Unification of Gravitation, Gauge Field and Dark Energy

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Abstract

This paper is composed of two correlated topics: 1. unification of gravitation with gauge fields; 2. the coupling between the daor field and other fields and the origin of dark energy. After introducing the concept of “daor field” and discussing the daor geometry, we indicate that the complex daor field has two kinds of symmetry transformations. Hence the gravitation and SU(1,3) gauge field are unified under the framework of the complex connection. We propose a first-order nonlinear coupling equation of the daor field, which includes the coupling between the daor field and SU(1,3) gauge field and the coupling between the daor field and the curvature, and from which Einstein’s gravitational equation can be deduced. The cosmological observations imply that dark energy cannot be zero, and which will dominate the doom of our Universe. The real part of the daor field self-coupling equation can be regarded as Einstein’s equation endowed with the cosmological constant. It shows that dark energy originates from the self-coupling of the space-time curvature, and the energy-momentum tensor is proportional to the square of coupling constant $\lambda$. The dark energy density given by our scenario is in agreement with astronomical observations. Furthermore, the Newtonian gravitational constant $G$ and the coupling constant $\epsilon$ of gauge field satisfy $G = \lambda^2 \epsilon^2$.

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1 Introduction

Recent astronomical observations on Type Ia supernovae [1, 2, 3] and the cosmological microwave background radiation [4, 5, 6] indicate that our universe is spatially flat and accelerating at present, which supports for a concordant cosmological model of inflation + cold dark matter + dark energy. Furthermore all updated cosmological observations do not conflict with a group of cosmological parameters, namely $(\Omega_\Lambda, \Omega_M) \approx (0.7, 0.3) [7]$, which implies that the doom of our universe will be dominated by dark energy [8]. The properties of dark energy and its equation of state [9] have evoked much controversy in both astronomy and particle physics communities [10, 11]. In this paper we find that dark energy originates from the self-coupling of the space-time curvature. Firstly we will explain why we must introduce the daor field in trying to unify gravitation with other interactions.

In the history there have been attempts to unify gravitation with other interactions, notably the Kaluza-Klein approach of compactifying higher dimensional space, and the Einstein-Strauss-Schrödinger [12, 13] approach of considering a Hermitian metric tensor and interpreting the antisymmetric field as that of the Maxwell field strength. The Kaluza-Klein approach has been widely developed into modern extra-dimensional theories, e.g., ADD scenario [14] and Randall-Sundrum models [15], while the scenarios of the complex space-time metric are less familiar [16]. It is now well known that the antisymmetric part of the Hermitian metric can not be explained as the electromagnetic field strength but rather as an antisymmetric tensor where the theory is consistent only if the field is massive [17]. Following the method of complexified geometry, Penrose proposed the twistor theory [18] to study the quantization of gravitation in 1970s. In modern physics there are mainly two popular paradigms on the unification of gravitation and quantum theory, that is, the string theory [19] and the loop quantum gravity [20]. The string theory is constructed on a 10-dimensional space-time (11-dimensional space-time for M theory). Therefore there are enough extra dimen-
sions to accommodate the gauge fields. The most challengeable problem in the string theory is how to compactify the extra dimensions to give observable predictions. The loop quantum gravity theory originates from Ashtekar’s re-expression [21] of general relativity, and inherits the geometric viewpoint from general relativity. The loop quantum gravity has yielded several interesting results, such as the discrete spectrum of the volume operator.

Half a century ago, Yang and Mills constructed the non-Abelian gauge field theory [22]. Based on the Yang-Mills theory and the Higgs mechanism [23], Glashow, Salam and Weinberg set up a renormalizable electroweak gauge theory [24]. Combining this theory with quantum chromodynamics yields the so-called $SU(3)_C \times SU(2)_L \times U(1)_Y$ standard model of particle physics. Recently, it is claimed that a great unified theory (GUT) endowed with SO(10) gauge group can give the neutrino masses and mixing [25] which are not contradictory to modern experiments [26]. If physicists believe that the standard model or GUT should be a low-energy approximation of a high energy unified quantum theory which incorporate gravitation with gauge fields, then those inner symmetry such as $SU(3)_C, U(1)_Y$ should be reasonably accommodated in a higher energy theory. In this paper, we propose a possibility to give those inner symmetries without introducing extra-dimensions. Hence in our framework there is no difficulty on compactification. After introducing the complex “doar field” and discussing the daor geometry, we obtain a complex spin connection in which the space-time connection and SU(1,3) gauge field are combined together. In the paper [27], we have argued that quantizing gravitation needs to reformulate Einstein’s equation such that the new formalism must at least have the properties of a complex field and the first-order differential equation. Therefore we propose a first-order nonlinear equation for daor field, and prove that Einstein’s equation can be deduced from it. The real part of the daor field self-coupling equation is equivalent to Einstein’s equation with the cosmological constant term. It shows that dark energy originates from the self-coupling of the space-time curvature.
The paper is organized as follows. In section 2 basic principles are given, from which the complex doar field appears naturally. The following section is devoted to the discussion of the doar geometry, where the complex connection is acquired. In section 4 the unification of gravitation with SU(1,3) gauge field is implemented, and a first-order nonlinear field equation is given, from which Einstein’s equation can be deduced. The origination of dark energy is discussed in section 5. Conclusions and final remarks are presented in the final section 6.

