Phenomenological Consequences of Soft Leptogenesis

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Abstract

Soft supersymmetry breaking terms involving heavy singlet sneutrinos can be the dominant source of leptogenesis. The relevant range of parameters is different from standard leptogenesis: a lighter Majorana mass, $M \lesssim 10^9$ GeV (allowing a solution of the gravitino problem), and smaller Yukawa couplings, $Y_N \lesssim 10^{-4}$. We investigate whether the various couplings of the singlet sneutrinos, which are constrained by the requirement of successful ‘soft leptogenesis’, can have observable phenomenological consequences. Specifically, we calculate the contributions of the relevant soft supersymmetric breaking terms to the electric dipole moments of the charged leptons and to lepton flavor violating decays. Our result is that these contributions are small.

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I. INTRODUCTION

The discovery of neutrino masses and mixing makes the see-saw mechanism [1, 2, 3, 4] highly attractive. The existence of heavy singlet neutrinos with Majorana masses $M$ and Yukawa couplings $Y_N$ to the active neutrinos becomes very likely. This framework can also dynamically generate the baryon asymmetry of the universe by Leptogenesis [5]. The supersymmetric extension of this model (SSM+N) is well motivated because it protects the Higgs mass from large radiative corrections involving the heavy neutrino. Supersymmetry must be broken, hence the SSM+N includes soft supersymmetry breaking terms for the SSM fields and for the singlet sneutrino field $\tilde{N}$. The singlet sneutrino soft terms, in turn, can affect leptogenesis and even give the dominant contribution for low enough masses ($M \lesssim 10^9$ GeV) and small Yukawa couplings ($Y_N \lesssim 10^{-4}$) [6, 7, 8].

In general, only the lightest singlet (s)neutrino contributes to leptogenesis, since its interactions wash out any lepton asymmetry created by the heavier singlet sneutrinos. However, this lightest singlet (s)neutrino, even with $M \sim \mathcal{O}(1$ TeV), is practically impossible to discover at colliders, since the Yukawa couplings need to be small enough in order for the decay to be out of equilibrium. Singlet (s)neutrinos contribute, however, to electric dipole moments (EDM) of leptons, $d_\ell$, and to lepton flavor violating (LFV) interactions $Br(\ell_i \rightarrow \ell_j \gamma)$ and $R(\mu \rightarrow e$ in Ti). These measurements already constrain various SSM soft breaking terms [15, 16], especially their phases which should be $< \mathcal{O}(10^{-2})$ to be consistent with the non observation of EDMs [17] (however, this limit is significantly weakened if $m_{SUSY} \sim 1$ TeV [18]). Much work has been devoted to the possible connection between leptogenesis and EDM and LFV measurements [19, 20, 21, 22, 23, 24, 25].

In this paper, we briefly describe the SSM+N model in section II. Then we estimate the contribution of soft breaking terms that can induce leptogenesis to $d_\ell$ in section III and to the LFV interactions $Br(\ell_i \rightarrow \ell_j \gamma)$ and $R(\mu \rightarrow e$ in Ti), in section IV. We conclude in section V. Our main result is that soft leptogenesis gives small low-energy phenomenological effects, so that present and near future experiments of EDM and LFV do not constrain it.

In our previous works [6, 8], we considered for simplicity a one generation model. Since soft leptogenesis constrains only the couplings of the lightest sneutrino, $\tilde{N}_1$, we only consider its contribution to the EDM and to LFV interactions and not all $\tilde{N}_i$ contributions.
II. THE MODEL

The relevant superpotential terms of the SSM+N are

\[ W = Y_N L H_u \tilde{N} + M \tilde{N} \tilde{N} + Y_L L H_d \tilde{E} + \mu H_u H_d , \]  

(1)

where \( L \) is the supermultiplet containing the left handed lepton, \( N, E \) are the SU(2)-singlet superfields of the neutrino and charged lepton respectively, and \( H_u, H_d \) are the Higgs superfields. The relevant soft breaking terms are

\[ -\mathcal{L}_{\text{soft}} = B \tilde{N} \tilde{N} + m_2 \tilde{W}^a \tilde{W}^a + m_L^2 \tilde{L} \tilde{L} + A_N L H_u \tilde{N}^\dagger + A_L L H_d \tilde{E}^\dagger + \text{H.c.} . \]  

(2)

