PERTURBATIONS IN BOUNCING
COSMOLOGICAL MODELS

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Abstract
I describe the features and general properties of bouncing models and the evolution of cosmological perturbations on such backgrounds. I will outline possible observational consequences of the existence of a bounce in the primordial Universe and I will make a comparison of these models with standard long inflationary scenarios.

1 INTRODUCTION

The singularity theorems [1] show that, under reasonable physical assumptions, the Universe has developed an initial singularity, and will develop future singularities in the form of black holes and, perhaps, of a big crunch. Until now, singularities are out of the scope of any physical theory. If we assume that a physical theory can describe the whole Universe at every instant, even at its possible moment of creation, which is the best attitude because it is the only way to seek the limits of physical science (in the words of Wheeler about singularities [2], ‘physics stops, but physics must go on’) then it is necessary that the ‘reasonable physical assumptions’ of the theorems be not valid under extreme situations of very high energy density and curvature. We may say that General Relativity (GR), and/or nongravitational field theory,
must be changed under these extreme conditions. In fact, one must view such theorems as indications of the limits of GR and/or intervention of new types of matter.

The many possibilities to circumvent the theorem hypotheses (quantum effects in geometry and/or matter, non minimal couplings, curvature squared terms, etc...) have generated many cosmological models without singularities which can be classified in two types:

1) **Models with a beginning of time.**

As examples belonging to this class, one can cite the ones coming from euclidean quantum cosmology [3], and models tunneling from nothing [4], both in the framework of quantum cosmology.

2) **Eternal universes.**

One can divide these models in two subclasses.

i) *Always expanding models.*

To avoid the singularity, they must have a phase with $\ddot{a} > 0$. As examples one can cite the pre Big-Bang model [5] and the more recent emergent universe [6].

ii) *Bouncing models.*

In these models, there is a contraction phase preceding the expansion phase and a minimal value of the scale factor where $\ddot{a} > 0$ and $a(t_0) = 0$.

Within GR, as we will see in the next section, realistic bounces without a long inflationary period after it can occur only for matter contents which violate not only the SEC but also the Null Energy Condition (NEC, $\rho + p < 0$). Examples of bouncing models with these properties have as matter content quantum fields or nonlinear vector fields described by nonlinear electrodynamics [7] (which can be modelled by two fluids with $p_1 = 1/3\rho_1$, $p_2 = 5/3\rho_2$, with $\rho_2 < 0$, yielding $a(t) = a_0(1 + t^2)^{1/4}$), radiation plus a negative energy free massless scalar field [8] yielding $a(\eta) = a_0\sqrt{1 + \eta^2}$ ($\eta$ is conformal time).

Outside GR, bouncing models may be obtained without fluids violating the NEC condition in models with non minimal couplings [9], using Weyl geometries [10], with curvature squared terms [11], and branes within charged ADS black holes [12]. Bouncing models can also be obtained in string motivated tree-level actions [13] and in quantum cosmology [14].

In this contribution I will describe the features of the bouncing models and the evolution of cosmological perturbations on them. In the next section
the general properties of bouncing models will be outlined. In Section III I will describe the evolution of cosmological perturbations in such backgrounds. Finally, in the Conclusion, I will outline possible observational consequences of the existence of a bounce in the primordial Universe and I will make a comparison of these models with standard long inflationary scenarios.

2 PROPERTIES OF BOUNCING MODELS

Around the bounce, the scale factor behaves generally as

\[ a = a_0 + b \eta^{2n} + d \eta^{2n+1} + e \eta^{2n+2} + \ldots, \]  

(1)

where \( \eta \) is conformal time, \( b > 0 \) and \( n \geq 1 \).

Defining \( \beta \equiv \rho + p \) and \( \Upsilon \equiv p'/\rho' \) (which is not necessarily the sound velocity if the fluid violates the NEC condition), one can write the cosmological equations for homogeneous and isotropic geometries in the Einstein frame as

\[ H^2 + K \propto a^2 \rho, \]  

(2)

\[ \beta \propto H^2 - H' + K, \]  

(3)

where \( H \equiv a'/a \). Within GR, \( \rho \) and \( p \) are, respectively, the energy density and pressure of the matter content of the model. Outside GR, \( \rho \) and \( p \) may also depend on gravitational degrees of freedom.

The characteristics of the fluid in the bounce can be obtained by combining Eqs. (1,2,3), yielding (see Ref. [15] for details)

1) For \( n > 1 \) and \( K \neq 0 \), \( \Upsilon \propto 1/\eta^2 \) and \( \beta \propto K \).

2) For \( n > 1 \) and \( K = 0 \), \( \Upsilon \propto 1/\eta^{2n} \) and \( \beta < 0 \).