2 Basic Principles

In this paper, let us suppose an ideal universe, in which there are no fermion fields. Einstein’s gravitational equation with the cosmological constant term can be written as \[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu} , \tag{2.1} \]
where $G$, $\Lambda$ are the Newtonian gravitational constant and the cosmological constant respectively, and $T_{\mu\nu}$ is the total energy-momentum tensor, in which the energy-momentum tensor of gauge fields is denoted by $T^g_{\mu\nu}$. In the case of electromagnetic field, $T^\text{em}_{\mu\nu}$ is given by
\[ 4\pi T^\text{em}_{\mu\nu} = \frac{1}{4} g_{\mu\nu} f_{\alpha\beta} f^{\alpha\beta} - f_\mu^\alpha f_{\nu\alpha} , \tag{2.2} \]
where $f_{\alpha\beta}$ is the strength of electromagnetic field. The sign convention adopted in this paper is the same as in Misner-Thorne-Wheeler’s book[28]. The metric tensor of Minkowski space-time $\eta_{ab}$ is written as follows
\[ \eta^{00} = -1 , \quad \eta^{11} = \eta^{22} = \eta^{33} = +1 , \quad \eta^{ab} = 0 \text{ for } a \neq b . \tag{2.3} \]

We have proposed the following two basic principles in the paper [27]:

\[ \text{In this paper, we use the summation convention of Einstein: if an index occurs as both a subscript and superscript in the same term, then the term is summed over the range of the repeated index, and the summation sign is omitted. Using Roman suffixes to refer to the bases of local Minkowski frame; using Greek suffixes to refer to curvilinear coordinates of space-time.} \]
1. Physical space-time is a 3 + 1 manifold, which looks like a Minkowski space-time around each point.

2. The intrinsic distance
\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad \mu, \nu = 0, 1, 2, 3, \] (2.4)
keeps invariant under any physical transformations. One kind of local transformations keeping \( ds^2 \) invariant is corresponding to one kind of physical interactions.

In the literature, decomposing the metric into vierbeins or tetrads \( e^a_\mu(x) \) has been adopted extensively [29, 30, 31]. The vierbein decomposition of the space-time metric is expressed as
\[ g_{\mu\nu} = \eta_{ab} e^a_\mu(x) e^b_\nu(x), \] (2.5)
which keeps \( ds^2 \) invariant under local SO(1,3) group transformation. But it is not the largest symmetry group as we will show. Here we give the larger symmetry group transformations which keep \( ds^2 \) invariant. To do so, we introduce the complex vierbein (or tetrad) [32] field \( k^a_\mu \) or \( K^a_\mu \), which satisfies
\[ 2g_{\mu\nu} = \bar{k}^a_\mu \eta_{ab} k^b_\nu + k^a_\mu \eta_{ab} \bar{k}^b_\nu, \] (2.6)
\[ 2G^{\mu\nu} = \bar{K}^a_a \eta^{ab} K^b_\nu + K^a_a \eta^{ab} \bar{K}^b_\nu, \] (2.7)
\[ g_{\mu\nu} G^{\nu\lambda} = G^{\lambda\nu} g_{\nu\mu} = \delta_\mu^\lambda, \] (2.8)
where bar denotes complex conjugation, the matrices \( g, G, k \) and \( K \) are all nonsingular. It is obvious that the matrix \( g \) is still real and symmetrical in Eq.(2.6).

We have stressed in the introduction that the object studied in this paper is the complex vierbein. To embody the linking property of \( k^a_\mu \) between matter fields and the space-time structure, also between gravitation and gauge interactions, here we suggest giving the complex vierbein (or tetrad) field \( k^a_\mu \) a new name, daor field‡.

‡“Dao” is a basic and important concept in ancient Chinese philosophy. “Dao” is used to refer not only the unobservable existence from which everything originate but also the laws which dominate the doom of everything. “Dao” is also used to demonstrate the abstract relationship between the dual things such as “Yin” and “Yang”, nihility and existence. Because \( k^a_\mu(x) \) plays such a similar role in physics as will be shown in the following sections, we suggest calling \( k^a_\mu(x) \) the “daor field”.

5
As the space-time metric is real and symmetrical in general relativity, there are at most 10 free-parameters in $g_{\mu\nu}$. But Eq.(2.6) and Eq.(2.7) demonstrate that there are at most 32 free-parameters in the daor field. It is too much for describing the space-time metric from the viewpoint of general relativity. To cancel nonphysical freedom, the following covariant constraint should be satisfied

$$\bar{k}_\mu^a \eta_{ab} k^b_\nu = k_\mu^a \eta_{ab} \bar{k}^b_\nu, \quad \bar{K}_\mu^a \eta^{ab} K^b_\nu = K_\mu^a \eta^{ab} \bar{K}^b_\nu, \quad (2.9)$$

then Eq.(2.6) and Eq.(2.7) become

$$g_{\mu\nu} = \bar{k}_\mu^a \eta_{ab} k^b_\nu, \quad G^{\mu\nu} = \bar{K}_\mu^a \eta^{ab} K^b_\nu. \quad (2.10)$$

Define a tensor $N_{\mu\nu}$ with respect to curvilinear coordinate as follows

$$2N_{\mu\nu} = \bar{k}_\mu^a \eta_{ab} k^b_\nu - k_\mu^a \eta_{ab} \bar{k}^b_\nu. \quad (2.11)$$

The above equation shows that the tensor $N_{\mu\nu}$ is antisymmetrical, namely $N_{\mu\nu} = -N_{\nu\mu}$. Thus the covariant constraint $N_{\mu\nu} = 0$ only provides 6 independent constraint equations to the components of daor field. That is to say, the daor field can at most have 26 independent free-parameters.

Consider general real coordinate transformations $x \rightarrow x'(x)$, since

$$dx'^\mu = \partial x'^\mu/\partial x^\nu dx^\nu, \quad k'^b_\mu = k^b_\nu \partial x^\nu/\partial x'_{\mu}, \quad (2.12)$$

the intrinsic distance is invariant under general coordinate transformations.

