unchanging between $10^{18}$ and $10^{19}$ eV. Another interpretation is discussed below.

3.4. Mass estimates from the lateral distribution function

The rate of fall of particle density with distance from the shower axis provides a further parameter that can be used to deduce the mass composition. Showers with steeper lateral distribution functions (LDFs) than average will arise from showers that develop later in the atmosphere, and vice versa. A detailed measurement of the LDFs of showers produced by primaries of energy greater than $10^{17}$ eV was made at Haverah Park using a specially constructed ‘infilled array’ in which 30 additional water tanks of 1 m$^2$ were added on a grid with spacing of 150 m. When the experimental work was completed in 1978, the data could not be fitted with the shower models then available for any reasonable assumption about the primary mass. Recently [10], these data have been re-examined using the QGSJET98 model. The choice of model was justified by showing that it adequately described data on the time spread of
the Haverah Park detector signal over a range of zenith angles and distances near the core (< 500 m). Here the difference predicted between the average proton and iron shower is only a few nanoseconds and the fit achieved is good. Density data were fitted by a function $\rho(r) \sim r^{(\eta + r/4000)}$, where $\eta$ is the steepness parameter. The spread of $\eta$ was compared with predictions for different primary masses. The proton fraction, assuming a proton-iron mixture, is found to be independent of energy in the range $3 \times 10^{17}$ to $10^{18}$ eV and is (34 ± 2)%. When this fraction is evaluated with QGSJET01, in which a different treatment of diffractive processes is adopted from that in QGSJET98, then the fraction rises to 48%. The fraction is larger because the later model predicts shower maxima that are higher in the atmosphere and accordingly, to match the observed fluctuations, the proton fraction must be increased. The difference in the deduced ratio thus has a systematic uncertainty from the models that is larger than the statistical uncertainty. Although the necessary analysis has not been made, it is clear that the Sibyll 2.1 model would be consistent with an even smaller fraction of protons.

A similar analysis has been carried out using data from the Volcano Ranch array. As with the Haverah Park information, no satisfactory interpretative analysis was possible when the measurements were completed. Using 366 events for which Linsley left detailed information, and QGSJET98, the fraction of protons is estimated as $(11 \pm 5(\text{stat}) \pm 12(\text{sys}))$% at $\sim 1$ EeV \cite{17}. With QGSJET01 the flux of protons would increase to $\sim 25\%$, indicating again that model uncertainties remain a serious barrier to interpretation.

3.5. Mass from the thickness of the shower disk

The particles in the shower disc do not arrive at a detector simultaneously, even on the shower axis. The arrival times are spread out because of geometrical effects, velocity differences, and because of delays caused by multiple scattering and geomagnetic deflections. The first particles to arrive (except very close to the shower axis) are the muons as they are scattered rather little and geometrical effects dominate. At Haverah Park four detectors, each of 34 m$^2$, provided a useful tool for studying how the thickness of the shower disc depends upon the development of the cascade. Recently, an analysis of 100 events of mean energy $\sim 10^{19}$ eV has shown that the magnitude of the risetime is indicative of a large fraction (~80%) iron nuclei at this energy \cite{18}. This type of study will be considerably extended with the Pierre Auger Observatory in which the photomultipliers within each water tank are equipped with 25 ns flash ADCs.

3.6. Summary of data on primary mass above 0.3 EeV

In figure \ref{fig:3} taken from \cite{17}, the results taken from various reports of the Fe fraction are shown. It is disappointing that the data from Volcano Ranch and from Haverah Park are not in better agreement as a similar quantity, the lateral distribution function of the showers, was measured at each array and the same model - QGSJET98 - was used for interpretation, although with different propagation codes (AIRES and CORSIKA respectively). We cannot explain this difference: at $10^{18}$ eV the estimates of the fraction of Fe are separated by over 2 standard deviations.

In figure \ref{fig:3} data from the Akeno/AGASA and the Fly’s Eye experiments are also shown. The Akeno/AGASA groups measured the muon densities in showers, normalised at 600 m. The energy thresholds for Akeno and AGASA were 1 and 0.5 GeV respectively. The Fly’s Eye data are deduced from measurements of the depth of shower maximum. In an effort to reconcile differing claims made by the two groups of the trend of mass composition with energy, Dawson et al. \cite{19} reassessed the situation using a single model, SIBYLL 1.5 on both data sets. SIBYLL 1.5 was an early version of the SIBYLL family that evolved to SIBYLL 1.6 and 1.7. It is the estimates of the Fe fractions from \cite{19} that are shown in figure \ref{fig:3}. There are major discrepancies between these estimates and between those from Volcano Ranch and Haverah Park. However, the predictions of the muon density and of the depth of shower maximum made with the version of SIBYLL used in \cite{19} differ significantly from those
that would be derived now using QGSJET98 or 01 (or with the later SIBYLL version, 2.1). We now discuss this point in some detail.