3) For \( n = 1 \), \( d \neq 0 \) and \( \forall K \), \( \Upsilon \propto 1/\eta \) and \( \beta \propto K - 2b/a_0 \).

4) For \( n = 1 \), \( \forall K \), and \( d = 0 \), \( \Upsilon = \text{const.} \) and \( \beta \propto K - 2b/a_0 \).

For a bouncing model to be realistic, it must be connected to the standard cosmological model at nucleosynthesis, where \( a \propto \eta \propto t^{1/2} \). In this case, \( 2b >> a_0 \) (see Ref.[15]) and, as one can see from above, only the fist case with \( K = 1 \) does not violate the NEC condition in GR.

As in general there is a NEC violation \( (\rho + p < 0) \) and as at nucleosynthesis the NEC is satisfied \( (\rho + p > 0) \), then there is a conformal time, \( \eta_{\text{NEC}} \), called
NEC transition time, where $\beta \equiv \rho + p = 0$. At this time one can show that, in all cases,

$$\Upsilon \propto \frac{1}{(\eta - \eta_{\text{NEC}})}.$$  \hfill (4)

Hence, in all cases, there is a divergence in $\Upsilon$ either in the bounce itself or in the NEC transition time.

Note that for non interacting fluids in GR where $p_i = \omega_i \epsilon_i$, $\omega_i = \text{const.}$, with $\epsilon = \sum_i \epsilon_i$ and $p = \sum_i p_i$ then, as a consequence of the energy-momentum conservation laws, $\rho = \rho(a)$ implying that $a(\eta)$ even, with $n = 1$ \cite{16}. Hence these classes belong to the fourth case above, with \Upsilon regular at the bounce but divergent at NEC transition.

3 THE EVOLUTION OF COSMOLOGICAL PERTURBATIONS ON BOUNCING MODELS.

The scalar perturbations of homogeneous and isotropic spacetimes are given by

$$ds^2 = a^2(\eta) \left\{ (1 + 2\phi) d\eta^2 - 2B_{ij} d\eta dx^i \right\} - \left[ (1 - 2\psi) \gamma_{ij} + 2E_{ij} \right] dx^i dx^j \right\}.$$ \hfill (5)

Using the gauge-invariant Bardeen potentials are $\Phi = \phi + [(B - E')a]/a$ and $\Psi = \psi - a'(B - E')/a$ and the splitting of the gauge invariant pressure perturbation into adiabatic and entropy components,

$$\delta p = \left( \frac{\partial p}{\partial \epsilon} \right)_S \epsilon + \left( \frac{\partial p}{\partial S} \right)_\epsilon S = \Upsilon \delta \epsilon + \tau \delta S,$$ \hfill (6)

where $\delta S$ is usually (but not always, specially for NEC violating fluids) the entropy fluctuation, the perturbed Einstein’s equations yield (we are now restricted to GR)

$$\Phi'' + 3\mathcal{H}(1 + \Upsilon)\Phi' - \Upsilon \nabla^2 \Phi + [2\mathcal{H}' + (1 + 3\Upsilon) \times (\mathcal{H}^2 - K)]\Phi = 4\pi G a^2 \tau \delta S$$ \hfill (7)
Note the presence of \( \Upsilon \) in these equations.

For ‘adiabatic’ perturbations, \( \delta S = 0 \), one obtains

\[
\Phi'' + 3\mathcal{H}(1 + \Upsilon)\Phi' - \Upsilon\nabla^2\Phi + [2\mathcal{H}' + (1 + 3\Upsilon) \times (\mathcal{H}^2 - K)]\Phi = 0.
\]

As \( \Upsilon \) diverges either at the bounce and/or in the NEC transition, \( \Phi \) diverges [15]. For scale factors behaving as \( a = a_0 + b\eta^2 + c\eta^4 \ldots \), this happens only in the NEC transition.

However as we will now show, this fact does not sign an instability of these models [16, 17], but simply indicates that entropy perturbations cannot be neglected at the NEC transition time.

As an explicit example, take two non interacting fluids satisfying \( p_1 = \omega_1 \rho_1 \), \( p_2 = \omega_2 \rho_2 \) and define

\[
s \propto \left( \frac{\delta \rho_1}{\rho_1(1 + \omega_1)} - \frac{\delta \rho_2}{\rho_2(1 + \omega_2)} \right).
\]

One can show, using the perturbed Einstein’s equations, that \( s \) and \( \Phi \) satisfy

\[
s'' + \mathcal{H}(1 - 3c_z)s' + k^2c_zs = \frac{k^2}{\beta}(k^2 - 3K)\Phi,
\]

\[
\Phi'' + 3\mathcal{H}(1 + \Upsilon)\Phi' - \Upsilon\nabla^2\Phi + [2\mathcal{H}' + (1 + 3\Upsilon) \times (\mathcal{H}^2 - K)]\Phi = \frac{f(a)}{\beta}s,
\]

where, as before, \( \beta = \rho + p \).