Under the rotation of the locally complexified Minkowski frame, the daor field $k^b_\nu$ transforms as follows

$$k^a_\nu \rightarrow k'^a_\nu = S^a_b k^b_\nu. \quad (2.13)$$

If the matrix $S^a_b$ satisfies

$$\bar{S}_a^c \eta_{cd} S^d_b = \eta_{ab}, \quad (2.14)$$

namely, $S^a_b$ being the element of SU(1,3) group, then the intrinsic distance is invariant under the rotation of the locally complexified Minkowski frame.
Hence we can draw a conclusion: by introducing the complex daor field, we find that the intrinsic distance and the covariant constraint are invariant under two kinds of transformations: one is the general real coordinate transformation \( x \rightarrow x'(x) \); the other is the SU(1,3) group transformation of the Roman suffixes.

### 3 Daor Geometry and Its Connection

Obviously the daor field \( k^a_\mu \) satisfying the constraint Eq.(2.9) can be reexpressed in terms of the real vierbeins as

\[
k^a_\mu (x) = l^a_b(x) e^b_\mu (x),
\]

where \( e^b_\mu (x) \) is the real vierbein defined by Eq.(2.5), and the matrix \( l^a_b(x) \) satisfies Eq.(2.14), namely, \( l^\dagger \eta l = \eta \).

To provide the mathematical tools for an advanced study in the daor field, we would like to introduce the main results of daor geometry given by Ref.[27]. It can be defined that\(^5\)

\[
e^a = e^a_\mu \, dx^\mu , \quad k^a = l^a_b(x) e^b_\mu , \quad \bar{k}^a = \bar{k}^a_b \, \partial \mu , \quad K^a = K^a_\mu \partial \mu,
\]

where \( k^a \) and \( \bar{k}^a \) are the daor field 1-forms, \( K^a_\mu \) and \( K^a_\mu \) are given by

\[
K^\dagger = k^{-1} , \quad K^{-1} = k^\dagger.
\]

It is proved that the set \( \{K^0_0, K^1_1, K^2_2, K^3_3\} \) is the basis of locally complexified Minkowski frame \( \mathbf{M} \) and the set \( \{k^0, k^1, k^2, k^3\} \) is a basis of its dual frame \( \mathbf{M}^\ast \). Hence a vector \( U \) and a covector \( V \) are denoted by \( U = U^a K^a_\mu \) and \( V = V_a k^a \) respectively. Therefore an \((r,s)\)-type tensor \( T \) can be uniquely expressed as

\[
T = T_{i_1\ldots i_r}^{j_1\ldots j_s} K^\dagger_{i_1} \otimes \cdots \otimes K^\dagger_{i_r} \otimes k^{j_1} \otimes \cdots \otimes k^{j_s},
\]

where \( T_{i_1\ldots i_r}^{j_1\ldots j_s} \) are the components of the tensor \( T \).

\(^5\)We use \( \partial_\mu \) to denote the partial differential operator \( \frac{\partial}{\partial x^\mu} \) for brevity.
The Cartan’s exterior product and exterior differentiation can also be built up in daor geometry. For a $p$-form $\alpha_p = \frac{1}{p!} f_{a_1 \ldots a_p} k^{a_1} \wedge \ldots \wedge k^{a_p}$, its exterior differentiation is defined as the following

$$d\alpha_p = \frac{1}{p!} f_{a_1 \ldots a_p; l} k^l \wedge k^{a_1} \wedge \ldots \wedge k^{a_p}. \quad (3.5)$$

Let $\alpha_p \in \Lambda^p(M)$ and $\beta_q \in \Lambda^q(M)$, their exterior product satisfies $\alpha_p \wedge \beta_q = (-1)^{pq} \beta_q \wedge \alpha_p \ [33, 34]$.

For a covector field $\alpha_1 = f_a(x) k^a(x)$, when the daor field is chosen, then

$$\nabla \alpha_1 = df_a \otimes k^a + f_a \nabla k^a. \quad (3.6)$$

The above equation shows that $\nabla \alpha_1$ can be calculated if the covariant differentiation $\nabla k^a$ of the daor field is given. $\nabla k^a$ denote the infinitesimal variance of the daor field $k^a$ at the neighborhood of a point and can be expressed as

$$\nabla k^a = (\nabla l^a_b) e^b + l^a_b (\nabla e^b) = -l^a_c B^c_b e^b - l^a_b \theta^b_c e^c = -l^a_b \omega^b e^c, \quad (3.7)$$

where

$$\omega^a_b = B^a_b + \theta^a_b = -<K^i_b, \nabla k^a> = \omega^a_{ib} k^i. \quad (3.8)$$

Since $\omega^a_b$ is the complex matrix valued 1-form, we suggest calling $\omega^a_b$ the daor connection 1-form.

It is well known that $\theta^a_b$ defined in Eq.(3.7) is the spin connection introduced first by Cartan. From the viewpoints of Yang-Mills gauge field, the daor connection $\omega^a_b$ is the SU(1,3)$ \times $SO(1,3) gauge field. Or equivalently, in the language of differential geometry, we can say that $\omega^a_b$ is the connection on SU(1,3)$ \times $SO(1,3) principal bundle. The curvature of this principal bundle is thus expressed as

$$\Omega^a_b = d\omega^a_b + \omega^a_c \wedge \omega^c_b. \quad (3.9)$$

Because of Eq.(3.8), the curvature $\Omega^a_b$ can be written as

$$\Omega^a_b = R^a_b + F^a_b + \theta^a_c \wedge B^c_b + B^a_c \wedge \theta^c_b, \quad (3.10)$$
where $R^a_b$, $F^a_b$ are the curvature of SO(1,3) principal bundle and the curvature of SU(1,3) principal bundle respectively. The definitions of $R^a_b$ and $F^a_b$ are

$$
R^a_b = \frac{d\theta^a_b + \theta^a_c \wedge \theta^c_b}{2}, \quad (3.11)
$$

$$
F^a_b = \frac{dB^a_b + B^a_c \wedge B^c_b}{2}. \quad (3.12)
$$

The equation (3.7) shows that there are two categories of local gauge transformations: One is the local SU(1,3) group transformation, the covariant principle naturally leads to a necessary input of SU(1,3) gauge field; The other is the local SO(1,3) group transformation. The spin connection $\theta^a_b$ represents the effects of gravitation.