An extremely useful set of comparisons of the predictions from SIBYLL 1.7 and 2.1 with those from QGSJET98 has been given in [13]. We understand that SIBYLL 1.6 and SIBYLL 1.7 differ only in that the neutral pions were allowed to interact in the latter model and it is not believed that this will make a serious difference to the predictions at energies below $10^{19}$ eV [20]. Therefore, in what follows, we regard the SIBYLL 1.7 and the QGSJET98 differences as being identical to those that exist between SIBYLL 1.6 (or 1.5) and QGSJET98, for which no similar comparisons are available. It is convenient to compare conclusions at $10^{18}$ eV. More detailed cross-checks, over a range of energies, would require more extensive knowledge of features of the Fly’s Eye and Akeno/AGASA systems than we possess.

Turning first to the data from the depth of maximum, we note that at $10^{18}$ eV the measured value of $X_{max}$ is $\sim675$ g cm$^{-2}$, with an error that is less than the size of the data point ($<10$ g cm$^{-2}$). The predictions for proton primaries made with SIBYLL 1.7 and QGSJET98 are 760 and 730 g cm$^{-2}$ respectively [13]. Thus, a mass composition less dominated by Fe is favoured compared with the $\sim90\%$ estimated in [19]. The choice of SIBYLL 2.1 would alter this argument rather little as the predicted depth at $10^{18}$ eV is 740 g cm$^{-2}$ [13]. Further study of this matter could be made but the data from Fly’s Eye has now been complemented by data from the HiRes stereo system [10] (although the differences between the data sets have not been explained) and there will also be data from the Auger instruments.

A qualitative statement about the shift expected in the Fe fraction, as estimated from the measurement of muon densities at 600 m, from changes in the model can be made using information in [13]. Although the calculations do not exactly match the energies of the Akeno/AGASA measurements ($>0.3$ GeV is computed and $>0.5$ GeV measured), ratios between the predictions of different models are not strongly dependent upon energy threshold. What is of importance is the ratio of the number of muons predicted, at $10^{18}$ eV, for SIBYLL 1.7, SIBYLL 2.1 and QGSJET98. At $10^{18}$ eV, the numbers are in the ratios 1: 1.17: 1.44. The difference in muon number between SIBYLL 1.7 and QGSJET98 is comparable to that expected between proton and Fe primaries ($\sim50\%$, but also model dependent). It is clear that the more recent models, if applied to the Akeno/AGASA data after the manner of the analysis of [19], would lead to a significant reduction in the predicted fraction of Fe nuclei. To pursue this further would require knowledge of the predicted densities at 600 m, information that is presently lacking. We note that the shift in the Fe fraction from the muon data is probably substantially larger than it is when using the data on $X_{max}$.

We are not able to use the information reported from the HiRes-MIA experiment [21] in which muons and $X_{max}$ were observed simultan-
ously. As with Akeno/AGASA, the muon density at 600 m was determined. The problem we have is that while the papers describe the data as being consistent with a mass composition that becomes lighter with energy, this appears, on close scrutiny of figures 1 and 2 of [21], to be true only for the $X_{\text{max}}$ data. The muon data, which are compared with predictions of QGSJET98, look to be consistent with a constant and heavy mass from $5 \times 10^{16}$ to beyond $10^{18}$ eV. It would be very interesting to establish that the same model gives different predictions for the mass variation with energy for different measured quantities: this might lead to further understanding of the models, or of the systematic errors in measurements of $X_{\text{max}}$, as discussed above.

This discussion is intended to demonstrate the difficulties with which one is faced with when trying to compare data. Measurements from different experiments are rarely analysed contemporaneously and the shifts in the inferences from the use of different models can be substantial.

4. Possibilities of Identifying Photon and Neutrino Primaries

4.1. Super-heavy relic particles

An idea to explain the UHECRs that have been reported beyond 100 EeV is that super-heavy relic particles with masses of $\sim 10^{12}$ GeV, produced in the early Universe, may decay to produce UHECR [22]. While details of the fragmentation of these particles remains a matter of debate, it is generally accepted that the resulting UHECR beam would contain copious fluxes of neutrinos and photons.