Here \( \tilde{W}^a \) \( (a = 1, 2, 3) \) are the \( SU(2)_L \) gauginos, \( \tilde{N}, \tilde{L}, H_u, H_d \) are scalar fields (and \( N, L, h_u, h_d \) are their fermionic superpartners). For a one generation model the Lagrangian derived from eqs. (1) and (2) has three independent physical CP violating phases:

\[ \phi_N = \text{arg}(A_N Y_N^* M B^*) , \]
\[ \phi_W = \text{arg}(m_2 Y_N A_N^*) , \]
\[ \phi_L = \text{arg}(A_L Y_L^* M B^*) . \]  

(3)

After spontaneous symmetry breaking, another phase is added:

\[ \theta = \text{arg}(m_2 \mu v_u v_d) , \]  

(4)

where \( v_i \), for \( i = u, d \) are the vacuum expectation values of the Higgses, which are complex in general. In [6, 8] we investigated the contributions of \( \phi_N \) and \( \phi_W \) to leptogenesis.

III. THE EDM OF THE ELECTRON

The CP violation in lepton interactions, that is necessary to generate leptogenesis, is likely to induce electric dipole moments for the charged leptons (for a review on EDMs, see [26]). We estimate the size of these contributions, and compare them to experiment results. The present bounds on the EDM of charged leptons are currently

\[ d_e < 1.5 \times 10^{-27} \text{ e cm} , \]
\[ d_\mu < 2.8 \times 10^{-19} \text{ e cm} , \]
\[ d_\tau < 3 \times 10^{-16} \text{ e cm} . \]  

(5)
FIG. 1: The two leading contributions to the EDM of the electron from sneutrino mixing.

The near future expected sensitivities are

\[ d_e < 10^{-33} \text{ e cm} \] 30,  
\[ d_\mu < 10^{-26} \text{ e cm} \] 31 .  

(6)

We use here the mass eigenstate formulas of [32] to calculate the one loop contribution to the electron EDM using the mass basis of the fields. Given an interaction of a lepton \( \ell \) with a sneutrino \( \tilde{\nu} \) and a chargino \( \chi \) of the form

\[
\mathcal{L} = - \sum_{ija} \bar{\ell} (C_{L\alpha}^{x} P_{L} + C_{R\alpha}^{x} P_{R}) \chi_{a} \tilde{\nu}_{x} + \text{h.c.} ,
\]

(7)

where \( \ell = e, \mu, \tau \), one obtains the following one-loop contribution to \( d_e \):

\[
d_e = \frac{m_{\chi}}{16\pi^{2}m_{\tilde{\nu}}^{2}} I_{4} \left( \frac{m_{\chi}}{m_{\tilde{\nu}}^{2}}, \frac{m_{\chi}}{m_{\tilde{\nu}}^{2}} \right) \text{Im} (C_{L\alpha}^{x} C_{R\alpha}^{x}) ,
\]

(8)

where

\[
I_{4}(r, s) = \int_{0}^{1} dx \frac{x^{2}}{1 - x - rx - sx(1 - x)} .
\]

(9)

The two leading contributions that involve the interactions of the singlet sneutrinos come from the diagrams of fig. 1. In the sneutrino mass matrix, we neglect terms of order \( |v_u \mu Y_N/M^2| \), assume the hierarchy \( |v_u A_N|, |v_u M Y_N| \ll |B| \ll |M|^{2} \), and write the sneutrino mass eigenstates up to order \( |B v_u (A_N \pm M Y_N)/M^4| \). We obtain:

\[
d_e \approx \frac{Q_e |m_e Y_N| \tan \beta}{16\pi^{2} m_{\tilde{\nu}}^{2}|M|^{2}} \times \]

\[
\left\{ |A_N| m_{\chi_{1}} I_{\chi_{1}} \sin \theta_{U} \sin \theta_{V} \sin \left( \varphi_{n} + \frac{\varphi_{U} - \varphi_{V}}{2} \right) + m_{\chi_{2}} I_{\chi_{2}} \cos \theta_{U} \cos \theta_{V} \sin \left( \varphi_{n} + \frac{\varphi_{U} + \varphi_{V}}{2} \right) \right\} 
\]

\[
+ \left| B Y_N \right| M \left| m_{\chi_{1}} I_{\chi_{1}} \sin \theta_{U} \sin \theta_{V} \sin \left( \varphi_{n} + \frac{\varphi_{U} - \varphi_{V}}{2} \right) + m_{\chi_{2}} I_{\chi_{2}} \cos \theta_{U} \cos \theta_{V} \sin \left( \varphi_{n} + \frac{\varphi_{U} + \varphi_{V}}{2} \right) \right\}
\]