Firstly, let us consider an intrinsic rotation of the daor field

$$
k^a \rightarrow k'^a = l^a_k e^b = S^a_b k^b, \quad (3.13)
$$

where $S^a_b$ satisfies Eq.(2.14), namely, $S^a_b$ is a faithful representation of SU(1,3) group. $l^a_k$ is defined by $l^a_k = S^a_c l^c_k$. From Refs.[27, 35], it is known that, under the intrinsic SU(1,3) rotation of the daor field, the daor connection 1-form $\omega^a_b$ transforms as follows

$$
\omega'^a_b = S^a_c \omega^c_d (S^{-1})^d_b + S^a_c (dS^{-1})^c_b. \quad (3.14)
$$

Since $B^a_b$ is the connection of SU(1,3) principal bundle, under the SU(1,3) gauge rotation of the daor field, $B^a_b$ transforms into

$$
B'^a_b = S^a_c B^c_d (S^{-1})^d_b + S^a_c (dS^{-1})^c_b, \quad (3.15)
$$

and $\theta^a_b$ satisfies

$$
\theta'^a_b = S^a_c \theta^c_d (S^{-1})^d_b. \quad (3.16)
$$

Furthermore, it is easy to prove that under this rotation $F^a_b$ and $\Omega^a_b$ become

$$
F'^a_b = S^a_c F^c_d (S^{-1})^d_b, \quad \Omega'^a_b = S^a_c \Omega^c_d (S^{-1})^d_b. \quad (3.17)
$$

Secondly, consider an orthogonal rotation of the real orthonormal vierbein

$$
e^a \rightarrow e'^a = \Phi^a_b e^b, \quad (3.18)
$$
where $\Phi^a_{\ b}$ satisfies
\[ \Phi^a_{\ c}\eta_{cd}\Phi^d_{\ b} = \eta_{cd}. \]  
(3.19)

Eq.(3.19) demonstrates that $\Phi^a_{\ b}(x)$ is a representation of SO(1,3) group. The daor field transforms as
\[ k^a \rightarrow k'^a = l^a_{\ c}e'^c = \Phi^a_{\ b}k^b, \]  
(3.20)

where $l^a_{\ c} = \Phi^a_{\ b}l^b_{\ e}(\Phi^{-1})^e_{\ c}$. After this transformation, the new daor connection is
\[ \omega'^a_{\ b} = \Phi^a_{\ c}\omega^c_{\ d}(\Phi^{-1})^d_{\ b} + \Phi^a_{\ c}(d\Phi^{-1})^c_{\ b}, \]  
(3.21)

Similarly, we acquire
\[ \theta'^a_{\ b} = \Phi^a_{\ c}\theta^c_{\ d}(\Phi^{-1})^d_{\ b} + \Phi^a_{\ c}(d\Phi^{-1})^c_{\ b}, \]  
(3.22)
and
\[ B'^a_{\ b} = \Phi^a_{\ c}B^c_{\ d}(\Phi^{-1})^d_{\ b}. \]  
(3.23)

The transformation formulae for the curvatures $R^a_{\ b}$ and $\Omega^a_{\ b}$ are given by
\[ R'^a_{\ b} = \Phi^a_{\ c}R^c_{\ d}(\Phi^{-1})^d_{\ b}, \quad \Omega'^a_{\ b} = \Phi^a_{\ c}\Omega^c_{\ d}(\Phi^{-1})^d_{\ b}. \]  
(3.24)

4 Unification of Gravitation with Gauge Fields

Consider a kind of gauge groups, which are the subgroups of SU(1,3) group, and the element of which can be written as
\[ Z^a_{\ b}(x) = e^{it^a_{\ b}(x)}. \]  
(5.1)

Where $t^a_{\ b}$ is a $4 \times 4$ matrix, which is traceless and Hermitian, and, of course, should be the function of curvilinear coordinates. Therefore $Z^a_{\ b}$ is a 4-dimensional representation of gauge group. Some groups such as U(1), SU(2) all satisfy the condition (5.1).

In this paper we only discuss the SU(1,3) gauge field. Covariance of the daor field equations under the local SU(1,3) group rotation directly leads to the introduction
of Yang-Mills gauge field [22], say, denoted by $\tilde{B}_{bc}^a k^c$. Comparing Eq.(3.15) with the formulae in Yang-Mills theory, one obtains

$$B^a_b = i\epsilon \tilde{B}^a_b , \quad \tilde{B}^a_b = Z^a_c \tilde{B}^c_d (Z^{-1})^d_b + \frac{1}{i\epsilon} Z^a_c (dZ^{-1})^c_b ,$$

(5.2)

where $\epsilon$ is the coupling constant of the SU(1,3) gauge field. Both $\tilde{B}^a_b$ and $B^a_b$ are 1-forms. The gauge field strengths $\tilde{F}^a_b$ corresponding to the gauge field $\tilde{B}^a_b$ reads

$$\tilde{F}^a_b = d\tilde{B}^a_b + i\epsilon \tilde{B}^a_c \wedge \tilde{B}^c_b .$$

(5.3)

It is obvious that $F^a_b = i\epsilon \tilde{F}^a_b$.