4.2. Limits to the fraction of photon primaries

It is unlikely that the majority of the events claimed to be near $10^{20}$ eV have photons as parents as some of the showers seem to have normal numbers of muons, the tracers of primaries that are nuclei\(^a\). (see paper in these proceedings by Shinosaki). It has been argued that the cascade profile of the most energetic Fly’s Eye event [23] is inconsistent with that of a photon primary [24]. However, in a recent paper by Risse et al. [25] (and Risse, these proceedings) have used calculations made with the QGSJET01 and SIBYLL2.1 models to show that the profile of this event can be explained under the assumption of any baryonic primary between a proton and iron nucleus, and that the primary photon hypothesis, although not favoured, cannot be rejected.

An alternative method of searching for photons has been developed using showers incident at very large zenith angles. Deep-water tanks have a good response to such events out to beyond 80°. At such angles the bulk of the showers detected are created by baryonic primaries but they are distinctive in that the electromagnetic cascade stemming from neutral pions has been almost completely suppressed by the extra thickness of atmosphere penetrated. At 80° the atmospheric thickness is $\sim 5.7$ atmospheres at sea-level. At Haverah Park, showers at such large zenith angles were observed and the shower disc was found to have a very small time spread. A complication for the study of inclined showers is that the muons, in their long traversal of the atmosphere, are very significantly bent by the geomagnetic field. A study of this has been made and it has been shown that the rate of triggering of the Haverah Park array at large angles can be predicted [26]. In addition, it was found that the energy of the primaries could be estimated with reasonable precision so that an energy spectrum could be derived. The concept of using the known, and mass independent, spectrum deduced from fluorescence detectors to predict the triggering rate as a function of the mass of the primary has led to a demonstration that the photon flux at $10^{19}$ eV is less than 40% of the baryonic component [27], a conclusion similar to that of the AGASA group, made by searching for showers which have significantly fewer muons than normal [28].

The flux of photons expected at $10^{19}$ eV from super-heavy relic particles has recently been reassessed [29] and is expected to be lower than originally predicted. This arises because of a more detailed examination of the fragmentation process. A large flux of photons is now predicted\(^a\)

\(^a\)This assumes that the photo-pion production cross-section behaves ‘normally’
at 100 EeV and can be sought in the Auger data using the method developed at Haverah Park.

4.3. Neutrino Primaries:
Neutrino primaries may be detectable by studying very inclined showers. This idea was first proposed by Berezinsky and Smirnov [30] and was re-examined in the context of the Auger Observatory by Capelle et al. [31]. A discussion of the potential of the Pierre Auger Observatory to detect such events is given in the paper by K-H Kampert in the proceedings of this meeting. A neutrino can interact anywhere in the atmosphere with equal probability.

5. The Zatsepin effect
One method capable of giving an estimate of the primary mass, that makes no assumptions about the particle physics at extreme energies, is that proposed by Zatsepin [32] in 1951. He pointed out that heavy nuclei would undergo photodisintegration in the radiation field of the sun. This is the same process, but with the 2.7 K and IR radiation fields, that would affect the spectrum of Fe nuclei at the very highest energies. The process in the solar photon field is important for $^{56}$Fe nuclei at energies of around $10^{18}$ eV and the resulting fragments ($^{55}$Fe and a neutron) would produce showers at roughly the same time and at nearly the same place in the atmosphere. In a later calculation, Gerasimova and Zatsepin [32] overlooked the effect of the interplanetary magnetic field and predicted a separation of fragments of only a few 100 metres. When the effect of the interplanetary field was included, core separations of many kilometres were estimated. The importance of Zatsepin’s idea is that the ratio of the sizes of the two correlated showers would be proportional to the mass of the fragmented primary. No assumptions about particle physics are needed.

In view of the scale of the Pierre Auger Observatory ($\sim$ 31 km radius) and the improved knowledge of the interplanetary field, a further study was made of this effect [32]. However, it was found that core separations of less than 10 km (a scale relevant to the Auger surface array and to AGASA) were infrequent (0.3 per year on Auger) and hard to detect, as the optimum energy of about $6 \times 10^{17}$ eV is rather low. However, there is a stronger signal for separations of $\sim 10^3$ km, the approximate distance between two sites being considered for the northern Auger Observatory in Utah and South Eastern Colorado: this prospect may be worthy of more detailed scrutiny.

Conclusions
To make full use of forthcoming information on the energy spectrum and arrival direction distribution at the highest energies, and to interpret what already exists, it is necessary to improve our knowledge of the mass of the cosmic rays above $10^{19}$ eV. Such evidence as there is does not support the widely adopted assumption that all of these cosmic rays are protons: there may be a substantial fraction of iron nuclei present, even at $10^{20}$ eV. Photons do not appear to dominate at $10^{19}$ eV.

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