(10)
where $Q_e = -1$ is the electric charge and

$$I_{\chi_i} = I_4 \left( \frac{m_{\chi_i}^2}{m_{\tilde{\nu}}^2}, \frac{m_{\tilde{\nu}}^2}{m_{\tilde{\nu}}^2} \right) \sim I_4 \left( \frac{m_{\chi_i}^2}{m_{\tilde{\nu}}^2}, 0 \right).$$

where $I_4(r,s)$ defined in eq. (9). Here $m_{\tilde{\nu}}^2$ is the mass-squared of the light sneutrino, $\chi_{1,2}$ are the two chargino mass eigenstates, $\theta_U$ and $\varphi_U$ ($\theta_V$ and $\varphi_V$) are the mixing angle and phase in the matrix that transforms between the mass eigenbasis and interaction eigenbasis of the charge $-1(+1)$ charginos. A strong suppression arises from singlet-doublet sneutrino mixing. It is given by

$$-v_u^* A_N^* + \frac{2B^* v_u^* M Y_N^*}{|M|^2}.$$  

These terms contribute to $\varphi_n$ and $\varphi'_n$. For $m_\chi \sim \mu$:

$$\varphi_n \approx \phi_W - \theta \quad \varphi'_n \approx \phi_W - \theta + \phi_N$$

However, for a general $m_\chi$, the CP violating phases are more complicated combinations of $\phi_W$, $\phi_N$, $\theta$, and $\varphi_{U,V}$, where the latter is the only CP violating phase that appears in $\varphi_{U,V}$. The $A_N$ contribution is analogous to the $A_u$-chargino contribution to the EDM of the up quark, with the replacement $N \leftrightarrow d$ and $L \leftrightarrow u$.

In our framework, $\theta_{U,V} = O(1)$. The result is then

$$|d_e| \approx \frac{em_e \tan \beta}{16\pi m_{\tilde{\nu}}^2} \left| \frac{m_\chi Y_N}{2M^2} \right| \left( |A_N| + \frac{|B Y_N^*|}{M} \right) \lesssim 6 \times 10^{-35} \text{ e cm}.$$  

We use here $A_N/Y_N \sim m_\chi \sim m_{\tilde{\nu}} \sim m_{\text{SUSY}}$, $M > 1 \text{ TeV}, \tan \beta < 100$, and the bound from leptogenesis for out of equilibrium decay:

$$\frac{Y_N^2}{M} \lesssim 3 \times 10^{-15} \text{ GeV}^{-1}.$$  

The contribution of $B$ is significant only if $M \lesssim 10^5 \text{ GeV}$, when $B$ can assume its natural value, $B \sim M m_{\text{SUSY}}$, since only then it has no restriction from soft leptogenesis. Note that for large $M$, $M \sim 10^9 \text{ GeV}$, there is a two loop contribution of the same order, where $B$ generates an $A_L$ term through 1-loop diagrams. For such a high scale for $M$, however, all contributions are much smaller than the estimate of due to the large sterile sneutrino mass. We conclude that the upper limit on the contributions to the EDM of the electron related to $A_N$ and $B$ is well below the sensitivities of current and future experiments.
Assuming that $\tilde{N}_1$ has comparable mixing with all three flavors of $\tilde{\nu}$, we can estimate the contribution to $d_\mu$ ($d_\tau$) by simply replacing $m_e$ with $m_\mu$ ($m_\tau$) in eq. (13). The result is:

\[ d_\mu \lesssim 10^{-32} \text{ e cm}, \]
\[ d_\tau \lesssim 2 \times 10^{-31} \text{ e cm}. \]  

These bounds are far too strong even for the next generation of experiments.

IV. LEPTON FLAVOR VIOLATION OF CHARGED LEPTONS

The Yukawa interactions of the sterile sneutrino contribute to $\text{Br}(\ell_i \to \ell_j \gamma)$ in two ways: through the RGE of the slepton masses and through two diagrams similar to fig. but replacing the incoming (outgoing) electron with $\ell_i$ ($\ell_j$) and $\tilde{\phi}^+_u$ with Wino. We use here the formulas of \cite{9} to calculate both contributions, estimate their size, and compare them to experimental current limits,

\[ \text{Br}(\mu \to e\gamma)_{\text{exp}} < 1.2 \times 10^{-11}, \]
\[ \text{Br}(\tau \to e\gamma)_{\text{exp}} < 3.7 \times 10^{-7}, \]
\[ \text{Br}(\tau \to \mu\gamma)_{\text{exp}} < 3.1 \times 10^{-7}. \]  

and near future expected sensitivities,

\[ \text{Br}(\mu \to e\gamma)_{\text{future}} < 10^{-14}, \]
\[ \text{Br}(\tau \to e\gamma)_{\text{future}} \lesssim 10^{-8}, \]
\[ \text{Br}(\tau \to \mu\gamma)_{\text{future}} \lesssim 10^{-8}. \]  