The energy-momentum tensor of the SU(1,3) gauge field can be written in the form [36, 37]

$$4\pi T^g_{\mu\nu} = \frac{1}{4} g_{\mu\nu} \text{tr}(\tilde{F}_{\alpha\beta} \tilde{F}^{\alpha\beta}) - \text{tr}(\tilde{F}_{\mu}^{\alpha} \tilde{F}_{\nu\alpha}) ,$$

(5.4)

where ‘tr’ means operation of acquiring the trace of a matrix, and $T^g_{\mu\nu}$ is, as can easily be seen, traceless, $g^{\mu\nu} T^g_{\mu\nu} = 0$, but $\nabla^{\mu} T^g_{\mu\nu} \neq 0$. In Eq.(5.4), we have introduced the physical gauge fields $f^I_{\alpha\beta}$, $I = 1, 2, \ldots, 15$, as follows

$$\tilde{F}_{\alpha\beta} = \frac{1}{2} \sum_I T^I f^I_{\alpha\beta} ,$$

(5.5)

where $T^I$, $I = 1, 2, \ldots, 15$, are fifteen $4 \times 4$ generators of SU(1,3) group. The Lie algebra of SU(1,3) group is of the form

$$[T^I, T^J] = iC^{IJ}_K T^K ,$$

(5.6)

here $C^{IJ}_K$ being the structure constants of SU(1,3) group. Furthermore, the generators $T^I$’s are selected so that $T^I$ satisfy

$$T^I T^I = 1 ,$$

(5.7)

where $1$ denotes $4 \times 4$ unit matrix. Since we only consider the coupling between the daor field and the SU(1,3) gauge field, the energy-momentum tensor in Einstein’s equation (2.1) must include the term of Eq.(5.4).
In the case of Einstein’s gravitational theory, the Levi-Civita connection $\Gamma^\mu_{\alpha\beta}$ is adopted. In Riemannian geometry the Levi-Civita connection is determined by two conditions, the covariant constancy of the metric and the absence of torsion. In our case, the daor connection is complex. We need not the torsion-free condition. In the paper [27] we have argued that quantizing gravitation needs to rebuild Einstein’s gravitational theory such that the new formalism at least includes two properties: 1, the complexified field; 2, the first-order nonlinear field equation. In this paper, we try to unify gravitation with gauge fields and pave the way for quantizing this theory. Therefore, the daor field equation should have the following properties:

1. Einstein’s general relativity is a special result of this theory.

2. Gauge fields are naturally introduced and incorporated with gravitation.

3. The equation is consistent with the basic principles of quantum theory, e.g., the physical operators appearing in the theory are Hermitian.

4. The theory can give some new predictions.

Because of these considerations, we propose the fundamental doar field equation as follows

$$dk^a + (\omega^a_b + \lambda \mathcal{R}^a_b) \wedge k^b = 0.$$  \hspace{1cm} (5.8)

Where $\lambda$ is the real coupling constant, which will be given at the end of this section.

Where $\mathcal{R}^a_b$ is defined by

$$\mathcal{R}^a_b \equiv iX\Omega^a_b = <X,\Omega^a_b> = <\gamma^c K^\dagger_c, \frac{1}{2} \Omega^a_{bij} k^i \wedge k^j> = \gamma^c \Omega^a_{bcj} k^j.$$  \hspace{1cm} (5.9)

Here $X = \gamma^a K^\dagger_a$ is a vector field, $\gamma^a$ denotes the standard flat-space Dirac matrices, which satisfies $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$.

Now we would like to demonstrate the properties embodied by Eq.(5.8). First, Eq.(5.8) is covariant under both the local SU(1,3) gauge transformations and the local SO(1,3) rotations of the moving frame. Secondly, the covariant constancy of the metric
makes sure that $\theta^a_b = -\theta^b_a$, hence the operator $i\text{d} + i(\omega^a_b + \lambda R^a_b)\wedge$ is Hermitian, which is consistent with the requirement of quantum theory. Finally, in quantum field theories, all gauge fields are introduced by the local gauge invariance of the spinor fields. Therefore, in the daor field framework, the spinor fields should be included also. Obviously the daor field described by Eq.(3.1) can be naturally extended to

$$k^a_\mu(x) = l^a_b(x)c^b_\mu(x) , \quad l^a_b(x) = [\psi(x)]^a_b , \quad (5.10)$$

where $\psi(x)$'s are sixteen $4 \times 1$ matrices, which are also constrained by

$$[\psi^\dagger(x)]_a^c \eta_{cd} [\psi(x)]^d_b = \eta_{ab} . \quad (5.11)$$

The above extension of the daor field implies that Dirac matrices $\gamma_a$ may appear in the Eq.(5.8), or we can say, should uniquely appear in the vector $X$. This point reasonably demonstrates why the vector $X$ should be selected as $\gamma^a K^+_a$.

The covariant exterior derivative of a $(1,1)$-type tensor valued differential form $V^a_b$ of degree $p$ is defined as[35]

$$DV^a_b = dV^a_b + \omega^a_c \wedge V^c_b - (-1)^p V^a_c \wedge \omega^c_b . \quad (5.12)$$

Exteriorly differentiating Eq.(3.9), we can find the Bianchi identity

$$d\Omega^a_b + \omega^a_c \wedge \Omega^c_b - \Omega^a_c \wedge \omega^c_b = 0 , \quad (5.13)$$

namely, $D\Omega^a_b = 0$.