Inspection of Eq. (9) shows that entropy fluctuations cannot be neglected, even for arbitrarily small but non vanishing values of the wavelength \( k \), at the NEC transition time as long as its source term diverges there. Hence, adiabatic perturbations cannot be defined at this point and the divergence detected in Eq. (7) for \( \delta S = 0 \) is not physically meaningful. In fact, taking the two equations together, one can show that there are no divergences in the full Bardeen potential [16]. Hence, in the NEC transition time, entropy
fluctuations are crucial. In the bounce itself, the same sort of process may happen.

The evolution of cosmological perturbations in bouncing models has other characteristic features. In general, bouncing models do not have particle horizons. Hence, as in inflationary models, one can impose reasonable physical initial conditions for the perturbations, contrary to the Standard Cosmological Model without inflation, where initial conditions are arbitrary due to the lack of physical causal interactions when perturbations begin to evolve. In the far past of general bouncing models the Universe is almost flat. Hence, one can impose vacuum initial conditions for the perturbations without any transplanckian problem based on simple quantum field theory in flat space, yielding a quantum mechanical origin for them.

In general, after performing suitable changes of variables like $u = g(a)\Phi$, the perturbation equations can be put in the simple form $u'' + [\Upsilon k^2 - V(a)]u = 0$ in power law cosmologies, $V(a) \propto a^2/l_H^2$, where $l_H$ is the Hubble radius. and hence the transition from oscillatory regimes ($k^2 > V(a)$) to growing and decaying regimes ($k^2 < V(a)$) through potential crossing is equivalent to horizon crossing as long as $k^2 \approx V(a)$ is equivalent to $\lambda_{\text{phys}} \approx l_H$, where the physical wavelength $\lambda_{\text{phys}}$ is given in terms of the comoving wavelength $\lambda$ through $\lambda_{\text{phys}} = a\lambda$ and $k \propto 1/\lambda$. In bouncing models, it is definitely not true [8, 18], and what is relevant is the potential crossing.

Another important remark is that, defining the transfer matrix as $\tilde{A}_+ = \mathbf{T} \tilde{A}_-$, with $\tilde{A} \equiv (A_d(k), A_s(k))$, $\Phi = A_d(k)f_d(k, \eta) + A_s(k)f_s(k, \eta)$, where the indices $(+, -)$ refers to (after, before) the bounce and the indices $(d, s)$ refers to (dominant, subdominant) modes, respectively, then, contrary to intuition even for very short bounces, $\mathbf{T}$ may depend on $k$. A consequence of this is that matching conditions through a bounce must be treated with care, there are no general rules, and analysing in detail each particular case is preferable, without risks of mistakes.

Bounces can also magnify perturbations which oscillates after it, and bouncing models can be constructed in which a scale invariant spectrum for large wavelengths can be obtained [8].

For gravitational waves, the spectrum is much more complicated then in inflationary models and highly model dependent, yielding different possible observational predictions [18, 19].
4 CONCLUSIONS.

Bouncing models appear in many instances of physics. They have no singularities, no horizon problem, and they may yield a causal explanation and quantum origin of structures in the Universe, with possible scale invariant spectrum. If the bounce occurs below Planck energies, there is no transplanckian problem [20].

In realistic models within GR without inflation, bounces occur only with violation of the NEC. In the NEC transition time, entropy fluctuations are important and cannot be neglected at all.

Many general claims about perturbations on bouncing models, specially concerning matching conditions through it, were proven to be erroneous due to counter examples which have been found [8, 18]. Hence, it is safer to study specific well motivated models.

Some possible observational consequences of a bounce are
a) Oscillations in the primordial spectrum.

b) Different effects in the polarization due to gravitational waves.

These effects taken together may discriminate bounces from many-fields inflation and transplanckian physics.

Bouncing models may solve issues of inflationary models like the singularity and transplanckian problem. However, with respect to initial conditions issues like the flatness problem and the isotropization problem, bouncing models are silent 1. Perhaps they must be complemented with quantum cosmological ideas, or be joined with a long inflationary phase after the bounce, adding the good features of both. But one must take care: plausible vacuum initial conditions in the pre bouncing phase may not be transferred to vacuum initial conditions at the onset of inflation, a necessary condition to get a Harrison-Zeldovich spectrum.

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\footnote{1About the homegeneization problem, both bouncing models and inflation are silent}
References