We assume universality of the soft breaking terms at the Planck scale. Only in the framework of universality, flavor changing couplings come only from $Y_N$ and $A_N$. In a general SSM, there are arbitrary flavor changing mass-squared terms for sleptons already at high scale, and their contributions cannot be separated from the contribution of $Y_N$ and $A_N$. Thus only in models of universality, there is a correlation between LFV observables and leptogenesis parameters. We use here renormalization group equation (RGE) to find the low scale values of the soft breaking terms.
The leading contribution to the branching ratio comes from gaugino interactions, and is denoted here by $\text{Br}(\ell_i \to \ell_j \gamma)_g$. It depends on the off-diagonal elements of the doublet slepton mass-squared matrix as follows:

$$\text{Br}(\ell_i \to \ell_j \gamma)_g = \frac{\alpha m_{1/2}^5}{(8\pi)^2 m_{\tilde{\nu}}^2} \left[ \frac{g^2}{m_{\tilde{\nu}}^2} \left( \frac{m_{\chi}^2}{m_{\tilde{\nu}}^2} \right) \right]^2 \sim \frac{\alpha^3 |m_{Lij}|^2}{G_F m^8} \tan^2 \beta. \quad (18)$$

where the last expression is the known approximation and

$$f_1(x) = \frac{2 + 3x - 6x^2 + x^3 + 6 \ln x}{6(1 - x)^4}.$$

We take the off-diagonal elements of the slepton mass matrix $m_{Lij}^2$ to vanish at the Planck scale, and use RGE to evolve it to low energy. The relevant one-loop RGE is

$$8\pi^2 \mu \frac{d}{d\mu} (m_{L}^2)_{ij} = (m_{L}^2 + m_{N}^2 + m_{H_2}^2)(Y_N^\dagger Y_N)_{ij} + (A_N^\dagger A_N)_{ij}. \quad (19)$$

Assuming that $|A_N| \sim |Y_N m_{\text{SUSY}}|$, one gets $(m_{L}^2)_{ij}/m_{\text{SUSY}}^2 \sim Y_N^2$ for $i \neq j$. From soft leptogenesis we can estimate:

$$(Y_N^\dagger Y_N)_{11} = \sum_{\ell} |(Y_N)_{\ell 1}|^2 < 10^{-8} \Rightarrow |Y_N|_{11} \lesssim 10^{-4}. \quad (20)$$

This gives $(m_{L}^2)_{ij}/m_{\text{SUSY}}^2 \lesssim 10^{-8}$ with $i \neq j$, and consequently

$$\text{Br}(\mu \to e\gamma)_g \lesssim 10^{-21},$$

$$\text{Br}(\tau \to e(\mu)\gamma)_g \sim \frac{m_{\tau}^5}{m_{\mu}^5} \text{Br}(\mu \to e\gamma)_g \lesssim 2 \times 10^{-15}. \quad (21)$$

where we used $m_{\text{SUSY}} \sim 100$ GeV to maximize the possible effects. Note that this contribution does not involve CP violation. Comparing this result to (16, 17), we conclude that this contribution is unobservably small. The main reason is that $\text{Br}(\mu \to e\gamma) \propto Y_N^4$, and soft leptogenesis requires a very small Yukawa coupling $Y_N$ (see (20)).

The direct contribution of the sneutrino to $\mu \to e\gamma$ involves Yukawa couplings instead of one of the gauge interactions, and is denoted here as $\text{Br}(\mu \to e\gamma)_Y$. Using the expressions of [9] and considering, for simplicity, a two generation model of neutrinos, we get
for $\tilde{N}_1$ contribution

$$Br(\mu \to e\gamma)_Y = \frac{\alpha m^5_\mu}{\tau^1_\mu} \left[ \frac{g^3 \sin 2\theta_Y}{2(4\pi)^4 m^2_\tilde{\nu}} \right] \frac{m^2_{L\mu e}}{m^2_{SU3Y} \sqrt{2}|M|^2} \frac{1}{(\cos \theta_{\tilde{\nu}} - \sin \theta_{\tilde{\nu}})}$$

$$\times \text{Re} \left[ (f_{x2} e^{i\varphi_Y} - f_{x1} e^{-i\varphi_Y}) \left( -v^*_u A^*_N Y^*_{N\mu} \right. \right.$$  
$$\left. + \frac{2B^* v^*_u M}{|M|^2} \sum_{i,x} (Y^*_N Y_{Nix}) \right)$$  