In general relativity, the connection of space-time manifold, $\theta^a_b$, is determined in terms of the vierbeins and inverse vierbeins and is related to the Christoffel symbol $\Gamma^\mu_{\nu\rho}$, where $\Gamma^\nu_{\nu\rho}$ is uniquely determined by two conditions, the covariant constancy of the metric and the absence of torsion. In our framework, there is not torsion-free condition. The connection $\omega^a_b$ can be determined by the condition that the covariant exterior derivative of $R^a_b$ must be vanishing. That is to say,

$$D R^a_b = D(i_X \Omega^a_b) = d \cdot i_X \Omega^a_b + \omega^a_c \wedge i_X \Omega^c_b + i_X \Omega^a_c \wedge \omega^c_b = 0 . \quad (5.14)$$

Whether these $4 \times 1$ matrices can be treated as the spinor fields will be studied in the forthcoming paper.
From the above equation, it is easy to prove that the complex connection $\omega^a_b$ must satisfy the following constraint equation

$$\left(\omega^a_{bj,e,i} - \omega^a_{bi,j,e} + \omega^a_{ce,j,i} - \omega^a_{be,j,i} + \omega^a_{be,i,j} - \omega^a_{ce,i,j} + \omega^a_{cj,i} - \omega^a_{cj,e,i} + \omega^a_{ci,e} - \omega^a_{ci} + \omega^a_{bj} - \omega^a_{cj} + \omega^a_{be} - \omega^a_{ce} + \omega^a_{bj} - \omega^a_{cj} + \omega^a_{be} - \omega^a_{ce}ight) k^i \wedge k^j = 0,$$

(5.15)

because $\omega^a_{ci,e} \equiv \partial_e \omega^a_{ci}$ and $i_x \Omega^a_b = i_x \cdot d\omega^a_b = (\omega^a_{bj,i} - \omega^a_{bi,j}) \gamma^i k^j$.

Conversely, the complex curvature $\Omega^a_b$ can be directly obtained from $\mathcal{R}^a_b$ as follows

$$\Omega^a_b = \frac{1}{2} \gamma_i k^i \wedge \mathcal{R}^a_b.$$

(5.16)

In the following we will demonstrate that Einstein’s gravitational equation (2.1) can be deduced from Eq.(5.8) and Eq.(5.15). Define the Hermitian operator 1-form

$$\hat{W} \equiv i \delta^a_b d + i (\omega^a_c + \lambda \gamma^c \Omega^a_{bc}) \wedge,$$

(5.17)

then Eq.(5.8) is rewritten as $\hat{W} \mathcal{R}^b = 0$. Multiplying both sides of Eq.(5.8) by the operator $\hat{W}$ yields

$$0 = \hat{W} \mathcal{R}^b$$

$$= \left[ d(\omega^a_b + \lambda \mathcal{R}^a_b) + (\omega^a_c + \lambda \mathcal{R}^a_c) \wedge (\omega^a_b + \lambda \mathcal{R}^a_b) \right] \wedge \mathcal{R}^b$$

$$= \left[ \Omega^a_b + \lambda (d\mathcal{R}^a_b + \omega^a_c \wedge \mathcal{R}^c_b + \mathcal{R}^a_c \wedge \omega^c_b) + \lambda^2 \mathcal{R}^a_c \wedge \mathcal{R}^d_b \right] \wedge \mathcal{R}^b.$$

(5.18)

Since we have required that $D\mathcal{R}^a_b = d\mathcal{R}^a_b + \omega^a_c \wedge \mathcal{R}^c_b + \mathcal{R}^a_c \wedge \omega^c_b = 0$ in Eq.(5.14), the above equation reduces to

$$\left( \Omega^a_b + \lambda^2 \mathcal{R}^a_c \wedge \mathcal{R}^c_b \right) \wedge \mathcal{R}^b = 0.$$

(5.19)

A sufficient, but not necessary, solution for Eq.(5.19) is expressed as

$$\Omega^a_b = -\lambda^2 \mathcal{R}^a_c \wedge \mathcal{R}^c_b.$$

(5.20)

Now we will demonstrate that the above solution is not self-contradictory. Eq.(5.12) and Eq.(5.14) make sure that

$$D(\mathcal{R}^a_e \wedge \mathcal{R}^c_b) = (D\mathcal{R}^a_e) \wedge \mathcal{R}^c_b - \mathcal{R}^a_e \wedge (D\mathcal{R}^c_b) = 0.$$

(5.21)
Hence Eq.(5.20) is consistent with Bianchi identities.

Componentwise Eq.(5.20) can be written as

\[
\frac{1}{2} \Omega^a_{\ bij} k^i \wedge k^j = -\lambda^2 \gamma^c \gamma^d (\Omega^a_{\ eci} k^i) \wedge (\Omega^e_{\ bdj} k^j) = -\frac{1}{2} \lambda^2 (\Omega^a_{\ eci} \Omega^e_{\ bci} - \Omega^a_{\ eci} \Omega^e_{\ bci}) k^i \wedge k^j ,
\]

(5.22)

namely

\[
\Omega^a_{\ bij} = \lambda^2 (\Omega^a_{\ eci} \Omega^e_{\ bci} - \Omega^a_{\ eci} \Omega^e_{\ bci}) .
\]

(5.23)

It is well known, in gauge field theory the freedom of gauge field is too much to describe the physical system. Theorists must introduce some kind of gauge fixing condition, such as Coulomb gauge in QED or Landau gauge in QCD. In our framework we propose the following gauge fixing condition

\[
\theta^a_{\ c} \wedge \tilde{B}^c_{\ b} + \tilde{B}^a_{\ c} \wedge \theta^c_{\ b} = 0 .
\]

(5.24)

In this gauge the curvature \( \Omega^a_{\ b} \) reduces to

\[
\Omega^a_{\ b} = R^a_{\ b} + i \epsilon \tilde{F}^a_{\ b} .
\]

(5.25)

Inserting above equation into Eq.(5.22), we acquire two equations which are respectively real part and complex part of an equation. They are expressed as follows

\[
R^a_{\ bij} = \lambda^2 \left( R^a_{\ e j} R^e_{\ bci} - R^a_{\ e i} R^e_{\ bci} \right) + \lambda^2 \epsilon^2 \left( \tilde{F}^a_{\ e i} \tilde{F}^e_{\ bci} - \tilde{F}^a_{\ e j} \tilde{F}^e_{\ bci} \right) ,
\]

(5.26)

and

\[
\tilde{F}^a_{\ bij} = \lambda^2 \left( \tilde{F}^a_{\ e j} R^e_{\ bci} + R^a_{\ e i} \tilde{F}^e_{\ bci} - R^a_{\ e j} \tilde{F}^e_{\ bci} - \tilde{F}^a_{\ e i} R^e_{\ bci} \right) .
\]

(5.27)

Contracting \( a \) with \( i \) in \( R^a_{\ bij} \) yields

\[
R_{\mu\nu} = R^i_{\ \mu i} = \lambda^2 \left( R^i_{\ e i} R^e_{\ \mu ci} - R^i_{\ e i} R^e_{\ \mu cv} \right) + \lambda^2 \epsilon^2 \left( \tilde{F}^i_{\ e i} \tilde{F}^e_{\ \mu ci} - \tilde{F}^i_{\ e i} \tilde{F}^e_{\ \mu cv} \right) ,
\]

(5.28)

where \( R_{\mu\nu} \) is called Ricci tensor.
Einstein’s gravitational equation (2.1) can be rewritten as follows

\[ R_{\mu\nu} = 8\pi G \left( T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^\rho_\rho \right) + \Lambda g_{\mu\nu} , \]  

(5.29)

where \( T^\rho_\rho \) denotes the contraction of the stress-energy tensor \( T_{\mu\nu} \). In the case of SU(1,3) gauge field, inserting Eq.(5.4) into Eq.(5.29) yields

\[ R_{\mu\nu} = G \left[ \frac{1}{2} g_{\mu\nu} \text{tr}(\tilde{F}_{\alpha\beta} \tilde{F}^{\alpha\beta}) - 2 \text{tr}(\tilde{F}_{\mu}^\alpha \tilde{F}_{\nu}^\alpha) \right] + \Lambda g_{\mu\nu} . \]  

(5.30)

From Eq.(5.28), the Einstein tensor \( R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R^\rho_\rho \) leads to

\[
\lambda^2 \epsilon^2 \left[ \left( \tilde{F}^e_{e i} \tilde{F}^e_{\mu e c} - \tilde{F}^i_{e i} \tilde{F}^e_{\mu e c} \right) - \frac{1}{2} g_{\mu\nu} \left( \tilde{F}^i_{e i} \tilde{F}^{e\rho}_{\epsilon p} - \tilde{F}^i_{e i} \tilde{F}^{e\rho}_{\epsilon i} \right) \right] = \lambda^2 \epsilon^2 \left[ \tilde{F}^e_{e i} \tilde{F}^e_{\mu e c} - \text{tr} \left( \tilde{F}^c_{e i} \tilde{F}^c_{\mu e} \right) - \frac{1}{2} g_{\mu\nu} \tilde{F}^i_{e i} \tilde{F}^{e\rho}_{\epsilon p} + \frac{1}{2} g_{\mu\nu} \text{tr} \left( \tilde{F}^c_{e i} \tilde{F}^c_{\rho} \right) \right].
\]  

(5.31)

We have indicated that the energy-momentum tensor of SU(1,3) gauge field should be included in the total energy-momentum tensor in Einstein’s gravitational equation. Comparing Eq.(5.31) with Eq.(2.1), Eq.(5.4) directly yields a simple identity

\[ G = \lambda^2 \epsilon^2 , \]  

(5.32)

where the Newtonian gravitational constant \( G \), the coupling constant of gauge field \( \epsilon \) and the coupling constant of the daor field \( \lambda \) are connected.

We can also draw a conclusion that the term \( \lambda^2 \left( R^i_{e i} R^e_{\mu c i} - R^i_{e i} R^e_{\mu c e} \right) \) in Eq.(5.28) plays the same role as that of the term \( \Lambda g_{\mu\nu} \) in Eq.(5.30) by comparing Eq.(5.28) with Eq.(5.30), namely

\[ \lambda^2 \left( R^i_{e i} R^e_{\mu c i} - R^i_{e i} R^e_{\mu c e} \right) \rightarrow \Lambda g_{\mu\nu} . \]  

(5.33)

In this section we have proposed the first-order nonlinear daor field equation (5.8), from which Einstein’s gravitational equation (5.29) can be deduced. We have argued that the cosmological constant term should be substituted by a Ricci squared term. This fact will be studied further in the next section.
5 Dark Energy Originates from the Self-coupling of the Space-time Curvature

Recent cosmological observations [1, 2, 3, 4, 5, 6] have not only strengthened and expanded the big bang model, but they have also revealed some surprises. In particular, most of the universe seems to be made of something fundamentally different from the matter of which we are made. About 23% of the total energy is dark matter, composed of particles most likely formed early in the Universe. About seventy percents is in a smooth dark energy whose gravitational effects began causing the expansion of the Universe to speed up just a few billion years ago. However, the nature of this energy and, therefore, the meaning of the non-zero cosmological constant which is needed in the equations that describe the nowaday acceleration of the expansion is still a mystery.