$$+ (f_{x2} e^{-i\varphi_Y} - f_{x1} e^{i\varphi_Y}) \left( -v_u A_N Y^*_{N\mu} \right.$$  
$$\left. + \frac{2B v_u M^* \sum_{i,x} (Y^*_N Y_{Nix})}{|M|^2} \right) \right]$$

\[ \lesssim 4 \times 10^{-30} \] (22)

where $f_{x_i} = f_i \left( m^2_{\chi_i}/m^2_{\tilde{\nu}_x} \right)$, and $\theta_{\tilde{\nu}}$ is the mixing angle between the light generations. The CP violating phases are $\theta$ and generalizations of $\phi_W$ and $\phi_N$ for the case of a two generations model. Here we approximated $M_2 \sim M_1$, $m^2_{\tilde{\nu}_2} \sim m^2_{\tilde{\nu}_1}$ and $B_{ij} \propto M_{ij}$. Note that (22) also depends on $Y^4_N$, similar to (18). This contribution is much smaller than (21), since it is inversely proportional to $M^2$, which enables us to use the stricter bound of (14) instead of (20). We learn that both contributions are much smaller than present limits and future sensitivity.

In the SSM+N, the gaugino contribution to $\mu \to e\gamma$ is almost always dominant in the $\mu \to e$ conversion, and hence there is a relation between the predicted $Br(\mu \to e\gamma)$ and $R(\mu \to e \text{ in Ti})$:

$$\frac{R(\mu \to e \text{ in Ti})}{Br(\mu \to e\gamma)} \approx 5 \times 10^{-3} ,$$  (23)

where $R(\mu \to e \text{ in Ti}) \equiv \Gamma(\mu^{-Ti} \to e^{-Ti} \text{ g.s.})/\Gamma(\mu^{-Ti} \text{ capture})$ (g.s. stands for ground state). The current limit [38] and near future expected sensitivity [39] are

$$R(\mu \to e \text{ in Ti})_{\text{exp}} < 6.1 \times 10^{-13} ,$$

$$R(\mu \to e \text{ in Ti})_{\text{future}} < 10^{-18} .$$  (24)

We learn from (21) that our estimate,

$$R(\mu \to e \text{ in Ti}) \lesssim 6 \times 10^{-25} ,$$  (25)

is below the near future expected sensitivity.

V. CONCLUSIONS

In this work we estimated the contribution to low energy observables of soft supersymmetry breaking terms within the range that leads to successful soft leptogenesis. The
direct contribution of \( \tilde{N} \) to both the EDM and \( Br(\ell_i \rightarrow \ell_j \gamma) \) is inversely proportional to \( M^2 \). Therefore, the lighter is \( M \), the less suppressed is the singlet sneutrino contribution. One may think therefore that soft leptogenesis, which requires \( M \) to be lighter than standard leptogenesis, can have more significant phenomenological consequences. However, both the electron EDM and the branching ratio of \( \ell_i \rightarrow \ell_j \gamma \) strongly depend on \( Y_N^2 \):

\[
d_e \propto Y_N^2, \quad Br(\ell_i \rightarrow \ell_j \gamma) \propto Y_N^4.
\]  

(26)

Since the Yukawa couplings of \( \tilde{N}_1 \) must be small, \( Y_N \lesssim 10^{-4} \), for out of equilibrium decay \[14\], these contributions are strongly suppressed. We learn that the contribution of soft supersymmetry breaking terms that induce soft leptogenesis to low energy observables is much smaller than other contributions in the SSM.

The analysis here is model independent, in the sense that we do not use any flavor model for the structure of the Yukawa matrix. We considered only \( \tilde{N}_1 \) contributions, and the constraints from successful soft leptogenesis on \( M_1 \), \( Y_{N1k}^2 \) and soft breaking terms of \( \tilde{N}_1 \). The contributions to EDMs and LFV processes from the heavier singlet sneutrinos are not directly constrained by leptogenesis, and therefore can be larger. \(^1\)

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\(^1\) In \[40\], GUT models are used to find \( Y_N \) and \( M \) for the heavier singlet neutrinos. Then soft leptogenesis as the dominant source of the baryon asymmetry and \( Br(\mu \rightarrow e\gamma) \) within the sensitivity of future experiments can be simultaneously obtained. However, the dominant contribution to the LFV interactions there comes from heavier sneutrinos and not from \( \tilde{N}_1 \).


[28] H. Deng, for the Muon $(g – 2)$ Collaboration, Talk presented at WIN-03, October 2003,
[39] PRISM Collaboration, letter of intent to the J-PARC experiment (L25),