Recently there have been a number of different attempts [38] to modify gravity to yield accelerating universe at late times. Specially the models [39] in which the higher order curvature invariants is directly added to the Einstein-Hilbert action have been widely investigated. The cosmological models in Ricci squared gravity has also been studied in Ref.[40]. In last section we have showed that in our scenario of the daor field, the cosmological constant term in Einstein’s equation must be substituted by the term that describes the effect of the self-coupling of the space-time curvature. It is well known that the notion of dark energy is an evolutionary expansion of the cosmological constant. Hence, from this point of view, one can say that dark energy originates from the self-coupling of the space-time curvature in our scenario.

To demonstrate the physical meaning of Eq.(5.33) explicitly, we consider a simple case that the space-time is a maximally symmetric manifold. The mathematical theory of symmetric space is elaborate and has been used extensively in modern physics. Anti de Sitter, de Sitter and Minkowskian space-times are all maximally symmetric space. The Maldacena conjecture on the AdS/CFT correspondence has become an important part of string theory [41]. The curvature tensor of a four dimensional maximally
symmetric space-time is given by
\[ R_{\sigma\rho\nu\lambda} = \frac{R^\lambda}{12} \{ g_{\nu\rho} g_{\sigma\alpha} - g_{\alpha\rho} g_{\sigma\nu} \} . \] (6.1)

The Ricci tensor is easily acquired by contracting the suffix \( \sigma \) with the suffix \( \alpha \), that is
\[ R_{\mu\nu} = \frac{R^\lambda}{4} g_{\mu\nu} . \] (6.2)

It is well known that \( R^\lambda_{\lambda} \) is a constant in the case of maximally symmetric space-time, namely \( R^\lambda_{\lambda} = R \), \( R \) being the curvature of the space-time \([42]\).

Inserting Eq.(6.1) into Eq.(5.33) yields
\[ \frac{\lambda^2}{24} R^2 g_{\mu\nu} \Rightarrow \Lambda g_{\mu\nu} . \] (6.3)
then the above equation reduces to
\[ \Lambda = \frac{\lambda^2}{24} R^2 . \] (6.4)

Similar to the definition of the vacuum energy density, \( <\rho> = \Lambda/(8\pi G) \), we can define the dark energy density as follows
\[ \rho = \frac{\lambda^2}{24} \frac{R^2}{8\pi G} = \frac{R^2}{192\pi \epsilon^2} , \] (6.5)
the above equation exactly sets up a relationship between the dark energy density and the curvature of maximally symmetric space-time. Hence the dark energy parameter \( \Omega_{\Lambda} \) is given
\[ \Omega_{\Lambda} = \frac{8\pi G \rho}{3H_0^2} = \frac{\lambda^2 R^2}{72H_0^2} . \] (6.6)
We can express the dark energy density in terms of a mass scale,
\[ \rho = M_{de}^4 , \] (6.7)
so the updated observational result is
\[ M_{de} \sim 10^{-3} \text{eV} . \] (6.8)
In our scenario the coupling constant $\epsilon$ is the unique coupling constant of all gauge fields, so the mass scale of $\frac{1}{\epsilon}$ should be chosen as $M_{GUT} \simeq 2 \times 10^{16}$GeV. The main motivation for this selection is that, at least in supersymmetric models, the running gauge couplings of the standard model unify at the scale $M_{GUT}$ [43], hinting at the presence of a grand unified theory involving a higher symmetry with a single gauge coupling. Therefore, we can get the curvature of maximally symmetric space-time, that is

$$R \simeq \frac{M_{de}^2}{M_{GUT}} \simeq 1 \times 10^{-23} \text{ m}^{-1}. \quad (6.9)$$

What we obtained here is in agreement with the result of recent astronomical observations [1, 2, 3, 4, 5, 6].

After A.H. Guth, it is widely believed that the horizon problem and the flatness problem in modern cosmology can be solved once we accept that our universe underwent an inflationary evolution in the early epoch [44, 45]. The recent observations on the CMB radiation [4, 5, 6] confirmed the existence of the early inflationary cosmological epoch as well as the accelerated expansion of the present universe. The cosmological observations [4, 5, 6] also demonstrated that the large scale structures in present university possibly originate from the quantum fluctuation in the epoch of inflation [46]. In our scenario, dark energy originates from the self-coupling of the space-time curvature. Obviously

$$\chi^2 \left( R_{\epsilon}^{\alpha} \epsilon_{\alpha}^{\nu} R_{\mu \nu}^{\epsilon} - R_{\epsilon}^{\alpha} \epsilon_{\alpha}^{\nu} R_{\mu \nu}^{\epsilon} \right) \propto R^2. \quad (6.10)$$

This equation indicates that our scenario maybe has provided a natural mechanism for the early inflation when the curvature of our universe is very large. This mechanism will have no difficulties on the exit of inflation which has been confronted by many inflationary scenarios. This topic will be explicitly discussed in our forthcoming papers.
6 Conclusion

The locally complexified vierbein (or tetrad) field has been discussed. We renamed it and suggested calling it the daor field. Under this expanded symmetry, we introduced the daor geometry where the connection is complex. After setting up the geometric tools, we proposed a first-order nonlinear equation, from which Einstein’s gravitational equation can be deduced. The real part of the daor field coupling equation can also be regarded as Einstein’s equation endowed with the cosmological constant term. Our equation has shown that dark energy originates from the self-coupling of the space-time curvature. The energy-momentum tensor of dark energy is affected by the scale of the grand unified theory. The dark energy density obtained in our scenario is in the same order as that given by the astronomical observations.

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